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Aristotelian finitism

Tamer Nawar

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Abstract It is widely known that Aristotle rules out the existence of actual infinities but allows for potential infinities. However, precisely why Aristotle should deny the existence of actual infinities remains somewhat obscure and has received relatively little attention in the secondary literature. In this paper I investigate the motivations of Aristotle's finitism and offer a careful examination of some of the arguments considered by Aristotle both in favour of and against the existence of actual infinities. I argue that Aristotle has good reason to resist the traditional arguments offered in favour of the existence of the infinite and that, while there is a lacuna in his own 'logical' arguments against actual infinities, his arguments against the existence of infinite magnitude and number are valid and more well grounded than commonly supposed.

Keywords Aristotle · Aristotelian commentators · Infinity · Mathematics · Metaphysics

1 Introduction

It is widely known that Aristotle embraced some sort of finitism and denied the existence of so-called 'actual infinities' while allowing for the existence of 'potential infinities'. It is difficult to overestimate the influence of Aristotle's views on this score and the denial of the (actual) existence of infinities became a commonplace among philosophers for over two thousand years. However, the precise grounds for Aristotle's finitism have not been discussed in much detail and, insofar as they have received attention, his reasons for ruling out the existence of (actual) infinities have often been

✉ Tamer Nawar
tamernawar@gmail.com

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deemed obscure or ad hoc (e.g. Hussey 1983, pp. xxiii–xxiiv, 79). In this paper, I examine why Aristotle (and some later Aristotelians) rejected the existence of (actual) infinities and argue that, at least in several cases, Aristotle's reasons for rejecting the existence of the infinite are more principled than commonly supposed.

Aristotle's main discussion of the infinite (*to apeiron*) occurs in the third book of the *Physics* (chapters 4 to 8) within a broader inquiry into the nature of change (*kinēsis*) and those things (such as place and time) which are deemed necessary for change.[1] Change is something continuous (*sunechēs*; cf. *Cat.* 4b20–5) and so the student of nature must, Aristotle thinks, get clear on the nature of continuity. But getting clear on continuity requires, he thinks, getting clear on the infinite (*Phys.* 200b16–20) and so one should reflect upon the infinite and consider whether the infinite exists (*einai*), and, if it exists, what, precisely, it is (202b35–6). To this end, Aristotle: offers a critical survey of the earlier Greek philosophers (who had often made the infinite a prominent part of their physics, 203a4–b15); notes that there are problems in supposing that the infinite exists, but also problems in supposing that it does not exist (203b31–2; cf. 206a9–10); discusses various arguments for and against the existence of infinity (204a8–206a8); and concludes that it is not the case that the infinite exists unqualifiedly (*haplōs*, 206a9; cf. *Metaph.* 1077b15–17). Instead, there is a sense in which the infinite exists and a sense in which it does not exist (206a12–14). Aristotle goes on to draw a distinction between existing potentially (*dunamei*) and actually (*entelecheia*) (206a14–15) and notes that the infinite exists through addition (*prosthesei*) or division (*diairesei*) (206a15–16). Having established that magnitude is infinitely divisible but is not actually (*kat'energeian*) infinite, he infers that, in general, the infinite exists only potentially (*dunamei*) (e.g. 206a18–19; cf. 206b12–14, 207b11–12) and concludes: 'the infinite exists in virtue of one thing's constantly being taken after another—each thing taken is finite, but it is always one followed by another' (206a27–9). He then offers a revisionary characterisation of the infinite—'it is not that of which no part is outside, but that of which something [i.e. some part] is always outside' (207a1–2, 7–8)—and discusses what the infinite's potential mode of existence amounts to in the case of magnitude (*megethos*) and number (*arithmos*) (207a33–207b27). Finally, Aristotle offers a response to some of the previously considered arguments in favour of the (actual) existence of the infinite (208a5–24).

The philosophical literature on Aristotle's account of the infinite has focused primarily and almost exclusively on attempting to clarify and make consistent Aristotle's brief and somewhat sketchy remarks concerning the infinite's (potential) mode of being in chapter 6 (which is seemingly likened both to that of processes and of matter).[2] In

[1] I have followed the standard translations in generally translating '*apeiros*' as 'infinite' (rather than, e.g., 'unlimited') and '*peperasmenos*' as 'finite' (rather than, e.g., 'limited'), but have found it necessary to render '*perainein*' as 'to limit' because of the lack of an appropriate verb (and it is worth keeping in mind, especially with regard to (D) and (F) [see below], that rendering '*apeiros*' as 'unlimited' is often more perspicuous).

[2] The literature is largely divided between two camps. One view, associated especially with Hintikka (1966) and Bostock (1972), focuses on Aristotle's remarks likening the infinite's manner of existence to that of processes (e.g. *Phys.* 206a18–25, a31–3, b13–14). On this view, only processes may be infinite (i.e. unending) and, in claiming that the infinite exists only potentially, Aristotle means to express either: (a) that the processes in question are composed of temporal parts which are never there all at once and

contrast, precisely why Aristotle should rule out the existence of (actual) infinities in the first place has received less in the way of attention and has not been adequately understood. In what follows, I shall clarify the often highly elliptical arguments Aristotle discusses both in favour of and against the existence of infinity and argue that, despite a significant lacuna in his account when it comes to ruling out infinite sets or pluralities, Aristotle's reasons for ruling out the existence of actual infinities in the case of magnitude and number are in fact less ad hoc and more well grounded than often thought.

2 Aristotle's response to arguments for infinity

Aristotle takes the question of whether the infinite exists to be of the utmost importance for students of the natural world (cf. *Cael.* 271b4–9). However, the question constitutes a conundrum (*aporia*) which takes the form of a destructive dilemma: 'If one supposes the infinite not to exist, many impossibilities follow; and if one supposes it to exist, many impossibilities also follow' (*Phys.* 203b31–2; cf. 206a9–12).[3] Aristotle's general strategy, when facing an *aporia*, is to critically examine the various established beliefs (*endoxa*) and arguments on the matter at hand (cf. *Eth. Nic.* 1145b2–7). Accordingly, he first outlines five reasons which have been offered by other philosophers for supposing that the infinite is or exists (*Phys.* 203b16–25):

(A) from considering time, for it is infinite (203b16–17);
(B) from considering the division or divisibility (*diairesis*) of magnitudes, for mathematicians do in fact use the infinite (203b17–18);
(C) 'that only so will coming-to-be and ceasing-to-be not give out, i.e. only if there is an infinite from which that which comes to be is subtracted' (203b18–20);
(D) 'that what is finite (*peperasmenos*) always reaches a limit (*peras*) in relation to something [else] (*pros ti*), so that necessarily there must be no limit [i.e. the infinite exists], if everything is always limited (*perainein*) by something other than itself' (203b20–22); and
(E) because numbers, mathematical magnitudes, and what is beyond the heavens do not 'give out in thought' (*hupoleipein tē noēsei*) and so are all thought to be infinite (203b22–25).

Footnote 2 continued
whose parts do not endure (Hintikka); or (b) that the processes in question are unending and cannot be completed (Bostock). Another view, associated especially with Lear (1979), focuses on Aristotle's remarks likening the infinite's manner of existence to that of matter (e.g. 206b14–16; cf. 207a21–5, 207a35–b1, 207b34–208a4). On this view, something is potentially *infinite iff* 'there will *always* be possibilities that remain unactualized' (Lear 1979, p. 191). Thus, for instance, magnitude is potentially infinite because while its capacity (*dunamis*) for division may be (partially) actualized through someone dividing it, it can always be further actualized through further divisions. The same thought applies to number being potentially infinite through addition. For further discussion, see Charlton (1991), Coope (2012).

[3] Translations of *Physics* 3–4 follow Hussey (1983) with occasional modifications. Translations of Aristotle's other texts follow Barnes (1984). The Greek text of the *Physics* used is that of Ross (1936). Translations of Euclid, Alexander of Aphrodisias, Simplicius, Philoponus, and Aquinas rely upon Heath (1926), myself, Edwards (1994), Urmson (2002), and myself respectively.

Given their elliptical presentation in the text, the warrant these arguments are meant to provide for believing that the infinite exists and Aristotle's own assessment of their merit has often seemed less than clear. However, broadly speaking, it seems that Aristotle wishes to accept the force of (A) and (B) (cf. 206a9–12), but (re)describe the sort of infinity they introduce as merely potential and not actual, and wishes to largely deny the force of (C), (D), and (E). In what follows, I shall clarify each of the arguments and Aristotle's response to them so far as possible.

With regard to (A), we should notice that when Aristotle talks of time being infinite he has in mind not so much its continuity or divisibility, but that there is no first or last moment of time. However, why he should think time is infinite in this sense is not entirely clear. It is sometimes thought that Aristotle relies upon his arguments against a first (moment of) change in book 8 of the *Physics* (251a8–252a5) (e.g. Hussey 1983, p. 76), or else simply upon the weight of scientific opinion (nearly all his predecessors took time to be infinite; cf. *Phys.* 251b17–18; *Cael.* 279b4–280a35).[4] However, later Aristotelians took Aristotle to rule out a first or final moment of time by employing a *reductio* which relied upon the notion(s) that: for any time t, it must be conceivable that *now* takes place at or within time t, and that for any *now*, there must be a before and after, or that for any time t, there must be a before and after (Aquinas *In Phys.* 8, lect. 2, par. 982–4; cf. Avicenna *Physics of the Healing* 2.11.3–5, 12.1; 3.7.3);[5] or simply that proponents of the finitude of time must (incoherently) suppose that there is a time at which there is no time (Simpl. *In Phys.* 466, 3–6; cf. *Phys.* 226b6–7; 251b10–11; *Metaph.* 1021b13–14).

With regard to (B), Aristotle takes the continuous to be such that it is divisible into parts which are themselves continuous. Every magnitude is divisible into a magnitude (207b26–7) and is thus infinitely divisible. Something with magnitude, such as a line, cannot be composed of (or divided into) (indivisible) items without magnitude, such as points (e.g. *Gen. Corr.* 316a29–34). Aristotle takes continuous entities to be composed of (and divisible into) parts, the limits of which 'are touching and have become one and the same and are, as the word ('continuous', ['*sunechēs*']) signifies, contained in each other' (*Phys.* 227a10–12). Indivisible items, such as points, are simples; they do not have parts or, for that matter, limits, and thus cannot be continuous with other indivisible items and thus cannot compose continua (*Phys.* 231a24–6, b15-16; cf. *Metaph.* 1002a34–b11; Euclid *Elements* 1, def. 1).[6] While Aristotle takes mathematics to require the infinite divisibility of magnitudes (*Cael.* 271b9–11; cf. Simpl. *In Phys.* 466, 7–27; Philop. *In Phys.* 404, 13–405, 3), he does not think it requires infinitely extended magnitudes (207b27–29).[7]

[4] Plato's *Timaeus* is, seemingly, the notable exception (cf. *Cael.* 279b4–280a35; *Phys.* 251b17–18).

[5] The gloss I provide concerns Aquinas's remarks and is tentative. Avicenna offers a more detailed discussion. Note that, for Aristotle, the now is an instantaneous point in time without duration (analogous to a point on a line) which serves as a limit to divide the time before and the time after (cf. *Phys.* 218a3–30; 233b33–234a31; Waterlow 1984).

[6] For further discussion, see White (1992, pp. 28–30, 193–220).

[7] On the prospects of Aristotle's finitist mathematics with reference to discussions of parallel lines and other phenomena, see Hussey (1983, pp. 93–96, 178–179).

With regard to (C), Aristotle is describing the putative need, posited by certain of his philosophical predecessors (notably Anaximander and Thales), for the existence of something infinite or boundless from which those things which come-to-be (and cease-to-be) emerge. On this conception of the infinite, the infinite is some sort of primordial fuel which provides the material for natural processes, dictates their nature, and is responsible (in some putatively active sense of 'responsible', cf. *Phys.* 203b4–15) for them. However, as Aquinas notes, the thinkers in question seem to have supposed that for there to be perpetual bread-making across time, an infinitely large mass of dough is required (Aquinas, *In Cael.* 1, lect. 9, par. 96). In contrast, Aristotle observes that as long as those things which come to be are not destroyed *ad nihilum*, things can continue to come into existence from the recycled elements of previous existents. Thus a finitely large universe can provide the materials for perpetual coming-to-be and ceasing-to-be (*Phys.* 208a8–11) and there is no need to posit the existence of the infinite along the lines just mentioned. Thus this putative reason for supposing that the infinite exists can easily be rejected.

In (D), we are offered an argument. The argument seems to have been important in motivating the view that there exists something infinite but precisely how the argument is meant to go, or what Aristotle's response to it was, has not—as far as I am aware—been perspicuously discussed. In the text, (D) proceeds from the view that anything 'limited' (i.e. finite) requires a limiter to the conclusion that there exists something infinite. Though elliptical—Simplicius notes that he finds the reasoning difficult (*In Phys.* 466, 31–467, 4)—I take it that the strongest form of (D) proceeds roughly as follows:

(1) for any x, x is either infinite or finite;

(2) for any x, if x is finite, then there exists some y, which is beyond x (i.e. which is numerically distinct from x and not a part of x or occupying the same place as x), such that y limits x;

(3) a is either infinite or finite;

(4) | a is infinite;

(5) | there exists something infinite;

(6) | a is finite;

(7) | b, which is beyond a, is such that b limits a;

(8) | b is either infinite or finite;

(9) | | b is infinite;

(10) | | there exists something infinite;

(11) | | b is finite;

(12) | | c, which is beyond b, is such that c limits b;

(13) | | c is either infinite or finite;

...

On this informal reconstruction, the argument takes the form of an infinite regress and aims to establish that something infinite does exist (or that an infinite number of Fs do exist). It proceeds as follows. Treat as true that there is something in the world and call this thing 'a'. On the basis of (1), we may infer that a is either infinite or finite. Supposing that a is infinite, it follows that there exists something infinite, i.e. that the infinite exists. Supposing that a is finite, since everything finite ('limited', *peperasmenos*) is limited (*perainein*) by something else, i.e. a limiter, which is distinct from itself and not a part of itself, it follows that a is limited by something else, b. But b is either infinite or finite. Supposing that b is infinite, it follows that there exists something infinite. Supposing that b is finite, it follows that b is limited by c, and so on. Thus, either some limiter will be infinite, in which case something infinite exists; or else there will be an endless (i.e. infinite) sequence of (finite) limiters (or so the thought goes, more on this in a second). If we take there being an infinite sequence of limiters to amount to the infinite existing, then in this case also the infinite exists.

If that is the correct understanding of the argument, then it has several problems. First, one might complain that since the assumptions are never discharged the argument has no conclusion. Secondly, even if this issue were remedied, (D) (or some close variant) cannot be used to establish an infinite number of *distinct* limiters. Although new constants (e.g. 'b', 'c',...) are continuously introduced, one should not assume that they each refer to distinct individuals. In fact, (D) establishes the existence of only at least two distinct individuals, a and b; however, it is perfectly possible that $a = c$, $b = d$, etc. Thus, one thing might be limited by another, but the second may be limited by the first (or so the thought goes on the argument as presented here). Hence, even if we suppose that the premises are true, (D) fails to establish that there is something infinite or an infinite number of individuals.

Aristotle's own response to (D) comes towards the end of book 3 of the *Physics* and focuses on a different issue. The argument, he thinks, is to be rejected on the following grounds:

> To be touched (*haptesthai*) and to be limited (*peperanthai*) are different. The former is relative (*pros ti*) and is [the touching] *of something* (for everything touches something), and is an accident of some of the finite things. However, what is limited [i.e. finite] is not relative (208a11–13).

Aristotle's response has been deemed deficient (e.g. Hussey 1983, p. 97) but I think that, having better understood the logic of (D) (the argument to which Aristotle is responding), we can see that Aristotle's response turns upon denying (2), which is necessary for generating the regress. Aristotle takes the proponents of (D) to have been misled by the surface grammar of the expression 'being finite' or 'being limited' (*peperasmenos*). Whereas one might take these expressions to express relations and imply the existence of some 'limiter' (i.e. something that does the limiting, *perainein*) in much the same way that 'being touched' implies the existence of some 'toucher' (i.e. something doing the touching), Aristotle denies that being finite or being limited is or requires a relation or, better put, as Simplicius subtly notes (*In Phys.* 516, 18–

20), an *external relation*.[8] Aristotle takes a finite thing's limit to be *in* the finite thing (e.g. *Phys.* 212b28). Thus, a thing can instantiate the property of being finite purely in virtue of the way it and its parts are (without reference to anything else). Accordingly, it seems that *even if* there were only one substance in the world, it could instantiate the property of being finite (contrast this with the response to (D) offered above which required at least two things in the world) Insofar as we are tempted to construe magnitude, size, or shape as intrinsic properties (and this has seemed plausible to many), Aristotle's claim has force and (*pace* Hussey 1983) is to the point in rebutting (D) for if (2) is denied, the regress cannot get started.[9]

Finally, we come to (E). The thought goes that because numbers, mathematical magnitudes, and what is beyond the heavens do not 'give out in thought' (*hupoleipein tē noēsei*, 203b24) they are all infinite. This was taken to be an especially important argument in favour of accepting the existence of the infinite (203b22–3; cf. Avicenna *Pointers and Reminders* 2.1.4). While it is unclear in several respects, it seems that we are here offered an argument from conceivability. Thus, for instance, numbers do not 'give out in thought' because, for any number x, one can conceive of a number y such that y is greater than x (and something similar holds for magnitudes and the cosmos). From this it is meant to follow that there exists such a number and (somehow) that thereby the infinite exists.

Simplicius observes that (E) relies upon the notion that certain acts of thinking are constituted by or require a relation to certain objects of thought (Simpl. *In Phys.* 467, 12–14). That is to say, roughly speaking, that if one conceives of x, then there must be some x, of which one is conceiving. Aquinas notes the wider currency of variants of this thought among some of the ancient Greeks (*In Phys.* 3, lect. 7, par. 341) but the thought is not exclusively ancient.[10] When put plainly, the notion seems mistaken; nonetheless, especially when implicit, it has often been assumed and it seems that something similar is at work in (E). Aristotle's own response to (E), like his response to (D), comes at the end of book 3 of the *Physics* and turns on denying the presupposition upon which (E) seems to rely (*Phys.* 208a16–19). Conceiving of x as F does not imply that x is F or that x exists. However, there lies a significant complication in the fact that (E) is meant to apply to the objects of mathematics, such as numbers and mathematical magnitudes. It has seemed plausible to many mathematicians and philosophers that such objects exist in a manner quite different from that of everyday, concrete objects. Aristotle's own views on the nature of mathematical objects and their

[8] 'But if someone were to say that a limit is relative (*pros ti*) (since a limit is a limit *of something*), still it is not relative to something else (*ou pros allo*), but relative to itself (*pros auto*)' (Simpl. *In Phys.* 516, 18–20).

[9] Further, we should notice that while Aristotle accepts that every body is either finite or infinite (e.g. *Cael.* 274a30–31), Aristotle denies (1) in its current form or else restricts the domain of the quantifier to a limited domain as it is not the case that anything whatsoever is finite or infinite. For instance, points (*stigmai*) and *pathē* are neither (*Phys.* 202b32–4). Since a point is the limit of a line (*Top.* 141b19–22), some limits will be neither finite nor infinite (cf. Alex. Aphr. *In Top.* 30, 27–31, 26).

[10] For instance, in early and mid-twentieth century discussions of perception, a lot of mileage was gained from the notion that if, in perceptual experience, I am aware of something as being red, then there must be something, of which I am aware, that is red.

existence are difficult to interpret (for discussion of number, see below);[11] however, insofar as he seems to think that the existence of (at least some) mathematical objects is mind-dependent in some strong sense (this is sometimes interpreted as the force of *Phys* 223a21–9, e.g. Hussey 1983, p.183) or else that thinking about (at least some) mathematical objects brings into actual existence what was previously only potential (cf. *Metaph.* 1051a21–33; *De Anima* 430b10), his response to (E) is either problematic or, at the very least, requires something further in the way of elaboration. Thus, as it stands, Aristotle's response to (E) seems less than adequate. However, in what follows I will suggest that Aristotle does have the resources to give a fuller response to (E).

3 Aristotle's arguments against infinity

Following his discussion of the traditional arguments cited in favour of the infinite's existence, Aristotle goes on to offer his own arguments against the existence of the infinite. If the infinite were to exist, Aristotle remarks, then it would have to be either a substance (*ousia*) in its own right, a per se accident (*sumbebēkos kath' auto*) of something else, or else exist in some other way (203b32–5; cf. 200b26–8, 206a12–14). However, not the first; the infinite cannot, Aristotle thinks, exist as a substance. Just as the number four is not a substance because it does not exist in its own right (instead we must instead have four *of something*, *Metaph.* 1092b19–20; *Phys.* 221b14–15), so too with magnitude and also, Aristotle thinks, the infinite (204a17–20). It is not a 'this' (*tode ti*, 206a29–30) and does not exist independently as a substance, but is more plausibly supposed to be feature (*pathos*) of number (*arithmos*) or magnitude (*megethos*) (203b34–5; cf. 204a10, 17–19, 28–9).

What then, of the second option (i.e. that the infinite exists as a per se accident)? Aristotle's discussion of this point is more complex. Given the focus of the *Physics*, Aristotle initially directs his inquiry to the natural world and focuses primarily on whether there is a physical, perceivable magnitude which is infinite (204a1–2, a34–204b4, 205a7–8). More concretely, he asks whether there is, or could be, a body which is infinitely extended in length (*sōma apeiron epi tēn auxēsin,* 204b3–4; cf. *diestēkos eis apeiron*, 204b22; *sōma apeiron*, 205a8; 207a33–5; *apeiros kata to megethos*, *Cael.* 268b11–13.). While most of his predecessors thought that there was an infinitely extended body (e.g. *Cael.* 271b1–3), in the *Physics* Aristotle offers two strands of argument as to why this cannot be right.

One strand of thought, by far the more lengthy, proceeds 'physically' (*physikōs*, 204b10) in that the arguments rely upon assumptions particular to Aristotelian physics.

[11] On the one hand, Aristotle seems to takes (at least some) mathematical claims to be (literally) true (*Metaph.* 1077b31–33). On the other hand, he seems to take (at least some) mathematical claims, notably those which treat the relevant objects as being 'separated', as being best understood along fictionalist lines (*Metaph.* 1078a17–31; cf. *Phys.* 193b31–5). (These claims are perfectly consistent.) He also seems to claim that (at least some) mathematical objects do *not* exist in the manner typically supposed (*Metaph.* 1077b15–17) and that (at least some) mathematical objects *do* exist in the manner typically supposed (1077b32–4). Aristotle here appeals to the fact that existence is spoken of in different ways (either in actuality [*entelecheia*] or in the manner of matter [*hulikōs*], 1078a28–31) in a manner which parallels the discussion of the infinite in the *Physics* (see above).

Thus, for instance, one argument relies upon the thought that the elements, due to their natures, must stand in certain determinate relations to each other and that these relations require a finite number of elements and a finite amount of each (204b13–14); another argument supposes that each element has its proper place (205a10–11); and another appeals to something like explanatory economy, proposing that we can explain the same phenomena without positing infinite entities (cf. *Cael.* 302b20–9). These arguments are not without interest, but a clear exposition requires a detailed treatment of Aristotle's physics and of the complementary arguments offered in *De Generatione et Corruptione* and *De Caelo* (and this is more than I am able to offer here). Instead, I shall here focus upon the other stand of argument against the existence of the infinite body which Aristotle offers in the *Physics*. It is much briefer (204b4–10) and proceeds 'logically' or 'dialectically' (*logikōs*), relying upon assumptions *not* particular to Aristotelian physics. While Aristotle does *not* generally regard 'logical' arguments as entirely conclusive for inquiries in natural science, such arguments were seemingly deemed necessary for philosophical inquiry into the sciences (Alex. Aphr. *In Top.* 30, 9–14) as well as pedagogy and persuasion of naysayers, and these particular 'logical' arguments have been identified as highly important in motivating Aristotle's finitism (Hussey, pp. xxiii–xxiiv, 79). Yet, in spite of this presumed importance, they have received surprisingly little discussion.[12] Aristotle's text is as follows:

(F) If the definition of body is that which is bounded (*horizein*) by a surface (*epipedos*), then there cannot be an infinite body, either as an object of thought (*noētos*) or of sense-perception (*aisthētos*) (204b5–7).

(G) Neither can there be a separated (*kechōrismenos*) infinite number: for number (*arithmos*), or what has number, is countable (*arithmētos*), and so, if it is possible to count (*arithmein*) what is countable, it would be possible to traverse (*diexerchomai*) the infinite (204b7–10).

In (F) and (G), Aristotle argues (elliptically) from the 'boundedness' of body and the countability of number to the conclusion(s) that there is no infinite body or infinite number. With regard to (F) we may notice that Alexander of Aphrodisias (*In Top.* 30, 14–18), Philoponus (*In Phys.* 417, 1–4), and Averroes [*In Phys.* 3m t,c, 40, f. 103raB (cited by Trifogli 2000, p. 91)] seem to have understood the argument roughly as follows:

(1) every x is such that if x is a body, then x is bounded by a surface;
(2*) every x is such that if x is bounded by a surface, then x is finite;

therefore:

(3*) every x is such that if x is a body, then x is finite.

While this can, with minor modifications, be put into very neat (Aristotelian) syllogistic form, I think that the following reconstruction is possible and may better support Aristotle's broader argument(s) against infinitude[13]:

12 Thus, for instance, while Hussey offers a brief discussion of (G) (1983, pp. 79–80), he does not offer an analysis of (F) or, indeed, say much about it beyond that it is 'too quick' (1983, p. 79).

13 The inference to (7) presupposes that not being bounded entails not being bounded by a surface. Despite not being a logical truth, this seems true, necessary, analytic, and knowable a priori. If necessary, it can be

(1)	every x is such that if x is a body, then x is bounded by a surface;	[premise]
(2)	every x is such that if x is infinitely extended, then x is not bounded;	[premise]
(3)	there is an x such that x is an infinitely extended body;	[supposition]
(4)	a is an infinitely extended body;	[from 3]
(5)	a is bounded by a surface;	[from 1, 4]
(6)	a is not bounded;	[from 2, 4]
(7)	contradiction;	[from 5, 6]
(8)	there is no x such that x is an infinitely extended body	[*reductio*]

Thus construed, (F) offers a *reductio* wherein a contradiction is shown to follow from supposing the contradictory of the intended conclusion. That is to say, taking (1) and (2) to be true, a contradiction follows from supposing that there exists some infinitely extended body. It was already observed above that Aristotle's principal response to the argument against conceivability (see (E) above) lies in resisting the use of conceivability in these circumstances (*Phys.* 208a16–19). However, since we cannot, Aristotle thinks, conceive of contradictions (cf. *Metaph.* 1005b23–4, 29–30), the reconstruction suggested allows one to better see not only why Aristotle thinks that there are no particular infinitely extended bodies, but also why Aristotle might be inclined to claim (as he does here, *Phys.* 204b6) that an infinitely extended body (or magnitude) is not even conceivable as an object of thought.

On either of the construals of (F) offered the argument is valid. However, as Philoponus (*In Phys.* 417, 14–21) and Aquinas (*In Phys.* 3, lect. 8, par. 352) noticed, one might well deny the premises, especially (1) (which is shared by both interpretations of (F) which I have given above). After all, why need one accept that all bodies are bounded by surfaces? Being bounded by a surface doesn't seem to be part of any widely accepted definition of body and Aristotle himself often defines or characterises body (*sōma*) simply as a magnitude divisible (*diaireton*) or extended (*diastaton*) in three dimensions (e.g. *Phys.* 202b20; *Cael.* 268a7–10, b6–7, 274b19–20). If that is what it is to be a body, then it is not immediately clear why the notion of a body infinitely extended in three dimensions is incoherent (cf. *Phys.* 204b20–2).[14] Accordingly, one may legitimately worry whether (1), which is necessary for the argument in (F) (however we decide to interpret (F)), is in fact well motivated and not simply an ad hoc stipulation designed to render infinitely extended bodies impossible.

One proposal, advanced by certain medieval Oxford commentators, supposed that a body must be bounded by a surface because bodies are composed of matter and

Footnote 13 continued
supplied as a premise (e.g. ⟨if a is not bounded, then a is not bounded by a surface⟩). Simplicius *may* be making an observation in this vicinity (*In Phys.* 477, 17–19).

14 Unlike some later philosophers (consider, for instance, the discussions of Al-Kindī and Avicenna mentioned below), Aristotle does not here seem to consider in any detail the possibility of bodies possessing certain boundaries but which are infinitely extended in only one or two dimensions (such as an infinitely tall skyscraper). Instead, he considers only bodies which are infinitely extended in all dimensions (cf. *Phys.* 204b20–1).

form and that a body unbounded by surface requires the existence of matter without form (which is, the thought goes, impossible); however, precisely how the argument is meant to go is not entirely clear (similar concerns apply to the other seemingly less perspicuous proposals put forward by said commentators, see Trifogli 2000, pp. 92–95). What then might be said? While some sort of inductive argument might be offered in support of (1) (cf. Alex. Aphr. *In. Top.* 30, 27–31), it seems more promising to suppose that Aristotle would justify (1) by focusing on the difficulties stemming from allowing a body which is not bounded by any surface or, more generally, of a magnitude which is not bounded by any limit.[15] The sort of vagueness that would be instantiated by such unbounded entities is troubling, especially when we consider the identity conditions of these putative entities (and one need not embrace the Quinean slogan—no entity without identity—to feel unease). Aristotle, does, I think, have principled (and independently plausible) reasons for resisting the existence of such entities. Notably, he characterises a thing's limit or boundary thus: 'we call a limit (*peras*) the last point (*eschaton*) of each thing, i.e. the first point beyond which it is not possible to find any part [of that thing], and the first point within which every part [of that thing] is' (*Metaph.* 1022a4–5). In being infinitely extended, an object would thereby lack limits or boundaries and this would give rise to several worrying oddities. Since a limit separates what is 'inside' an object and what is 'outside' it—both 'constitutionally' and spatially—there would not only be nothing 'outside' such an object, but also, seemingly, nothing 'inside' it. Lacking limits or boundaries, there would be nothing to separate what constitutes an infinitely extended object (i.e. its proper parts) from what does not constitute it, and such infinitely extended objects would be such that not only would there would be no location they did not occupy, but also seemingly no location which they did occupy (since a boundary is also 'the first point within which every part [of that thing] is').[16] While these thoughts are not (as far as I am aware) explicitly signalled by Aristotle or later commentators in the relevant discussions concerning infinite bodies, they are maintained by Aristotle elsewhere (and are independently plausible) and provide insight into a possible motivation for (1). We may thus observe that not only is Aristotle's argument in (F) valid (on either of the construals offered), but that the denial of the existence of infinitely extended bodies need not be seen as ad hoc and may plausibly be seen to rest upon other (seemingly sensible) metaphysical views.

[15] Al-Kindī and Avicenna gave substantial attention to arguing against the possibility of infinite magnitudes or bodies and offer alternative arguments. However, they depart from Aristotle on a number of scores (for instance, Avicenna is more tolerant of actual infinities than most working in the Aristotelian tradition) and, unlike the 'physical' and 'logical' arguments offered by Aristotle, the arguments they offer against infinitude proceed 'geometrically', i.e. they rely upon mathematical demonstrations and premises which Aristotle does not mention in these contexts. See Al-Kindī *On the Explanation of the Finitude of the Universe*; Avicenna *Pointers and Reminders* 2.1.11; *Salvation* 2.2; cf. *Physics of the Healing* 3.8.1; Rescher and Khatchadourian (1965), McGinnis (2010).

[16] We can thus see why Aristotle characterises the infinite as that of which something [i.e. some part] is always outside' (*Phys.* 207a1–2, 7–8) and contrasts it with what is 'complete' (*teleios*) or 'whole' (*holos*) (i.e. 'that of which nothing [i.e. no part] is outside', *Phys.* 207a8–9, 11–15; cf. *Metaph.* 1021b12–14). As regards spatiality, recall that Aristotle takes commonsense to be (at least largely) correct in supposing that everything there is must be somewhere [e.g. *Phys.* 205a10; 208a29, 208b32–3 (though he may make an exception for the unmoved mover; *Cael.* 279a11–18; cf. *Phys.* 267b6–9)].

Finally, then, we come to (G), Aristotle's argument that there is no infinite number.[17] The argument seems to proceed as follows:

(1)	if x is a number, then x is countable;	[premise]
(2)	if x is countable, then x is traversable;	[premise]
(3)	if x is infinite, then x is not traversable;	[premise]
(4)	there is some x such that x is a number and x is infinite;	[supposition]
(5)	a is a number and a is infinite;	[from 4]
(6)	a is countable;	[from 1, 5]
(7)	a is traversable;	[from 2, 6]
(8)	a is not traversable;	[from 3, 5]
(9)	*contradiction*	[from 7, 8]
(10)	there is no x such that x is a number and x is infinite	[*reductio*]

The argument *seems* valid (assuming no shift in sense between the relevant terms, but more on this in a moment) but requires some clarification. First, we should notice that many of the Greeks conceived of a number as 'a plurality (*plēthos*) composed of units (*monadōn*)' (Euclid *Elements* 7, def. 2). Aristotle shared this conception (e.g. *Metaph.* 1053a30; cf. *Phys.* 207b7; *Metaph.* 1039a12–13; 1085b22; Simpl. *In Phys.* 477, 36), and that is why, for instance, he takes two to be the smallest number (since one is not a *plurality* of units) (e.g. *Phys.* 220a27; *Metaph.* 1056b25–8; 1085b10; 1088a6–8; cf. Frege *Grundlagen* §25).[18] Secondly, something should be said about

[17] Hussey offers the following reconstruction (1983, p. 80), but little in the way of further discussion:

(1) Any actual numbered totality is countable (taken as self-evident).
(2) For any countable totality it is possible for someone to have counted that totality (by definition of 'countable').
(3) To have counted an infinite is to have traversed an infinite.
(4) What is infinite cannot have been , 204a5–6).
∴ (5) No infinite totality can have been counted (by (3) (4)).
∴ (6) No infinite totality is countable (by (2) (5)).
∴ (7) No actual numbered totality is infinite (by (1) (6)).

For reasons that will immediately become clear I don't think that this interpretation allows for a clear or straightforward diagnosis of the argument.

[18] For moderns, number terms (e.g. 'three', 'eight', etc.) are usually viewed as singular terms which refer to abstract, unique objects (e.g. Frege *Grundlagen* Introduction, §38, §61). In contrast, for the Greeks, there were many twos, many threes (and so forth) because there were many sets of two units, of three units (and so forth). Felix, Tibbles, and Tigger constitute a number and each unit in this number is a cat (for sets or collections of *seemingly* heterogeneous units, cf. *Metaph.* 1088a8–14). It seems that, on Aristotle's view, when we have a set (collection, etc.) of three cats and a set of three dogs, the number of cats is *equal* to the number of dogs but the numbers are not—contra Frege (e.g. *Grundlagen* §62–3)—(numerically) identical because the numbers (i.e. the sets) in question are composed of different units (*Phys.* 220b8–12, 224a2–10; cf. Mayberry 2000, pp. 52–59). It has been thought prudent *not* to use the notion of a set in approaching such issues (e.g. Hussey 1980, p. xx), but it seems that, on the ancient conception described here, numbers (*arithmoi*) are in fact akin to collections or sets as conceived by moderns in several important respects (for discussion, see Burnyeat 1987, pp. 220–221, 234–238; Mayberry 2000, pp. 17–63).

the sense of ‘traversable’ and ‘countable’. In this context, ‘traverse’ (*dierchomai*) must express either counting or an activity which is constituted primarily by counting (LSJ s.v., ‘go through’; ‘pass through’; ‘go through in detail, recount’) and this is indeed the way it has been traditionally understood (e.g. Aquinas *In Phys.* 3, lect. 8, par. 351). However, the modality and aspect at issue in both ‘traversable’ and ‘countable’ (and whether it remains uniform across the argument) is not entirely clear. In these contexts, Aristotle distinguishes between several senses in which a thing might not be ‘traversable’ (204a2–6, 13–14). The two relevant senses are as follows:

(i) sometimes something is not traversable because traversing cannot *get started*. For instance, try counting or ‘going through’ how many drops of water are in a lake. Given that there are no well-defined units, the task cannot be done because it cannot even be begun.

(ii) sometimes something is not traversable because traversing cannot be *completed*. For instance, try counting or ‘going through’ the natural numbers until you reach a number which has no number greater than it. In this case the task may be begun (for we may indeed begin counting) but not completed.

These observations being made, we may proceed to the argument. At first glance, (G) seems valid and dialectically effective. However, the main issue with (G) and those variants upon it which we find in the commentaries (e.g. Simpl. *In Phys.* 477, 26–8) is that for the argument to be valid there must *not* be a shift in the sense of ‘countable’ and ‘traversable’, but, in order for the premises to seem obviously true, there must be a shift in the sense of ‘countable’ and ‘traversable’.

On the conception of number (*arithmos*) at issue here not everything which is countable is a number (for it seems that one is countable and yet is not a number). However, due to being composed of determinate *units*, it does seem that every number is countable in the sense that one could, at the very least, begin ‘going through’ (i.e. counting) the units which compose it (this is the sense described in (i)).[19] If that is the sense of ‘countable’ in (1) and the sense of ‘traversable’ and ‘countable’ in (2), then both (1) and (2) seem uncontroversially true. However, if that is the sense of ‘traversable’ in (3), then it is not clear why (3) need be accepted as true and it in fact seems false. After all, why could one not even begin counting the elements of an infinite collection or set (e.g. the set of natural numbers)? Instead, for (3) to seem an obvious truth, it seems that the sense of ‘traversable’ at issue must be that according to which the counting or ‘going through’ can be begun but not completed (i.e. the sense described in (ii)). But, if that sense of ‘traversable’ and ‘countable’ is uniform across the argument, then it is not immediately clear why (1) and (2) should be accepted as true (cf. Philop. *In Phys.* 417, 19–21).[20] Thus, for instance, with regard to (1), why think that a collection or set need be such that one must be able to complete the counting or ‘going through’ of the units which compose it?

[19] There is a close connection in Greek language and thought between counting (*arithmein*), being countable (*arithmētos*), and number (*arithmos*). The translation of ‘*arithmos*’ as ‘number’, although ubiquitous, slightly obscures this fact.

[20] This must, I think, be the sense of ‘countable’ Philoponus has in mind when he complains, against Aristotle, that advocates of the existence of the infinite would not grant that all number (*arithmos*) is countable (*arithmētos*) (*In Phys.* 417, 19–21).

Aristotle would, I think, have regarded (G) as valid and would have opted for the sense of 'traversable' and 'countable' described in (ii), insisting that, in the case of number, one must be able to complete the counting or 'going through' of the units which compose the relevant set or collection. He takes number not just to be a plurality of units (as described above), but to be a *finite* plurality (e.g. *plēthos peperasmenon, Metaph.* 1020a13) and this makes it plausible to suppose that he would think that number is countable in the sense that its counting can be *completed*. On this interpretation, (1) (which would be the controversial premise), would stipulate that if x is a number, then x is such that one *can* complete counting or going through the units which compose it. Thus construed, (G) is valid and, in defence of (1), we may notice that Aristotle was not alone in conceiving of number (*arithmos*) as finite and countable in this way.[21]

However, that Aristotle should not be alone in holding this view provides us with less in the way of substantive reasons than we might wish for accepting (1) and, indeed, the argument as a whole is somewhat limited in its effectiveness. The conclusion is, perhaps, simply 'too close' to the crucial premise (for if it is part of the definition of number that it be finite or that is can be completely counted or 'traversed', then the argument does little work) and, in any case, when the sense of 'countable' and 'traversable' is specified (and Aristotle himself is at pains to mark the distinction in senses, see above), the argument loses much of its dialectical effectiveness against naysayers. Proponents of the existence of the infinite could simply deny (1), or, as Aquinas notices, they could accept that *number* cannot be infinite, but still hold that the infinite exists by arguing that a multitude or plurality (*plēthos*) can be infinite without claiming that it is a number (Aquinas *In Phys.* 3, lect. 8, par. 352).

It is not entirely clear what Aristotle would have to say on this score. One possible response (similar to what was raised with regard to (F)) would advert to the oddness of what the proponent of the infinite suggests. Thus, for instance, if one assumes that pluralities are like sets in the relevant ways (see above), and also follows Cantor in thinking that what it is for a set A to be infinite is for there to be some set B such that B is a proper subset of A and the cardinality of A is the same as that of B(so that there is a one-to-one correspondence between the elements of A and the elements of B), then an infinite plurality would go against what Euclid describes as a common notion: that the whole is greater than the (proper) part (*Elements* 1, comm. not. 5). However, in contrast with the response offered to Aristotle when discussing (F), the notion contravened in this case is less intuitive and one might, after all, characterise what it is for a plurality to be infinite differently (e.g. such that the counting of it could never be completed). Accordingly, this strikes me as a less persuasive rejoinder, but it is not immediately clear how else Aristotle might hope to rule out the existence of an infinite plurality. It is here then that we find the lacuna in Aristotle's account. On the interpretation proposed of (G), the argument is fine as far as it goes. However, *even if* one were to accept (1) (the controversial premise, i.e. that number need be such that

[21] Iamblichus seems to attribute this conception of number (as a finite plurality) to Eudoxus (Iambl. *In Nicom.* 10, 17–20). Pritchard (1995, pp. 23–30) argues that the conception of number as necessarily finite had wider currency among the Greeks but the evidence is not straightforward.

one can complete its counting), then it is not clear on what basis one might rule out the existence of infinite pluralities.

4 Conclusion

This paper has sought to advance our understanding of Aristotle's finitism by examining the arguments Aristotle considers both in favour of and against the existence of the infinite. These arguments have often been found wanting and have generally not received much in the way of discussion. However, I hope to have shown that, despite there being a lacuna in Aristotle's reasons for rejecting the existence of infinite pluralities or sets, Aristotle's reasons for ruling out the existence of actual infinities are, in general, less ad hoc and more deeply grounded in his other commitments (and independently plausible notions) than commonly supposed.

In Sect. 2, I examined those traditional arguments in favour of the existence of the infinite which Aristotle describes in the *Physics*, (A)–(E), and Aristotle's own response to them. While Aristotle largely accepts the force of (A) and (B) (and seeks to incorporate the infinite divisibility of magnitudes and that there is no first or last moment of time into his own account of the potential infinite), he rejects the force of (C), (D), and (E) and, against some scholars, I argued that Aristotle does so (at least in the case of (C) and (D)) for good reasons.

In Sect. 3, I gave careful consideration to Aristotle's own positive 'logical' or 'dialectical' arguments against the existence of infinite magnitude and number, (F) and (G). I argued that, when properly understood, Aristotle's argument against the existence of an infinite magnitude is valid, dialectically effective, and relies upon plausible premises. Aristotle's argument against the existence of infinite number is also valid but does, I suggested, rely upon more controversial premises and the conclusion it establishes seems to fall short of what Aristotle needs. For *even if* we accept the argument as sound, it only rules out the existence of infinite *numbers* and does not rule out the existence of infinite pluralities or sets. Having better understood Aristotle's arguments for and against the infinite we can see not only that his finitism is better grounded than commonly supposed, but also precisely where the gap in the warrant for his finitism lies.

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