DOMAINS OF DISCOURSE

By Philip HUGLY and Charles SAYWARD

1. Consider the following argument. Suppose there is a domain of discourse of English. Then everything of which any predicate of English is true is a member of that domain. But if English has a domain of discourse, then, since 'is a domain of discourse of English' it itself a predicate of English and true of that domain, that domain is a member of itself. But nothing is a member of itself. Thus, English has no domain of discourse.

We can put the argument this way. First note that the schema

S $(x)(y)((Fx \ \& \ y$ is a domain of discourse of English$) \rightarrow x$ is a member of $y)$

holds for any replacement of 'F' by a predicate of English (an expression of English true or false of anything). For if English has a domain of discourse then it includes everything of which any predicate of English is true. Now, the expression 'is a domain of discourse of English' is a predicate of English for it is true or false of things, e.g., it is false of the set of arabic numerals. Thus, we obtain from the above schema the truth

(1) $(x)(y)((x$ is a domain of discourse of English $\& \ y$ is a domain of discourse of English$) \rightarrow y$ is a member of $x)$

But,

(2) (x) x is not a member of x

And from (1) and (2) it follows that

(3) (x) x is not a domain of discourse of English

i.e., English has no domain of discourse.

2. Essential to the above argument is that 'is a domain of discourse of English' is a predicate of English, otherwise (1) is not obtainable from schema S. This might be challenged. After all, a fairly common view is that if a sentence X belongs to a certain language L a sentence saying X is true in L or false in L does not itself belong to L, but to another

language, L's metalanguage. Sometimes this view is attributed to Tarski. For example, Prior says:

> Further, Tarski argues, a sentence asserting that some sentence S is a true sentence of some language *L* cannot itself be a sentence of the language *L*, but must belong to a metalanguage in which the sentences of *L* are not used but mentioned. ([1], p. 223)

We think Prior is wrong in attributing this view to Tarski. Tarski held natural languages to be inconsistent in part because he held that natural languages do contain their own semantical locutions.(1)

In any event, the view Prior articulates (whether held by Tarski or not) is relevant to the argument in section 1. If natural languages do not contain their own semantical locutions and if 'is a domain of discourse of English' is a semantical locution, then that expression does not belong to English, and, consequently, (1) is not obtainable from S.

This view seems to us to be out of touch with reality. Whatever the lessons of the semantical paradoxes, that 'is true in English' and 'is a domain of discourse of English' do not belong to English cannot be one of those lessons. Just consider

> 'Two is even' is true in English. It is not true in German, though, since it is not a sentence of German.

These sentences are true. True in what language? Well, what language are they written in? English, of course. These sentences are true in English. There is nothing sophisticated about this point. Indeed, it really amounts to pointing out that what is written in a particular natural language is written in that language.

3. Is the supposition that English is a language which contains the predicate 'is a domain of discourse of English' *crucial* to seeing that English has no domain of discourse? To be sure, this supposition (or, as it seems to us, this fact) is used in getting (3), the conclusion that English has no

(1) Alfred TARSKI 'The Concept of Truth in Formalized Languages', *Logic, Semantics and Matamathematics*, Oxford University Press (1956), 164-166. This putative consequence of the liar paradox has been challenged by Hugly and Sayward in 'Is English Inconsistent'?, *Erkenntis*, Vol. 15, No. 3, 343-347.

domain of discourse. But might one get to the same conclusion without having to rely on this supposition?

Here is an argument for an affirmative answer. Let L be any language which can express the membership relation. Now, if the axiom of regularity is true, then L has no domain of discourse. For if L did have such a domain that domain must member all the classes there are. But that domain is itself a class. So it must member itself. This is contradicted by the axiom of regularity. Now English is certainly a language which can express the membership relation. Thus English has no domain of discourse.

From this it looks as if one might get to (3) without relying upon the supposition that 'is a domain of discourse of English' is a predicate of English. For the argument just given does not rely upon that supposition.

But it relies upon another supposition which is a good deal less plausible. This is the supposition that English can express the membership relation, or, more deeply, that there is such a relation as *the* membership relation. The idea here is that we so use the language of classes that (i) our variables range over all the classes there are, and (ii) the epsilon denotes the membership relation defined on all the classes there are. But is there any such maximal range or any such maximal relation? There is a good deal of plausiblity in the contrary view of the matter, namely, that for any range there is a more inclusive one and that for any membership relationship there is a more inclusive one.

In saying this we are not taking sides on this very complicated issue. Our point is simply that the argument for (3) that relies upon (i) and (ii) is a good deal more problematic than the argument articulated in section 1.

A concluding remark. Both arguments just reviewed assume what the axiom of regularity implies, that no class is a member of itself. But this is not a point of logic. Nor is it settled by consistency conditions on set theory. How is it settled? A realist about sets would claim that how things are with sets settles the matter even though we are currently or even permanently unable to see what the sets there are themselves disclose on this point. But we are not realists. The *point* of our argument really is to show that *if* we appeal to domains of discourse in our formal semantical theorizing about a natural language such as English, then we must employ a set theory which not only fails to affirm the axiom of regularity, but actually denies it. And *that* will have further consequences. For example, reflect on the fact that the 'all' of last resort is the natural language 'all' – our 'all' of English. Here 'all' means *all*. So suppose we allow self-membering

classes, with the domain of discourse of English as one such class. Then there is a universal class, namely the domain of discourse of English. So, the set theory we must employ if we are to grant English a domain of discourse will, e.g., deny either that each set yields its power set or that each set is outnumbered by its power set. Further, that theory must deny the axiom of separation. For 'x is not a member of x' will be admissable and yet cannot, on pain of contradiction, separate out a class from a universal class. It is to such accumulations of inconveniences of theory that we would ultimately appeal in arguing that our formal semantical theorizing about natural languages such as English must forgo domains of discourse.

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REFERENCE

[1] Prior, A. N., 'Correspondence Theory of Truth', *The Encyclopedia of Philosophy*, Vol. 2, edited by Paul Edwards, Collier-Macmillan (1967): 223-232.