Paradox without Self-Reference

Why are some sentences paradoxical while others are not? Since Russell the universal answer has been: circularity, and more especially self-reference.[1]

Not that self-reference suffices for paradox. Such a view is refuted by the work of Gödel and Tarski, and by various commonsense examples, such as "For the last time, stop that racket!" and "So dear Lord to Thee we raise, this our hymn of grateful praise." What many do seem to think is that some sort of self-reference, be it direct or mediated, is necessary for paradox. So one often hears that the surest way of keeping a language paradox-free is to impose an absolute ban on all self-reference. "This may be using a cannon against a fly," it is said, "but at least it stops the fly."

Except that it does not stop the fly: paradoxes like the Liar are possible in the complete absence of self-reference. Imagine an infinite sequence of sentences S_1, S_2, S_3,....., each to the effect that every subsequent sentence is untrue:

(S_1) for all $k > 1$, S_k is untrue

(S_2) for all $k > 2$, S_k is untrue

(S_3) for all $k > 3$, S_k is untrue

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[1] Some semantical paradoxes, for instance Grelling's, trade not on self-reference but on circularity of other kinds. Self-reference has seemed essential to Liar-like paradox, however. This note gives an example of a Liar-like paradox that is not in any way circular.

Suppose for contradiction that some S_n is true. Given what S_n says, for all $k>n$, S_k is untrue. Therefore (a) S_{n+1} is untrue, and (b) for all $k>n+1$, S_k is untrue. By (b), what S_{n+1} says is in fact the case, whence contrary to (a) S_{n+1} is true! So every sentence S_n in the sequence is untrue. But then the sentences subsequent to any given S_n are all untrue, whence S_n is true after all! I conclude that self-reference is neither necessary nor sufficient for Liar-like paradox.

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