On falsifying empirical contradictions

A proof of the unfalsifiability of contradictory observation sentences

Abstract

The possibility of testing contradictory statements about the factual world has been suggested but barely discussed in the relevant literature. Here I argue that if we assume that there are contradictory observation sentences, it would be logically impossible to falsify them. Accordingly, the extension of the dialetheist programme into empirical science would be non inadvisable for it would introduce logically unfalsifiable claims.

1 Logical assumptions

According to Priest [9], there exist contradictory sentences describing observable states of affairs. In this paper I prove that all contradictory sentences $\phi \wedge \neg\phi$, where both ϕ and $\neg\phi$ are observation sentences, are unfalsifiable if we assume, as Priest, that they are verifiable. In order to prove this we need a set of FO-wffs $\mathbf{L}$ defined as usual plus the clause:

Def. $$\alpha \in \mathbf{L} \Rightarrow \mathsf{F}\ulcorner\alpha\urcorner \in \mathbf{L} \tag{L}$$

The first logical assumptions I need to introduce are: *elimination of* $\leftrightarrow$ (from $\vdash \alpha \leftrightarrow \beta$ deduce $\vdash \alpha \rightarrow \beta$ and $\vdash \beta \rightarrow \alpha$), *transitivity of* $\rightarrow$ (from $\vdash \alpha \rightarrow \beta$ and $\vdash \beta \rightarrow \gamma$ deduce $\vdash \alpha \rightarrow \gamma$), *reflexivity of* $\rightarrow$ ($\vdash \alpha \rightarrow \alpha$) and *simplification* ($\vdash \alpha \wedge \beta \rightarrow \alpha, \vdash \alpha \wedge \beta \rightarrow \beta$). I also need a falsity predicate F satisfying the following axioms:

Ax. $$\vdash \alpha \rightarrow \mathsf{F}\neg\alpha \tag{F1}$$

Ax. $$\vdash \neg\alpha \rightarrow \mathsf{F}\alpha \tag{F2}$$

Ax. $$\vdash \mathsf{F}\alpha \rightarrow \mathsf{F}\ulcorner\alpha \wedge \beta\urcorner, \vdash \mathsf{F}\beta \rightarrow \mathsf{F}\ulcorner\alpha \wedge \beta\urcorner \tag{F3}$$

In the next section I define the class of observation sentences of our language, which are the kind of sentences that can falsify and corroborate a theory. The concepts of potential falsifiers and corroborators for sentences are formally introduced in section 3. The thesis of this paper is formally proved in section 4 with the help of three further assumptions. Finally, section 5 discusses the significance of this proof for empirical dialetheism.

2 Observation

Let $\mathbf{S}$ be the set of singular sentences of $\mathbf{L}$, which excludes all formulae constructed using clause (L). The function $\mathbf{O}\text{B} : \wp\mathbf{L} \longrightarrow \wp\mathbf{S}$ returns the set of observation sentences of any subset of $\mathbf{L}$, and satisfies the following axioms.

$$Ax. \qquad \alpha \in \mathbf{O\scriptstyle B}(\mathbf{A}) \Rightarrow \neg\alpha \in \mathbf{O\scriptstyle B}(\mathbf{A}) \tag{O4}$$

$$Ax. \qquad \alpha \in \mathbf{O\scriptstyle B}(\mathbf{A}) \;\;\&\;\; \vdash \alpha \leftrightarrow \beta \Rightarrow \beta \in \mathbf{O\scriptstyle B}(\mathbf{A}) \tag{O5}$$

In what follows, ϕ and ψ will range over $\mathbf{O\scriptstyle B}(\mathbf{L})$.

Axiom (O4) is justified as long as we restrict the logical form of observation sentences to that of singular sentences, instead of also including existential sentences. If ϕ is a singular sentence, so can be $\neg\phi$ because it would also be a singular sentence. But if it were existential, $\neg\phi$ would be equivalent to a universal sentence, which would make it inaccessible to us by observation.[1]

There are good reasons for considering that the negation of any singular observation sentence is an observation sentence too. In order to do this we must assume, as Bobenrieth [1, 2], that observing $\neg\phi$ is simply observing a situation incompatible with what is described by ϕ. Hence, we could observe what $\neg\phi$ describes only by observing what is described by another observation sentence ψ whose empirical content is incompatible with that of ϕ.

Accordingly, $\neg\phi$ would not be an observation sentence iff no ψ whatsoever could describe a situation incompatible with ϕ. This might be because (*i*) ϕ is compatible with all observable situations, in which case it is simply unfalsifiable, or because (*ii*) it is falsifiable but $\mathbf{L}$ has no formula expressing the observable situation incompatible with ϕ, in which case $\mathbf{L}$ is inappropriate for our formalization. Since we want ϕ to be falsifiable, and $\mathbf{L}$ to be appropriate for its falsification, we must conclude that $\neg\phi$ is an observation sentence. This justifies axiom (O4).

I will dismiss any concerning about the impossibility of constructing a universal set of observation sentences (cf. [4]) in order to make my argument more straightforward. I will address those concernings in a future work. Now let us turn to the classes of potential corroborators and falsifiers of sentences.

3 Falsifiable sentences

A thorough characterisation of falsifiability is possible only if we introduce the concepts of *occurrence* and *event* (cf. [6], §23). Since this would unnecessarily

[1] It is not clear to me whether Popper's *Basissätze* —which are equivalent to my observation sentences— have the logical form of singular or existential statements. Although he characterises them as *Tatasachenfestellungen* ([6], p. 45), he explicitly requires them to be existential statements in order to reproduce the asymmetry between scientific laws and its potential falsifiers: "[W]ir müssen die logische Form der Basissätze so bestimmen, daß die Negation eines Basissatzes seinerseits kein Basissatz sein kann" [6, p. 58]. 'Basissatz' is here synonymous with 'potential falsifier'.

complicate my exposition, I will use a more 'direct' approach stating that an observation sentence is a potential falsifier of a theory iff it implies that is false. In the same way, if an observation sentence ϕ implies that a sentence α is false, then the former is a potential falsifier of the latter.

Def. $$\mathbf{FA}(\alpha) = \{\phi \mid \vdash \phi \rightarrow \mathsf{F}\alpha\} \quad (Fa)$$

From where we have:

{F1} $$\phi \in \mathbf{FA}(\neg\phi) \quad (6)$$

{F2} $$\neg\phi \in \mathbf{FA}(\phi) \quad (7)$$

{F3} $$\mathbf{FA}(\alpha) \subseteq \mathbf{FA}(\alpha \wedge \beta), \mathbf{FA}(\beta) \subseteq \mathbf{FA}(\alpha \wedge \beta) \quad (8)$$

Proof. Definition (Fa) guarantees that $\vdash \phi \rightarrow \mathsf{F}\alpha$, for all $\phi \in \mathbf{FA}(\alpha)$. From this and (F3) follows, by transitivity of $\rightarrow$, that $\vdash \phi \rightarrow \mathsf{F}\lceil\alpha \wedge \beta\rceil$, which by definition (Fa) implies that $\phi \in \mathbf{FA}(\alpha \wedge \beta)$. The proof for β is similar.

All of this leads to the following corollary:

{F1, F2, F3} $$\phi, \neg\phi \in \mathbf{FA}(\phi \wedge \neg\phi) \quad (9)$$

Now, although empirical theories cannot be verified, it is possible to corroborate them through observation sentences. To corroborate a theory means to verify or confirm a case where it holds. Accordingly, I call ϕ a potential corroborator of α iff ϕ implies an observation sentence ψ implied by α.

Def. $$\mathbf{Co}(\alpha) = \{\phi \mid \vdash \phi \rightarrow \psi, \text{ where } \vdash \alpha \rightarrow \psi\} \quad (Co)$$

We could have alternatively said that all ϕ implied by some ψ such that $\vdash \alpha \rightarrow \psi$ are potential corroborators of α. But this definition is too weak. Suppose we want to justify a sentence β, and we are given the chance to have evidence for only one of α or γ, where $\vdash \alpha \rightarrow \beta$ and $\vdash \beta \rightarrow \gamma$. Which evidence would be more reliable? Obviously, the evidence for α because, whereas γ gives us abductive evidence for β, α logically implies β.

Definition (Co) has some weird consequences in the classical logic framework. Since $\vdash \phi \wedge \neg\phi \rightarrow \phi$ and $\vdash \phi \rightarrow \phi$, it may be said that $\phi \wedge \neg\phi$ is a potential corroborator of ϕ. This makes more sense than it looks like because in classical logic anything follows from a contradiction; hence, it is not weird at all that logical absurdities be potential corroborators of any sentence. Nevertheless, we need not to conclude that since we have not stipulated that $\phi \wedge \neg\phi$ is an observation sentence —nor any conjunction of observation sentences for that matter. On the other hand, although I would exclude logically absurd wffs from the set of observation sentences, nothing prevents us from considering $\phi \wedge \neg\phi$ an observation sentence in a dialetheic framework, since $\phi \wedge \neg\phi$ is not absurd in such framework.

The following is an important consequence of definition (Co).

{} $$\phi, \neg\phi \in \mathbf{Co}(\phi \wedge \neg\phi) \quad (10)$$

Proof. Reflexivity of $\to$ and simplification entail $\vdash \phi \to \phi$ and $\vdash \phi \wedge \neg\phi \to \phi$. These imply that $\phi \in \mathbf{Co}(\phi \wedge \neg\phi)$. The proof for $\neg\phi$ is similar.

Suppose, now, that some ϕ were both a potential corroborator and falsifier of α. What would it mean to verify ϕ in terms of our epistemic attitude towards α? It seems that we would be both corroborating and falsifying α. This situation is similar to the one of both accepting and rejecting the same sentence, which Priest himself deemed as irrational —although with the qualification that "the ideal rational agent may be an impossible object" ([8], p. 274). Hence, it is mandatory that $\mathbf{Co}(\alpha) \cap \mathbf{Fa}(\alpha)$ be empty, regardless of the α. To this I will call the *requirement of non overlapping*. This requirement, though, is not satisfied by contradictory observation sentences, as theorems (9) and (10) immediately imply:

{F1, F2, F3} $$\phi, \neg\phi \in \mathbf{Co}(\phi \wedge \neg\phi) \cap \mathbf{Fa}(\phi \wedge \neg\phi) \quad (11)$$

In other words, for all contradictory sentences $\phi \wedge \neg\phi$, there is some observation sentence that both corroborates it and falsifies it, regardless of the ϕ. This may be taken as the ultimate argument against any form of empirical dialetheism. However, it is still possible to save the programme by defining a set of potential refuters $\mathbf{Re}(\alpha)$ of a sentence α as follows:

Def. $$\mathbf{Re}(\alpha) = \mathbf{Fa}(\alpha) - \mathbf{Co}(\alpha) \quad (Re)$$

If we transfer the requirement of non overlapping from potential falsifiers to potential refuters, we would only need that no observation sentence be both a potential refuter and corroborator of the same sentence, which trivially follows from the previous definition.

{} $$\mathbf{Re}(\alpha) \cap \mathbf{Co}(\alpha) = \{\} \quad (12)$$

With this result at hand, I define a sentence to be *falsifiable* or *refutable* iff its class of potential refuters is not empty.

Def. $$\alpha \text{ is } \textit{refutable} \Leftrightarrow \mathbf{Re}(\alpha) \neq \{\} \quad (Ref)$$

This leads to the question of whether it is possible for $\mathbf{Re}(\phi \wedge \neg\phi)$ to be non empty. I address this in the next section.

4 The unfalsifiability of contradictions

Since contradictory sentences are conjunctions, it is worth seeing how can we falsify a conjunction. If ϕ and ψ are not dialetheias, then both $\neg\phi$ and $\neg\psi$ are potential falsifiers of the conjunction $\psi \wedge \phi$. Nevertheless, this does not mean that $\neg\phi$, $\neg\psi$ and their equivalents are the only members of $\mathbf{Fa}(\phi \wedge \psi)$.

Consider the case where p is the proposition "Aldo killed John", and q, the proposition "Bertha killed John". It's clear that "Aldo *did not* kill John" ($\neg p$) and "Bertha *did not* kill John" ($\neg q$) are potential falsifiers of "Aldo and

Bertha killed John" $(p \wedge q)$. But so is a sentence r, stating "John was killed by exactly one person", for the falsity of $p \wedge q$ follows from r. This means that some ψ, which is not equivalent to neither ϕ nor $\neg\phi$, could imply that $\phi \wedge \neg\phi$ is false. Such ψ would be not only a potential falsifier of $\phi \wedge \neg\phi$, but also its potential refuter, which would make $\phi \wedge \neg\phi$ falsifiable.

This seems to be the case of one of Priest's observable contradictions ([7], §3.3). Since theory of colour forbids that something be both green and red —in the same way that it can be both green and yellow—, it may be considered contradictory to state that some object is both green and red. In that case, the verification that such object is completely black —assuming that is possible— would be enough to falsify that it is both green and red.

This, nonetheless, is a misconception of the problem because we are not falsifying a proper contradiction here: we are not falsifying that something is both green and not green (or both red and not red). Verifying that something is black results, instead, in the verification that it is not green, which means that *we are corroborating that it is both green and not green*. This means that any potential falsifier of a contradictory observation sentence will also be its potential corroborator and that $\mathbf{R\textsc{e}}(\phi \wedge \neg\phi)$ is empty.

Now, although I have not stipulated that any instance of $\phi \wedge \neg\phi$ is in $\mathbf{O\textsc{b}}(\mathbf{L})$, I have been assuming it so far in order to see whether it is falsifiable. I can now prove that such is not the case. In order to do that I need to add three further axioms, some of which are more problematic than the previous ones, but that happen to be endorsed by Priest [8, §5.2].

Ax. $$\vdash \mathsf{F}\alpha \rightarrow \neg\alpha \tag{F13}$$

Ax. $$\vdash \neg(\alpha \wedge \neg\beta) \leftrightarrow \beta \vee \neg\alpha \tag{T14}$$

Ax. $$\vdash \alpha \wedge \beta \rightarrow \alpha \vee \beta \tag{T15}$$

From where we obtain our main theorem.

{} $$\mathbf{R\textsc{e}}(\phi \wedge \neg\phi) = \{\} \tag{16}$$

Proof. By definition (*Re*) we just need to prove that $\psi \in \mathbf{C\textsc{o}}(\phi \wedge \neg\phi)$ holds for all $\psi \in \mathbf{F\textsc{a}}(\phi \wedge \neg\phi)$. This follows iff (a) for some observation sentence χ (b) such that $\vdash \phi \wedge \neg\phi \rightarrow \chi$, (c) assuming $\vdash \psi \rightarrow \mathsf{F}\lceil\phi \wedge \neg\phi\rceil$ results in $\vdash \psi \rightarrow \chi$. Such χ is just $\phi \vee \neg\phi$. (a) Since $\phi \wedge \neg\phi$ is assumed to be in $\mathbf{O\textsc{b}}(\mathbf{L})$, it follows by (O4) that $\neg(\phi \wedge \neg\phi) \in \mathbf{O\textsc{b}}(\mathbf{L})$. And since, by (T14), $\vdash \phi \vee \neg\phi \leftrightarrow \neg(\phi \wedge \neg\phi)$, (O5) implies that $\phi \vee \neg\phi \in \mathbf{O\textsc{b}}(\mathbf{L})$. (b) $\vdash \phi \wedge \neg\phi \rightarrow \phi \vee \neg\phi$ trivially follows from (T15). (c) By (F13) we have $\vdash \mathsf{F}\lceil\phi \wedge \neg\phi\rceil \rightarrow \neg(\phi \wedge \neg\phi)$ and $\vdash \neg(\phi \wedge \neg\phi) \rightarrow \phi \vee \neg\phi$ follows from (T14) by elimination of $\leftrightarrow$. From a double application of transitivity of $\rightarrow$ on $\vdash \psi \rightarrow \mathsf{F}\lceil\phi \wedge \neg\phi\rceil$ and the previous results we obtain $\vdash \psi \rightarrow \phi \vee \neg\phi$.

Although this sufficiently proves the unfalsifiability of contradictory observation sentences, it may also be taken as evidence that falsificationism is not a good epistemological theory at least for dialetheism. I address this possible objection in the following and last section.

5 Final remarks

In this paper I have given no argument against the possibility of verifying contradictory observation statements. Priest [10] has good arguments for that, which are quite persuasive even to the orthodox. What I have proved instead is that *if there were contradictory observation sentences, they would be unfalsifiable*. More generally, this means that *it is not be possible to falsify an inconsistent theory qua inconsistent*. Hence, empirical dialetheists would be in the very dogmatic situation where it would be logically impossible for them to falsify at least one such 'contradictory observation statement'.

This also affects da Costa's argument for dialetheism ([3], §III.3), or Hegel's thesis as he calls it, according to which it is easier to verify that there are true contradictions about the world than to falsify it. Furthermore, this thesis cannot possibly be falsified since we would need to verify only one contradictory statement in order to prove it, but to falsify it we would need to falsify all possible contradictory statements. However, since contradictory statements are not even falsifiable, it is logically impossible to just corroborate the negation of Hegel's thesis, which is the least we could ask of any statement worth of empirical science.

Empirical dialetheists cannot argue, as Neurath [5] or Reichenbach [11], that falsificationism proposes an excessively simplified model of science. Empirical dialetheism does not satisfy even this simplified model, and it is not to be expected that it satisfy a more sophisticated one. What they can argue, instead, is that very concept of falsification is irrelevant to empirical dialetheism. This can be done if they take contradictory sentences expressing dialetheias to be false, as well as true, in which case there would be no epistemic gain in falsifying them. In that case, we would have to ask the more general question of whether contradictory observation sentences are acceptable or rejectable. But that is a topic for future research.

References

[1] Andrés Bobenrieth Miserda. *Inconsistencias, ¿Por qué no? Un Estudio Filosófico Sobre la Lógica Paraconsistente*. Premios Nacionales de Cultura 1995. Bogotá: Colcultura, 1996. ISBN: 9789586122573.

[2] Andrés Bobenrieth Miserda. "Paraconsistency and the Consistency or Inconsistency of the World". In: *Handbook of Paraconsistency.* Proceedings of the 4th World Congress on Paraconsistency. (IRIT, Toulouse, July 28–31, 2003). Ed. by Jean-Yves Béziau, Walter Alexandre Carnielli, and Dov M. Gabbay. Logic and Cognitive Systems 9. London: College Publications, 2007, pp. 493–512. ISBN: 9781904987734.

[3] Newton Carneiro Affonso da Costa. *Ensaio sobre os fundamentos da lógica.* 2nd ed. São Paulo: Hucitec, 1994.

[4] Paul Karl Feyerabend. "Explanation, reduction and empiricism". In: *Philosophical Papers.* Vol. 1: *Realism, rationalism and scientific method.* Cambridge: Cambridge University Press, 1981. Chap. 4, pp. 44–96. ISBN: 0521228972.

[5] Otto Neurath. "Pseudorationalismus der Falsifikation". In: *Erkenntnis* 5.1 (Dec. 1935), pp. 353–65. ISSN: 1572-8420. DOI: 10.1007/BF00172326.

[6] Karl Raimund Popper. *Logik der Forschung. Zur Erkenntnistheorie der modernen Naturwissenschaft.* Wien: Springer, 1935. ISBN: 9783709120217. DOI: 10.1007/978-3-7091-4177-9.

[7] Graham Priest. *Doubt Truth to be a Liar.* Oxford: Clarendon Press, 2006. ISBN: 9780199263288.

[8] Graham Priest. *In Contradiction. A Study of the Transconsistent.* Second, expanded edition. Oxford: Clarendon Press, 2006. ISBN: 9780199263295.

[9] Graham Priest. "Inconsistency and the Empirical Sciences". In: *Inconsistency in Science.* Ed. by Joke Meheus. Origins: Studies in the Sources of Scientific Creativity 2. Dordrecht: Springer, 2002, pp. 119–28. ISBN: 9789401700856. DOI: 10.1007/978-94-017-0085-6_7.

[10] Graham Priest. "Perceiving contradictions". In: *Australasian Journal of Philosophy* 77.4 (1999), pp. 439–46. DOI: 10.1080/00048409912349211.

[11] Hans Reichenbach. "Über Induktion und Wahrscheinlichkeit. Bemerkungen zu Karl Poppers „Logik der Forschung“". In: *Erkenntnis* 5 (1 1935). ISSN: 0165-0106. DOI: 10.1007/BF00172315.