Informal Reasoning & Logical Formalization

Michael Baumgartner

1. Introduction

In ever so many philosophical introductions to formal logic the latter is presented as the philosopher’s *ars iudicandi*, i.e. as the instrument that evaluates the quality of philosophical arguments or, more generally, of informal reasoning.\(^1\) Irrespective of its content, an argument is only worthy of consideration if it is valid, i.e. if the truth of its premise(s) necessitates the truth of its conclusion. As the validity of arguments hinges on the latter’s form, validity tests must be conducted by abstracting from the content of the argument. According to a prevalent conception of logic among philosophers, the instrument best suited for that task is formal logic.

Philosophical arguments are commonly formulated in natural language. Thus, in order to put logical formalisms to work when it comes to evaluating the validity of philosophical arguments, natural language first must be translated into a suitable formalism. Such translations call for stringent justification.\(^2\) Proofs involving the transformation of ordinary language to a formalism are convincing only if they rely on a systematic understanding of the adequacy of the formalizations resorted to. Not any logical formula can be considered an adequate formal representation of a given statement or argument.

Standard criteria of adequate formalization, however, indirectly presuppose that the validity of an argument be determined *prior* to formalizing it. This, in turn, casts doubts on the putative power of logic to reverse informal reasoning, i.e. to expose a seemingly valid argument as invalid after all or vice versa. The constraints imposed by criteria of adequate formalization and, hence, by a precondition of evaluating informal reasoning by means of logical formalisms seriously question the usefulness of formal

---

1. Cf. e.g. Brun (2004: sect. 1.1).
languages to argument evaluation. Even though the problematic interdependence of criteria regulated logical formalization and formal argument evaluation has been acknowledged repeatedly in the literature, the prevalence of the *ars iudicandi* conception of logic among philosophers has remained virtually unaffected by this finding. Pre-theoretical intuitions as the one professing that the quality of informal arguments must be somehow assessable by means of logical formalisms seem to be immune to theoretical counterarguments, regardless of the latter’s strength.

Therefore, rather than once more criticizing the *ars iudicandi* conception from a purely theoretical viewpoint, the main part of this paper discusses a famous and very well documented dispute over the validity of a certain sort of arguments, viz. of arguments involving definite descriptions, in order to illustrate – from a practical viewpoint, so to speak – that logical formalisms indeed are of no help when it comes to settling the question whether a given argument is valid or not. A philosopher that has regularly been involved in such validity disputes is P. F. Strawson. His opposition to Russell’s widely accepted analysis of definite descriptions is but one example of a controversy that, in the end, revolves around the question whether pertaining arguments are valid or not.\(^3\) Russell and Strawson are at odds over the validity of arguments featuring definite descriptions in at least one of their premises, thus, of arguments as “The present king of France is wise. Therefore, there is a present king of France”. Even though logical formalisms have played a central role in this dispute, it shall be shown that formalizing such arguments contributes nothing whatsoever to settling their controversial validity. Rather than determining whether arguments involving definite descriptions are valid or invalid, Russell and Strawson, upon discussing the proper logical analysis of definite descriptions, merely contrast converse informal validity assessments rendered explicit by nonequivalent formalizations.

Before we look at the contentious validity of arguments featuring definite descriptions in section 3, section 2 is first going to introduce the

\(^3\) Another example is Strawson’s unconventional claim that universal affirmative predication cannot be formalized within first-order logic, but only by means of Aristotelian syllogisms. For Strawson’s logical analysis of definite descriptions cf. Strawson (1950), for his analysis of universal affirmative predication cf. Strawson (1952: 173-179).
two most discussed criteria of adequate formalization in order to, then, briefly rephrase the theoretical reasons indicating the incompatibility of the *ars iudicandi* conception, on the one hand, and criteria regulated logical formalization, on the other.

2. Adequate Formalization and Informal Validity

The starting point for systematic investigations into logical formalization traditionally has been the formalization of natural language within classical first-order logic. The literature concerned with criteria of adequate first-order formalization comprises only a handful of studies. In a nutshell, the debate over adequacy criteria for formalizations turns on two core criteria: *correctness* and *completeness*. Concisely put, a formula $\Phi$ is correctly assigned to a statement $A$ iff whatever formally follows from $\Phi$ informally follows from $A$, and whatever formally implies $\Phi$ informally implies $A$. In contrast, formalizations $\Phi_1, \Phi_2, \ldots, \Phi_n$ are said to be *complete* for statements $A_1, A_2, \ldots, A_n$ iff every informal dependence among $A_1, A_2, \ldots, A_n$ is mirrored by a formal dependence among the corresponding formalizations $\Phi_1, \Phi_2, \ldots, \Phi_n$. For instance, if $A_1$ informally implies $A_2$, $\Phi_1$ and $\Phi_2$ are complete for $A_1$ and $A_2$, respectively, iff $\Phi_1$ formally implies $\Phi_2$.

---

4 E.g. Blau (1977), Epstein (1990) and (1994), Sainsbury (1991/2001), Brun (2004) or Baumgartner & Lampert (2008). – There is another important thread in the literature on logical formalization. Davidson’s theory of meaning, Chomsky’s generative grammar and, most of all, Montague’s universal grammar are the best known approaches to formalization that, rather than settling for mere formalization *criteria*, implicitly or explicitly subscribe to the ambitious project to define an *effective formalization procedure* (cf. e.g. Davidson, 1984; Chomsky, 1977; Montague, 1974). Yet, the project of developing such a procedure that would link natural languages and first-order logic is not even close to a successful completion. Moreover, in view of the ambiguities and context-dependency of natural language the successful completability of this project can be doubted in principle. For more details on this procedure-driven thread in the formalization literature and on the reasons why it still is far from being completed cf. Baumgartner & Lampert (2008: sect. 1).

5 Several different variants of completeness can be found in the literature. The one given here corresponds to completeness as defined in Baumgartner & Lampert (2008: sect. 3.1).
correctness is defined for single formalizations, completeness, in turn, is relativized to propositional complexes. While any statement is correctly formalized by \( p \) along with a suitable realization, formalizations can only be considered complete if they represent the inner structure of a formalized text, i.e. if they mirror the informal dependencies among the text’s component statements \( A_1, A_2, \ldots, A_n \). The latter motivation is of particular importance when it comes to formally representing the validity of arguments. Famous formalization efforts such as the formalization of Aristotelian syllogisms, Russell’s analysis of definite descriptions or Davidson’s account of action sentences were all motivated by the urge to formally represent the informal validity of arguments. Accordingly, both correctness and completeness are taken to be necessary conditions of adequate formalization in the following.

Correctness and completeness render the adequacy of formalizations dependent on the same two notions: formal and informal inferential dependencies among formulae and statements, respectively. While formal dependence among formulae is to be understood relative to a given calculus, where that notion normally is straightforwardly defined in terms of formal implication, two statements are said to be informally dependent if one of the statements or its negation is judged to necessitate the truth or falsity of the other statement or its negation without compulsory recourse to any criterion of this necessitation. Similarly, we often use expressions correctly without being able to define them or justify their application. Hence, while \( p \land q \) and \( p \) are formally dependent, “Cameron is a mother”

---

6 A realization is an assignment of expressions of natural language to the categorematic parts of a formula. The categorematic expressions contained in a formula are its propositional variables, proper names and predicate letters (cf. Epstein, 1994: 13).

7 Even though common formalization practice often calls for complete formalizations, completeness is sometimes denied the status of a necessary condition of adequate formalization because the most prevalent versions of completeness cannot be applied in a finite number of steps to concrete formalizations. However, as the variant of completeness developed in Baumgartner & Lampert (2008) and resorted to in this paper is unproblematically applicable to formalization candidates, adequate formalizations can readily be required to be complete in the context at hand.
and “Cameron is a woman” are informally dependent, as the truth of the former statement necessitates the truth of the latter. Analogously, arguments are said to be *informally valid* if the truth of their premises necessitates the truth of their conclusions, whereas corresponding inference schemes are said to be *formally valid*.  

Spelling out correctness and completeness in terms of informal and formal dependencies yields two syntactical criteria of adequate formalization. Correctness and completeness can equivalently be defined semantically: A formalization on $\Phi$ of a statement $A$ is correct and complete iff the verbalization of every model of $\Phi$ expresses a truth condition of $A$ and the verbalization of every counter-model of $\Phi$ stands for a falsehood condition of $A$. To illustrate, consider the following simple example:

(a) Swiss like cheese.  
\[ \forall x(Fx \to Gx) \quad (1) \]
\[ F: \ldots \text{is Swiss}; \ G: \ldots \text{likes cheese} \quad (2) \]

Every interpretation of (1) such that the extension assigned to $F$ is a subset of the extension assigned to $G$ is a model of (1), all other interpretations are counter-models. By means of the realization (2) the models and counter-models of (1) can be *verbalized*, i.e. translated back into natural language. If such a verbalization describes a situation or constellation in which (a) is true, that verbalization expresses a truth condition of (a), otherwise a falsehood condition. All in all, thus, a correct and complete formalization shares its truth and falsehood conditions with the statement it formally represents.

That means clarity on formal and informal dependencies or truth and falsehood conditions of natural language texts is a precondition of adequately formalizing these texts. Logical formalisms represent truth conditions of statements in a syntactically transparent way. Such as to decide which formula adequately captures a colloquial statement, the latter’s truth conditions must be informally determinable. Put trivially, natural language texts must be understood before they can be formalized – logical formalization cannot clarify what is informally indeterminate.
Formalizing natural language texts, in turn, is a precondition of putting logical formalisms to work in the course of argument evaluation. The fact that the two central conditions of adequate formalization presuppose informal clarity about formalized texts, thus, is very consequential for the widespread *ars iudicandi* conception of logic, i.e. the view that takes logic to be the philosopher’s primary tool to determine the validity of arguments. For informally assessing the truth conditions of arguments prior to adequately formalizing them amounts to informally judging whether they are valid or not. Yet, if informal validity judgements are a precondition of adequately formalizing pertaining arguments, these judgements cannot be revised by logical formalisms. If an informally valid (invalid) argument is captured by a formally invalid (valid) inference scheme, it is not the informal validity (invalidity) of the argument that is rendered doubtful, but the adequacy of the corresponding formalization. Hence, on the one hand, if logical formalisms are resorted to in order to answer the question whether a given argument is valid or not, the adequacy of the involved formalizations is in need of stringent justification. On the other hand, such justification presupposes clarity about the validity of the respective argumentative context. This is a paradoxical finding that strongly conflicts with standard intuitions as to the *ars iudicandi* conception of logic.

In *Adequate Formalization* we conclude from this justification paradox that formalizations cannot be said to serve the validation of informal reasoning.\(^{10}\) Rather, formalizations transparently represent, i.e. *explicate*, the validity or invalidity of arguments.\(^{11}\) A formalization of an argument replaces an ambiguous and mistakable expression by an unambiguous and unmistakable formula that transparently represents the formal structure on which the argument is based. Accordingly, we argue that logic should not be seen as an *ars iudicandi*, but as an *ars explicandi*.

---

\(^{10}\) Cf. Baumgartner & Lampert (2008).

\(^{11}\) Note that the term “explication” is here not used in Carnap’s sense (cf. Carnap 1971: §§ 2-3). While for Carnap explicandum and explicans may differ in meaning or truth conditions, respectively, a logical or formal explicans is required to coincide with its informal explicandum with regard to truth conditions. In virtue of its syntax alone the formalization explicates the informally assessed truth conditions of the formalized text.
Informal Reasoning & Logical Formalization

Even though there is no disagreement over the fact that standard criteria of adequate formalization presuppose informal validity judgements, it is not normally conceded that logic consequently cannot serve the identification of valid arguments. Ordinarily, such as to maintain the *ars iudicandi* conception and evade the justification paradox, the relationship between natural and formal languages is conceived in terms of a so-called *(wide)* *reflective equilibrium*, which notion derives from Rawls and has been introduced into the context at hand by Goodman:

I have said that deductive inferences are justified by their conformity to valid general rules, and that general rules are justified by their conformity to valid inferences. But this circle is a virtuous one. The point is that rules and particular inferences alike are justified by being brought into agreement with each other. A rule is amended if it yields an inference we are unwilling to accept; an inference is rejected if it violates a rule we are unwilling to amend. The process of justification is the delicate one of making mutual adjustments between rules and accepted inferences; and in the agreement achieved lies the only justification needed for either.\(^\text{12}\)

According to this conception, informal validity is the gauge that measures the quality of formal definitions of what valid derivations are and formal validity systemizes and regulates its informal counterpart. Formal validity analyzes and theoretically represents informal reasoning, while informal (in)validity assessments may be reversed for reasons of systematics, conceptual simplicity, or incompatibility with accepted background theories. Logical formalization and its reversal – verbalization, i.e. the transformation of formulae into statements – are localized at the core of this equilibrium as they mediate between the implementation of informal and formal validity.

Accounting for the interplay between informal and formal dependencies by drawing on such equilibrium considerations has been criticized on many grounds. Thagard and Siegel argue that, contrary to Goodman’s claim, the interdependence between informal and formal dependencies is

not virtuous but vicious.\textsuperscript{13} If the interplay between informal and formal reasoning indeed is to be modelled in terms of a reflective equilibrium neither pole of that equilibrium is in any way justifiable. In \textit{Adequate Formalization} we contend that, if informal reasoning really always risked to be revised by formal constraints, the standard criteria of adequate formalization would be deprived of their status as necessary conditions for that adequacy, for they presuppose informal validity judgements.\textsuperscript{14} Thus, as an immediate consequence of a methodology of formalization embedded in equilibrium considerations, criteria of adequate formalization become mere rules of thumb that are sometimes resorted to upon formalizing natural language and sometimes neglected. A forteriori, without strict criteria of adequate formalization logical formalisms cannot reliably be applied as instruments to identify valid arguments. In case of contradicting informal and formal validity assessments, it would be completely undetermined which assessment to abandon. The attempt to ground the \textit{ars iudicandi} conception of logic in equilibrium considerations does not lead the way out of the justification paradox.

The justification paradox can only be avoided if one of its horns is taken for granted, i.e. if either informal or formal reasoning is exempt from the requirement of being justified by its respective counterpart. The constraints imposed by the problem of translating a natural into a formal language clearly suggest which horn that should be: informal reasoning. The formalization of natural language arguments essentially presupposes informal assessments of the truth conditions of the component statements of a formalized argument. Whenever internal dependencies among the statements in a text or the truth conditions of these statements cannot be ascertained informally, there is no criterion that would determine the adequacy of a respective formalization. Moreover, whenever there is a conflict between informal considerations and corresponding formalizations there do not exist two feasible sources of error – defectiveness of the informal judgement or inadequacy of the formalization –, but only one: In case of conflict it is always the formalization that is in need of revision.

\textsuperscript{13} Cf. Thagard (1982) and Siegel (1992).
\textsuperscript{14} Cf. Baumgartner & Lampert (2008).
To somebody professing the traditional *ars iudicandi* conception of logic this consequence must seem highly counterintuitive. Indeed it seems odd to concede that no argument can ever be identified as a fallacy by means of formal logic and that what is ordinarily called a “fallacy” is not mistaken informal reasoning, but rather a misunderstanding of informal arguments expressed by inadequate formalizations. Apparently, intuitions to the unacceptability of such consequences tend to be immune to theoretical objections, notwithstanding the strength of these objections. Therefore, that informal reasoning, in fact, cannot be proven to be wrong or right by way of formalizing pertaining arguments shall, in what follows, be illustrated by a famous exemplary dispute over the validity of arguments taken from the literature.

3. Arguments Involving Definite Descriptions

3.1 Diverging Analyses

A prominent controversy that essentially centers on the question whether arguments of a certain sort are valid or not has originated from P. F. Strawson’s unorthodox logical analysis of definite descriptions. There are at least two reasons as to why the validity disputes provoked by Strawson’s account of definite descriptions are illustrative when it comes to casting doubts on a logical formalism’s power to reverse informal validity judgements: First, Strawson explicitly subscribes to common criteria of logical formalization to the effect that the reason why this controversy has evolved in the first place cannot be seen in diverging accounts of logical formalization;\(^{15}\) second, Strawson distinctly deviates from ordinary validity assessments and authors defending the latter do not succeed in backing their standards in ways that would not be question-begging.

In *On Referring* Strawson vehemently opposes Russell’s widely accepted logical analysis of definite descriptions, which, as a direct consequence of Strawson’s criticism, is then no less vehemently defended by Russell.\(^{16}\) While, according to Russell, statements involving definite de-

\(^{15}\) Cf. e. g. Strawson (1952: 55-56, 148).

\(^{16}\) Cf. Russell (1957). For details on Russell’s Theory of Descriptions cf. e. g. Russell (1905).
criptions are to be formalized in terms of existentially quantified formulae that do no feature referring expressions, Strawson analyzes definite descriptions as referring expressions and, thus, statements containing definite descriptions as subject-predicate statements. The dispute revolves around the validity of arguments as the following notorious exemplar:

(b) The present king of France is wise. Therefore, there is a present king of France.

As is well known, Russell analyzes the premise of (b) in terms of “There is exactly one present king of France which is wise”, or formally:

\[
\exists x(Fx \land \forall y(Fy \rightarrow y = x) \land Gx)
\]

\(F: \ldots\text{is a present king of France; } G: \ldots\text{is wise}\)

(3)

As the conclusion of (b) is uncontroversially adequately formalized by \(\exists xFx\), Russell takes (b) to be a valid argument that can be formally captured in first-order logic by the following formal implication which, again, is to be understood relative to realization (4):

\[
\exists x(Fx \land \forall y(Fy \rightarrow y = x) \land Gx) \rightarrow \exists xFx
\]

(5)

Prima facie, this seems to be a fairly cogent implementation of the first-order formalism in order to formally prove the validity of (b): First, both premise and conclusion are formalized and, second, the formula assigned to the premise and the formula assigned to the conclusion are concatenated by means of a subjunctor which yields a formal implication and, therefore, can be claimed to prove the informal validity of (b). Yet, a closer look reveals that (5), rather than proving the validity of (b), presupposes that validity. For (5) can only be revealing with respect to formal properties of (b), if (5) is an adequate first-order representation of (b). Such adequacy is in need of justification. A formal implication as (5) can only be considered adequate for an argument as (b) if (5) is correct for (b). As (5) is tautological, every well-formed first-order expression formally implies (5). Hence, in order for (5) to be correct for (b) every (atomic or complex)

---

17 For brevity, I shall only be concerned with formalizations within first-order object language and, thus, ignore formalization candidates involving operators as “\(\vdash\)” or “\(\therefore\)” that belong to metalanguage.
Informal Reasoning & Logical Formalization

statement must informally imply (b), which, obviously, is only the case if (b) is tautologous, hence, informally valid. In short, (5) is adequate for (b) only if (b) is informally valid.

That a Russellian analysis of (b) presupposes rather than proves the validity of that argument can also be seen if we contrast it with Strawson’s analysis. According to Strawson, the premise of (b) is an ordinary subject-predicate statement predicating of the present king of France that he is wise – “The present king of France” being a singular term referring to the present king of France. He thus formalizes (b)’s premise by \( Ga \) where \( a \) refers to the present king of France and \( G \) stands for “…is wise”. As Strawson agrees with Russell that the conclusion of (b) is adequately formally captured by an existential statement \( \exists x Fx \), the formula adequately representing (b) in first-order logic, according to Strawson, is the following material implication:

\[
Ga \rightarrow \exists x Fx \tag{6}
\]

\( F: \) ...is a present king of France; \( G: \) ...is wise; \( a: \) the present king of France

As in case of Russell’s analysis, at first sight, it might be thought that the non-tautologous nature of (6) formally proves the invalidity of (b). As in Russell’s case, however, (6), rather than proving the informal invalidity of (b), presupposes it, for \( Ga \) and \( \exists x Fx \) are complete formalizations of (b) only if (b) is informally assessed to be an invalid argument. If “The present king of France is wise” and “There exists a present king of France” were informally judged to be dependent, that dependence would have to be mirrored by a complete formalization. In the following passage from On Referring Strawson indeed explicitly presupposes that (b) is informally invalid:

We might put it as follows. To say, “The king of France is wise” is, in some sense of “imply”, to imply that there is a king of France. But this is a very special and odd sense of “imply”. “Implies” in this sense is certainly not equivalent to “entails” (or “logically implies”). And this comes out from the fact that when, in response to his statement, we say (as we should) “There is no king of France”, we should certainly not say we were contradicting the statement that the king of France is wise. We are certainly not saying that it’s false. We are,
rather, giving a reason for saying that the question of whether it’s true or false simply doesn’t arise.\textsuperscript{18}

If the conclusion of (b) is false, its antecedent, according to Strawson, does not express a proposition whatsoever – it does not constitute a well-formed statement.\textsuperscript{19} The utterance or sign sequence “The present king of France is wise”, \textit{presupposes} – but does not imply – that there actually exists a referent of the definite description. As is well known, Strawson’s notion of a presupposition has given rise to many questions and, accordingly, has provoked manifold reactions.\textsuperscript{20} The intricate details of that notion, however, are of no importance to the context at hand. For our purposes, Strawson’s notion of a \textit{presupposition} can simply be seen as a relation between utterances or grammatically well-formed sign sequences, on the one hand, and propositions, on the other: A sequence $S$ presupposes a proposition $A$ iff, $S$ expresses a proposition iff $A$ is true.\textsuperscript{21} The informal judgement that the sequence constituting (b)’s premise in this sense presupposes the truth of (b)’s conclusion is perfectly captured by formalizing (b) in terms of (6). Formal semantics of first-order logic requires that names be assigned exactly one object in the domain. If that is not the case, the sign sequence $Ga$ cannot be considered an expression of first-order logic. In that case, (6) would not be a first-order expression either. As (b) is an informally invalid argument whose premise presupposes that there is a king of France, it can be completely formalized by (6).

All in all, Russell and Strawson apply the same standards of adequate formalization. Nonetheless, they arrive at totally different formalizations of

\textsuperscript{18} Strawson (1950: 330).

\textsuperscript{19} Strictly speaking, in the quoted passage Strawson says that if there is no present king of France, the antecedent of (b) still constitutes a statement (or a proposition), but one that lacks a truth value. However, apart from violating the law of excluded middle (cf. Russell, 1905: 485), such an account “conflicts with Strawson’s view that descriptions are devices used for \textit{referring}. Strawson’s position, then, is that no proposition is expressed.” (Neale, 1990: 26). Strawson clarifies this in Strawson (1974/2004: 50-54).

\textsuperscript{20} Cf. e. g. Sellars (1954) and Strawson (1954).

\textsuperscript{21} Note that in the paper at hand \textit{presupposition} is used in this specific sense only in the context of Strawson quotes. Everywhere else throughout this text \textit{presupposition} is non-technically used to indicate that something is taken for granted.
an elementary and seemingly self-explanatory argument as (b). The two authors so strikingly diverge with respect to logically analyzing (b) because they disagree on the informal validity of (b) prior to formalizing that argument. More specifically, they are at odds as to the informal truth conditions of (b)’s premise. Or put differently, the sign sequence constituting (b)’s premise does not express the same proposition for Russell and Strawson. By formalizing (b) both authors, rather than evaluating the validity of (b), render their particular understandings of (b)’s premise formally transparent, i.e. they explicate their readings of (b)’s premise.

3.2 The Debate

Clearly though, Russell and Strawson do not perceive themselves as merely explicating different readings of “The present king of France is wise”. If explication were all there was to the matter, there would be no reason for dispute. Rather, they maintain to be discussing whether definite descriptions \textit{de facto} are referring expressions or not. In case of arguments that amounts to the question whether arguments of type (b) \textit{de facto} are valid or not. The previous section has shown that standard criteria of adequate formalization as correctness and completeness are of no avail when it comes to determining the informal validity of arguments. Indeed, relative to two different informal validity assessments the question as to which of two non-equivalent formalizations is correct and complete does not even arise in the first place.

After having triggered the debate in \textit{On Referring}, Strawson recognizes the lack of a conclusive argument that would decide between his own and Russell’s position in \textit{Identifying Reference and Truth-Values}.\textsuperscript{22} In sharp contrast, Russell remains very irritated by Strawson’s reluctance to

\textsuperscript{22} Cf. Strawson (1964). In \textit{Direct Singular Reference: Intended Reference and Actual Reference} (1986) Strawson even concedes that Russell’s analysis of definite descriptions may sometimes be considered adequate. However, as he does not offer any reasons for this change of opinion and as the paper at hand is not primarily concerned with reproducing all the different stances Strawson has ever taken towards definite descriptions, this concession is not discussed further here.
accept his analysis of definite descriptions. Apart from his repeated insistence that (b)’s premise is “plainly” and “certainly” false, if there is no present king of France, Russell – more or less explicitly – offers three main reasons as to why his analysis is the proper one. In what follows, I discuss these reasons in ascending order of relevance.

(I) Russell develops the Theory of Descriptions by implicitly endorsing what we may call a principle of informal equivalence: If there exists a statement $B$ not comprising a definite description such that $B$ is informally equivalent to a statement $A$ which features a definite description $d$ and $B$ has a well-defined truth value irrespective of whether there exists an object conforming to $d$ or not, then $A$ must have the same truth value as $B$ whenever there is no object $d$. Against the background of this principle he argues that, since “The present king of France is wise” is informally equivalent to “There is exactly one present king of France which is wise” and the latter is false if there is no present king of France, the former must be false too. In a similar vein, it might be held that “The present king of France is wise” is informally equivalent to “France presently has exactly one wise king” which, again, is false if France has no king. It is evident that the principle of informal equivalence in no way supports Russell’s claim that (b) is informally valid. Rather than supporting that claim, it presupposes it. Formally proving the validity of (b) presupposes a positive answer to the question whether (5) is an adequate formalization of (b) which, in turn, presupposes clarity on whether “The present king of France is wise” really is informally equivalent to “There is exactly one present king of France which is wise” or to “France presently has exactly one wise king”. Therefore, in order to counter (I), Strawson does not have to reject the principle of informal equivalence, which indeed seems very persuasive. Instead,

---

23 E. g. in Russell (1957).
24 Russell (1905: 484 and 490).
25 Cf. Russell (1905). In (1957) Russell actually even offers a fourth reason. He blames Strawson for not respecting the law of the excluded middle. Indeed, as indicated in footnote 19 above, Strawson’s analysis of definite descriptions, at times, seems to violate that law. However, in accordance with Neale (1990), I have spelled out Strawson’s position in a way that is not affected by this criticism. Hence, I do not further discuss this issue here.
Strawson simply denies the informal equivalencies professed by Russell.\textsuperscript{26} Thus, the debate is back to a mere confrontation of different readings of “The present king of France is wise”.

(II) Russell argues that negative existential statements involving definite descriptions are undoubtedly true if there does not exist an object corresponding to the description. He illustrates this with the following example: Assume objects $a$ and $b$ do not differ in any respect. Then the statement “The difference between $a$ and $b$ does not exist” is clearly true.\textsuperscript{27} That means the non-existence of an object corresponding to a definite description does not generally prohibit sign sequences comprising definite descriptions from expressing a (true or false) proposition. Accordingly, so the argument continues, the falsehood of the conclusion of (b) does not suspend the propositional status of (b)’s premise either. Strawson, however, does not claim that all sign sequences consisting of a definite article followed by a noun phrase in effect amount to definite descriptions. Such sign sequences, according to Strawson, often are parts of predicates and not of referring expressions, as for instance in “The exhibition was visited by the present king of France”. Strawson takes this statement to be false if there is no king of France.\textsuperscript{28} It is the whole argumentative context in which a statement appears that determines its logical analysis, not its grammatical surface.\textsuperscript{29} Moreover, in \textit{Introduction to Logical Theory}, Strawson explicitly denies statements as “The difference between $a$ and $b$ does not exist” the status of subject-predicate statements.\textsuperscript{30} Therefore, even though Strawson does not explicitly address Russell’s exemplary statement, he would certainly join Russell in holding that this statement is true and, consequently, spell it out somehow along the lines of “There is no difference between $a$ and $b$”. And relative to such an informal assessment of truth conditions “The difference between $a$ and $b$” indeed, as Russell claims, is no referring expression. Yet, this finding has no bearing whatsoever on the logical analysis of (b).

\begin{itemize}
\item \textsuperscript{26} Cf. Strawson (1964: 86-87).
\item \textsuperscript{27} Cf. Russell (1905: 485). Many different examples of the same sort can be found in Russell (1918: 212-221).
\item \textsuperscript{28} Cf. Strawson (1964: 89-90).
\item \textsuperscript{29} Cf. Strawson (1964: 92).
\item \textsuperscript{30} Cf. Strawson (1952: 191).
\end{itemize}
Finally and most importantly, both in *On Referring* and in *Mr. Strawson on Referring* Russell takes one of the most noteworthy qualities of his analysis of argument (b) to be its universality. He maintains that definite descriptions must be formalized analogously whenever and wherever they occur in a statement. According to his Theory of Descriptions, every occurrence of a definite description indeed is to be analogously formalized, viz. in terms of a uniquely existential expression. Thus, Russell’s account satisfies the universality requirement, whereas Strawson’s account, as we have seen above, appears not to be universal in this sense. Strawson might try to straight-out reject this objection by claiming that he, just as well, treats all definite descriptions alike, viz. in terms of referring expressions. The fact that e. g. “The difference between a and b does not exist” can be said to be true does not show that “the difference between a and b” is a definite description that is not to be treated as referring expression, rather, it shows that “the difference between a and b” is no definite description in the context under consideration. However, Russell’s universality claim is not to be understood such that all logically identified definite descriptions are to be formalized alike. Rather, Russell has a grammatical notion of a definite description in mind: Expressions composed of the definite article “the” and a noun phrase in singular form are to be formalized alike.31 In fact, Strawson explicitly denies that singular noun phrases preceded by “the” can all be formalized alike.

Does that tip the scales against Strawson’s referential treatment of definite descriptions or, more particularly, against his ascription of a non-tautologous formula to argument (b)? Is (b) a valid argument because formalizing it in terms of a formal implication allows for a more general treatment of expressions composed of “the” and singular noun phrases? I doubt that a positive answer to this question would be very compelling. Indeed, the so-called misleading form thesis is a generally accepted doctrine in the literature on formalization stating that the grammatical form of an expression or statement is misleading as to its logical form.32

---

32 Cf. e. g. Brun (2004: ch. 7.1). Even though Strawson subscribes to the misleading form thesis (cf. 1952: 50-53), a lot of his later work is (mis)guided by the idea that, contrary to the misleading form thesis, there is a tight connection between grammatical and logical forms after all (cf. e. g. Strawson, 1974/2004).
natural language is ambiguous to the extent that logical forms of statements and arguments cannot be identified based on syntactic or grammatical criteria. The literature abounds with examples that illustrate such ambiguities. For instance, contrast the “The present king of France is wise” with “The whale is a mammal”. Both statements have the same grammatical form – “the” followed by a singular noun phrase, or subject term, followed by a predicate – yet only the first statement features a definite description, “The whale is a mammal” normally being formalized in terms of a universally quantified conditional. Russell might try to defend his claim as to the universality of the Theory of Descriptions by denying that “the” whale is a denoting expression in the second statement and, in consequence, no definite description. However, such a manoeuvre would presume that the notion of a definite description would no longer be spelled out in purely grammatical terms, which, in turn, would render Strawson’s account no less universal in regard to analyzing definite descriptions. That means formalization practice provides no rationale for Russell’s universality requirement. Expressions composed of “the” and singular noun phrases simply cannot all be formalized alike.

Russell might concede that there is no rationale for generally requiring grammatically similar expressions to be formalized analogously. Still, he could insist that the Theory of Descriptions is more general with respect to the formalization of expressions composed of “the” and singular noun phrases than Strawson’s account. Thus, Russell could impose the following maxim of analogy on adequate formalizations: Whenever possible, if Φ is an adequate formalization of a statement A and A is of the same grammatical type as a statement B, then Φ is also an adequate formalization of B. Put differently, of two different formalization strategies for expressions of a grammatical type t that strategy is preferable which allows for a more unified formalization of expressions of type t. Clearly, such a maxim involves a lot of vagueness. By what scale is the unification of formalizations to be measured or what criteria decide on grammatical typing? Nonetheless, maxims along these lines can be found in many studies on logical

---

formalization. Hence, can the dispute over the validity of arguments as (b) be settled if adequate formalizations are not only required to be correct and complete but, moreover, to comply with the maxim of analogy?

As a matter of fact, Strawson repeatedly – explicitly and implicitly – subscribes to a maxim of analogy as well. While Russell succeeds in backing up his analysis by means of the maxim of analogy provided that expressions composed of “the” and singular noun phrases are taken to constitute a grammatical type that calls for a maximally unified formalization, the maxim can be used in favor of Strawson’s analysis given that subject-predicate statements are seen as a grammatical type requiring a maximally unified formalization. The characteristic grammatical feature of “the present king of France” in (b)’s premise, for Strawson, is not the definite article followed by a singular noun phrase, but the fact that it constitutes the subject phrase in “The present king of France is wise”. The latter, according to Strawson, is a subject-predicate statement, i.e. a well-formed juxtaposition of a subject and a predicate phrase. Now, subject-predicate statements are normally formalized by means of singular terms and predicates, the paradigmatic example being $Fa$. Therefore, the maxim of analogy demands that (b)’s premise be formalized in that manner as well, which favors formalization (6).

While in On Referring, Russell agrees that $Fa$ ordinarily is the adequate formal representation of subject-predicate statements, he later declares the prevalent eliminability of proper names:

> Common words, even proper names, are usually really descriptions. That is to say, the thought in the mind of a person using a proper name correctly can generally only be expressed explicitly if we replace the proper name by a description.

---

35 Cf. e.g. Brun (2004: ch. 12.4.1).
37 In several texts, Strawson takes great pains to spell out the notion of a subject-predicate statement in purely grammatical terms (cf. Strawson, 1959: part II; or Strawson, 1974/2001). However, the misleading form of natural language prohibits an entirely grammatical notion not only of a definite description, but also of a subject-predicate statement.
39 Russell (1911: 152).
As is well known, Quine radicalized and generalized this idea by professing the general eliminability of singular terms:

We no longer labor under the delusion that the meaningfulness of a statement containing a singular term presupposes an entity named by the term. A singular term need not name to be significant.\(^{40}\)

Hence, in light of the Russell-Quine elimination of proper names, it might be argued that a Russellian analysis of (b) exceeds Strawson’s account with respect to generality or unification as it not only covers arguments involving definite descriptions, but even arguments featuring proper names. In other words, it could be claimed that Russell’s analysis is preferable because formalizing (b) in terms of (5) endorses a formalization strategy that allows for an elimination of proper names or singular terms in general and, hence, for a unified formalization of subject-predicate statements and uniquely existential statements. Prima facie, it certainly seems odd to hold that a specific argument is valid because formalizing it in terms of a formal implication yields a most unified formalization practice. The validity of a particular argument appears to be independent of the formalization of other statements. Nonetheless, such an argumentative backing of Russell’s analysis could be claimed to be an illustrative exemplification of the wide reflective equilibrium allegedly regulating the interplay between natural and formal languages.\(^{41}\)

In order to determine whether Russell’s account can really be backed by such equilibrium considerations, it first must be clarified what exactly the claim as to the eliminability of proper names amounts to. Russell’s and Quine’s eliminability claim does not involve a logical notion of a proper name, i.e. they do not hold that expressions that must be assigned exactly one object of reference are eliminable from natural language. According to Quine, such existential presuppositions are not represented by “modern formal logic”:

A substantial offshoot of Mr. Strawson’s reflexions on truth-value gaps is a theory […] in which a distinction is made between the referential and the predicative role of a singular term. This distinction, little heeded in logical literature, is important for an appreciation of ordinary language; and, as Mr. Strawson well

---

\(^{40}\) Quine (1948: 8-9).

\(^{41}\) Cf. sect. 2 above.
brings out, it reveals a marked failure on the part of Russell’s theory of descriptions to correspond to the ordinary use of ‘the’.

Normally, if the role of a singular term in a given statement is referential, the question of the truth of the statement does not arise in case the purported object of the term is found not to exist. Since modern formal logic closes all such truth-value gaps, it is not to be wondered that there is nothing in modern logic to correspond to the referential role, in Mr. Strawson’s sense, of terms.\(^{42}\)

That means the eliminability claim involves a grammatical or linguistic notion of a proper name according to which proper names are those expressions constituting the subject matter of onomatology. The eliminability claim says that onomatologically identified proper names are no logical proper names, i.e. expressions that must be assigned exactly one object in the domain of quantification. Rather, onomatologically identified proper names are (parts of) uniquely existential expressions. Is a thus understood eliminability of proper names sufficient grounds on which to prefer Russell’s analysis of (b) for its more unified formalization approach embedded in equilibrium considerations and, hence, to profess the informal validity of (b)?

Strawson is far from being impressed by the eliminability of proper names within a Russell-Quine framework. On the contrary, he takes this to be an absurd consequence of Russell’s analysis of statements as (b).\(^{43}\) To Strawson “singular predication lies at the core of logic”\(^{44}\). Subject-predicate statements constitute the fundamental type of statements. How is it that Strawson, on the one hand, accepts the maxim of analogy, yet, on the other, fails to recognize the superior generality of Russell’s formalization of (b)? The answer is at hand: Strawson takes his analysis to be more general than Russell’s. Definite descriptions, according to Strawson, are referential expressions that presuppose the existence of the object referred to. This characteristic allows for treating them on a par with proper names or singular terms. Thus, even though Strawson never explicitly subscribed to that claim himself, his account could be interpreted to foster the elimination of definite descriptions: Expressions composed of “the” and singular

\(^{42}\) Quine (1953: 439).
\(^{43}\) Cf. e.g. Strawson (1952: 189-190), or (1974/2001: 40).
\(^{44}\) Strawson (1952: vii).
noun phrases are never to be formalized in terms of uniquely existential formulae.

In light of this finding the question now becomes: What are the better candidates for being eliminated in the name of a maximally unified and general account of logical formalization? Quine would opt for singular terms because their elimination yields logical formulae that do not presuppose the existence of particular objects or, in his words, that are free from truth-value gaps. Strawson would opt for definite descriptions because expressions involving definite descriptions in effect presuppose the existence of corresponding objects. They can thus not be adequately formalized by uniquely existential formulae. At this point, we can easily tell that we are back where we started from: the question whether the premise of (b) presupposes the existence of the present king of France or whether it entails that existence – hence, back to the question whether (b) is informally valid or not.

The whole debate over the proper logical analysis of definite descriptions rests on different informal validity assessments with respect to arguments as (b). Without such different validity assessments there would be no controversy in the first place. The arguments pushing the debate forward all center on and presume these validity assessments. The debate does not shed any light on whether arguments as (b), in fact, are valid or not and whether definite descriptions, in fact, are uniquely existential expressions or singular terms. All in all, logical formalisms and logical analyses are of no avail when it comes to settling the validity of arguments involving definite descriptions. (5) and (6), rather than proving the validity or invalidity of (b), explicate different readings of (b).

4. Conclusion

At the heart of the Russell-Strawson debate concerning the proper logical analysis of definite descriptions lies the question whether arguments as (b) are valid or not. The discipline ordinarily considered authoritative for evaluating the validity of arguments is formal logic. However, we have seen that adequately formalizing an argument A presupposes a determinate validity judgement with respect to A. That judgement cannot be revised by any formal validity or invalidity proof, because all such proofs presume the
adequacy of employed formalizations. Consequently, Russell’s tautologous and Strawson’s non-tautologous formalization of the argument “The present king of France is wise. Therefore, there is a present king of France” turn out to be formal explications of Russell’s and Strawson’s informal validity assessments prior to formalizing that argument.

Friends of the *ars iudicandi* conception of logic might concede that formal logic indeed is not serviceable to answer the question whether arguments of type (b) are valid or not. Nonetheless, it could be insisted that formal logic may well be resorted to in order to evaluate the validity of ever so many other types of arguments. After all, it might be claimed, purported instances of *modus ponens* can be shown to be valid by formal means. Take the following example: “All of John’s children are asleep. Shamus is one of John’s children. Therefore, Shamus is asleep”. While the second premise and the conclusion are subject-predicate statements, the first premise is ordinarily said to be of universally quantified conditional form. The concatenation of the three statements by a subjunctior then yields a formal implication which can be claimed to prove the validity of that argument. Yet, as indicated in the introduction, Strawson would even reject this line of reasoning, as he takes the first premise of this alleged instance of *modus ponens* to presuppose that John actually has children which is not adequately formally captured by a universally quantified conditional. The argument under consideration, for Strawson, thus is not an instance of *modus ponens* after all.

Irrespective of whether one finds Strawson’s grounds on which he takes specific cases of universal affirmative predication to presuppose that the extensions of certain predicates are non-empty convincing or not, this paper should have made it clear that the reason why the controversial validity of arguments of type (b) cannot be settled by formal means in no way hinges on the particularities of arguments involving definite descriptions. Logical formalisms can only be put to work to identify valid arguments if pertaining formalizations have been firmly justified and such justifications are only to be had if the validity of pertaining arguments has been determined prior to formalizing them. The justification paradox does not

---

only affect arguments featuring definite descriptions. It lies at the core of the whole problematic interplay between formal and natural languages.*

Literature


* This paper is greatly indebted to Timm Lampert in collaboration with whom the theory of formalization resorted to here has been developed. Moreover, I thank the editors of this volume for helpful comments on an earlier draft and the Swiss National Science Foundation for generous support of this work (grant # PP001-114812/1).


Informal Reasoning & Logical Formalization

Autorenangaben und Abstract

Michael Baumgartner is currently a lecturer and postdoctoral research associate at the University of Konstanz, Germany. From 2008 to 2009 he was a member of the philosophy of language research group meaning.ch at the University Bern, Switzerland, and from 2007 to 2008 a Visiting Fellow at the Center for Philosophy of Science of the University of Pittsburgh. In 2005 he earned his doctorate at the University of Bern with a dissertation on causation entitled *Complex Causal Structures*. His publications include a monographic introduction to the philosophy of causation and a number of papers on causation, regularity theories, interventionism, causal reasoning, and logical formalization.

According to a prevalent view among philosophers formal logic is the philosopher’s main tool to assess the validity of arguments, i.e. the philosopher’s *ars iudicandi*. By drawing on a famous dispute between Russell and Strawson over the validity of a certain kind of argument – of arguments whose premises feature definite descriptions – this paper casts doubt on the accuracy of the *ars iudicandi* conception. Rather than settling the question whether the contentious arguments are valid or not, Russell and Strawson, upon discussing the proper logical analysis of definite descriptions, merely contrast converse informal validity assessments rendered explicit by non-equivalent logical formalizations.

***
Viele Philosophen sehen in logischen Formalismen das zentrale Werkzeug zur Beurteilung der Gültigkeit philosophischer Argumente, d. h. die formale Logik gilt weitum als die *ars iudicandi* des Philosophen. Anhand einer berühmten Debatte zwischen Russell und Strawson über die Gültigkeit einer bestimmten Klasse von Argumenten – von Argumenten, deren Prämissen singuläre Kennzeichnungen enthalten – wird in diesem Artikel die Richtigkeit der *ars iudicandi* Auffassung in Zweifel gezogen. Anstatt die Frage zu beantworten, ob Argumente des strittigen Typs gültig sind oder nicht, kontrastieren Russell und Strawson im Verlauf ihrer Debatte um die richtige logische Analyse singulärer Kennzeichnungen lediglich unterschiedliche informelle Gültigkeitsurteile, welche sie durch nicht äquivalente Formalisierungen explizit machen.