Three Dogmas of 'If'

Rani Lill Anjum

Introduction

In this paper I want to argue that a truth functional account of conditional statements 'if A then B' not only is inadequate, but that it eliminates the very conditionality expressed by 'if'. Focusing only on the truth-values of the statements 'A' and 'B' and different combinations of these, one is bound to miss out on the conditional relation expressed between them. But this is not a flaw only of truth functionality and the material conditional. All approaches that try to treat conditionals as mere functions of their antecedents and consequents will end up in some sort of logical atomism where causal matters simply are reduced to the joint occurrence of A and B. What we need is a non-extensional approach to conditionals that can account for hypotheticality, potentiality, and dependency, none of which can be understood by looking to the antecedent or consequent *per se*.

The importance of being earnest about 'if'

Why bother about the logic of 'if'? Well, first of all because so much is at stake if we *fail* to grasp the logic of conditionals. Without an adequate understanding of 'if', we cannot account for some of the most basic matters in life, namely potentiality, hypotheticality and dependency. Unless we know what it means that something might happen, or that something would prevent or trigger something else to happen, or that something happens because of something else; how would we even be able to hope, fear, expect or regret anything? A language without conditionals cannot be the language of a world where we make predictions, choices, calculations or even explanations. A language without conditionals would be the language of a world that is nothing but a collection of unrelated particulars (events, facts, properties, or whatever). But this is not the world as we know it. Our world is all about causal relations between such particulars, whether the particulars themselves are taken to be actual, potential or purely hypothetical. In our world we need 'ifs', and we need them badly. If successful, a logic of conditionals can help us understand matters like causation, dispositions and laws. If failed, it can dissolve the very conception of conditionality.

Three dogmas of conditionals

What is the logic of 'if'? Although this is a matter of controversy, there is a tendency to let certain anticipations motivate and limit the scope of the debate. Instead of discussing particular positions, I want to argue against three dogmas that I find to be more of less basic in any proposed logical analyses of conditionals:

1. **The dogma of truth functionality**: The material conditional is an adequate representation of conditionals with respect to *truth conditions*, though we may need some further conditions of assertability, probability or modality.

2. **The dogma of counterfactuals**: The material conditional is an adequate representation of *indicative* conditionals, though counterfactual or subjunctive conditionals must be given a different account.

3. **The dogma of functionality**: A conditional's truth, probability, assertability or modality is calculable from the truth, probability, assertability or modality of its antecedent and consequent.

These principles all point in the direction of an extensional logic. In the following I will try to make clear why an adequate logic of conditionals cannot be based in any of these dogmas, and that each of them will inhibit an account of the conditionality expressed by ‘if’.

1. The dogma of truth functionality
If conditionals have truth functional truth conditions, there seems to be no alternative to the material conditional ‘A ⊃ B’. The material conditional is defined as true whenever we have one of the following combinations of truth-values for the antecedent and the consequent: (TT), (FT) or (FF). This means that a conditional would be truth functionally defined as false only when the antecedent is true and the consequent is false (TF); otherwise true:

A	**B**	**A ⊃ B**
T	T	T
T	F	F
F	T	T
F	F	T

That the truth conditions of a conditional cannot be truth functional should be clear from the fact that the truth-values of ‘A’ and ‘B’ as such cannot determine the truth-value of ‘if A then B’:

(TT) A conditional can be true when the antecedent and the consequent are both true, as in ‘If Socrates is a man, then he is mortal’, or it can be false under these circumstances, as in ‘If Socrates is mortal, then he is a man’.

(TF) A conditional is false whenever the antecedent is true and the consequent is false, as in ‘If Socrates is a man, then he is a cat’.

(FT) A conditional can be true when the antecedent is false and the consequent is true, as in ‘If Socrates is a cat, then he is mortal’, or it can be false under these circumstances, as in ‘If Socrates is a stone, then he is mortal’.

(FF) A conditional can be true when the antecedent and the consequent are both false, as in ‘If Socrates is a stone, then he is inanimate’, or it can be false under these circumstances, as in ‘If Socrates is inanimate, then he is a stone’.[1]

Accordingly, the truth conditions of ‘if A then B’ do not seem to coincide with the truth conditions of ‘A ⊃ B’ in other cases than when ‘A’ is true and ‘B’ is false.

So why would anyone think that the material conditional represents an adequate account of a conditional’s truth conditions? Well, the thing is that there are some strong indications of a logical equivalence, or at least compatibility, between ‘⊃’ and ‘if’, namely the fact that they both allow *contraposition*, *modus ponens* and *modus tollens*:

1. ***Contraposition***
If A then B ⇔ If not-B then not-A

[1] The point made here is taken from Garland, a medieval logician. Cf. Ivan Boh (1993) *Epistemic Logic in the Later Middle Ages*, London, Routledge, pp. 4-5.

2. *Modus ponens*
If A then B
A
$\therefore$ B

3. *Modus tollens*
If A then B
not-B
$\therefore$ not-A

This means that for both '$\supset$' and 'if' there seems to be a sufficient/necessary relation between the antecedent and consequent, with the antecedent being a sufficient condition for the consequent, and the consequent being a necessary condition for the antecedent. But not all conditionals have these logical properties. Consider for instance expressions like 'If you are hungry, there's food in the kitchen', 'If he made a mistake, it wasn't a big mistake' and 'If you want my opinion, you should divorce him'. None of these allow contraposition. For instance, if there is no food in the kitchen, this is not sufficient for you not being hungry.

Conditionals that do not allow contraposition should be taken as a separate class. We can call these conditionals *rhetorical*. In these conditionals, 'if' does not express a sufficient/necessary relation between antecedent and consequent, but is used either to doubt the antecedent or assert the consequent in various ways. Conversely, whenever a conditional expression allows both *modus ponens* and *modus tollens*, it also allows contraposition. In those cases 'if' expresses a sufficient/necessary relation between the antecedent and the consequent. This characterizes the set of what are called *ordinary* conditionals. The ordinary conditionals are of two kinds, (i) The ones expressing causal relations ('If I drop this pen, it will fall', 'If the interest rises, the property market slows down') and (ii) the ones expressing categorical facts of classification ('If Socrates is a man, he is mortal', 'If *n* is an even number, it is divisible on 2').

We thus have a case where ordinary and material conditionals have some basic logical properties in common, allowing *modus ponens*, *modus tollens* and contraposition. This is usually taken as an indication of the exactness of the material conditional as an interpretation of 'if'. But this is to jump to conclusions. Consider for instance the fact that the following inferences are equally valid for the material conditional, though not for ordinary conditionals:

A	$\neg$ A	$\neg$ A
B	$\neg$ B	B
$\therefore$ A $\supset$ B	$\therefore$ A $\supset$ B	$\therefore$ A $\supset$ B

From this we would not be tempted to conclude that the material conditional gives us the logical properties of 'if'. The reason why the material conditional seems to grasp the logical properties of ordinary conditionals is that the material conditional is *parasitic* on ordinary conditionals: It gets its meaning and plausibility entirely from the use of 'if'. We keep forgetting that the material conditional is nothing but a function, allowing the pairs of truth-values (TT), (FT), (FF) for any ordered pair of arbitrary chosen statements (AB).

So unless we read ordinary conditional properties into the material conditional, we would not be able to make any sense out of saying things like: '*If* the antecedent is true, *then* the consequent must be true as well' or '*If* the premises are true, *then* the conclusion must also be true'. And how would we then be able to explain the validity of *modus ponens* and *modus tollens* in a meaningful way? In fact, the only reason why the material conditional seems to be intuitively well defined via *modus ponens* and *modus tollens* is that the conditional is already assumed in the premises. And then of course we would never interpret 'if' as material, meaning only (TT), (FT), (FF).

All this indicates that *modus ponens*, *modus tollens* and contraposition are not first of all properties of the material conditional. Instead, these are actually properties of ordinary conditionals that, if completely handed over to material conditional treatment, would be reduced to meaning absolutely nothing but a certain combination of truth-values.

2. The dogma of counterfactuals

The case of counterfactuals is generally considered as representing the main problem for a truth functional account of conditionals, namely that if the antecedent 'A' is false, the conditional 'If A then B' is defined as true (and so is 'If A then not-B'). And this is of course a problem if we want a logic that preserves the truth conditions of 'if'. But the problem with counterfactuals actually points to the general problem of truth functionality, and not particularly to a problem related to the nature of conditionals with false antecedents. Note that the following inferences are valid according to the truth functional definition of conditionals, though none of they hold if '⊃' is replaced by 'if':

1. $(A \& B) \Rightarrow ((A \supset B) \& (B \supset A))$
2. $(\neg A \& \neg B) \Rightarrow ((A \supset B) \& (B \supset A))$
3. $B \Rightarrow ((A \supset B) \& (\neg A \supset B))$
4. $\neg A \Rightarrow ((A \supset B) \& (A \supset \neg B)$

Then why should the case of (4) be taken as exceptional and more in need of a solution than (1), (2) or (3)? Getting a true conditional from a true consequent or from any pair of true (or false) statements seem equally problematic as getting a true conditional from a false antecedent. In fact, what all this points to is that the truth functional approach to conditionals is an inadequate and reductive one that in the end leads to a triviality of conditional relations.[2]

By treating all conditionals containing a false antecedent (typically referred to as counterfactuals or subjunctives) as being special cases with different logical properties from the remaining class of conditionals (typically called factual or indicative conditionals), one expects to be able to preserve the truth functional interpretation of at least indicative conditionals. The problem is that truth functionality fails in accounting for the truth conditions of conditionals *in general*, and not only for counterfactuals.

To introduce an alternative approach to conditionals with false antecedents is therefore a dead-end that obscures the nature of conditionals rather than clarifying it. A conditional is neither factual nor counterfactual in the traditional sense, according to the truth or falsity of the antecedent. If I am wondering if it's true that I will get sick if I drink a whole bottle of whisky, then it is completely irrelevant to ask myself whether or not I am actually going to drink it. The conditions under which the conditional is true will not change just because I decide not to drink the whisky. To treat counterfactuals as special cases hence misses the

[2] See also the triviality proofs of Robert Brandom (1981): 'Semantic Paradox of Material Implication', *Notre Dame Journal of Formal Logic* 22, April, pp. 129-132, and David Lewis (1976): 'Probability of Conditionals and Conditional Probabilities' (I & II), reprinted in F. Jackson (ed.): *Conditionals,* 1991, pp. 76-101 & 102-110.

point that the truth or falsity of ‘A’ and ‘B’ as such is not sufficient for asserting a conditional relation between them.

Now Adams’ detection of the difference in truth conditions for ‘If Oswald didn’t kill Kennedy, then someone else did it’ and ‘If Oswald hadn’t killed Kennedy, then someone else would have’ seems to have motivated the general expectation of a logically relevant distinction between indicatives and counterfactuals. Since we would accept the first conditional but not necessarily the second, they do seem to call for a different treatment. But again, this is not because of some problem of *counterfactuals*. Rather, it is related to the distinction between indicatives and subjunctives. Note that the factual/counterfactual distinction is about the truth or falsity (or for other logical systems, possibility, probability, or assertability) of the antecedent, while the indicative/subjunctive distinction concerns the grammatical mood of the conditional expression. Whether or not the antecedent is false, that is, whether or not we are dealing with a counterfactual, is not revealed by surface syntax or grammar.[3] Instead of saying that all conditionals of the subjunctive mood are counterfactual, therefore, maybe one could say that the subjunctive mood indicates a higher degree of hypotheticality than the indicative mood.

A property usually associated with subjunctivity, namely *hypotheticality,* seems to be an essential part of all ordinary conditionals (that is, those conditionals that are used to express a sufficient/necessary relation between antecedent and consequent). As long as the background conditions remain unchanged, ordinary conditionals can appear with various degrees of hypotheticality without a change in truth conditions. This means that a conditional’s truth-value is not affected simply by changing mood from the indicative to the subjunctive. For instance, the same conditional relation is expressed in ‘If I drink a bottle of whisky, I’ll get sick’, ‘If I were to drink a bottle of whisky, I would get sick’ and ‘If I had drunk a bottle of whisky, I would have gotten sick’. They should therefore not be treated as logically different.

Let’s go back to Adams’ pair of conditionals. The divergence in truth conditions between the two conditionals is not due to a logically relevant distinction between factuals and counterfactuals, nor between indicatives and subjunctives, but to a change of background conditions. In the first conditional, it is assumed that Kennedy was killed. Then of course he must have been killed by Oswald or someone else. In the second conditional we are asked to disregard this fact and imagine what would have followed if Oswald hadn’t killed Kennedy. Another change of background conditions is that while the first conditional does not express anything about whether it actually was Oswald who killed Kennedy, the second seems to have such an implication.

Now many of the alternative proposals for a logic of ‘if’ seem to be motivated by problems related to counterfactuals. So even though the material conditional is not regarded as a perfect, adequate, or as an exhaustive interpretation of ‘if’, it might still express the truth conditions of *indicative* conditionals. Counterfactuals will on this view need a separate account. If the material conditional is thought to account for the truth conditions of indicative conditionals, then, we would need some further conditions of assertability, probability or modality to account for counterfactuals. I have tried to show that, if the move from the indicative to the subjunctive mood does not represent a change in truth conditions for the conditional, a separate logical treatment of counterfactuals is not even called for.

[3] See for instance Bernard Comrie (1986): ‘Conditionals: A typology’, in Ferguson C.A., A. Meulen, J.S. Reilly, E. Traugott (eds.): *On Conditionals*, Cambridge, Cambridge University Press, pp. 77-99, and David K. Johnston (1996): ‘The Paradox of Indicative Conditionals’, *Philosophical Studies* 83, 1996, pp. 93-112.

3. The dogma of functionality

Regardless of whether the distinction between indicatives and subjunctives (or factuals and counterfactuals) is taken as logically significant, there seems to be general agreement that the truth, probability, assertability or modality of 'If A then B' is in some way *calculable* from the truth, probability, assertability or modality of 'A' and 'B'. But this will in general take us no further than the material conditional. For instance, the attempt to improve the material conditional by bringing in modality seems to amount to nothing more than replacing truth with necessity. One would still get a problem with counterfactuals, saying that 'not-A' being necessary would imply 'If A then B' being necessary. One would also get that a necessary 'B' implies a necessary conditional 'If A then B'.

Another option is to replace truth with probability, saying that the probability of 'B given A' is equal to the probability of 'A & B' divided on the probability of 'A'. This is Adams' thesis for conditional probability:[4]

AT Pr(If A then B) = Pr(B/A) = Pr(AB)/Pr(A)

Although AT avoids a problem associated with counterfactuals, namely that a conditional gets a high probability on no other grounds than the antecedent having low probability, it still inherits some problems faced by the material conditional: i) If the probability of 'A' is zero, the probability of 'B given A' will also be zero, and: ii) If the probability of 'B' is 1, the probability of 'B given A' will also be 1. But in addition to i) and ii), that at least must be admitted to be closely related to problems with the material conditional, AT faces a third and distinct problem: iii) If 'A' and 'B' are probabilistically independent, we will get that Pr(B/A) = Pr(B). What i), ii) and iii) point to is that Adams' thesis is a purely compositional interpretation of conditionals, treating them as products of the probabilities of 'A', 'B' and 'A & B'.

There are lots of other theories of conditionals as well, but no one that treats conditionals as primitive. Actually, there seems to be a general reluctance against recognizing the actual cause of the problems related to truth functionality: To treat conditionals as mere functions of their antecedent and consequent will necessarily involve dissolution of the conditional relation between the antecedent and consequent; the hypotheticality, potentiality and dependency that 'if' is supposed to express. And why even consider 'A', 'B' or 'A & B' when the conditional 'If A then B' is in question? When someone says 'If you fall down from that cliff, you'll die', the question about whether or not I will fall is completely irrelevant for the conditional. Our interest is in the causal relation, and in what enables us to establish, predict, and explain such a relation.

So if a logical system allows us to infer a conditional relation 'If A then B' from the truth, probability, assertability, possibility or necessity of 'A' and 'B', we seem to have fallen victim of the logical mistake pointed out by Hume. According to Hume, the joint occurrence (or even constant conjunction) of two events A and B is not *per se* sufficient for inferring a relation between them: We can observe that A and B occur, and we can observe that A occurs before B. We can even observe some contact between A and B. But none of these observations are in themselves ample grounds for inferring that A is a *cause* of B, or that there is any connection between them whatsoever. Allowing 'if A then B', 'if B then A', 'if not-A

[4] Ernest W. Adams (1975): *The Logic of Conditionals: An application of probability to deductive logic,* Dordrecht, Reidel.

then not-B' and 'if not-B then not-A' to follow from any pair of true statements 'A' and 'B', the material conditional seems to permit inferences that even Hume wouldn't think of warning us against.

So how can a conditional relation 'If A then B' be inferred from the joint occurrence of two particulars A and B? Or even worse, how can it be inferred from the joint occurrence of two truth-values (TT)? Isn't it commonly accepted that it is a logical mistake to verify a hypothesis, but not to falsify it, based on single observations? When a logical system allows us to verify all hypotheses that are not falsified, we are left with an epistemological mismatch between when we are justified in affirming a conditional and when we are justified in affirming a scientific hypotheses.

One way to explain away this seemingly inconsistency between logic and philosophy of science would be to say that logical truth does not, and is not meant to have, epistemological or metaphysical bearing. Logic is just a consistent system that spells out analytic truths. It isn't concerned with how the world actually is, or with how we get knowledge about the world. And such an answer certainly takes the edge off my criticism. But it also makes logic irrelevant for a philosophical analysis of 'if'. And maybe that is all to the best. A system that can deal only with factuality and counterfactuality, indicating the occurrence or non-occurrence of some particulars A and B, will necessarily prevent us from ever getting down to the very nature of causal matters.

Seeking a way to calculate the truth, probability or modality of a conditional from the truth, probability or modality of the antecedent or consequent, all the main logics of conditionals turn out to be extensional and reductionist. Hence they are unsuitable for dealing with hypothetical matters such as potentiality, dispositions, predictions and causation. In short, they cannot deal with *if*s. So although a logical system originally was intended as a scientific tool, free from metaphysical presuppositions, the three dogmas of 'if' seem to give rise to a very specific metaphysics, namely one in which the world is a collection of unrelated particulars. Thus truth functional or any extensional logic actually presupposes a philosophical position very similar to logical atomism.

Ten lessons about 'if'

So what is the alternative for a logic of 'if'? Are there any positive results in all this criticism? Can we even have a logic of conditionals, or should we give up on the whole project? These are the questions I usually get. And of course it is much easier to tear down than it is to build up. But in all my criticism, there are also some positive lessons to be learned, I think. In an attempt to sum up this paper, I offer ten:

1. A conditional is not a truth function, but a form of statement in our language, used to assert a certain relation in the world. A truth function (of two variables) is nothing but an assignment of a truth-value to each combination of truth-values for any ordered pair (AB).

2. A conditional is not factual or counterfactual, but hypothetical.

3. A conditional does not change truth conditions from the indicative to the subjunctive mood, unless there is a change in background conditions.

4. A conditional does not only have one fixed set of logical properties; it is an expression that can be used in various ways for various rhetorical purposes. There is thus not one single logic of conditionals, but several.

5. A conditional that allows contraposition will always allow both *modus ponens* and *modus tollens*, hence it will be an ordinary conditional where the antecedent is taken as a sufficient condition for the consequent and the consequent is taken as a necessary condition for the antecedent.

6. A conditional that does not allow contraposition will only allow either *modus ponens* or *modus tollens*, hence it will be a rhetorical conditional, generally used to doubt the antecedent or assert the consequent in various ways.

7. A conditional cannot be accounted for by a purely extensional logic, but we must always first look at the content of the conditional expression and interpret what kind of logical relation is asserted.

8. A conditional cannot be inferred from the truth, probability, assertability, possibility or necessity of its antecedent or consequent, but must be justified and explained in the same way that we justify and explain our scientific or other hypotheses: outside the immediate context of the antecedent and consequent of the conditional itself. (The causal relation between me dropping a pen and it falling cannot be found by testing and affirming the dropping and falling by some constant conjunction. What we must do is to look to external and underlying causes, like gravitation, mass, friction, and what have you.)

9. A conditional is not a function of its antecedent and consequent, but must be taken as conceptually basic, irreducible and primitive.

10. That a system of logic is consistent doesn't mean that it can tell us anything about how we reason or ought to reason. Expecting an extensional logic to teach us something about conditionals is like expecting sudoku to teach us something about mathematics.