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ORIGINAL RESEARCH

Non-propositionalism and the Suppositional Rule

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Abstract
It can often seem like the attitude we hold towards a conditional should be our attitude in the consequent on the supposition of the antecedent. Following by Williamson (Suppose and Tell: The Semantics and Heuristics of Conditionals. Oxford University Press, 2020), we call this The suppositional rule (SR). The Adams-style non-propositional theories of indicatives upholds some key implications of SR, allowing, for instance, our credence in a conditional to be the probability of the consequent given the antecedent. Williamson (Suppose and Tell: The Semantics and Heuristics of Conditionals. Oxford University Press, 2020) has recently provided a series of inconsistency arguments against SR. He thereby intends to undermine non-propositional views as well as other rivals to his favoured material conditional account. I outline a strategy which theorists of all stripes can employ to avoid Williamson's arguments. I then show how non-propositionalists can implement this strategy. I show how they can uphold SR when it is intuitively compelling, whilst allowing it to fail when it is not.

1 Introduction

Williamson (2020) has recently argued that we assess conditionals primarily through using the following rule:

> The Suppositional Rule (SR)**:** Take an attitude unconditionally to if $P > Q$ just in case you take it conditionally to Q on the supposition of P.

SR gives particularly plausible predictions when applied to certain epistemic attitudes:

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> The Suppositional Rule for Acceptance (SRA): Unconditionally accept $P > Q$ just in case you conditionally accept Q on the supposition of P.
> The Suppositional Rule for Credences (SRC): Take the same credence unconditionally in $P > Q$ that you do conditionally to Q on the supposition of P.
> The Suppositional Rule for Epistemic Necessity (SRmust): Regard $P > Q$ as unconditionally epistemically necessary just in case one regards Q as conditionally epistemically necessary on the supposition of P.

For example, it seems like we should accept "if I strike the match, it will light" just in case we accept that the match will light on the supposition that it is struck; it seems our credence in "if the die lands on an even number, it will land on a heads" should be the credence we assign to the die landing on a 6 given it lands on an even number; it seems we should regard "if Sarah studies hard, she will pass" as epistemically necessary just in case we regard it as epistemically necessary that Sarah will pass on the supposition that she studies hard.

Much of the literature on indicatives has focused on SRC. Despite its intuitive plausibility, it has long been known to be in tension with standard truth-conditional theories of indicatives. Imagine you have a fair coin in front of you. You are considering the conditional "if I flip the coin, it will land heads". SRC tells us that the credence we should assign to this conditional is 0.5. However, this conflicts with several popular semantics for indicatives. Suppose you have a fair coin in front of you. Since there is almost certainly an epistemically possible world where you flip the coin and it lands tails, strict and Kratzer-style restrictor theories seem to make this conditional almost certainly false.[1] Material conditional analyses will tend to overestimate the probability of this conditional. If you are just as likely to flip the coin as not, the material conditional analysis tells you its probability is 0.75. Moreover, the triviality results provide a general argument showing that conditionals cannot express propositions (sets of possible worlds) whilst SRC is upheld in all rational probability distributions. This presents a dilemma for theorists, reject SRC or disavow the standard propositional treatment for conditionals and/or some standard tenets of probability theory.

Advocates of Adams-style non-propositionalism opt for the latter option. They reject the idea that conditionals express propositions and that the probability of a conditional is its probability of truth. This allows them to uphold SRC without triviality (at least, for certain kinds of statements).[2] The non-propositional framework is usually accompanied by the idea that conditionals are vehicles for expressing our conditional attitudes. So, for example, the purpose of an assertion of "if it rains, the match will be delayed" is to express our confidence in the match being delayed on the supposition that it rains.

Although, as we will see, its questionable whether non-propositionalists are committed to upholding SR in its entirety, the fact that they uphold SRC makes it relatively easy and natural for them to uphold other instances of SR such as SRA and

[1] Given this conditional contains no overt modal operators, standard Kratzer-style restrictor theories agree with strict theories with respect to its truth-conditions.

[2] I will tend to drop this caveat in the rest of the paper.

SRmust. Given it seems we generally evaluate conditionals in accordance with these principles, this provides non-propositionalism with a distinct advantage over theories. It can maintain a close match between the attitudes in conditionals we think are rational and the ones which the theory predicts.

In his recent book, Williamson seeks to undermine the justification for SR and the judgments it licenses. He thereby aims to pull the rug out from under the feet of non-propositional theories of indicatives as well as truth-conditional rivals to his favoured material conditional account. Although Williamson holds that SR is the primary way we evaluate conditionals, he contends that it is inconsistent. He produces three main arguments for this. The first applies SR to complex attitudes, the second applies it to our patterns of credence, and the third to our deductive attitudes.[3] He embraces what is, in effect, an error theory in which the way in which we evaluate conditionals is riddled with inconsistency and error.

Dialectically, Williamson's inconsistency arguments are intended to lessen our confidence in the judgments arising from SR, thereby taking away the main source of support for rivals to the material conditional account.[4] The thought is that the intuitive appeal of these theories is largely derived by our instinctive desire to uphold this principle. But once we see that it is inconsistent, we will thereby come to see this goal as inherently misguided.[5]

Once we see that SR is inconsistent, Williamson thinks that this will cause us to rethink the status of this rule. He thinks that we will thereby be pushed towards his view that SR is a heuristic, one designed to gives us a quick and dirty way of approximating the correct attitude to hold towards a conditional. He thereby attempts to explain away the data that is inconvenient to the material conditional account such as why the probability of $P > Q$ is seemingly the probability of Q given P rather than the probability of the material conditional $P \rightarrow Q$. He supplements his negative arguments with a subtle positive case for the material conditional analysis based on its ability to mediate between the various heuristics we use to assess conditionals. He also provides an account of counterfactuals which involves the material conditional compositionally interacting with a necessity operator expressed by "would".

In the following, we will put aside Williamson's positive case for the material analysis of indicatives as well as his theory of counterfactuals. We will instead focus on the negative part of his arguments, the ones aimed at undermining rivals to his analysis of indicatives.[6] We will focus on defending one particular kind of theory, Adams-style non-propositionalism. However, the general strategy outlined here will be available to other kinds of theories as well.

[3] For expositional reasons, these arguments are presented in reverse order to their presentation in Williamson's book.

[4] The material conditional theory of indicatives holds that an indicative $P > Q$ is truth-functionally equivalent to the material conditional $P \rightarrow Q$ which itself is truth-functionally equivalent to the disjunction $\neg P \vee Q$ and negated conjunction $\neg(P \wedge \neg Q)$.

[5] Williamson thinks that certain principles such as the law of conditional excluded middle only seem valid because of SR. He thinks that we would be misguided to favour truth-conditional theories (e.g. Stalnaker's) on the basis that they validate these principles.

[6] See Rothschild (2021) for criticism of his positive case for the material conditional theory.

In Sect. 2 I outline my general strategy for avoiding Williamson's arguments. In Sect. 3 I outline a standard Adams-style non-propositional theory. In Sects. 4, 5, and 6 I outline Williamson's inconsistency arguments against SR pertaining to our complex, probabilistic, and deductive attitudes respectively. In each of these sections, I identify a principle which the non-propositionalist has good reason to reject. I show how a natural extension of the Adams-style framework leads to failures of each principle, thereby avoiding inconsistency. In Sect. 7 I draw some broader lessons concerning the viability of non-propositionalism.

2 The Strategy

Our general strategy will be to concede that SR is inconsistent, but reject Williamson's claim that our intuitive procedures for assessing conditionals are inconsistent as well.[7] A key part of our approach (in Sects. 4 and 6) will be to argue that our intuitions do not always align with SR when it is applied to certain attitudes. To take one illustrative example, suppose I am considering buying a ticket for an upcoming lottery. On the supposition that I win the lottery, I am glad that I bought a ticket. However, I am not particularly glad about the obvious fact that if I win the lottery, I bought a ticket. I trust the reader will agree that my conditional and unconditional attitudes are entirely appropriate here, despite conflicting with SR.

The fact that the predictions of SR are not intuitively plausible for certain attitudes does not immediately undermine Williamson's view of this rule. Williamson can simply retort that, as a heuristic, it is to be expected that there will be cases where we find SR's predictions uncompelling. In these cases, it may be that other considerations override the heuristic (see Williamson, 2020, p.92–3) or that it's inoperative for some other reason. Thus, the existence of intuitive counterexamples to SR doesn't automatically mean that we use this principle in a consistent way or that we aren't led seriously astray by the data that it feeds us.[8]

However, Williamson's case that SR is a heuristic depends heavily on his claim that our actual practices of assessing conditionals are inconsistent. The fact that we do not always intuitively assess conditionals in line with SR, allows for a way of undermining this claim. Let's suppose we weaken SR so that it is *descriptively adequate*. That is, it upholds (at least the best part of) our intuitive judgements about the attitudes which are appropriate to hold in conditionals. We will call this suitably weakened principle, SR_{DA}. Given it is descriptively adequate, SR_{DA} will imply SR's intuitively plausible instances (for attitudes like acceptance, credence, and regarding something as epistemically necessary), but not its intuitively implausible ones (for attitudes like being glad about something). This principle would thus provide a better representation of our practices of competently assessing conditionals than SR. That is, it would be SR_{DA} and not SR which would provide the most accurate predictions about the attitudes we regard as intuitively appropriate to hold in conditionals.

[7] I am heavily indebted to two anonymous reviewers for prompting me to think through the ideas discussed in this section. Both helped me clarify the strategy pursued in this paper.

[8] I am very grateful to an anonymous reviewer for suggesting this line of response.

If this principle is consistent (relative to some plausible background theory), then we would have reason to hold that our intuitive practices of assessing conditionals are consistent as well. This would thereby undermine Williamson's claim that our procedures of assessing conditionals are deeply inconsistent. Williamson's case that SR is a systematically unreliable heuristic would then look rather thin.

A comparison will illustrate the benefits of this approach. Williamson (ibid, p.60–3) suggests the disquotational schemas '"P" is true iff P' and '"P" is false iff not P' govern our competence with the predicates "true" and "false". However, he notes we have reason to regard these principles as inconsistent given we can derive a contradiction from them using classical logic when "P" is substituted for the liar sentence. Williamson takes this to inductively support his claim that inconsistent principles can govern our competent linguistic practices, thereby supporting his claim that SR is an inconsistent and unreliable heuristic.

I think Williamson draws the wrong lesson from this comparison. The inconsistency of these disquotational principles does not give us reason to reject their intuitively unproblematic instances. It does not give us reason to reject "it's snowing in Oslo" is true iff it's snowing in Oslo and "it's snowing in Oslo" is false iff it's not snowing in Oslo. In fact, Williamson (1996, p.162–4) has previously criticised supervaluationist theories of vagueness for failing to validate these unproblematic instances of the disquotational principles. He (ibid, p.197) has further argued that we can uphold most ordinary instances of the disquotational schemas in the face of the liar paradox by endorsing slightly weaker versions of these principles. This strategy promises to accommodate the intuitive appeal of the disquotational principles, without committing us to widespread inconsistency.

Why then does he does not apply the same approach to SR as he does to the disquotational principles? It is surely just as counterintuitive to hold that the probability of "if I flip the (fair) coin, it will land heads" is significantly higher than 0.5 as it is to reject many ordinary instances of the disquotational principles. Weakening SR in a way that avoids such counterintuitive consequences promises to provide a more attractive solution to his inconsistency arguments than one which requires a wholesale revision of our intuitive judgements.

Of course, sometimes our intuitions are inconsistent, and it is not possible to preserve them all. If this is so, we have to pick and choose the ones we want to preserve. But rather than disregarding substantial bodies of intuitive data, it is generally good theoretical practice to construct theories which preserve the maximum number of intuitions possible. Suffice to say, it is doubtful the material conditional account of indicatives makes optimally good sense of our intuitive judgements compared to other theories.

Williamson's selectively dismissive attitude towards the intuitive data may be justified if there was strong independent evidence for the claim that SR is a heuristic. For example, we might find that people generally override the data that it feeds us after slow, deliberative reasoning. But as acknowledged by Williamson (2020, p.113), such evidence is lacking with respect to SR. If we find an instance of SR compelling, we generally do so even after careful reflection.[9] In fact, I venture careful reflection tends to *lead* to agreement with intuitively compelling instances of SR like SRA, SRC, and SRmustnot. Williamson's case that SR is a heuristic would also be supported if it was based on research from cognitive science or psychology. However, as he (ibid, p.22) also acknowledges, his account does not significantly draw from these disciplines.[10]

Even though I do not believe that SR is either consistent or descriptively adequate, I will sometimes describe us as competently applying SR when evaluating conditionals. Williamson sometimes seems to suggest that employing inconsistent rules automatically commits one to inconsistency in action/behaviour.[11] This is not so. Following a rule once does not commit one to following it at all times. We can consistently apply inconsistent rules provided we judiciously restrict our usage to cases that do not get us into trouble. For example, we can consistently apply the disquotational principles (or rather, the capture and release rules for truth and falsity) as long as we are careful to avoid applying them to sentences which commit us to a contradiction. We can consistently apply the rule that tells us to be kind and honest to one another as long as we don't try and apply it in cases where it is impossible to be honest and kind. Similarly, we can consistently apply SR in certain cases without this necessarily committing us to inconsistency.[12]

In the rest of this paper, we will implement the strategy outlined above. We will argue that the non-propositional theory of conditionals can endorse a weaker version of SR that is both consistent and descriptively adequate.[13] It is worth stressing at this point that it is far from a forgone conclusion whether this strategy will succeed. Williamson's inconsistency arguments may only rely on principles that are intuitively justified. If we can only restore consistency by making substantial intuitive sacrifices, then we would then be forced to acknowledge that our procedures for assessing conditionals *are* irredeemably inconsistent. We would then need to follow Williamson in taking a far more critical eye to the intuitive data. It may also be that

[9] Williamson (2020, p.114). thinks that there are some cases where we can come to the override SR. The cases which he draws on are ones which involve conditionals with impossible antecedents. I will argue later that these are cases which are best treated as ones in which we are *employing* SR, not overriding it.

[10] This feature of Williamson's account is criticised in Rothschild (2021, p.22–3).

[11] For example, he (2020, p.60) says: "It is consistent to hold that people implicitly rely on inconsistent rules in speaking and understanding their native language. In that sense, linguistic competence may be inconsistent.".

[12] It can often be a delicate matter as to whether someone is following a rule R^1 or a weaker rule R^2. In fact, it can often be appropriate to describe someone as following both R^1 and R^2. For example, when a driver stops at some traffic lights, we can seemingly describe them as following both the rule that tells us not to run red lights *and* the rule that tells us to obey a country's laws. Let's suppose subject S always acts in accordance with R^1, but sometimes violates R^2. Particularly if S's decisions are largely implicit, describing S as following R^1 with certain *restrictions* and describing them as following the rule R^2 seem to amount to much the same thing (although there may be differences in emphasis). For instance, if R^1 = SR and R^2 = SR_{DA}, there does not seem to be much difference in describing S as following SR_{DA} or following SR with certain restrictions. Neither does there seem to be much difference in describing S as following SR_{DA} or its instances like SRA, SRC, and SRC.

[13] We won't formulate a principle that gives descriptively accurate predictions for *all* attitudes. This would obviously require a lot of work. We will only consider the attitudes mentioned in Williamson's arguments.

Williamson's arguments relies on principles that non-propositionalists specifically are committed to (whether they are intuitive or not). In this is so, there would be no principled way for them to avoid his arguments. The challenge will be to demonstrate that this is not so, the non-propositionalist can defuse Williamson's arguments without significant costs.

3 Adams-Style Non-propositionalism

The non-propositional theory was first outlined in Adams (1965, 1966) and was elaborated in his (1975, 1998) books.[14] We will outline this framework here.

First, take a standard propositional language closed under the logical connectives. We call a statement in this language a *factual statement*. We use the letters A, B, C as metavariables ranging over factual statements. Adapting a convention from Williamson (2020), we use repeated letters AA, BB, CC to stand for sets of these sentences. Now consider a language which allows free embeddings of the indicative connective, $>$. We call statements containing the indicative conditional operator $>$ *nonfactual statements.* We call conditionals with factual antecedents and consequents simple conditionals. We use P, Q, R, S letters as metavariables ranging over nonfactual and factual statements. We use the double letters PP, QQ, RR, SS to stand for sets of these sentences.

Factual statements are assigned truth values relative to possible worlds subject to the usual truth-functional clauses (we use "→ ε to stand fo the material conditional connective). We will assume here for the sake of simplicity that the set of worlds is finite. We denotate the set of worlds on which A is true as $[\![A]\!]$. A probability mass function cr assigns real numbers between 0 and 1 to possible worlds with the constraint that our credence in each world sums to 1. Our credence in a proposition (proposition) X is just the sum of the credence assigned to each world in X. We can then define the probability of X conditional on Y as $\mathrm{Cr}(X|Y)$. We define this as the ratio $\mathrm{Cr}(X \cap Y)/\mathrm{Cr}(Y)$ when $\mathrm{Cr}(Y) > 0$. Whilst it is relatively common to leave Cr(X|Y) undefined when $\mathrm{Cr}(Y) = 0$, we will follow Adams (1998, p.57) in defining this to be 1 when $\mathrm{Cr}(Y) = 0$. As we will, later argue (in Sects. 4 and 6), this is not merely a matter of convenience. There are some substantive philosophical reasons for defining conditional probability like so.[15] We now assign credences to

[14] This form of non-propositionalism has been taken up by several, including Edgington (1995), Gibbard (1981), and Bennett (2003).

[15] In standard infinite probability spaces, an event can be assigned 0 probability despite being possible and an event can be assigned probability 1 even though it is not absolutely guaranteed to come to pass. So, if we were to uphold this definition in infinite probability spaces, then the conditional probability of C given A will always be 1 even when A is possible but has probability 0. This is *no more* problematic than leaving conditional probabilities undefined when A is assigned probability 0. Nonetheless, there are cases where this definition of conditional probability seemingly goes awry. For example, suppose that a needle could fall on any of the uncountable number of points in a circle, including π_1 and π_2. That the needle falls on any one point is no more probable than it falling on another. It will standardly follow that the probability that it falls on either point is 0. It seems we nonetheless want the probability that it falls on π_1 given it falls on π_1 or π_2 to be .5, not 1. In response, we may want to employ some non-standard

sentences. We define the credence in a factual statement so that $\mathrm{Cr}(A) = \mathrm{Cr}([\![A]\!])$. In other words, the credence we assign to a factual sentence is just the credence assigned to the proposition it expresses. We define $\mathrm{Cr}(C|A)$ as $\mathrm{Cr}([\![C]\!]|[\![A]\!])$. In accordance with SRC, we define the credence in a simple conditional $A > C$ so that $\mathrm{Cr}(A > C) = Cr(C|A)$. In other words, our credence in a conditional is our credence in the consequent on the supposition of the antecedent.

Although entailment might be defined in a number of different ways (see Adams, 1996; p.57), typically it is defined like so:

- $P_1, \ldots, P_n \vDash Q$ iff it cannot be the case that $u(Q) > (u(P_1)+, \ldots, +u(P_n))$

Where $u(P) = 1 - \mathrm{Cr(P)}$. In other words, $P_1, \ldots, P_n$ entails Q iff the uncertainty in the conclusion cannot exceed the sum uncertainty of the premises. This is called *probabilistic entailment*. One of its principal advantages is that it avoids the so-called paradoxes of material implication. For instance, according to the probabilistic conception of entailment, we have failures of the sequents $P > Q \vDash (P \wedge R) > Q$ and $\neg P \vDash P > Q$. It is also well known that probabilistic entailment agrees with classical validity when $P_1, \ldots, P_n$ and Q are all factual statements.

In the traditional form of non-propositionalism, neither truth-conditions nor epistemic attitudes are assigned to conditionals embedded under the logical operators. They have no defined meaning within the non-propositional framework. The fact that compounds of conditionals are often difficult to parse is sometimes referenced as justification for this feature. However, unsatisfied with this aspect of Adams' framework, several theorists have sought to extend it to assign probabilities complex statements (see for example Jeffrey & Stalnaker (1994) and McGee (1994)).

We will grant here (at least for the sake of argument) that we can meaningfully take epistemic attitudes towards compounds of conditionals. After all, it is antecedently rather implausible that we cannot rationally believe or assign credence to such statements. We will thereby assume (whilst omitting full details) that the simple approach outlined here can be extended so that credences and other epistemic attitudes can be assigned to statements containing embedded conditionals in a coherent way. Given some of Williamson's arguments rely on this assumption, the aim is to thereby make Williamson's case against Adams-style non-propositionalism as strong as possible.

Footnote 15 (continued)
probability theory such as one employing Popper functions (see Popper (1959)) or nonstandard analysis. The first approach allows C given A to be assigned non-trivial probabilities when A is assigned probability 0. The second allows us to define probabilities on events which are assigned probabilities infinitesimally close to 0. For discussion of the benefits and demerits of these approaches see Lewis, 1981; Skyrms, 1980; Williamson, 2007; Pruss, 2015; Easwaran, 2014; Howson, 2021. If we adopt some non-standard conception of probability, all we require for our arguments to go through is that C is certain conditional on A when A is entirely ruled out (however this is defined). This is a very minimal requirement.

4 Complex Attitudes

Before we outline Williamson's first argument, we will need to introduce some of his notation. We use $PP||Q$ to represent the claim that we should take an arbitrary attitude towards Q on the supposition of PP. The following can thus be taken to be a formulation of SR:

- $\text{SR}_{||}$:$PP, Q||R \iff PP||Q > R$

We can then formulate particular instances of this principle such as SRA:

- $\text{SR}_{||^a}$:$PP, Q||^a R \iff PP||^a Q > R$

Williamson argues that The Suppositional Rule supports Reflexivity (Ref), Commutativity in the Consequent with Conjunction (CCC), and Commutativity in the Consequent with Negation (CCN)[16]:

- Ref :$||^a P > P$
- CCC:$SS||^a P > (Q \wedge R) \iff SS||^a (P > Q \wedge P > R)$
- CCN:$SS||^a (P > \neg Q) \iff SS||^a \neg(P > Q)$

He then shows that these principles are inconsistent through the following simple derivation:

(1) $||^a (P \wedge \neg P) > (P \wedge \neg P)$ by Ref
(2) $||^a ((P \wedge \neg P) > P) \wedge ((P \wedge \neg P) > \neg P)$ by CCC and (1)
(3) $||^a ((P \wedge \neg P) > P) \wedge \neg((P \wedge \neg P) > P)$ by CCN and (2)

We can see that this derivation commits us to accepting a contradiction based on no suppositions at all. Given that Williamson believes that Ref, CCC and CCN follow from The Suppositional Rule, he concludes that it is inconsistent.

One potential way for the non-propositionalist to block the above proof is to insist that compounds of conditionals are not meaningful and therefore are not appropriate candidates for our epistemic attitudes. However, as we remarked above, it's antecedently plausible that we *can* rationally take epistemic attitudes towards (at least some of) these kinds of statements. We will therefore grant Williamson this assumption going forwards.

[16] Williamson's argument formulates these principles as logical principles, not as claims about our attitudes of acceptance. So, for example, CCN is formulated as $(P > \neg Q) \iff \neg(P > Q)$. He takes his formulation of CCN to follow from his derivation of our formulation of CCN—$SS||^a (P > \neg Q) \iff SS||^a \neg(P > Q)$. I have simplified his proof slightly to avoid having to jump from claims about attitudes to claims about implication. This has the additional advantage that it is not immediately clear how to understand the '$\iff$' biconditional on the non-propositional picture when it connects nonfactual statements.

Williamson gives us proofs to show that REF, CCC, and CCN can be derived from SR. We will take for granted his proofs of Ref and CCC. We will seek to block his proof of CCN, and thereby, the above derivation.

In order to derive CCN from SR, he appeals to the following rule governing the negative attitude of rejection:

- $||^{not}:PP||^{not}Q \iff PP||^{a}\neg Q$

In his words, this principle tells us "to take the negative attitude to something is just to take the positive attitude to its negation (under the same suppositions)" (Williamson, 2020 p.45).

He shows that $SR_{||^a}$ and $||^{not}$ combine to imply CCN like so:

(1) $RR||^{a}P > \neg Q \iff$
(2) $RR, P||^{a}\neg Q \iff$ by $SR_{||^a}$
(3) $RR, P||^{not}Q \iff$ by $||^{not}$
(4) $RR||^{not}P > Q \iff$ by $SR_{||^{not}}$
(5) $RR||^{a}\neg(P > Q)$ by $||^{not}$

At first glance $||^{not}$ might seem uncontroversial. If one supposes that it rains (together with a consistent set of background assumptions), it seems obviously true that one should conditionally reject "I will go to the beach" iff we should conditionally accept "I will not go to the beach". As we will see, the same is not true when one considers statements under *inconsistent* suppositions, however.

Imagine I ask you to suppose some statement T which implies U and $\neg U$. Under that supposition, through *ex falso quodlibet* you come to accept both V and $\neg V$. Should you thereby come to reject V on the supposition of T? In a natural sense, no. If we want to maintain that rejection is universally incompatible with acceptance under any supposition, we must hold that we do not reject V under the supposition of T. Thus, we have $T||^{a}\neg V$ but not $T||^{not}V$ and thus the rule $||^{not}$ fails. Given this sense of rejection, we should accept all statements and reject no statement under an inconsistent supposition.[17]

As evidenced by lines 1 and 2 of Williamson's derivation, SR can sometimes tell us to accept opposite conditionals. Under the supposition of T, we derive V and thus by SR unconditionally accept $T > V$. Similarly, under the supposition of T we derive $\neg V$ and by SR, unconditionally accept $T > \neg V$. Thus, on the basis of SR, we accept both $T > V$ and $T > \neg V$. We are only committed to a contradiction if we are committed to CCN and we have seen this relies on the questionable assumption $||^{not}$. So providing rejection is always incompatible with acceptance, Williamson's argument does not show that SR is inconsistent.

The following excerpt from Russell is referenced by Williamson (2020, p.92) as a counterexample to The Suppositional Rule:

[17] Some philosophers argue that rejection should not be equated with accepting the negation even under consistent suppositions. My point here is that even philosophers who reject these arguments may naturally reject the equivalence of these attitudes under inconsistent suppositions.

"Naïve reason leads to physics and physics if true, shows that naïve realism is false. Therefore, naïve realism, if true, is false; therefore, it is false." (Russell, 2013, p.15)

It is clear from this passage that Russell accepts both of the following conditionals:

(A) If naïve realism is true, then naïve realism is false
(B) If naïve realism is true, then naïve realism is true

Thereby, Russell seems to accept a counterexample to CCN.

Williamson (2020, p.92–3) treats this as a case where Russell is overcoming the suppositional heuristic. He claims that "the heuristic can in practice be overridden by weightier epistemic sources, such as testimony and mathematical proof" (ibid, p.93).

This seems to me to be an unnatural conclusion. On the face of it, the way Russell comes to accept these conditionals is no different to any other. He comes to accept (A) and (B) because *on the supposition* that naïve realism is true, he accepts both that it is true and that it is false. It is strange to treat this as a case where a distinct non-suppositional means of evaluating conditionals takes precedent over our normal suppositional modes of assessment.[18]

Something similar can be said for another example he cites (ibid, p.21). Suppose you assert the following about somebody who you are certain is not a qualified doctor:

(C) If he's a qualified doctor, I'm the Pope.

The idea of this conditional is to demonstrate your incredulity in the antecedent is such that you would accept anything if you discovered it was true (including statements incompatible with "I'm the pope"). Again, Williamson thinks this is a case where we are overriding SR. But does it not seem more natural to hold that we accept (C) because *on the supposition* of him being a qualified doctor, we would

[18] We don't always treat conditionals with inconsistent antecedents as vacuously true. Consider the following *counterpossible* conditionals: If "dialetheism is true, then no contradiction is true" and "if dialetheism is true, then some contradiction is true". We intuitively seem to judge former as false and the latter as true. Does this itself show that our ways of evaluating conditionals are inconsistent? No. One plausible explanation of why we evaluate these conditionals differently, is that we are employing a different kind of supposition to our usual mode. We are employing a mode of supposition that permits non-vacuous supposition of impossible antecedents. When we evaluate Russell's conditional, we seem to keep certain logical principles fixed, when we evaluate these conditionals we don't. The fact we sometimes regard conditional with impossible antecedents as being true does not undermine consistency of our normal suppositional practices. On the other hand, the non-propositionalist might simply agree with Williamson (2021, Chapter 5; 2018) that both these conditionals are vacuously true and find an alternative way of explaining away this data. Counterpossibles are hard for everyone, not just non-propositional theories.

be willing to accept anything?[19] Seen this way, one comes to accept (C) through an application of SR (or rather SR_{DA}/SRA) not some distinct non-suppositional source of belief.

We noted before that on one natural sense of rejection, we do not reject any statement under the supposition of an impossible statement. But what if one insists that rejection of a statement is just acceptance of its negation under any supposition? This also seems to be a reasonably natural way of defining "rejection". We would then need to hold that we should both accept and reject any statement under an inconsistent supposition. Acceptance would then be compatible with rejection under inconsistent suppositions. In this case, $||^{not}$ clearly holds, and Williamson's derivation of CCN goes through. In fact, it doesn't really matter how we define "rejection". As long as we grant that there is an attitude of "accepting the negation", $||^{atn}$, we will then obviously have $PP||^{atn}Q \iff PP||^{a}\neg Q$ and thus we will be able to rerun the proof of CCN by replacing $||^{not}$ with $||^{atn}$. In combination with Ref and CCC, will then be able to show that SR is inconsistent.

But at this point, we should recall the purpose of Williamson's proof. The purpose is to show that our actual practices of evaluating conditionals are inconsistent. He is thereby attempting to disabuse us of the notion that we should bring our theories and our intuitions into close alignment. To achieve this purpose, it is not enough to show that SR leads to a contradiction. Rather he must show that instances of SR which we regularly and seemingly competently follow are inconsistent.

But the above derivation does not show this. When we competently accept the negation of V under an inconsistent supposition T, competence does not always require us to accept the negation of $T > V$. For example, whilst Russell may competently accept "if Naïve realism is true, it's false", no one would regard him as thereby committed to "it's not the case that if naïve realism is true then it's true". Likewise, acceptance of (C) clearly does not commit one to acceptance of "it's not the case that if he's a qualified doctor, I'm a monkey's uncle".[20]

This allows us to implement the strategy we outlined above. Irrespective of whether it is equated with rejection, accepting the negation (like being glad of something) is clearly not an attitude for which SR generally gives intuitively plausible predictions. Suppose we formulate a weaker descriptively adequate version of SR, $SR_{DA,}$ which agrees with SR when it is intuitively compelling, but diverges from SR when it is not. It seems this would be the better representation of our practice of assessing conditionals. It would, moreover, evidently diverge from SR with respect to the attitude of accepting the negation. By endorsing a semantics that upholds SR_{DA} but not SR, we thus have a principled way of blocking his proof of CCN.[21]

We will shortly show how the non-propositionalist can uphold the intuitively justified principles relied upon in this argument, whilst allowing the counterintuitive

[19] One might think there is something non-literal about conditionals like (C). This might explain why we sometimes assert conditionals like (C) when the probability we assign to the consequent given the antecedent is low, but not zero. The aim is to humorously act like we are absolutely certain the antecedent is false.

[20] I'm presuming here that being a monkey's uncle is incompatible with being the pope.

[21] We block the application of $SR_{||^{not}}$ (or rather, $SR_{||^{atn}}$) on line (3) of his proof of CCN.

ones to fail. However, before we do so it's worth emphasising that the inconsistency of SR shouldn't be surprising given its immense generality. SR makes a claim about *all* attitudes, including highly esoteric ones. Thus, it seems inevitable that SR will turn out to be inconsistent. Moreover, it should be immediately evident that non-propositional theories are committed to failures of SR. Consider attitudes of the form: regarding the probability of *truth* of something being x. Whilst it's clearly rational to assign 0.5 probability to the truth of "the coin lands heads" under the supposition that Cat tosses the fair coin, on the non-propositional view (and several others) it would be irrational for us to assign 0.5 probability to the *truth* of the conditional "if Cat tosses the coin, it will land heads". Thus, restrictions to SR are already baked into the non-propositional framework.

Recall that the Adams-style theory outlined above assigns credence 1 to $Cr(C|A)$ when $Cr(A) = 0$. If this feature is maintained in a suitably expanded framework where credences are assigned to compounds of conditionals, it will naturally predict failures of CCN.[22] To illustrate this, note we can extend Adams' framework to assign credences to conjunctions of conditionals with well-defined credences by defining $Cr(P > Q \wedge P > R)$ as $Cr(P > (Q \wedge R))$. We can also assign credences to negated conditionals with well-defined credences by taking $Cr(\neg(P > Q)$ to bel $- Cr(P > Q)$.[23] If acceptance is certainty or credence above a certain threshold, it's easy to see that our system will validate SRC, SRA, Ref, CCC, but not CCN and SRatn. It thus upholds SR's intuitively plausible predictions, but not its counterintuitive ones. The non-propositionalist can thus avoid Williamson's argument without cost.[24]

5 Triviality

Many triviality arguments have been provided since Lewis's (1976) first bombshell result.[25] These are taken to undermine any theory that combines SRC with a standard propositional treatment of conditionals. These proofs are not typically taken to undermine non-propositional theories of conditionals. In fact, we outlined above a consistent non-propositional framework where SRC holds in all rational probability distributions. However, Williamson outlines a triviality argument that is especially tailored to undermine the non-propositional theory.

To formulate his target, Williamson outlines a standard Adams-style framework like the one given above. In this framework we have two different kinds of

[22] The Adams'-style propositional framework outlined above will uphold the probabilistic version of the principle of conditional excluded middle: $Cr(A > C) + Cr(A > \neg C) = 1$ when $Cr(A) \neq 0$. This is not so when $Cr(A) = 0$

[23] Of course, this move only allows us to assign credences to a limited selection of compounds of conditionals on its own. However, with some additional machinery, the non-propositionalist can assign credences to a broader range of statements in a way that agrees with these clauses.

[24] As indicated in §2, the non-propositionalist can still talk of us applying SR even though it is inconsistent. Thus, we can still describe ourselves as coming to accept (A), (B) and (C) through applications of SR.

[25] See for example, Stalnaker (1976), Hajek (1994), Bradley (2000), and Fitelson (2015).

statements, factual statements, and nonfactual ones. He presumes that in our language we have three *mutually inconsistent* factual sentences D, E and F as well as their truth-functional combinations. It is presumed that $Cr(Q|P)$ obeys standard principles of conditional probability whenever $Cr(P) > 0$.[26] In particular, he presumes the following principle (the name is my own):

- Conditional Certainty Lower Bound (CCLB). If $Cr(Q|P) = 1$ and $\mathrm{Cr}(P) > 0$, then $Cr(P) \leq Cr(Q)$.

Williamson takes SR to justify the following generalisation[27] of SRC[28]:

Suppositional Rule for Conditional Credences (SRCC): $Cr(Q > R|P) = Cr(R|Q \wedge P)$ where $Cr(Q \wedge P) > 0$.[29]

As Williamson points out, this principle seems to be supported by our practices of assigning credences to conditionals under suppositions. For example, on the assumption that I have a die in front of me, it seems we should assign 1/3 credence to the conditional "if I roll an even number, it will be a 2". The Adams-style theory we outlined before did not tell us how to assign a credence to a conditional under suppositions—$Cr(Q > R|P)$ is left undefined. However, the non-propositional theorist presumably needs some account of how this should be done.[30] Given SRCC gives intuitively plausible predictions, it seems the non-propositionalist has good reason to adopt SRCC. We will thus follow Williamson in adding it as a primitive to the non-propositional framework.

Williamson (2020, p.42–5) then shows the following:

Theorem. If Cr is a credence function where $Cr(D) > 0, Cr(E) > 0$, and $Cr(F) > 0$, Cr does not satisfy SRCC.

Assume the antecedent of Theorem. The credence assigned to D, E and F is thus more than 0 but less than 1. Now assume towards contradiction that SRCC holds. Then we have:

(1) $Cr((D \vee E) > D)|((D \vee E) \rightarrow D) =$
$Cr(D|(D \vee E) \wedge ((D \vee E) \rightarrow D)) = Cr(D|D) = 1$

But from CCLB and SRC we have:

[26] Williamson treats the conditional probability of C given A is treated as undefined when $\Pr(A) = 0$. This makes no material difference to his argument, however.

[27] Just substitute P for a tautology in SRCC to derive SRC.

[28] According to Williamson, taking credence x under the supposition of P is a kind of attitude. SR thus implies that we should take this attitude conditionally towards R under the supposition of Q iff we take that same attitude unconditionally towards $Q > R$.

[29] This principle is key in many of the triviality results, including Lewis' original one.

[30] At least, for factual background assumptions.

(2) $Cr((D \vee E) \rightarrow D) \leq \mathrm{Cr}((D \vee E) > D)) = \mathrm{Cr}(D|D \vee E)$

But it follows given standard probability theory that if $Cr(A \rightarrow C) \leq Cr(C|A)$ then $Cr(A) = 1$ or $Cr(A \rightarrow C) = 1$. Therefore, we have:

(3) $Cr(D \vee E) = 1$ or $Cr((D \vee E) \rightarrow D)) = 1$.

But given D, E, F are mutually incompatible $D \vee E$ entails $\neg F$, $(D \vee E) \rightarrow D$ entails $\neg E$. It follows from standard probability theory that if A entails C then $Cr(A) \leq Cr(C)$. So,

(4) $Cr(D \vee E) \leq Cr(\neg F) < 1$ and $Cr((D \vee E) \rightarrow D) \leq Cr(\neg E) < 1$

But this contradicts (3). Therefore, SRCC cannot hold for Cr, establishing our theorem. It follows that if SRCC holds, it can only do so in probability spaces with less than 3 live, mutually exclusive possibilities. That is, the only probability spaces in which SRCC can hold are trivial.

Where should the non-propositional fault the proof? The natural place is CCLB. There is no obvious reason why the non-propositionalist is committed to this principle. If all statements expressed propositions, then they would be committed to it.[31] However, they deny this explicitly. We will now see that a natural way of extending the standard non-propositional framework leads to failures of CCLB.

As we remarked above, the standard Adams-style framework we outlined above leaves $\mathrm{Cr}(B > C|A)$ undefined. If we wish to define the conditional probability of conditional statements, we cannot continue to define $\mathrm{Cr}(P|A)$ as the ratio formula $\mathrm{Cr}(A \wedge P)/\mathrm{Cr(A)}$ since we have not defined $\mathrm{Cr}(A \wedge P)$ when P is a conditional. Nonetheless, there is a natural way of extending Adams' framework to assign conditional probabilities to simple conditionals whilst avoiding this complication. We define the conditional probability of C given A as normal, but we define the conditional probability of $B > C$ given A like so:

SRCC*: $\mathrm{Cr}(B > C|A) = \mathrm{Cr}_A(B > C)$

Where Cr_A is Cr conditionalized on A.[32] It follows that $\mathrm{Cr}(B > C|A) = \mathrm{Cr}_A(B > C)$ $= \mathrm{Cr}_A(\mathrm{B}|\mathrm{C}) = \mathrm{Cr}(C|A \wedge B)$ in accordance with SRCC*, SRC, and some trivial probability theory. So, instead of taking SRCC as primitive, we derive it through SRCC* and SRC.

It is then easy to construct models with more than 2 live inconsistent possibilities. It is also easy to show that this system predicts failures of CCLB. Take a model with three worlds w_1, w_2 and w_3. We suppose that Cr assigns $1/3$ probability to each

[31] If this were so, then from $Cr(Q|P) = 1$ we could infer that there is no w where $w \in [\![P]\!]$, $w \notin [\![Q]\!]$, and $cr(w) > 0$ (for otherwise $\mathrm{Cr}(Q|P) < 1$). This implies $\mathrm{Cr}(P) \leq \mathrm{Cr}(Q)$. However, when Q does not express a proposition, this reasoning does not go through.

[32] That is, $\mathrm{Cr}_\mathrm{A}(C) = \mathrm{Cr}(C|A) = \mathrm{Cr}(A \wedge C)/\mathrm{Cr}(A)$ and (in accordance with SRC) $\mathrm{Cr}_A(B > C) = \mathrm{Cr}_A(B|C) = \mathrm{Cr}_A(B \wedge C)/\mathrm{Cr}_A(C)$.

world. The singletons of these three worlds correspond to 3 live inconsistent propositions such that $\{w_1\} = D$, $\{w_2\} = [\![E]\!]$, and $\{w_3\} = [\![F]\!]$. It is easy to check that $\mathrm{Cr}((D \vee E) > D)|((D \vee E) \to D) = 1$, yet $\mathrm{Cr}((D \vee E) > D) = \mathrm{Cr}(D|D \vee E) = .5$ which is less than $\mathrm{Cr}((D \vee E) \to D) = \frac{2}{3}$.

This slightly expanded non-propositional framework thus predicts failures of CCLB. This would be of little comfort if one thinks that CCLB (and the standard laws of conditional probability generally) *should* be upheld in languages with the indicative connective, >. However, I think intuitively this principle *should* fail.[33] To see this, take the following intuitive counterexample. Suppose that you have a lot of work to do tomorrow which will require you to be in the office. You therefore assign very high credence to:

(D) I will be in work tomorrow.

You are not certain of (D), however. There is a non-zero chance of you being in a serious car accident before tomorrow in which case it's highly unlikely you will be able to go to work. You thus assign a low credence to the following:

(E) If I am in a serious car accident, then I will be in work tomorrow.

Nonetheless, on the supposition that you are in work tomorrow, you are absolutely certain of (E). Under the supposition that you are in in work tomorrow, you know that, whatever else may happen, you will be in work tomorrow! Thus Cr(D) is high Cr(E) is low. and yet $Cr(\mathrm{E}|\mathrm{D}) = 1$. The pattern of credences here seems entirely rational, so we have an intuitive counterexample to CCLB.[34] Williamson argument thus rests on a principle that we intuitively have good reason to reject.

So, again it seems that the non-propositionalist can avoid Williamson's argument without cost. Note, unlike our response to his argument from complex arguments, we need not implement any restrictions to SR in this case.

6 Deduction

Finally, we will discuss Williamson's (2020, p.31–42) deductive argument for the inconsistency of The Suppositional Rule. Williamson first notes that SR supports the following special case of SRCC: $\mathrm{Cr}(Q > R|P) = 1 \iff \mathrm{Cr}(R|Q \wedge P) = 1$ when

[33] Recall that non-propositional theorists already reject aspects of standard probability theory such as the law of total probability for nonfactual statements.

[34] The counterexample is structurally analogous to the one in McGee (1985), except here we do not rely on right-nested conditionals. It is also similar to the example used to motivate the preservation condition in Bradley (2000).

$\mathrm{Cr}(Q \wedge P) > 0$.[35] He argues that by glossing the epistemic modal "must" in terms of probabilistic certainty, we can restate this like so[36]:

$\mathrm{SR}_{|\,|^{\mathrm{must}}}$: $Q > R|\,|^{\mathrm{must}}P \iff R|\,|^{\mathrm{must}}Q \wedge P$

This tells us to regard '$\varepsilon Q > R$" as epistemically necessary on the supposition P just in case we regard R as epistemically necessary on the supposition εQ and P". In fact, he argues that we can derive SRmust directly by applying SR to the attitude of taking something as epistemically necessary.

He then asks us to consider a context in which unconditional epistemic necessity reduces to logical truth, and conditional epistemic necessity to logical consequence. The thought is that in this context we can just interchange epistemic necessity with logical necessity. We thus can substitute $|\,|^{\mathrm{lnecc}}$ for $|\,|^{\mathrm{must}}$ in $\mathrm{SR}_{|\,|^{\mathrm{must}}}$giving us $Q > R|\,|^{\mathrm{lnecc}}P \iff R|\,|^{\mathrm{lnecc}}Q \wedge P$. But to say that some statement S is logically necessary on the supposition of TT is just to say that TT logically implies S. So, we can just rewrite this as follows:

$$\mathrm{SR}\vdash\, : P \vdash QR \iff P \wedge Q \vdash R$$

Given P is always equivalent to some (possibly degenerate) conjunction, which in turn can be replaced by the finite set of its conjuncts PP, this gives us:

$$\mathrm{SR}\vdash_{\Leftrightarrow} PP \vdash QR \Leftrightarrow PP, Q \vdash R$$

This makes $Q > R$ logically equivalent to the material conditional.[37] So far, this scheme of argument is similar to familiar arguments using the Or-to-If inference or the import/export schema to derive the equivalence of the indicative conditional with the material conditional. However, Williamson uses SR to derive the equivalence directly.

Williamson goes further than these arguments, however, by arguing that The Suppositional Rule is inconsistent. He argues that it implies other deductive principles inconsistent with $SR\vdash_{\Leftrightarrow}$. Williamson notes that SR supports the following special case of SRCC $\mathrm{Cr}(Q > R|P) = 0 \iff \Pr(R|Q \wedge P) = 0$ when $\Pr(Q \wedge P) > 0$. By interpreting epistemic impossibility in terms of credence 0, we can gloss this as follows:

SRmustnot: $Q > R|\,|^{\mathrm{eimp}}P \iff R|\,|^{\mathrm{eimp}}Q \wedge P$

This tells us to regard $Q > R$ as epistemically impossible on the supposition of P iff we regard R as epistemically impossible on the supposition of Q and P. We can also clearly derive this principle directly from SR by applying it to the attitude of regarding something as being epistemically impossible.

Similarly to before, we might think that SRmustnot provides support for:

[35] Our definition of conditional probability makes the qualification $\mathrm{Cr}(A \wedge B) > 0$ redundant.

[36] Actually, Williamson formulates this slightly differently. I have adapted the notation to ensure continuity with the notation introduced above.

[37] From $Q \rightarrow R, Q \vdash R$ we can infer $Q \rightarrow R \vdash Q > R$ by the right-to-left of $SR\vdash_{\Leftrightarrow}$. Conversely, from the left-to-right direction of $SR\vdash_{\Leftrightarrow}$ and the reflexivity of '$\vdash$' we have $Q > R, Q \vdash R$ from which we can infer $P > Q \vdash P \rightarrow Q$ by conditional proof for the material conditional.

SR $\vdash^{not}_{\Leftrightarrow} PP \vdash \neg (Q > R) \Leftrightarrow PP, Q \vdash \neg R$

Where PP is the possibly degenerate set of P's conjuncts. But then, with some other minimal logical principles, we can show that $SR\vdash_{\Leftrightarrow}$ and $SR\vdash^{not}_{\Leftrightarrow}$ are inconsistent:

(1) $Q \vdash \neg Q \rightarrow R$ by the semantics of $\rightarrow$
(2) $Q \vdash \neg Q \rightarrow \neg R$ by the semantics of $\rightarrow$
(3) $\neg Q \rightarrow R \vdash \neg QR$ by $SR\vdash_{\Leftrightarrow}$ with $\neg Q \rightarrow R, \neg Q \vdash R$
(4) $\neg Q \rightarrow \neg R \vdash \neg(\neg QR)$ by SR $\vdash^{not}_{\Leftrightarrow}$ with $\neg Q \rightarrow \neg R, \neg Q \vdash \neg R$
(5) $Q \vdash \neg QR$ by transitivity on (1) and (3)
(6) $Q \vdash \neg(\neg Q > R)$ by transitivity on (2) and (4)

This shows that Q is inconsistent, for arbitrary Q. If Q is a tautology, then the proof suffices to show the rules are inconsistent by themselves.

Again, a non-propositional theorist may attempt to avoid the proof by denying the meaningfulness of compounds of conditionals. However, Williamson blocks this response by formulating the above proof without any compounds of conditionals. We refer the reader to Williamson (2020) for this version.[38]

Let's first note that it's not obvious how the non-propositionalist should define the epistemic necessity of a conditional under a supposition. If $f^c(B)$ is the set of epistemically accessible worlds where B is true in context c, the epistemic necessity of (factual) C on the supposition of (factual) A is standardly defined such that C is epistemically necessary on the supposition of A iff $f^c(C) \subseteq f^c(A)$. Yet this definition does not work on the non-propositional picture if we substitute A or C for nonfactual statements.

One might think that this could constitute a response to Williamson's argument. Yet I do not think this strategy will work. For it seems we *can* regard conditionals as being conditionally epistemically necessary. For example, it seems we can regard it epistemically necessary that if I strike the match it will light under the supposition that the match will light. Thus, it seems the non-propositional theorist requires some way of defining the epistemic necessity of a conditional under a given supposition. An obvious way of dealing with this problem is to just equate the epistemic necessity of P under supposition BB as certainty in P given the evidence and the statements in BB. This is well-defined for simple conditionals given the minor adaptations to the non-propositional framework we made in the previous section. Let's therefore adopt this as a working definition of epistemic necessity going forwards.[39]

[38] His trick is to replace $RR \vdash S$ in the proof with $RR \vdash\_S$ where $'\vdash'\_$ stands for the relation 'is inconsistent with'. Together with some plausible rules that connect $\vdash\_$ with $\vdash$, he shows we can derive a contradiction. Given the non-propositionalist acknowledges that there is a logic of conditionals in which certain premises are inconsistent with others, they cannot avoid the proof by rejecting the meaningfulness of compounds of conditionals.

[39] In the finite models we are working with there's not much use for a distinction between epistemic necessity and certainty given the evidence. However, as indicated in fn.15 one might worry that in standard infinite models, the statements that are absolutely guaranteed to be true and the statements that are certain given the evidence can come apart. As remarked in the same footnote, in response, the non-

Secondly, note that regardless of how we define epistemic necessity, it's clear that we should reject SRmustnot. As Euclid showed, there must not be a largest prime on the supposition of there being a largest prime, and yet contrary to SRmustnot, the following must be true:

(F) If there is a largest prime, there is a largest prime.

This is predicted by the Adams-style non-propositional theory outlined above. Given the stipulation that $Cr(P|Q) = 1$ when $Cr(Q) = 0$, this framework tells us to be certain that both there is not a largest prime on the supposition of there being one as well as to be certain in (F). Thus, given our equation of epistemic necessity with certainty, it predicts that (F) is epistemically necessary and yet it is epistemically impossible that there is a largest prime on the supposition that there is one. We can also produce similar intuitive counterexamples using (A) and (C).[40]

Thus, it seems, as with the attitude of accepting the negation, we have an attitude for which SR's predictions are not intuitively plausible. This allows us to implement the strategy outlined above. We can weaken SR to give us a descriptively adequate version of this principle, one which gives more accurate predictions in terms of our intuitive judgements. Given the adapted non-propositional framework outlined above validates SRA, SRC, SRmust, and not SRatn or SRmustnot, it seems the non-propositionalist can uphold our intuitive judgments without inconsistency.

The rejection of SRmustnot takes away Williamson's justification for SR $\vdash^{not}_{\Longleftrightarrow}$.[41] However, this is not yet sufficient to take the bite out of Williamson's proof. For as we remarked above, SR$\vdash_{\Longleftrightarrow}$ by itself makes the indicative logically equivalent to the material conditional. This seems problematic by itself given the paradoxes

Footnote 39 (continued)
propositional theorist might adopt some non-standard probability theory. For example, they could adopt hyperreal-valued credences and define P to be epistemically necessary when P is assigned probability 1 given the evidence. Alternatively, they might eschew hyperreal-valued probability functions and define the epistemic necessity of a factual statement as normal and define a simple conditional $B > C$ to be epistemically necessary under supposition A just in case $[\![C]\!] \subseteq ([\![A]\!] \cap [\![B]\!])$.

[40] Analogously to the attitudes of rejection and accepting the negation, one might reject the equivalence of "it must not be that P" and "it is epistemically impossible that P" under inconsistent suppositions. Indeed, we must do so if we want the epistemic necessity of P to always be incompatible with the epistemic impossibility of P. It will then follow that nothing is epistemically impossible under an inconsistent supposition, but everything is epistemically necessary. If on the other hand, we always equate "it's epistemically impossible that P" with "it must not be that P", then every statement will be both epistemically necessary and epistemically impossible under an inconsistent supposition. Both definitions seem to me to be reasonably natural. The first definition would require us to restrict SR with respect to the attitude of regarding something as though it must not be the case, but not the attitude of regarding something as epistemically impossible.

[41] Even if SRmustnot did hold, the jump between this principle and SR $\vdash^{not}_{\Longleftrightarrow}$ would be problematic for reasons that are shortly to be explained.

of material entailment.[42] So, it seems the defender of non-propositionalism requires another way of blocking the proof.

The obvious way for them to do so is to reject Williamson's jump between SRmust and SR$\vdash_{\Leftrightarrow}$. Recall that in defining their conception of entailment, the non-propositional theorist was motivated to block the paradoxes of material inference and thereby invalidate the following intuitively invalid inferences:

If I strike the match, it will light; therefore, if I strike the match and its damp, it will light.
We won't get wet; therefore, if it's rainy, we won't get wet.

Yet if we took up a conception of entailment as strong as the preservation of epistemic necessity, the indicative would be logically equivalent to the material conditional. The above arguments would thus be valid. The non-propositional theorist avoids this by adopting the probabilistic conception of entailment. According to this conception of entailment, the fact that the epistemic necessity of *P* (here glossed as certainty) implies the epistemic necessity of *Q* does not entail that *P* logically implies *Q*.[43] This means that SR$\vdash_{\Leftrightarrow}$ fails and the above arguments are invalid. Therefore, according to the non-propositional picture, from SRmust we cannot infer SR$\vdash_{\Leftrightarrow}$ and thereby the logical equivalence of the indicative with the material conditional. It thus begs the question against the non-propositional theorist to assume that the jump from SRmust to *S*R$\vdash_{\Leftrightarrow}$ is legitimate.[44]

Williamson has a potential rejoinder. He (ibid, p.37) claims that we can derive SR$\vdash_{\Leftrightarrow}$ without SRmust by applying SR to the attitude of treating something as a logical consequence with side premises *PP*. He can thus circumvent the inference from SRmust to *SR*$\vdash_{\Leftrightarrow}$ and derive the latter directly from SR. However, this should not trouble the non-propositionalist. As we remarked above, they need not be committed to upholding SR in its entirety. They have reason to uphold the intuitively persuasive implications of SR and avoid its counterintuitive ones, however, they have no reason to uphold all its predictions. I conjecture that few will have strong intuitions about when they should adopt this attitude. So, from our perspective, its application conditions are fair game. The non-propositionalist can thus reject this instance of SR without cost. Indeed, presumably the reason why Williamson took a detour via SRmust in justifying *SR*$\vdash_{\Leftrightarrow}$ was that he believed he could use the intuitive pull of this principle to commit the non-propositional theorist to the intuitively neutral *S*R$\vdash_{\Leftrightarrow}$. But as we saw above, this leap does not work. It begs the question against the non-propositionalist to claim that the former commits them to the latter.

[42] This is not actually disastrous for the non-propositional theorist. They might adopt a certainty preservation conception of entailment. This would imply the logical equivalence of the indicative with the material conditional, but it would not imply a failure of SRC given their nonstandard conception of probability. They might employ some alternative means to explain away the paradoxes of material entailment.

[43] Although the fact that *PP* probabilistically entails *Q* implies that certainty in *PP* implies certainty in *Q*, the reverse direction does not follow.

[44] If the correct conception of entailment was truth preservation at all worlds, then we could take a context where conditional epistemic necessity reduces to entailment by taking a context in which all worlds which are not logically impossible are epistemically possible. However, if this is not the correct conception of entailment, it does not follow that we can find such a context.

7 Conclusion

We have seen that none of Williamson's arguments undermine Adams-style non-propositionalism. Each of the above argument rests on some principle that the non-propositional theorist has no reason to accept: The first relied on CCN, the second on CCLB, and the third relied on SRmustnot as well as the assumption that entailment is as strong as the preservation of epistemic necessity. For each of these principles, we showed their rejection could be intuitively justified. We moreover showed that natural extensions of the standard Adams-style framework led to failures of these principles.

Recall that one of the central advantages of non-propositionalism is the close match it maintains between the attitudes we intuitively think are appropriate to hold towards conditionals and the ones which the theory tells us are appropriate. Williamson's arguments are designed to take this advantage away, demonstrating that our ways of assessing conditionals are riddled with inconsistency and error. But whilst he has shown that SR is inconsistent, he has not shown that our procedures of assessing conditionals are too. SR does not always lead to intuitively plausible predictions. Our pattern of judgements conforms to SR in a range of cases, but sometimes do not. The non-propositionalist can avoid inconsistency by endorsing the intuitively plausible instances of SR, whilst rejecting it's the counterintuitive implications (as well as some of its intuitively neutral ones). Williamson has thus not shown that our procedures of assessing conditionals are inconsistent.

Of course, nothing I have said establishes that non-propositionalism is the one true theory of conditionals. There remain difficult problems for this view, not least its difficulty accommodating compounds of conditionals. Yet, it is one of the few theories that does justice to our suppositional ways of assessing conditionals. For anyone has shown, this is a good thing.

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Declarations

Conflict of interest The author declared that they have no conflict of interest.

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