Fakeons, Quantum Gravity And The Correspondence Principle

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Abstract

The correspondence principle made of unitarity, locality and renormalizability has been very successful in quantum field theory. Among the other things, it helped us build the standard model. However, it also showed important limitations. For example, it failed to restrict the gauge group and the matter sector in a powerful way. After discussing its effectiveness, we upgrade it to make room for quantum gravity. The unitarity assumption is better understood, since it allows for the presence of physical particles as well as fake particles (fakeons). The locality assumption is applied to an interim classical action, since the true classical action is nonlocal and emerges from the quantization and a later process of classicization. The renormalizability assumption is refined to single out the special role of the gauge couplings. We show that the upgraded principle leads to an essentially unique theory of quantum gravity. In particular, in four dimensions, a fakeon of spin 2, together with a scalar field, is able to make the theory renormalizable while preserving unitarity. We offer an overview of quantum field theories of particles and fakeons in various dimensions, with and without gravity.

1 Introduction

According to Bohr's correspondence principle, it must be possible to obtain the laws of classical mechanics from those of quantum mechanics in the limit of large quantum numbers, or, more generally, the *classical limit*. We can view the principle as a guideline for the selection of theories. Behind the necessity of such a selection is the fact that our observational power is considerably reduced when we explore the microscopic world.

At the classical level, we can uncover the physical laws relatively easily, because we can make a large number of experimental observations at the same time without disturbing the system, at least in principle. On the contrary, at the quantum level, our possibilities of observing the microscopic world are limited by several factors, including the laws of physics themselves, e.g. the uncertainty principle, as well as our physical constitution, the dimensions of the cells and atoms of which we are made. The human beings are clumps of atoms that are trying to "understand"[1] the laws that govern scales of magnitude that are billions of billions of times smaller than the smallest ingredient they are made of. Very probably, this is a vicious circle. Below certain scales of magnitude the universe may become unknowable to us.

Our thought is, so to speak, "classical", because it is shaped by the interactions between us and the classical environment where we live. The fundamental concepts of our logic (such as existence, origin, time, space, cause, effect, principle, consequence, etc.) are inherited from that environment. Unlikely they are absolute. They might just be useful approximations, or effective descriptions with limited ranges of applicability.

For these reasons, a quantum theory is not built from scratch, but instead guessed from another theory that is more familiar to us, which we call classical and which is later quantized. Unless there is a sort of correspondence between the two, our chances of understanding the quantum world are minimal.

Quantum mechanics forced us to waive determinism, which we used to take for granted. The lesson is that at any point we may have to modify the laws of physics and even the basic principles of our thinking in a profound way. Instead of assuming that our knowledge is "universal" and extends straightforwardly to the unknown portion of the universe, we must admit that the "principles" suggested by our classical experiences are just temporary work hypotheses.

With the help of the devices we build, we can extend the exploration of the world

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[1]When we "understand" something previously unknown to us, we just establish analogies, relations, correspondences with phenomena that are more familiar to us. Ultimately, "understanding" just means "getting used to".

way beyond the limits of our direct perception. However, the devices have limits as well, because they are macroscopic, like us. When we build them, we tacitly assume that the laws of nature derived from the observation of the known portion of the world remain valid when we explore the unknown portion. If everything works as expected, the assumption is validated. Yet, this does not prove that those laws are universal, i.e. hold for arbitrarily short time intervals, or arbitrarily large energies.

It is reasonable to expect that, when the energies we explore are not too high, the laws of nature remain to some extent similar to the laws obeyed by the phenomena occurring in the classical environment that surrounds us. Bohr's correspondence principle codifies such a similarity up to the atomic distances, which are the realm of quantum mechanics. What happens when we explore smaller distances? Conceivably, the correspondence will become weaker and weaker and our instruments will not help us indefinitely.

The first descent to smaller distances is quantum field theory. There, we talk about classical limit in a different sense, which may not even refer to true classical phenomena, as quantum chromodynamics shows. Yet, an upgraded version of the correspondence principle does emerge, summarized by the requirements of unitarity, locality and renormalizability. It leads us to build the standard model of particle physics.

At the same time, crucial limitations appear. For example, we are still unable to explain why the gauge group of the standard model is $U(1) \times SU(2) \times SU(3)$. Moreover, the matter sector is only weakly constrained. So far, all the attempts to unify the three interactions of nature encoded in the standard model have failed. One possible explanation is that we have not been clever enough, but another, sadder possibility is that the correspondence between the macroscopic world we live in and the microscopic world we wish to explore might be fading away. At some point all similarity will eventually disappear and we will remain blind and powerless. For this reason, we think that it is too risky to depart from the kind of correspondence that has worked so far: we have to be as conservative as possible.

The second descent, quantum gravity, may require to reconsider or refine the basic assumptions of the correspondence principle in nontrivial ways. In this paper, we show that further upgrades are indeed available and lead to an essentially unique solution to the problem of quantum gravity.

The basic new idea is the concept of fake particle (fakeon), which is able to reconcile renormalizability and unitarity. In four dimensions, quantum gravity is described by a triplet made by the graviton, a fakeon of spin 2 and a scalar field. The theory is very predictive and way more unique than the standard model. We infer the upgraded correspondence principle from its main properties and then give an overview of quantum field theories of particles and fakeons in various dimensions, with and without gravity.

The fakeons are introduced by means of a novel quantization prescription [1, 2] for the poles of the free propagators. In the Euclidean region of the space $\mathcal{P}$ of the complexified external momenta, a Feynman diagram is evaluated as usual, from the Euclidean version of the theory. Elsewhere, it is evaluated from the Euclidean region by analytic continuation up to the fakeon thresholds, which are the thresholds associated with the processes involving fakeons. Above those thresholds the diagram is evaluated by means of a nonanalytic operation, called average continuation, which amounts to take the arithmetic average of the analytic continuations that circumvent the threshold. Overall, we may view the procedure as a nonanalytic Wick rotation [3, 4]. Finally, to have unitarity, the fakeons must be projected away from the physical spectrum.

In a quantum field theory of particles and fakeons the quantization has to be understood in a new way. To mention one thing, the starting classical action, which is local, is just an interim action, because it is unprojected, i.e. it contains the (classical counterparts of the) fakeons. However, since the projection comes from the quantization, when we want to reach the classical limit we must first quantize the theory and then classicize it back. Only at the end of this procedure we obtain the true classical action, which is nonlocal. We see that the locality assumption must be understood anew and applied to the interim classical action.

In addition, the renormalizability requirement has to be formulated more precisely, because the existing definitions do not make us appreciate the peculiar role reserved to the gauge couplings. As far as the unitarity requirement is concerned, it does not need to be modified, but it is necessary to realize that it leaves room for both physical particles and fakeons.

Among the other approaches to the problem of quantum gravity appeared in the past decades, we mention string theory [5], loop quantum gravity [6], holography [7] and asymptotic safety [8]. Most of them follow from correspondence principles that sound very *ad hoc* and are based on assumptions that appear hard to justify. None of them is as close to the standard model as the solution based on the fakeon idea, which is a quantum field theory, admits a perturbative expansion in terms of Feynman diagrams and allows us to make calculations with a comparable effort (see refs. [9, 10]).

String theory is criticized for being nonpredictive [11]. Moreover, its calculations often require mathematics that is not completely understood. Loop quantum gravity is even more challenging, because it is at an earlier stage of development. The AdS/CFT correspondence and the asymptotic-safety program do not admit weakly coupled expansions. Our solution bests its competitors in calculatibity, predictivity and falsifiability. It is also rather rigid, because it contains only two new parameters.

A theory of quantum gravity is supposed to shed light on a new understanding of spacetime at the microscopic level. In the theories of particles and fakeons, this is the violation of microcausality: at energies larger than the fakeon masses, past, present and future lose meaning and there is no way to tell the difference between cause and effect. From the theoretical and experimental points of view, there is room for this prediction to be accurate. The new physics is expected to emerge at energies around the fakeon masses, which might be well below the Planck scale, possibly around 10^{12}GeV.

The paper is organized as follows. In section 2 we recall the fakeon idea. In section 3 we recall the formulation of quantum gravity it leads to. In section 4 we study the dressed propagators. In section 5 we classicize quantum gravity. In section 6 we summarize the lessons learned and upgrade the correspondence principle. Section 7 contains the conclusions.

2 Fakeons

In this section, we discuss the idea of fake particle introduced in ref. [1]. We start from the crucial property, which is unitarity. Once the S matrix is written as $1 + iT$, the unitarity equation $S^\dagger S = 1$ gives the optical theorem

$$2\mathrm{Im}T = T^\dagger T. \tag{2.1}$$

A diagrammatic version of this identity is provided by the so-called cutting equations [12], which express the real part of a diagram as a sum of "cut diagrams". The simplest cutting equations are

$$2\mathrm{Im}\left[(-i)\!\!\begin{array}{c}\rightarrow\!\!-\!\!\prec\end{array}\!\!\right] = \begin{array}{c}\rightarrow\!\!\!/\!\!\!\prec\end{array} = \int \mathrm{d}\Pi_f\left|\begin{array}{c}\rightarrow\!\!-\end{array}\right|^2, \tag{2.2}$$

$$2\mathrm{Im}\left[(-i)\!\!-\!\!\bigcirc\!\!-\right] = -\!\!\oslash\!\!- = \int \mathrm{d}\Pi_f\left|-\!\!\prec\right|^2, \tag{2.3}$$

where the integrals are over the phase spaces Π_f of the final states [13].

Let V denote the space of physical states. In various cases, it is necessary to work with a larger space W, which contains also unphysical states. The matrix element $\langle b|T|a\rangle$, where $|a\rangle, |b\rangle \in W$, is given by the connected, amputated diagrams, with external legs determined by $|a\rangle$ and $|b\rangle$. Under very general assumptions, it is relatively easy to prove a diagrammatic identity resembling (2.1) in W, which reads

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in W}\langle b|T^\dagger|n\rangle(-1)^{\sigma_n}\langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in W, \tag{2.4}$$

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where σ_n can be 0 or 1, the unphysical states being those with $\sigma_n = 1$. The identity (2.4) is called pseudounitarity equation.

The cut propagators of the cut diagrams carry information about the intermediate states $|n\rangle$ and the space W. Unitarity requires to prove that the identity (2.4) can be consistently projected onto V to give

$$\frac{1}{2i} \left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle \right] = \sum_{|n\rangle \in V} \langle b|T^\dagger|n\rangle \langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in V. \qquad (2.5)$$

What is crucial about the optical theorem is that it is not a linear equation, but a quadratic one, so it mixes different orders of the loop expansion. We can restrict the initial and final states $|a\rangle, |b\rangle$ of (2.4) at no cost, but it is not equally easy to restrict the sum over $|n\rangle \in W$ to a sum over $|n\rangle \in V$. Thus, a generic projection is inconsistent with unitarity: if we drop some states from the set of initial and final states, they are generated back as intermediate states $|n\rangle$ by the loop corrections. A projection that is consistent with unitarity must be a very clever one.

These simple remarks show us that unitarity is an essentially loop property. A tree-level action cannot be unitary *per se*, because the right-hand side of (2.3) is made of tree vertices only, but the left-hand side is made of loops.

At present, the fakeon projection is the only example of consistent projection, aside from the one that takes care of the Faddeev-Popov ghosts and the temporal and longitudinal components of the gauge fields. As we discuss below, the fakeon projection is actually very different from the projection of gauge theories, to the extent that it leaves an important remnant: the violation of microcausality.

Consider the propagator

$$G(p, m) = \frac{1}{p^2 - m^2}. \qquad (2.6)$$

Endowed with the Feynman prescription $(p^2 \to p^2 + i\epsilon)$, it becomes

$$G_+(p, m, \epsilon) = \frac{1}{p^2 - m^2 + i\epsilon} \qquad (2.7)$$

and describes a particle of mass m. Consider the identity (2.2), with vertices equal to $-i$. If P denotes the propagator of the intermediate line on the left-hand side, (2.2) gives the inequality $\text{Im}[-P] \geqslant 0$. Taking $P = G_+$, we get

$$\text{Im} \left[-\frac{1}{p^2 - m^2 + i\epsilon} \right] = \pi\delta(p^2 - m^2), \qquad (2.8)$$

which is indeed nonnegative.

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If we multiply (2.7) by a minus sign, we obtain a ghost, since $P = -G_+$ satisfies $\mathrm{Im}[-P] \leqslant 0$, in contradiction with the optical theorem. However, if we also replace $+i\epsilon$ with $-i\epsilon$, the right-hand side of (2.8) does not change and the optical theorem remains valid. The moral of the story is that we can in principle have both propagators

$$G_\pm(p, m, \epsilon) = \pm\frac{1}{p^2 - m^2 \pm i\epsilon},$$

since both fulfill the identity (2.2).

However, if we integrate directly on Minkowski spacetime, the presence of both G_+ and G_- in the same Feynman diagram originates nonlocal divergences at $\epsilon \neq 0$ [14] and worse problems for $\epsilon \to 0$. If, on the other hand, we start from the Euclidean version of the theory, we find that the Wick rotation is not analytic and must be defined anew [3, 4]. One way to uncover the concept of fake particle is precisely to make the Wick rotation work in a way that is compatible with unitarity.

Let us multiply (2.6) by $\pm$, to emphasize that what we are going to say applies irrespectively of the sign of the residue. Following [1], write

$$\pm\frac{p^2 - m^2}{(p^2 - m^2)^2}$$

and eliminate the singularity by introducing an infinitesimal width $\mathcal{E}$, to define the fakeon propagator as

$$\mathbb{G}_\pm(p, m, \mathcal{E}^2) = \pm\frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4} = \pm\frac{1}{2}\left[G_+(p, m, \mathcal{E}^2) - G_-(p, m, \mathcal{E}^2)\right]. \tag{2.9}$$

The propagator $\mathbb{G}_+(p, m, \mathcal{E}^2)$ describes a *fakeon plus*, while the propagator $\mathbb{G}_-(p, m, \mathcal{E}^2)$ describes a *fakeon minus*. Note that $\mathbb{G}_\pm(p, m, \mathcal{E}^2)$ vanishes on shell at $\mathcal{E} > 0$. This suggests that it does not truly propagate a particle.

Formulas (2.9) are not the end of the story: we still have to explain how to use them inside the Feynman diagrams. The matter is technically involved, but a shortcut, called *average continuation* [3, 2], allows us to jump directly to the final result.

Let $\mathcal{P}$ denote the hyperplane of the complexified external momenta. In the Euclidean region we can evaluate the diagram from the Euclidean version of the theory. No particular prescription or attention is needed there. The result can be analytically continued within $\mathcal{P}$ up to the "fakeon thresholds", i.e. the thresholds associated with the "would-be processes" involving fakeons. The fakeon thresholds are overcome by means of the average continuation, i.e. by taking the arithmetic average of the analytic continuations that circumvent the threshold. At the end, the probabilities of the processes that involve the fakeons vanish. This makes it possible to project the fakeons away and have unitarity.

Analyticity no longer holds in the usual sense. It is replaced by *regionwise analyticity*. For every Feynman diagram, $\mathcal{P}$ is divided into disjoint regions. In each region the diagram evaluates to an analytic function. The main region is the Euclidean one, where the Wick rotation is analytic. The other regions can be reached unambiguously from the Euclidean one by means of the average continuation.

3   Quantum gravity

The interim classical action of quantum gravity coupled to matter can be basically written in two ways. If we use higher derivatives, it reads

$$S_{\text{QG}} = -\frac{1}{2\kappa^2} \int \sqrt{-g} \left[2\Lambda_C + \zeta R + \alpha \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) - \frac{\xi}{6} R^2 \right] + S_{\mathfrak{m}}(g, \Phi), \qquad (3.1)$$

where Φ are the matter fields and $S_{\mathfrak{m}}$ is the covariantized action of the standard model, or an extension of it, once we equip it with the nonminimal couplings that are required by renormalization. The reduced Planck mass is $\bar{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi} = \sqrt{\zeta}/\kappa$ and α, ξ, ζ and κ are real positive constants, while Λ_C can be positive or negative. Here and below the integration measure $\mathrm{d}^4 x$ is understood.

The second way is obtained by adding extra fields to eliminate the higher derivatives. The result is, at $\Lambda_C = 0$,

$$\mathcal{S}_{\text{QG}}(g, \phi, \chi, \Phi) = S_{\text{H}}(g) + S_\chi(g, \chi) + S_\phi(\tilde{g}, \phi) + S_{\mathfrak{m}}(\tilde{g} e^{\kappa\phi}, \Phi), \qquad (3.2)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} + 2\chi_{\mu\nu}$ and

$$S_{\text{H}}(g) = -\frac{\zeta}{2\kappa^2} \int \sqrt{-g} R, \qquad S_\phi(g, \phi) = \frac{3\zeta}{4} \int \sqrt{-g} \left[\nabla_\mu \phi \nabla^\mu \phi - \frac{m_\phi^2}{\kappa^2} \left(1 - e^{\kappa\phi} \right)^2 \right],$$

$$S_\chi(g, \chi) = S_{\text{H}}(\tilde{g}) - S_{\text{H}}(g) - 2 \int \chi_{\mu\nu} \frac{\delta S_{\text{H}}(\tilde{g})}{\delta g_{\mu\nu}} + \frac{\zeta^2}{2\alpha\kappa^2} \int \sqrt{-g}(\chi_{\mu\nu}\chi^{\mu\nu} - \chi^2)\big|_{g\to\tilde{g}}. \quad (3.3)$$

The expression of (3.2) for $\Lambda_C \neq 0$ can be found in ref. [9].

In addition to the matter fields Φ, the theory describes the graviton, a scalar ϕ of squared mass $m_\phi^2 = \zeta/\xi$ and a spin-2 field $\chi_{\mu\nu}$ of squared mass $m_\chi^2 = \zeta/\alpha$. Making formula (3.3) more explicit, it is easy to show that the $\chi_{\mu\nu}$ quadratic action is a covariantized Pauli-Fierz action with the wrong overall sign, plus some nonminimal terms [9]. This means that, to have unitarity, the field $\chi_{\mu\nu}$ must be quantized as a fakeon. Instead, the ϕ action has the correct sign, so ϕ can be quantized either as a fakeon or a physical particle. Depending on which option we choose, we have a graviton/fakeon/fakeon (GFF) theory or a graviton/scalar/fakeon (GSF) theory.

Formula (3.2) shows that the matter fields Φ are sensitive to the whole triplet $\{g_{\mu\nu}, \chi_{\mu\nu}, \phi\}$ of quantum gravity, through the modified metric $\tilde{g}_{\mu\nu}\mathrm{e}^{\kappa\phi} = (g_{\mu\nu} + 2\chi_{\mu\nu})\mathrm{e}^{\kappa\phi}$. This means that the usual vertices that couple matter to gravity are accompanied by similar vertices that couple matter to $\chi_{\mu\nu}$ and ϕ. The theory predicts modified gravity-matter couplings. In particular, the effective graviton-matter vertices receive loop corrections due to the exchanges of $\chi_{\mu\nu}$ and ϕ, similar to the QED corrections studied in ref. [15].

Renormalizability can be straightforwardly proved from the action (3.1), because it does not depend on the quantization prescription [2]. Therefore, the beta functions coincide with those of the Stelle theory, which is the theory obtained by quantizing all the degrees of freedom by means of the usual Feynman prescription [16]. In the Stelle theory $\chi_{\mu\nu}$ is a ghost instead of a fakeon and unitarity is violated at energies larger than m_χ.

4 The dressed propagators

Thanks to the average continuation, calculating loop diagrams with the fakeon prescription does not require much more effort than calculating diagrams with the ordinary prescriptions [10, 9]. Among the first things to compute, we mention the one-loop self-energy diagrams, which give the physical masses $\bar{m}$ and the widths Γ.

If $p^2 - m^2$ is large enough, we can resum the bubble diagrams B and get the dressed fakeon propagators

$$\bar{\mathbb{G}}_\pm = \mathbb{G}_\pm + \mathbb{G}_\pm B \mathbb{G}_\pm + \mathbb{G}_\pm B \mathbb{G}_\pm B \mathbb{G}_\pm + \cdots = \frac{1}{\mathbb{G}_\pm^{-1} - B}. \tag{4.1}$$

After the resummation, we can take $\mathcal{E}$ to zero, which gives

$$\bar{\mathbb{G}}_\pm \sim \pm \frac{Z}{p^2 - \bar{m}^2 + i\bar{m}\Gamma_\pm} = \pm Z G_+(p, \bar{m}, \bar{m}\Gamma_\pm)$$

around the physical peak $p^2 = \bar{m}^2$, where Z is the normalization factor. The optical theorem implies

$$\mathrm{Im}[\mp Z G_+(p, \bar{m}, \bar{m}\Gamma_\pm)] = \frac{\bar{m}Z(\pm\Gamma_\pm)}{(p^2 - \bar{m}^2)^2 + \bar{m}^2\Gamma_\pm^2} \geqslant 0,$$

i.e. $\Gamma_+ > 0$, $\Gamma_- < 0$: a fakeon plus has a positive width, while a fakeon minus has a negative width. For $\Gamma_\pm \to 0^\pm$ we get

$$\lim_{\Gamma_\pm \to 0^\pm} \mathrm{Im}[\mp Z G_+(p, \bar{m}, \bar{m}\Gamma_\pm)] \sim \pi Z \delta(p^2 - \bar{m}^2). \tag{4.2}$$

In the case of a physical particle, we would find exactly the same result, which means that if we just watch the decay products of a fakeon, we have the illusion of a true particle.

As said, the resummation (4.1) is legitimate only if $p^2 - m^2$ is large enough. With physical particles, analyticity allows us to reach the peak straightforwardly. However, fakeons just obey regionwise analyticity, so we must be more careful. Indeed, the resummation misses the contact terms $\delta(p^2 - m^2)$, $\delta'(p^2 - m^2)$, etc. In general, the sum of such contact terms plus (4.2) gives

$$\sigma \pi Z \delta(p^2 - \bar{m}^2) \tag{4.3}$$

for $\Gamma_\pm \to 0^\pm$, where $\sigma = 1, 0, -1$ in the case of a physical particle, a fakeon and a ghost, respectively. Formula (4.3) tells us that if we try to detect the fakeons "on the fly", we do not see anything. With a physical particle, instead, what we obtain from the indirect observation, given by formula (4.2), coincides with what we obtain from the direct observation, given by formula (4.3). Finally, in the case of a ghost, we have the illusion of a particle if we observe its decay products, but get an absurdity (a "minus one particle"), when we try and observe it on the fly.

The properties just outlined appear to justify the name "fakeon", or fake particle. The fakeon can only be virtual, so the only way to reveal it is by means of the interactions it mediates.

Since $\chi_{\mu\nu}$ is a fakeon minus, its width Γ_χ is negative. In the case of the GFF theory, we find [9]

$$\Gamma_\chi = -C \frac{m_\chi^3}{M_{\text{Pl}}^2}, \qquad C = \frac{1}{120}(N_s + 6N_f + 12N_v), \tag{4.4}$$

where N_s, N_f and N_v are the numbers of (physical) scalars, Dirac fermions (plus one half the number of Weyl fermions) and gauge vectors, respectively. We are assuming that the masses of the matter fields are much smaller than m_χ, otherwise we have to include mass-dependent corrections. Note that the graviton and the fakeons do not contribute to Γ_χ. In the GSF theory there is another contribution due to ϕ, which depends on m_ϕ. The negative width is a sign that microcausality is violated. However, it is not the only way such a violation manifests itself, as we explain in the next section.

5   Projection and classicization

The generating functional $\Gamma(g_{\mu\nu}, \phi, \chi_{\mu\nu}, \Phi)$ of the one-particle irreducible correlation functions can be formally projected by integrating out the fakeons, using the fakeon prescription. This operation gives the physical Γ functional. In some sense, the fakeons can be viewed as auxiliary fields with kinetic terms.

For simplicity, consider an unprojected Γ functional $\Gamma(\varphi, \chi)$, where φ denotes the physical fields and χ denotes the fakeons. Solve the fakeon field equations $\delta\Gamma(\varphi, \chi)/\delta\chi = 0$ by means of the fakeon prescription and denote the solutions by $\langle\chi\rangle$. Then the physical, or projected, Γ functional Γ_{pr} is

$$\Gamma_{\mathrm{pr}}(\varphi) = \Gamma(\varphi, \langle\chi\rangle).$$

Since the fakeons are not asymptotic states, at the level of the functional integral it is sufficient to set their sources J_χ to zero:

$$Z_{\mathrm{pr}}(J) = \int [\mathrm{d}\varphi\mathrm{d}\chi] \exp\left(iS(\varphi, \chi) + i\int J\varphi\right) = \exp\left(iW_{\mathrm{pr}}(J)\right),$$

so $\Gamma_{\mathrm{pr}}(\varphi)$ is the Legendre transform of $W_{\mathrm{pr}}(J)$. Indeed, the unprojected formulas

$$\Gamma(\varphi, \chi) = -W(J, J_\chi) + \int J\varphi + \int J_\chi\chi,$$

$$\varphi = \frac{\delta W(J, J_\chi)}{\delta J}, \qquad \chi = \frac{\delta W(J, J_\chi)}{\delta J_\chi}, \qquad J = \frac{\delta\Gamma(\varphi, \chi)}{\delta\varphi}, \qquad J_\chi = \frac{\delta\Gamma(\varphi, \chi)}{\delta\chi},$$

turn into the projected ones

$$\Gamma_{\mathrm{pr}}(\varphi) = -W_{\mathrm{pr}}(J) + \int J\varphi, \qquad \varphi = \frac{\delta W_{\mathrm{pr}}(J)}{\delta J}, \qquad J = \frac{\delta\Gamma_{\mathrm{pr}}(\varphi)}{\delta\varphi},$$

when $J_\chi = 0$.

In the classical limit, the fakeon prescription and the fakeon projection simplify. In particular, the average continuation plays no role, because there is no loop integral, so we can take (2.9) as it stands, which gives the Cauchy principal value:

$$\frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4} = \mathcal{P}\frac{1}{p^2 - m^2}. \tag{5.1}$$

To illustrate the projection in a simple case, consider the higher-derivative Lagrangian

$$\mathcal{L}_{\mathrm{HD}} = \frac{m}{2}(\dot{x}^2 - \tau^2\ddot{x}^2) + xF_{\mathrm{ext}}(t), \tag{5.2}$$

where x is the coordinate, m is the mass and τ is a real constant. The unprojected equation of motion is

$$mK\ddot{x} = F_{\mathrm{ext}}, \qquad K = 1 + \tau^2\frac{\mathrm{d}^2}{\mathrm{d}t^2},$$

while the projected equation reads [17]

$$m\ddot{x} = \mathcal{P}\frac{1}{K}F_{\mathrm{ext}} = \int_{-\infty}^{\infty} \mathrm{d}u\frac{\sin(|u|/\tau)}{2\tau}F_{\mathrm{ext}}(t - u). \tag{5.3}$$

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We see that the external force is convoluted with an oscillating function, so the future ($u < 0$) contributes as well as the past. This is how the violation of microcausality survives the classical limit.

As for the classicization of quantum gravity in four dimensions, the unprojected field equations derived from (3.2) are

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \frac{\kappa^2}{\zeta}\left[\mathrm{e}^{3\kappa\phi}fT_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi) + fT_{\phi}^{\mu\nu}(\tilde{g},\phi) + T_{\chi}^{\mu\nu}(g,\chi)\right], \qquad (5.4)$$

for the metric tensor, and

$$-\frac{1}{\sqrt{-\tilde{g}}}\,\partial_\mu\left(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right) - \frac{m_\phi^2}{\kappa}\left(\mathrm{e}^{\kappa\phi} - 1\right)\mathrm{e}^{\kappa\phi} = \frac{\kappa\mathrm{e}^{3\kappa\phi}}{3\zeta}T_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)\tilde{g}_{\mu\nu},$$

$$\frac{1}{\sqrt{-g}}\frac{\delta S_\chi(g,\chi)}{\delta\chi_{\mu\nu}} = \mathrm{e}^{3\kappa\phi}fT_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi) + fT_{\phi}^{\mu\nu}(\tilde{g},\phi), \qquad (5.5)$$

from the variations of ϕ and $\chi_{\mu\nu}$, where $T_A^{\mu\nu}(g) = -(2/\sqrt{-g})(\delta S_A(g)/\delta g_{\mu\nu})$ are the energy-momentum tensors ($A = \mathfrak{m}$, ϕ, χ) and $f = \sqrt{\det\tilde{g}_{\rho\sigma}/\det g_{\alpha\beta}}$.

The fakeon projection of the GSF theory is obtained by solving the second line of (5.5) by means of the classical fakeon prescription, i.e. the Cauchy principal value, and inserting the solution $\langle\chi_{\mu\nu}\rangle$ into the other two equations. In the GFF theory, we have to solve both equations (5.5) by means of the classical fakeon prescription and insert the solutions $\langle\phi\rangle$, $\langle\chi_{\mu\nu}\rangle$ into (5.4). The projected equations can also be obtained from the finalized classical actions

$$\mathcal{S}_{\mathrm{QG}}^{\mathrm{GSF}}(g,\phi,\Phi) = S_{\mathrm{H}}(g) + S_\chi(g,\langle\chi\rangle) + S_\phi(\bar{g},\phi) + S_{\mathfrak{m}}(\bar{g}\mathrm{e}^{\kappa\phi},\Phi),$$

$$\mathcal{S}_{\mathrm{QG}}^{\mathrm{GFF}}(g,\Phi) = S_{\mathrm{H}}(g) + S_\chi(g,\langle\chi\rangle) + S_\phi(\bar{g},\langle\phi\rangle) + S_{\mathfrak{m}}(\bar{g}\mathrm{e}^{\kappa\langle\phi\rangle},\Phi), \qquad (5.6)$$

respectively, where $\bar{g}_{\mu\nu} = g_{\mu\nu} + 2\langle\chi_{\mu\nu}\rangle$.

The interim, unprojected actions (3.1) and (3.2) are local, while the finalized actions (5.6) are nonlocal. These properties remind us of the gauge-fixed actions, which are local, but unprojected, and become nonlocal (with most types of gauge-fixing conditions), once the Faddeev-Popov ghosts and the temporal and longitudinal components of the gauge fields are projected away. However, there is an important difference between the fakeon projection and the gauge projection, since the former acts on the initial, final and intermediate states [$|a\rangle$, $|b\rangle$ and $|n\rangle$ in formula (2.5), respectively], but not on the virtual legs inside the diagrams, while the latter also acts on the virtual legs. Thus, the gauge-trivial modes completely disappear, while the fakeons leave an important remnant, which is the violation of causality at energies larger than their masses.

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The masses of the fakeons are free parameters. If their values are sufficiently smaller that the Planck mass, we may be able to detect the violation of microcausality in the foreseeable future. Moreover, formulas (5.6) show that the violation of microcausality survives the classical limit.

As said, the fakeon prescription is not classical, but emerges from the loop corrections. The projected actions (5.6) must be understood perturbatively, since the parent quantum field theory that generates them is formulated perturbatively. Thus, the classicization is also perturbative and shares many features with the quantum theory it comes from, like the impossibility to write down "exact" field equations and the important roles played by asymptotic series and nonperturbative effects [18]. As far as we know, this backlash of the quantization on the classical limit is unprecedented.

The nonrelativistic limit can be taken after the fakeon projection and, possibly, the classicization. The fakeon propagator tends to the real part of the usual quantum mechanical kernel. Note that both fakeons and antifakeons contribute. For an analysis of nontrivial issues concerning the nonrelativistic limit of quantum field theory, see ref. [19].

6 The upgraded correspondence principle

In this section we summarize the lessons learned from the previous ones in connection with the correspondence principle and extend them to quantum field theories of particles and fakeons in arbitrary spacetime dimensions.

Unitarity   The unitarity requirement is unmodified, but better understood, since it makes room for both particles and fakeons.

Locality   The locality assumption must be upgraded, in the sense that it applies to the interim classical action. The finalized classical action is generically nonlocal, like the S matrix and the generating functional Γ of the one-particle irreducible diagrams.

Proper renormalizability   The renormalizability requirement, applied to the interim classical action, must be formulated more precisely, since the usual notions are too generic. We must demand *proper* renormalizability, which is a refinement of strict renormalizability. It means that the gauge couplings (including the Newton constant) must be dimensionless (with respect to the power counting governing the ultraviolet behaviors of the correlation functions), while the other physical parameters must have nonnegative dimensions in units

of mass. The standard model does show that the gauge couplings have this particular status among the couplings, so quantum gravity should conform to that.

We regard the three principles just listed as the cornerstones of the correspondence principle of quantum field theory, and in particular quantum gravity. If we remove the locality assumption, for example, we must guess the S matrix or the Γ functional directly, which are infinitely arbitrary. So doing, we have no way to determine the theory exhaustively, since, as stressed in the introduction, when we explore the infinitesimal world we cannot make infinitely many observations in a finite amount of time and/or without disturbing the system. If we remove unitarity, we open the way to the presence of ghosts, which leads to absurd behaviors. If we renounce renormalizability, then we can just be satisfied with the nonrenormalizable, low-energy theory of quantum gravity, obtained from the Einstein-Hilbert Lagrangian plus the counterterms turned on by renormalization [20].

In addition to the three basic requirements, we must include fundamental symmetries, like Lorentz invariance, general covariance and gauge invariance. Other properties are important, but not so much as to elevate them to fundamental principles. Among those, we mention causality and analyticity, which are downgraded to macrocausality and regionwise analyticity, respectively.

6.1 Uniqueness

Is the resulting correspondence principle sufficient to point to a unique theory? Various signals, like the arbitrariness of the matter sector of the standard model, tell us that this might be a utopian goal. However, we do have uniqueness in quantum gravity and a sort of uniqueness in form of the gauge interactions.

Let us start from flat space. In every even spacetime dimensions $d \geqslant 4$ the correspondence principle made of unitarity, locality, proper renormalizability and Lorentz invariance determines the gauge transformations [21] and the form of the interim classical action, which reads

$$S_{\text{YM}}^d = -\frac{1}{4} \int \mathrm{d}^d x \sqrt{-g} \left[F_{\mu\nu}^a P_{(d-4)/2}(D^2) F^{a\mu\nu} + \mathcal{O}(F^3) \right], \qquad (6.1)$$

where $F_{\mu\nu}^a$ denotes the field strength, D is the covariant derivative, $P_n(x)$ is a real polynomial of degree n in x and $\mathcal{O}(F^3)$ are the Lagrangian terms that have dimensions smaller than or equal to d and are built with at least three field strengths and/or their covariant derivatives. The quadratic terms have been simplified by means of Bianchi identities and partial integrations. As per proper renormalizability, the gauge coupling is dimensionless. The coefficients of the polynomial $P_{(d-4)/2}$ must satisfy suitable restrictions. In particular,

after projecting away the gauge modes, the massless poles of the propagators must have positive residues and must be quantized as physical particles. The other poles must have squared masses with nonnegative real parts. The poles with negative or complex residues, as well as those with positive residues but complex masses, must be quantized as fakeons. Finally, the poles with positive residues and nonvanishing real masses can be quantized either as fakeons or physical particles.

If we also demand microcausality, i.e. forbid the presence of fakeons, the set of requirements implies that the spacetime dimension d must be equal to four. Then the action is the Yang-Mills one,

$$S_{\text{YM}} = -\frac{1}{4} \int \mathrm{d}^4 x \sqrt{-g} F_{\mu\nu}^a F^{a\mu\nu}. \tag{6.2}$$

Although the interim classical actions (6.1) are essentially unique, i.e. they contain finite numbers of independent parameters, we emphasize that the gauge group remains free, as long as it is unitary and (together with the matter content) satisfies the anomaly cancellation conditions (which are other consequences of unitarity). In other words, the correspondence principle fails to explain why the gauge group of the standard model is the product of the three simplest groups, $U(1)$, $SU(2)$ and $SU(3)$, instead of anything else. For example, we cannot say why factors such as $SU(13)$, $SU(19)$, etc., are absent.

It also fails to predict the matter content of the theory that describes nature. Indeed, we are allowed to enlarge the standard model at will, to include new massive particles and/or massive fakeons, as long as they are heavy enough (to have no contradiction with experimental data) and the anomaly cancellation conditions continue to hold. The ultimate theory of nature could even contain infinitely many matter fields. In this respect, the correspondence principle is almost completely powerless. So far, every attempt (grand unification, supersymmetry, string theory and so on) to relate the matter content to the interactions, beyond the anomaly cancellation conditions, has failed. Probably, this is a sign of the fading correspondence.

Nevertheless, quantum gravity turns out to be more unique than any other theory. Indeed, its local symmetry (invariance under diffeomorphisms times local Lorentz invariance) is unique and the requirements of unitarity, locality, proper renormalizability and general covariance lead to the unique interim classical actions (3.1)-(3.2) in four dimensions.

We also have solutions in even dimensions $d \geqslant 4$. Their interim classical actions read

$$S_{\text{QG}}^d = -\frac{1}{2\kappa^2} \int \mathrm{d}^d x \sqrt{-g} \left[2\Lambda_C + \zeta R + R_{\mu\nu} \mathcal{P}_{(d-4)/2}(D^2) R^{\mu\nu} + R\mathcal{P}'_{(d-4)/2}(D^2) R + \mathcal{O}(R^3) \right], \tag{6.3}$$

where $\mathcal{P}_n$ and $\mathcal{P}'_n$ denote other real polynomials of degree n, while $\mathcal{O}(R^3)$ are the Lagrangian terms that have dimensions smaller than or equal to d, built with at least three curvature

tensors and/or their covariant derivatives. The free propagators must satisfy the same requirements listed above and be quantized as explained.

If we relax proper renormalizability into simple renormalizability, then we lose most uniqueness properties, because there exist infinitely many super-renormalizable theories of quantum gravity and gauge fields with fakeons in every spacetime dimensions d, with interim actions equal to (6.1) and (6.3), but polynomials $\mathcal{P}_n$ and $\mathcal{P}'_n$ having degrees $n > (d-4)/2$.

Summarizing, the upgraded correspondence principle is made of

$$
\begin{array}{c}
\text{unitarity} \\
\text{locality} \\
\text{proper renormalizability}
\end{array}
\tag{6.4}
$$

together with fundamental symmetries and the requirements of having no massless fakeons and finitely many fields and parameters. The combination (6.4) implies quantum gravity coupled to gauge and matter fields in four dimensions, with interim classical actions (3.1)-(3.2).

With respect to the version of the correspondence principle that is successful in flat space, the only upgrade required by quantum gravity amounts to better understand the meanings of the principles themselves, renounce analyticity in favor of regionwise analyticity and settle for macrocausality instead of full causality. As we wanted at the beginning, the final solution is as conservative as possible. The gravitational interactions are essentially unique, the Yang-Mills interactions are unique in form and the matter sector remains basically unrestricted.

6.2 Causality

Renouncing causality in quantum field theory is not a big sacrifice, because we do not have a formulation that corresponds to the intuitive notion [22]. What we have are off-shell formulations, such as Bogoliubov's definition [23], which also implies the Lehmann-Symanzik-Zimmermann requirement that fields commute at spacelike separated points [24]. The crucial issue is that it is not possible to accurately localize spacetime points by working with relativistic wave packets that correspond to particles that are on shell. This is more or less the reason why microcausality has not been treated as a fundamental principle in quantum field theory so far, maybe in anticipation that it was going to be renounced eventually. We could even say that the fate of causality was sealed from the birth of quantum field theory: quantum gravity just delivered the killing blow. For a more detailed discussion on these topics, see [25].

7   Conclusions

Various signals suggest that the correspondence between the macroscopic environment where we live, which shapes our thinking, and the microscopic world is doomed to become weaker and weaker as we explore smaller and smaller distances. The impossibility to predict the gauge group and the matter content of the theory of nature, as well as the fates of determinism and causality are signs that our predictive power is fading away. We have to cope with the fact that nature is not arranged to be understood or explained by us humans to an arbitrary degree of precision. The ultimate theory of the universe may look infinitely arbitrary to us. At the same time, the success of quantum field theory and the recent progresses in quantum gravity give us reasons to believe that we might still have a few interesting things to say before declaring game over.

In this paper we have studied the properties of quantum field theory of particles and fakeons in various dimensions. We have seen that the correspondence principle that worked successfully for the standard model admits a natural upgraded version that accommodates quantum gravity. It is encoded in the requirements of unitarity, locality of the interim classical action and proper renormalizability. The upgraded principle actually leads to an essentially unique theory of quantum gravity in every even dimensions greater than 2. In four dimensions, a fakeon of spin 2 and a scalar field are enough to have both unitarity and renormalizability. Causality breaks down at energies larger than the fakeon masses. The classical limit shares several features with the quantum theory it comes from, such as the impossibility to write exact field equations.

Our experience teaches us that determinism and causality dominate at large distances. On the other hand, when we explore smaller and smaller distances, we see a gradual emergence of "freedom", first in the form of quantum uncertainty, then in the forms of acausality and lack of time ordering. These facts suggest that the universe is radially irreversible, i.e. irreversible in the sense of the relative distances. When we move from the large to the small distances we see a pattern, pointing from the absolute lack of freedom to what we may call asymptotic anarchy.

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