

# Is Dialetheism an Idealism? The Russellian Fallacy and the Dialetheist's Dilemma

**Forthcoming in “Dialectica”, 2(2007).**

Francesco Berto  
Venice University – Ca'Foscari  
Department of Philosophy and Theory of Sciences  
E-mail: bertofra@unive.it

## *Abstract*

In his famous work on vagueness, Russell named “fallacy of verbalism” the fallacy that consists in mistaking the properties of words for the properties of things. In this paper, I examine two (clusters of) mainstream paraconsistent logical theories – the non-adjunctive and relevant approaches –, and show that, *if* they are given a strongly paraconsistent or dialethic reading, the charge of committing the Russellian Fallacy can be raised against them in a sophisticated way, by appealing to the intuitive reading of their underlying semantics. The meaning of “intuitive reading” is clarified by exploiting a well-established distinction between *pure* and *applied* semantics. If the proposed arguments go through, the dialetheist or strong paraconsistentist faces the following Dilemma: either she must withdraw her claim to have exhibited *true* contradictions in a metaphysically robust sense – therefore, inconsistent objects and/or states of affairs that make those contradictions true; or she has to give up realism on *truth*, and embrace some form of anti-realistic (idealistic, or broadly constructivist) metaphysics. Sticking to the second horn of the Dilemma, though, appears to be promising: it could lead to a collapse of the very distinction, commonly held in the literature, between a weak and a strong form of paraconsistency – and this could be a welcome result for a dialetheist.

This [Kantian] thought, that the contradiction which is posited by the determinations of the understanding in what is rational is *essential* and *necessary*, has to be considered one of the most important and profound advances of the philosophy of modern times. But the solution is as trivial as the viewpoint is profound; it consists merely in a tenderness for the things of this world. The stain of contradiction ought not to be in the essence of what is in the world; it has to belong *only* to thinking reason, to the *essence* of the *spirit*.

G.W.F. Hegel, *The Encyclopaedia Logic*

## 1. *Weak Paraconsistency, Dialetheism, and the Russellian Fallacy*

Paraconsistency is usually defined as a feature of any logic (derivatively, of any theory with an underlying logic) in which contradictions do not entail everything on the basis of some version or other of *ex falso quodlibet* (EFQ).<sup>1</sup> It is common in the relevant literature to admit (at least) two degrees of paraconsistency, by distinguishing between *weak* and *strong*

---

<sup>1</sup> (EFQ), suggestively called “Explosion” by many paraconsistent logicians, can be phrased in different ways: syntactically, as an axiom (e.g.,  $\neg\alpha \rightarrow (\alpha \rightarrow \beta)$  or, in the imported form,  $\alpha \wedge \neg\alpha \rightarrow \beta$ ); or as a rule of inference ( $\alpha, \neg\alpha / \beta$ , which is often called “Negation Elimination”); or in terms of logical consequence ( $\{\alpha, \neg\alpha\} \models \beta$ ), etc. I will not enter into the details, but one can check, e.g., Priest, Routley and Norman 1989, or the more recent Bremer 2005 and Berto 2006c, for comprehensive introductions to paraconsistency.

paraconsistency (see Priest and Routley 1989, 151-5; Beall 2004a, 6; Bremer 2005, *Introduction*). The motivations for the former are sometimes labelled – in fact, a bit misleadingly – as “proof-theoretic” (e.g., in Priest and Routley 1989, 151), and based upon the observation that there are interesting inconsistent but non-trivial theories and sets of beliefs. So-called intuitive semantics and naïve set theory, for instance, are notoriously inconsistent. The unrestricted Comprehension Principle entails, among other things, that Russell’s set both belongs and does not belong to itself,  $R \in R \wedge R \notin R$ . If, as it is frequently – though grudgingly - admitted, naïve theory does capture our natural intuitions on what a set is (that is to say, the extension of an arbitrary condition), it follows that our intuitions are inconsistent. Nevertheless, the theory does not appear to be trivial: the same intuitions lead us to reject many statements concerning sets – for instance, that  $\{\emptyset\} \in \emptyset$ , i.e., that the singleton of the empty set belongs to it. Furthermore, we all make the experience of discovering that our *beliefs* are inconsistent. As John Woods said, “inconsistency is no *rara avis*, not the defection from logical rectitude of an occasional abstract theory. Inconsistency routinely dogs us in the management of our beliefs, the manipulation of our memories, and the organization of our desires”; and “the [orthodox logician’s] canons are not, or not always, the ones that people resort to as strategies for inconsistency-*management*. The last thing that a competent reasoner would do is to submit himself to *ex falso* as such a strategy” (Woods 2003, 95-6). A weak paraconsistentist can reject (EFQ) by observing that inferences are commonly made from bodies of information containing inconsistencies, but people are clearly not at liberty to conclude anything they like. Typically, though, she assumes paraconsistent models as useful mathematical tools, but refuses the idea that contradictions can be *true*.

Strong paraconsistency is characterized, on the other hand, by the admission of true contradictions. It is also called *dialetheism*.<sup>2</sup> Motivations for dialetheism are well known: they are based upon the observation that some contradictions are provably true, in the sense that they are entailed by plain facts concerning natural language and our thought processes. For instance, extended Liar paradoxes like “This sentence is not true” are spelt in ordinary English. Their paradoxical characteristics, as dialetheists stress, are due exactly to the intuitive features of ordinary language: unavoidable self-reference; failure of metalinguistic hierarchies producing only languages that are expressively weaker than English; and the obvious presence of a truth predicate, “is true”, which is characterized (at least extensionally) by the Tarskian schema. Dialetheism claims to provide the most natural (if not the unique) real “solution” to semantic paradoxes: “accept them and learn to come to live with them” (Priest 1979, 219).

Now, Graham Priest (1987, 2000a and 2006, Ch. 2) argues that dialetheism is not by itself committed to a specific conception of truth. Nevertheless, if we accept even a mild form of *realism*,<sup>3</sup> the truth of some contradictions entails the existence of inconsistent

<sup>2</sup> Beall 2004a, 6, also distinguishes between a strong paraconsistentist and a dialetheist: the former admits “real possibilities” in which contradictions can be true; the latter makes the final step, and accepts true contradictions, that is to say, contradictions that are true in *this* world. This sub-distinction, though, is not very relevant for the purposes of this paper, whose key point is to investigate on the nature of a *world* that can make a contradiction true, be it our actual world or not.

<sup>3</sup> For the purposes of this paper, we do not need a technical notion of realism. In the broadly accepted use, to be a realist (as opposed to an idealist, or an anti-realist, or a constructivist) *about* some kind of entities is to maintain that such entities objectively exist apart from, and antecedently to, anyone’s thought of them; and, therefore, that our thoughts, beliefs and theories concerning such entities are made objectively true or objectively false by them, apart from what we think of them.

objects and/or states of affairs: those that make the contradictions true.<sup>4</sup> When we reach this point, a traditional reaction is available to the orthodox logician: the one that appeals to the *fallacy of verbalism*.

In his seminal paper on vagueness, Russell named this way “the fallacy that consists in mistaking the properties of words [or beliefs] for the properties of things” (Russell 1923, 62). Thus, I will henceforth call it the Russellian Fallacy. It makes no sense – so the objection goes – to talk of inconsistent objects, situations, or states of affairs. The world is all there, all together: how could some pieces of it contradict some other pieces? Consistency and inconsistency are properties of sentences, or theories (sets of sentences closed under logical consequence), or propositions (what sentences express), or maybe thoughts, or (sets of) beliefs, etc. Contradiction (*Widerspruch*, the Latin *contradictio*) has to do with discourse (diction, *sprechen*, *dicere*). The world, with its non-mental and non-linguistic inhabitants – armchairs, trees, people – is not the right *kind* of thing that can be consistent or inconsistent, and ascribing such properties to a part of the world is, to use Gilbert Ryle’s terminology, a category mistake.<sup>5</sup>

When the objection is phrased in these unrefined terms, an easy way out is available to the dialetheist: she can simply stress that consistency and inconsistency can be ascribed to (pieces of) the world in a *derived* sense. As Priest claims in *In Contradiction*, “to say that the world is consistent is to say that any true purely descriptive sentence about the world is consistent” (Priest 1987, 200). One may take also an intermediate stance and claim that, although talk of inconsistent (“non-propositional”, as it were) *objects* is a category mistake,<sup>6</sup> it does make sense to talk of inconsistent states of affairs or facts, e.g., of the world being inhabited both by the fact that  $P(a)$ , and by the fact that  $\neg P(a)$ . We may notice, however, that this may appear as a definitional matter: one may well call an object,  $a$ , inconsistent, or (self-)contradictory, precisely when such facts involving it as its being  $P$  and its not being  $P$  (or its being non- $P$ , which, as is well known, may not amount to the same) hold simultaneously. Consequently, and not accidentally, it is quite common in the current literature on (pro and against) dialetheism to straightforwardly speak of inconsistent objects, states of affairs, and entire inconsistent worlds.

However, the point can be framed in a more sophisticated manner; and this is the goal of my paper. In the following, I shall examine two (clusters of) theories that are commonly labelled as paraconsistent: the non-adjunctive and the relevant logic approaches. I will try to show how the charge of committing the Russellian Fallacy can be raised in a refined way, by appealing to the intuitive reading of the semantics underlying such theories. In the course of the exposition, the meaning of “intuitive reading” will be clarified by

---

<sup>4</sup> See for instance Beall 2000, for a dialetheist version of the correspondence theory of truth, providing a clear example of an explicitly realist dialetheism. There may be room for a further intermediate position, that is to say, an *anti-realist dialetheism*, which accepts true contradictions without inconsistent objects or states of affairs as their truth-makers. This position has been surprisingly little explored in the literature, but we can consider Kroon 2004 and Mares 2004 as early and very interesting efforts in this direction. One may also provide an anti-realist and quasi-instrumentalist account of “true” in terms of “told true” or “believed true”, and claim that *truth* is nothing more than this. For instance, see J.C. Beall’s *constructive methodological deflationism*, in Beall 2004b, 208: “after all, if truth is a mere (human) construction, introduced to play a given expressive role, then it is not surprising – indeed, it is likely – that the construction should turn out to be inconsistent”.

<sup>5</sup> See e.g. Bobenrieth 1998, 29: “There is not much sense in saying that facts ‘contradict’ each other [...] since facts do not ‘say’ anything, they do not ‘speak’ of each other or of themselves. [...] Therefore, the problem of contradictions does not lie in reality”.

<sup>6</sup> Which Priest sometimes does declare: see Priest 2006, 51.

exploiting a well-established distinction between *pure* and *applied* semantics, which is nowadays common even in textbooks of logic and philosophy of logic.

If the proposed arguments go through, the dialetheist or strong paraconsistentist faces the following Dilemma: either she must withdraw her claim to have exhibited true contradictions; or she has to give up realism on truth, and accept some form of idealistic, anti-realistic, or broadly constructivist metaphysics. From the dialetheist's point of view, embracing the second horn of the Dilemma might even appear as a promising strategy, leading to a collapse of the very *distinction* between a weak and a strong form of paraconsistency.

## 2. *Non-Adjunctive Approaches and the Italics Argument*

I have labelled as “non-adjunctive approaches” a cluster of theories, whose common feature is the refusal of Adjunction – roughly, of the idea that if sentences  $\alpha_1, \dots, \alpha_n$  separately taken hold, then their conjunction  $\alpha_1 \wedge \dots \wedge \alpha_n$  holds, too. In fact, “Adjunction” is spoken of in many ways: syntactically, it can be assumed as a primitive or derived rule of inference, or as an axiom, etc. Therefore, its denial can also take different shapes. The most intuitive way to characterize non-adjunctive approaches is probably a semantic one – that is to say, they (typically) reject the right-to-left direction of the standard homophonic semantic clause for conjunction:

$$(S\wedge) \quad T_w(\ulcorner \alpha \wedge \beta \urcorner) \Leftrightarrow T_w(\ulcorner \alpha \urcorner) \text{ and } T_w(\ulcorner \beta \urcorner),$$

where (with a bit of flexibility on the use/mention distinction)  $T_w$  is a (world-indexed) truth predicate applying to sentence names, and in general  $\ulcorner \alpha \urcorner$  is the name of sentence  $\alpha$ . So, (S $\wedge$ ) says that  $\alpha \wedge \beta$  is true (in world  $w$ ) if and only if  $\alpha$  is true (in that world) and  $\beta$  is true (in that world). According to most non-adjunctivists, this may fail for some quite unusual worlds.

One old example of non-adjunctive propositional logical calculus is Jaskowski's *discussive logic* (See Jaskowski 1948, and later developments by da Costa and Dubikajtis 1968; Kotas and da Costa 1979; Urchs 1995 and 2002). Its intuitive motivation is to refer the rise of contradictions to dialogues in which participants can contradict each other. Examples of situations suitable for a discussive treatment are incompatible evidences presented in a trial, or databases containing occasionally incompatible data coming from different sources. What holds in a non-adjunctive situation is what is put forward by some participant or source. Being assumed as self-consistent, this can be identified with the set of things true in a standard evaluation, or a *classical* possible world (See Priest and Routley 1989, 158, for such a presentation). The paraconsistent features of the system are due to the fact that the sum of all positions may contain inconsistencies.

The most developed essay in the field on non-adjunctive approaches, though, is *The Logic of Inconsistency. A Study in Non-Standard Possible Worlds Semantics and Ontology*, a classical 1980 book by Nicholas Rescher and Robert B. Brandom. What makes the book interesting is that authors are not interested in providing a merely syntactic non-adjunctive calculus: they directly face deep semantic and metaphysical issues. In fact, their work has been a

forerunner of the current wide range of studies on impossible worlds.<sup>7</sup> Non-standard worlds are obtained in the Rescher-Brandom approach by applying to standard possible worlds two ontological recursive operations, named *schematization* (or *world-conjunction*) and *superposition* (or *world-disjunction*), which can be symbolized, respectively, as  $\cap$  and  $\cup$  (not to be confused with the ordinary set-theoretic operations). A schematic world  $w_1 \cap w_2$  is a world in which hold all and only the states of affairs holding both in  $w_1$  and  $w_2$ . Dually, an inconsistent world  $w_1 \cup w_2$  is a world in which hold all and only the states of affairs holding either in  $w_1$  or in  $w_2$ . A world obtained by schematization can be, in general, locally ontologically underdetermined.<sup>8</sup> More relevant for our purposes are the inconsistent worlds obtained via superposition, which should be, dually, locally *overdetermined*.<sup>9</sup> An inconsistent world is an impossible world for sure.<sup>10</sup> Due to the rejection of (a half of) (S $\wedge$ ), though, the theory tolerates only *distributive* or, as the authors call them, *weak* inconsistencies:

$$T_w(\ulcorner \alpha \urcorner), T_w(\ulcorner \neg \alpha \urcorner),$$

but *collective* or *strong* inconsistencies never follow:

$$T_w(\ulcorner \alpha \wedge \neg \alpha \urcorner).$$

Details aside, the most common critique to non-adjunctive approaches, therefore, has to do exactly with the decidedly non-standard behaviour of conjunction. (S $\wedge$ ) is a non-negotiable clause, it is said, due to our truth-functional intuitions: conjunction just *is* the Boolean function whose meaning is given by that equivalency.<sup>11</sup> This has to do with the typical Quinean argument, that to change the logic is to change the subject. But Rescher and Brandom tried to justify the semantic anomaly of conjunction as mirroring an *ontological* anomaly of non-standard worlds:

There is something inherently non-truth-functional about such worlds: there is no general way of giving the truth-in- $w$  conditions of a molecular statement in terms of the truth-in- $w$  conditions of its components. [...] There are two ways of looking at this – either as a decisive blockage to the viability of such worlds, or as a recognition of the difficult facts of life outside the sphere of standardist simplicity. (Rescher and Brandom 1980, 20)

<sup>7</sup> Often defined as “worlds where the laws of logic are different” (Priest 2001, 171). One can look at the monographic issue 38 (1997) of the *Notre Dame Journal of Formal Logic*, entirely devoted to impossible worlds.

<sup>8</sup> “In the case of a schematic world, the situation is not just that *we* don’t know whether  $P$  or its contradictory  $\sim P$ , but that *the world itself is indeterminate* in this regard in its make-up [...]. The situation is one of *ontological underdetermination* – with regard to certain envisageable states of affairs the world is simply ‘incomplete’” (Rescher and Brandom 1980, 5).

<sup>9</sup> “The make-up of the [inconsistent] world embodies a synthesis or fusion of incompatible states of affairs. This can be thought of as taking a number of individually and separately altogether self-consistent worlds, and ramming them together” (*Ibid.*, 6).

<sup>10</sup> Actually, Rescher and Brandom prefer to keep calling such worlds “possible worlds” and to modify the notion of possibility, meaning by it something like “conceivability” or “descriptive construability” (*Ibid.*, 32). They have not been followed by the most recent literature in which, as anticipated, it is common to simply talk of “impossible worlds”.

<sup>11</sup> “For conjunction just is that connective which has the truth (holding) conditions:  $\ulcorner A \wedge B \urcorner$  is true (at a world) iff  $A$  is true and  $B$  is true (at that world). So something that fails adjunction is not then conjunction” (Priest and Routley 1989, 158).

Such a metaphysical motivation against the unconditional validity of Adjunction has been recently revitalized by Achille C. Varzi, in the context of a wider controversy against the truth-functional conception of connectives. Supervaluational treatment of disjunction is paradigmatic in this respect. Suppose both  $\alpha$  and  $\beta$  come out neither true nor false in some evaluation in a supervaluational framework, because different ways of filling the gaps due to vague predicates or non-denoting singular terms occurring in them yield different truth values. Then, it may be the case that  $\alpha \vee \beta$  is truth-valueless, but it may also be the case that it comes out true – particularly, if  $\beta$  is  $\neg\alpha$ . Thus, Kripke 1975 argued that such a failure of truth-functionality is a sign that supervaluationism misses the very point of the meaning of disjunction. When you say “ $\alpha \vee \beta$ ”, I am perfectly entitled to ask: “Ok, which one then, if not both?”. Adapting Tappenden 1993 and Varzi 2004, one may react by calling this the *Italics Argument*: “you claim that ‘either  $\alpha$  or  $\beta$ ’ holds, so *either  $\alpha$  or  $\beta$*  (stamp the foot, bang the table) must hold!”.

In a way, this sort of objection can be dismissed on the grounds of its unfair appeal to intuition. Change of semantics, change of subject – says the objection. Fair enough. But who got the semantics right in the first place? (Varzi 2004, 103)

A logic and its respective semantics, as Varzi’s rejoinder goes, are always based upon an *ontology*. If a logical law or consequence, by definition, must hold in all circumstances, then a systematic specification of what counts as an admissible circumstance is a part of what it takes to define a logic:

When we specify what counts as an admissible interpretation of the (object) language, we must rule out interpretations where ‘and’ and ‘or’ express something else than Boolean functions. [...] This point tends to be obfuscated by the fact that typically, as a matter of standard practice, the semantics of the logical operators is spelled out as being part and parcel of a recursive definition of truth: [...] the meaning of the logical operators is not specified by the structures used to interpret the language but rather fixed *indirectly* through a recursive definition of the truth-value of the statements in which they occur. It is imposed *ab initio* upon the entire semantic machinery. (Varzi 2004, 104)

Therefore, the status classically assigned to conjunction and disjunction *presupposes* our ontological stipulations on what an *admissible circumstance* is. And if we have a different and wider notion of admissible circumstance (such as the one of non-standard world in the Rescher-Brandom approach), we may change the status of our connectives. However, it would not be fair to say that we used the *change-of-subject* Quinean trick: we just got rid of a limited and restricted way of understanding the notion of circumstance, or world.

Now, such a wider notion of circumstance appears to be legitimate: admissible non-adjunctive circumstances are provided by the aforementioned cases of incompatible evidences in a trial; by corrupt databases; by tales and fictional worlds (for instance: in one of Conan Doyle’s stories, Watson limps because of a war wound in the leg; in different stories, the wound is in Watson’s shoulder and he does not limp. It does not follow, though, that Watson both limps and does not limp in any of Conan Doyle’s writings). Such “impossible” situations, as non-adjunctivists argue, *can* be analyzed and show a certain

amount of logical structure.<sup>12</sup> The obvious dualities between conjunction and disjunction also suggest the possibility of building a sub-valuational semantics, as a complementary approach with respect to supervaluationism: a semantics which makes  $\alpha \wedge \neg\alpha$  false, even if both  $\alpha$  and  $\neg\alpha$  are true, just like supervaluational semantics makes  $\alpha \vee \neg\alpha$  true, even if neither  $\alpha$  nor  $\neg\alpha$  are. Sub-valuational, non-adjunctive semantics have therefore been proposed by Varzi, Dominic Hyde, and others (See Hyde 1997; Varzi 1997, 2000; and prospective applications suggested in Beall 2004a, 10-11).

### 3. *Which World Are You Living In? – Part One*

The details of sub-valuational semantics can be left aside. A subject that often comes into play when we deal with the semantics of paraconsistent-related theories can now be addressed. The issue is a metaphysical one or, better, it deals with *the point at which we should draw a line* between ontology, on the one side, and epistemology and semantics, on the other. What is, in fact, the *metaphysical* status of the Rescher-Brandom non-standard worlds, or of Varzi's circumstances?

Let us begin with the former. In their book, Rescher and Brandom realize that their characterization of non-standard worlds is largely parasitic upon standard possible worlds: in order to understand what a non-standard, either schematic or inconsistent world is, we must (a) know what a standard (maximally consistent) possible world is, and (b) understand the recursive schematization and superposition operations,  $\cap$  and  $\cup$ .<sup>13</sup> Now, everyone knows that the ontological status of ordinary possible worlds is itself quite controversial. Positions range from a realistic stance, taking possible worlds at face value as real entities, independent of language and thought (see Lewis 1973, Ch. 4); to a conceptualist view, in which talk about possible worlds is considered as talk about ways in which we can conceive the world as different from what it is (see Kripke 1972); to a linguistic approach, which construes talk about possible worlds as talk of maximally consistent sets of sentences (see Hintikka 1969). *A fortiori*, one may cast serious doubts on the ontological status of worlds that appear to be conceptually dependent on ordinary possible ones, such as non-standard worlds. Rescher and Brandom propose, then, what they call the Parity Thesis:

We are not concerned to argue that possible worlds should be considered as part of the furniture of the universe, only that there is nothing to choose between standard and non-standard possible worlds in this regard. [...] Recall that we do not want to argue that we should treat any possible world as real, only that the considerations which can be advanced in favour of so treating standard possible worlds apply equally to non-standard possible worlds. (Rescher and Brandom 1980, 64-5)

---

<sup>12</sup> “If an impossible world is one in which there are discrepancies of the sort illustrated by the Watson example – a world in which certain facts both do and do not obtain – then we can keep such worlds under logical control exactly as we can keep the Holmes stories under control” (Varzi 1997, 622-3).

<sup>13</sup> The treatment of modal logic in Chapter 14 of their book proposes an adaptation to non-standard worlds of the binary accessibility relation of standard Kripke semantics, in which (a) from a non-standard world, only non-standard worlds of the same kind are accessible: for instance, only inconsistent worlds are accessible from inconsistent worlds; and (b) accessible worlds are ultimately made (via the recursive  $\cap$  and  $\cup$  operations) of standard worlds, which are accessible from the standard worlds we began with: for instance, from an inconsistent world  $w_1 \cup w_2$ , only inconsistent worlds made of worlds accessible from  $w_1$  and  $w_2$ , are accessible. Such a modified accessibility relation is therefore completely dependent upon the regular accessibility between standard worlds.

Various authors in the paraconsistent tradition, including Priest, have resumed the Parity Thesis.<sup>14</sup> In spite of their declared purpose, though, it seems to me that Rescher and Brandom do not manage to gain persuasive arguments in favour of Parity, mainly because non-standard, non-adjunctive worlds appear to be *representations* of worlds. I shall elucidate the point by exploiting a few more intuitive dualities between non-adjunctive sub-valuations and inconsistency, on the one side, and supervaluational semantics and other semantics of vagueness, on the other.

Supervaluationism, many-valued logics, etc., have demonstrated their usefulness in the treatment of vagueness. According to the vast majority of authors, though, there is no *de re* vagueness: vagueness can only belong to representations of the world, such as language and thought, not to the world itself. Or, at least, this is the dominant position – and it is compatible with most theories of vagueness.<sup>15</sup> For instance, according to supervaluationism vagueness arises from our failure to make decisions on the borders of such predicates as “tall”, “bald”; “rich”, “old”, etc. To turn this phenomenon into a *de re* vagueness, according to most philosophers, is to commit the Russellian Fallacy, and to confuse the *ordo repraesentationis* with the *ordo essendi*.

Now, “dualizing” back to the question of inconsistency: it may be observed that the examples of non-standard worlds, or circumstances, proposed in non-adjunctive approaches seem to actually be *theories*, or *cognitive* states, or *descriptions*. Jaskowski’s discussive contexts are dialogical situations, with participants in a discussion expressing opposite standpoints. Varzi’s paradigmatic circumstances crucially involve storytelling. Other cases appeal to databases obtained via different sources, which can end up in inconsistent information. It is important to emphasize that, in a sense, such an observation is not a direct objection against the non-adjunctive approach itself. On the contrary, a sub-valuational semantics may turn out to be useful in the treatment of linguistic inconsistency, as much as vagueness. Nevertheless, the point is that Rescher and Brandom at the outset promised us a theory capable of exhibiting *ontological*, not merely epistemic or linguistic inconsistencies. The failure of Adjunction in epistemic, and especially inductive contexts, is a recognized fact; but Rescher and Brandom had initially claimed that “this principle must be given up for *alethic* contexts as well, once one’s possible-world semantics envisages the prospect of inconsistent worlds” (Rescher and Brandom 1980, 18). The intuitive reading of non-standard worlds does not seem to reach the alethic level, though, and appears to be confined within epistemic or, in a wide sense, representational contexts. The models of non-adjunctive theories (what the theory should capture, express, or represent) seem to be theories in their turn, representations. As Rescher and Brandom themselves observe, “the consistent standardist can thus respond that non-standard models are representations of representings (namely beliefs)”, therefore “non-standard models and weakly inconsistent theories are just two different ways of representing beliefs” (*Ibid*, 128-9). This seems to be inconsistent (no pun intended) with the initial promise to exhibit “hard, *existential* inconsistency”, not just “soft, *epistemological* inconsistency” (*Ibid*, 2). To quote Timothy

---

<sup>14</sup> See e.g. his 1992 paper on non-normal worlds. A definitely realist position on non-standard worlds is endorsed, instead, by T. Yagisawa 1998, who calls it “extended modal realism”.

<sup>15</sup> It is fair to say that some, e.g., Tye 1990, Parsons and Woodruff 1995, support the view that the world itself can be vague, that is to say, there can be ontologically vague objects, sets, and/or states of affairs. The discussion on this point has focused on a famous argument against vague objects by Gareth Evans: see Evans 1978.

Smiley's dismay at the end of his examination of the Rescher-Brandom approach: "what else can one do but ask for one's money back?" (Smiley 1993, 19).

Now, one may propose to solve the problem by adopting some idealistic or constructivist position, thereby partly collapsing the distinction between "epistemic contexts" and "alethic contexts". In a metaphysically idealistic framework, the work to be done in order to establish the ontological furniture of the world coincides with an analysis of our conceptual scheme (and is sometimes taken as *relative* to it, to our ways of worldmaking).<sup>16</sup> This also appears to be the final conclusion of *The Logic of Inconsistency*: its last sections propose a retrieval of Peirce's methodological realism. Methodological realism makes of consistency and completeness regulative ideas for our knowledge;<sup>17</sup> and it is accompanied by an ontological idealism, consisting in "the claim that the signs in this sense which make up our beliefs at a time are what actually exist" (Rescher and Brandom 1980, 124-5). According to the metaphysical realist, real world is both consistent and complete, and inconsistent and/or incomplete circumstances actually consist in beliefs or representations in human minds. On the contrary, for the metaphysical idealist the world actually inhabited by humans *is* structured by their (occasionally inconsistent and/or incomplete) beliefs. Now, one does not need to be an admirer of Hegel (which I am: see Berto 2005, 2007) to consider metaphysical idealism as an absolutely respectable philosophical position. However, the evaluation of the claim, made by authors adhering to strong paraconsistency or dialetheism, of having exhibited *real* contradictions, has to be suspended, unless they consciously embrace such an idealistic position.

#### 4. *Pure Semantics, Applied Semantics*

Further evidence in this direction can be collected by considering the vast debate on the semantics for *relevance* (also called *relevant*) *logics*. The "problem of semantics" I will discuss in the following is not a purely technical one – that is to say, it is not the mere problem of finding an abstract structure with respect to which one can prove the adequacy of a given logical calculus. It is a philosophical question: the one of the intuitive interpretation of the formal structure itself. I have already referred to the issue of intuitive interpretation, in the discussion of non-adjunctive approaches. But the point can be made more precise now, by exploiting a distinction which has gained currency in the literature: the one between *pure* and *applied* semantics.

This kind of terminology has probably been introduced by Plantinga (see Plantinga 1974, 126ff). It is somehow misleading, since it gives the impression of having to do with the applications, or with particular uses, of a logic, whereas this is not what is at issue at all.<sup>18</sup> Nevertheless, it has become a standard one by being adopted in various textbooks of philosophy of logic, such as those by Kirwan 1978 and Haack 1978. Dummett talked – with his inimitable style – of "merely algebraic notion of logical consequence", as opposed to "semantic notion of logical consequence properly so called":

---

<sup>16</sup> On this point see, e.g., Goodman 1978, Sacks 1989, Sosa 1999, Talmy 2000 and, for an idealistic but non-relativistic position, Hirsch 1982.

<sup>17</sup> "One generally laudable characteristic of an inquiry is that over the long run it converges to a *consistent* world. That is, we have a bias towards wanting our inquiries to be progressive in the sense of weeding out inconsistent beliefs". Ditto for completeness; therefore, "the conjunction of these two conditions is the requirement that an inquiry converge to a standard, consistent *and* complete world" (Rescher and Brandom 1980, 111).

<sup>18</sup> As an anonymous referee appropriately pointed out to me.

Semantic notions are framed in terms of concepts which are taken to have a direct relation to the use which is made of the sentences in a language [...]. It is for this reason that the semantic definition of the valuation of a formula [...] is thought of as giving the meanings of the logical constants. Corresponding algebraic notions define a valuation as a purely mathematical object which has no intrinsic connection with the uses of sentences. (Dummett 1973, 204)

The history of the semantics for intuitionistic logic provides a good example of the difference. The early intuitionistic semantics – e.g., the topological interpretations provided by Tarski and others (see van Dalen 1986, 243ff) – may be reasonably qualified as pure semantics. As Dummett says, they “were developed before any connection was made between them and the intended meanings of the intuitionistic logical constants”. Therefore, even though intuitionistic calculus is complete with respect to these semantics, “no one would think of this as in any sense giving the meanings of the intuitionistic logical constants” (Dummett 1973, 205). However, it seems to me that the situation has changed with the development of Kripke semantics for intuitionistic logic, since in this case we appear to have an independent intuitive interpretation: intuitionistic worlds should represent cognitive situations, increasing over time along different possibilities, on the basis of the conception of mathematics as a mental activity. At each time  $t$ , the mathematician has different possibilities to increase his knowledge in the transition to time  $t_1$ . This can be framed formally, for instance, by considering the accessibility relation as a partial order and by assuming variable but never decreasing domains, that is to say, if  $w_2$  is accessible from  $w_1$ , then the domain of  $w_1$  is a subset of the one of  $w_2$  (see van Dalen 1986, 246ff). We have completeness proofs for intuitionistic logic with respect to such semantics, providing counter-models to the classical laws rejected by intuitionists – paradigmatically, Strong Double Negation and Excluded Middle. However, the technical details are not of great significance here. The important point is the underlying *heuristics*: such semantic structures provide, in my opinion, an intuitively justified representation of the mental activity of the mathematician, as an idealized subject constructing mathematical objects through her proofs.<sup>19</sup>

As a result, we seem to have moved from a pure to an applied semantics. Such a shift, according to Susan Haack, is essential for a formal system to qualify as a *logic* of some kind or other.<sup>20</sup> The general point is that, given *any* purely proof-theoretically described “logic” of a certain kind, it is always possible to get a cheap algebraic model for it: just build a sound and complete semantics from the Lindenbaum algebra of the logic itself. In Priest’s words, this “reminds us of the fact that we need to distinguish those semantics that are of a kind to be genuinely informative about the meanings of the notions involved, and

---

<sup>19</sup> Which does not mean that *Dummett* would approve the Kripke semantics for intuitionistic logic. He claimed in several works that the correct semantics for intuitionism should not be phrased in terms of possible worlds at all, and the meaning of the intuitionistic logical vocabulary should be entirely captured inferentially, that is, via introduction/elimination rules of inference in the style of Gentzen. I am indebted to an anonymous referee for this remark.

<sup>20</sup> “The identification of a system as a system of logic requires appeal to its (intended?) interpretation. To identify a system as a sentence calculus one does not only need to know the axioms/rules and their formal interpretation by means of matrices; one also needs to know that the values are to represent truth and falsity, the letters ‘ $p$ ’, ‘ $q$ ’, etc. to represent sentences, ‘ $-$ ’ negation, ‘ $\&$ ’ conjunction, ‘ $\vee$ ’ disjunction, and so forth. [...] I have already urged that the pure semantics, by itself, is not sufficient; to justify the claim of a formal system to be a modal logic (sentence logic) some intuitive account of the formal semantics, connecting that set-theoretical construction with the ideas of necessity and possibility (truth and falsity) seems essential” (Haack 1978, 30 and 189).

those that are not”, for “many logics have topological, algebraic or many-valued semantics which, though they are useful in establishing various things about the proof-theory [...] are of a purely ‘technical’ nature” (Priest 2006, 181). Therefore, some authors also make the stronger claim that, until the passage is made, no determinate meaning has been assigned to the logical vocabulary of a formal system. According to B.J. Copeland, for instance, any logical semantics follows these two stages in its development: the first step is the mere construction of the formal semantic apparatus; the second step is the interpretation of the mathematical formalism. At this point, logic cannot but defer to some extent to metaphysics, given that we need “an *explanation of the nature of the objects* which constitute the range of this [evaluation] function, of any indices occurring in the domain of the function, of any operations or relations on these indices, and so on” (Copeland 1986, 479; my italics).

## 5. *The Relevant Approach*

The distinction between pure and applied semantics is of great importance in order to evaluate some of the most discussed semantics for paraconsistent logics. During the Eighties and Nineties, authors like Copeland, van Benthem, Smiley, and others claimed that the semantics for relevance logics – possibly, the most developed paraconsistent approach – did not reach the status of applied semantics. Thus, providing an intuitive reading and, therefore, a not purely technical assessment for them was an open problem. It seems to me that the situation has changed nowadays. As we shall see, several authors have offered convincing intuitive interpretations of some relevant semantics. However, the point is that precisely *these* interpretations give us an idea of the *sense in which* a logical calculus that tolerates contradictions can have a model, or a set of circumstances, satisfying it. Let us see why.

### 5.1 *The American Plan*

We have both algebraic and model-theoretic semantics for relevance logics. A very simple semantics for the logic of First Degree Entailment (FDE) has been proposed by Michael Dunn 1976 and Nuel Belnap 1977, and developed by many authors. The approach has been called *American Plan*, since it has gained a lot of currency with relevant logicians in the United States. It is a many-valued semantics, whose key idea is to assume as truth values  $P(\{1, 0\})$ , that is to say, the power set of the classical set:  $\{\{1\}, \{0\}, \{1, 0\}, \emptyset\}$ . So we have gluts of truth values (sentences that are both true and false), and gaps too (sentences that are neither true nor false). The semantic clauses for extensional connectives – given here in the typical set-theoretic notation – are straightforward:

$$(S\neg 1) \quad 1 \in v(\neg\alpha) \Leftrightarrow 0 \in v(\alpha)$$

$$(S\neg 2) \quad 0 \in v(\neg\alpha) \Leftrightarrow 1 \in v(\alpha)$$

$$(S\wedge 1) \quad 1 \in v(\alpha \wedge \beta) \Leftrightarrow 1 \in v(\alpha) \text{ and } 1 \in v(\beta)$$

$$(S\wedge 2) \quad 0 \in v(\alpha \wedge \beta) \Leftrightarrow 0 \in v(\alpha) \text{ or } 0 \in v(\beta)$$

$$(Sv1) \quad 1 \in v(\alpha \vee \beta) \Leftrightarrow 1 \in v(\alpha) \text{ or } 1 \in v(\beta)$$

$$(Sv2) \quad 0 \in v(\alpha \vee \beta) \Leftrightarrow 0 \in v(\alpha) \text{ and } 0 \in v(\beta)$$

(See e.g. Dunn 1986, 192 and, for a slightly modified version, Priest 2001, 139ff). These truth conditions are somewhat familiar: the first one of each pair looks like the classical clause (though “ $\neg$ ” actually has more the aspect of so-called choice negation, than that of exclusion negation). The second one of each pair is classically redundant, but this is no longer the case when we have embraced the paraconsistent insight that things may be both true and false (or neither). The designated values are  $\{1\}$  and  $\{1, 0\}$ , and (EFQ), expressed, e.g., in the form

$$(EFQ) \quad \alpha \wedge \neg\alpha \rightarrow \beta,$$

can be refuted by assigning to  $\beta$  a non-designated value ( $\{0\}$ , or  $\emptyset$ ), and to  $\alpha$  (therefore, to  $\neg\alpha$ , and  $\alpha \wedge \neg\alpha$ ) the (designated) value  $\{1, 0\}$ : contradictions can sometimes be both true and false, although they can never be  $\{1\}$ , “simply true”.

## 5.2 *The Australian Plan and Impossible Worlds Again*

Much more debated is the (im)possible worlds perspective initially due to Routley and Meyer, and developed by various relevant logicians mainly in Australia, therefore called *Australian Plan* (as for such labels, see e.g. Meyer and Martin 1986; Restall 1995). It is presented in different versions.<sup>21</sup> For my purposes, I will adopt the following characterization. A *Routley-Meyer (im)possible worlds structure* is a quintuple  $\langle W, O, o, *, R \rangle$ , where  $W$  is a set of “situations (or worlds)” (Routley 1979a, 309);  $O$  is a subset of  $W$  including normal or “theorem-regular” situations, that is to say, worlds in which all the theorems of the logic hold;  $o$  is an element of  $O$ , representing the real world;  $*$  is a unary operation on  $W$ , i.e., a function from worlds to worlds (sometimes called “involution”);  $R$  is a ternary accessibility relation on  $W$ . The most controversial part of the story has to do with the intuitive reading of  $R$  and  $*$ , which are used in the semantic clauses for negation and the conditional. Let us focus on negation and the famous *Routley star*. Given a world  $w$ , the “involution” operation produces a world  $w^*$  (on whose nature I shall say something in the following). The clause for negation in this framework is something like:

$$(S\neg) \quad T_w(\ulcorner \neg\alpha \urcorner) \Leftrightarrow \text{Not } T_{w^*}(\ulcorner \alpha \urcorner).$$

The standard clause for classical negation claims that  $\neg\alpha$  is true (in  $w$ ) if and only if  $\alpha$  is not true (in  $w$ , i.e., in that very world). The Routley-Meyer negation is characterized by saying that  $\neg\alpha$  is true in  $w$  if and only if  $\alpha$  is not true in  $w^*$  (see Routley and Meyer 1976; Routley 1979a, 311).<sup>22</sup> This provides a further counter-example to (EFQ): we just have to consider

---

<sup>21</sup> For instance, we have a simplified variant proposed in Routley and Priest 1992 for the basic relevant system B. This shall not be dealt with here, though.

<sup>22</sup> Sometimes the clause is spelled saying that  $\neg\alpha$  is true in  $w$  if and only if  $\alpha$  is *false* in  $w^*$  (see e.g. Copeland 1979, 402; Restall 1995, 144). In this case, though, the distance between untruth and falsity is of minor importance.

the case in which  $\alpha$  is true in  $w$ ,  $\beta$  is not true in  $w$  and  $\alpha$  is not true in  $w^*$ : both  $\alpha$  and  $\neg\alpha$ , then, are true in  $w$  while  $\beta$  is not. We may notice that *ex falso* is neutralized without admitting a truth-value glut, as in the Belnap-Dunn semantics, or in similar paraconsistent many-valued logics, such as Priest’s Logic of Paradox, LP<sup>23</sup> (though intuitive formal dualities and correlations between the American and the Australian Plan have been extensively explored: see e.g. Routley and Priest 1992; Restall 1995). With a few more conditions we can validate various theorems of relevant logics and make the relevant negation have certain intuitive inferential features. For instance, the following clause on  $R$  and  $*$ , sometimes called Inversion,

$$R(w, w_1, w_2) \Rightarrow R(w, w_2^*, w_1^*),$$

is central to the validation of Contraposition; and the assumption that involution is of period two,

$$w^{**} = w,$$

is used in the validation of Double Negation (see e.g. Routley 1979a, 310; Dunn 1986, 207; Restall 1995, 144-5). The negation characterized by such clauses is usually called by relevantists *De Morgan negation*, since also De Morgan laws hold for it; therefore, it has almost all the inferential properties of standard negation (except that of being explosive, of course). But what kind of world is  $w^*$  with respect to  $w$ ? Here is a little bit of help from the literature:

The operation  $*$  is a reversal operation which takes a situation into its reverse and hence incompleteness in a situation into inconsistency, and inconsistency into incompleteness, i.e., the reverse of a situation  $[w]$  where both  $A$  and  $\sim A$  hold is a situation  $[w^*]$  where neither  $A$  nor  $\sim A$  hold. (Routley 1979a, 309)

One way of thinking of  $[w]$  and  $[w^*]$  is to regard them as ‘mirror images’ one of another reversing ‘in’ and ‘out’. Where one is inconsistent (containing both  $A$  and  $\sim A$ ), the other is incomplete (lacking both  $A$  and  $\sim A$ ), and *vice versa* (when  $[w] = [w^*]$ ,  $[w]$  is both consistent and complete and we have a situation appropriate to classical logic). Viewed this way the Routleys’ negation clause makes sense, but *it does require some anterior intuitions about inconsistent and incomplete set-ups*. (Dunn 1986, 191; my italics)

### 5.3 Relevant Troubles

The relevant problems for relevance come exactly at the level of the intuitive reading of the semantics. By appealing to the distinction introduced above, several authors have claimed that relevant semantics remain *pure*, not reaching the status of applied semantics. In particular, algebraic semantics have often been disqualified as “disguised syntax”, since the operators defined in the algebra seem to have a meaning just through their mirroring the

---

<sup>23</sup> Roughly, we obtain the basic LP by subtracting  $\emptyset$  from the set of truth-values of Dunn’s semantics for FDE, having thus a three-valued logic. As a result, all classical tautologies are maintained in LP, whereas the consequence relation is decidedly non-classical and non-explosive.

connectives of the syntax of calculus.<sup>24</sup> As van Benthem observed in his discussion of the relevant perspective, “there is no end to syntactical creativity”, but the real point is whether the calculus has an intuitive semantic interpretation (see van Benthem 1979, 340).

But something analogous may happen also with the Routley-Meyer (im)possible worlds semantics, which has been criticized, again, by van Benthem, Smiley, Copeland, and others. Its attractive features are due to its looking like a development of Kripke semantics, and this fact has probably allowed the former to take advantage of the respectability of the latter. However, such resemblance, as the detractors’ story goes, is essentially deceptive. The three-place relation  $R$  of the model is not similar to the accessibility or alternativeness relations due to Kripke, Hintikka and others, exactly because of the aforesaid reason: the latter do have (at least, according to most logicians) an intuitive meaning, whereas the former does not. The same lack of independent characterization affects the monadic operation  $*$ , which should give meaning to the relevant negation.<sup>25</sup> It is not clear, therefore, which is the “intended” meaning assigned to logical symbols by such structures.

Routley and Meyer attempted to make  $*$  more palatable, by attaching to it an intuitive meaning in terms of the notion of *assertion*, and by distinguishing between a *weak* and a *strong* assertion of a sentence. The weak way to assert  $\alpha$  is to omit the assertion of its negation; what is weakly asserted in  $w^*$  is precisely what is (strongly) asserted in  $w$ , and *vice versa*. In this sense, De Morgan-relevant negation should reduce classical negation to a particular case: under normal circumstances, that is to say, when we assert exactly what we do not deny,  $w$  and  $w^*$  coincide and we have a situation appropriate to classical negation (see Routley and Meyer 1973, 202). But let us listen to the justification provided by the Routleys in their paper:

The new negation is neither queer nor weak because it behaves exactly like classical negation over consistent and complete set-ups. Since its behaviour diverges from that of classical negation only over a class of situations not admitted by modal logic, it is not so much the negation introduced by our new rule  $[S\bar{~}]$  which is different as the class of situations that the rule entitles us to consider. (Routley and Routley 1972, 338)

Therefore, the key point concerns, again, the underlying *ontology*: classical logic simply *assumes* that, for each  $w$ ,  $w = w^*$ , that is to say, that all worlds are maximally consistent. Such an assumption lies beneath the standard semantic clause of exclusion negation. The reply to the Quinean *change-of-subject* charge for negation is analogue to the one provided by Rescher, Brandom and Varzi for conjunction: we (as paraconsistentists) have just broadened our horizons beyond the classical, narrow-minded way of understanding *worlds* and/or *circumstances*. Therefore, the position that defends classical-exclusion negation as the One

---

<sup>24</sup> Looking at the algebraic models used in completeness proofs for relevant logics, Dunn admits that “the above kind of soundness and completeness result is really quite trivial (though not unimportant), once at least the logic has had its axioms chopped so that they look like the algebraic postulates merely written in a different notation” (Dunn 1986, 187).

<sup>25</sup> “By itself this ‘star rule’ is merely a device for preserving a recursive treatment of the connectives [...] and it does nothing to explain their tilde until supplemented by an explanation of  $[w^*]$ ” (Smiley 1993, 17-8). “I will not say very much here about what intuitive sense (if any) can be attached to the Routleys’ use of the  $[*]$ -operator in their valutional clause for negation. Indeed this question has had surprisingly little extended discussion in the literature [...]. The Routleys’ [1972] paper more or less just springs it on the reader, which led me in Dunn [1976] to describe the switching of  $[w]$  with  $[w^*]$  as ‘a feat of prestidigitation’” (Dunn 1986, 190-1).

True Negation, claiming that a connective that does not play the game of the ordinary semantic clause is not *negation*, now looks like another case of the Italics Argument.<sup>26</sup>

Copeland and others found the distinction between weak and strong assertion simply unintelligible. First, the claim that De Morgan-relevant negation is an extension of classical negation because it includes both the classical case (i.e.,  $w = w^*$ ) and inconsistent-incomplete situations, seems to be an unwarranted one. In order to understand the interpretation given to a connective by a semantic structure we should look at the behaviour of that connective in *all* the worlds/circumstances of the model, not only in a subset of them. Secondly, now the problem is that the notion of weak assertion seems to have *itself* been introduced purely *ad hoc*, and its connection with the notion of assertion as usually understood appears obscure:

In exactly what sense of “affirm” could a babe-in-arms be said to have [weakly] affirmed Heisenberg’s Uncertainty Principle simply because it has omitted to assert the negation of the Principle? Talk of strong affirmation and weak affirmation remains mere jargon until it is explained how these supposed activities mesh in with the remainder of the network of psychological and behavioural transactions between humans and sentences. (Copeland 1979, 409)

Lacking further explanation on the nature of  $w^*$ , the star operation seems to be a formal trick. In addition, the structural conditions introduced above (that  $*$  is of period two, etc.) appear to be *ad hoc*, in the sense that their unique motivation is to validate the theorems giving to De Morgan negation the desired inferential features. But “if the only constraint on  $*$  is that the resulting theory should validate the right set of sentences, then we are indeed in the presence of merely formal model theory” (*Ibid*, 410).

The early relevant approach to paraconsistency has been, therefore, a very controversial one. Although technically feasible, the strategy has been considered by many as lacking philosophical importance: “judgment has to be suspended until the proposed formal semantics has received any kind of satisfactory explanation. (At the present stage, the sense in which contradictions may be said to be ‘true’ remains a purely formal one.)” (van Benthem 1979, 333).<sup>27</sup>

## 6. *Infos and Beyond (or Which World Are You Living In? – Part Two)*

Since the time van Benthem and Copeland wrote their reviews, much work has been done in the field of paraconsistency, and the situation has changed. Several authors provided not only further technical developments and results, but also plausible intuitive interpretations for the non-standard semantic structures described above. But the point is that, when we consider *these* interpretations, we are in a similar condition to the one we found ourselves in when we discussed non-standard worlds and/or circumstances of non-adjunctive approaches. The intuitive reading of such structures was in terms of theories, databases, cognitive frameworks, systems of beliefs, etc. Consequently, the claim that such worlds realize “ontic” inconsistencies is liable of Russellian Fallacy, *unless* we assume the idealistic-

---

<sup>26</sup> For a general discussion of the issue of negation within a broadly paraconsistent approach, see Berto 2006a, 2006b.

<sup>27</sup> See also 343: “can any, technical or philosophical, interest be attached to the proposed semantics? Precisely this question cannot be answered until the authors have told us much more about the intuitions behind their ‘worlds’, ‘ordering relation’ and especially their ‘reversal operation’”.

constructivist perspective according to which our actual world and its furniture are framed by our theories and conceptual schemes.

### 6.1 *Reinterpreting Dialethic Logics*

Recent attempts to justify some semantics of relevance logics seem to lead into this direction. It is true that various authors, including Priest, have given the four-valued semantics for FDE a strongly paraconsistent or dialethic interpretation. In Priest 2006, a dialethic *correspondence* theory of truth naturally follows exactly from the four-valued semantics for FDE, so that “the logic of the world would be First Degree Entailment” (*Ibid*, 52). But, first, this is not how the American Plan was intended by its creators Belnap and Dunn, who gave it a strictly epistemic reading. According to Dunn, the four-valued semantics seeks to depict the fact that “one can have inconsistent and or incomplete assumptions, information, beliefs, etc., and this is what we are trying to model to see what follows from them in an interesting (relevant!) way”. So “all this talk of something’s being both true and false or neither is to be understood epistemically and not ontologically” (Dunn 1986, 192-3; see also Dunn 1976; the epistemic reading was also implicitly subscribed by David Lewis, in Lewis 1982, 102).

Secondly, and more significantly, Bryson Brown has developed interpretations both of FDE and of Priest’s LP, which fully capture their consequence relations from the point of view of so-called *preservationism*. Now, the main aim of preservationist logics (see Schotch and Jennings 1989) is to provide accounts of consequence in which the property preserved by the consequence relation is not *truth* at all, but some other desirable feature (e.g., the level or degree of incoherence, or the number of ambiguous sentential variables) we want to maintain in situations of inconsistent *information*, without having to assume that the inconsistencies in question are true. This means that the FDE consequence relation can be read “without adopting a semantics that allows truth or any other ‘designated’ [...] values to be assigned to inconsistent sets of sentences or to contradictions” (Brown 2004, 141). According to Brown, this shows that a strongly dialethic reading of LP, or of FDE, is not mandatory at all: we can interpret the consequence relations of LP and FDE as preserving some (classically perfectly acceptable) semantic properties, rather than preserving dialethic truth-values. And this

blocks the slippery slope argument in Priest [2000b] that aims to drive us from the second level of paraconsistency (at which we accept the existence of interesting or valuable inconsistent theories that can be modelled with the help of paraconsistent consequence relations) to the third (at which we accept the possibility that some such theories are *true*). (Brown 2004, 146)<sup>28</sup>

That is, it blocks Priest’s “slippery slope” from weak paraconsistency to dialethicism.

### 6.2 *Interlude: Is Priest a Realist Dialetheist?*

---

<sup>28</sup> See also Brown 1998, 498-9: “At least some paraconsistent logics that have been developed using dialethic valuations can be reinterpreted in preservationist terms – in fact, I conjecture that they all can. [...] The philosophical question of the tenability of dialethicism is, in my own view, open, as is the closely related question of the tenability of dialethic interpretations of various paraconsistent logics”.

Priest is by far the author who has explored more extensively the general philosophical consequences of dialetheism. I already mentioned how Priest has often claimed that dialetheism is not tied to a specific conception of truth: not more to the coherence theory of truth (which would take us on the route to idealism, since “typically those who endorsed the [coherence] theory have held that it makes no sense to define truth in terms of some objective reality, independently of our cognitive functioning”) than to the correspondence theory (which is “likely to appeal to a metaphysical realist”: Priest 2006, 53). However, this looks like a strategic move, in order to avoid committing such a controversial theory as dialetheism to any specific position in the already controversial field of the competing theories of truth. Therefore, it is worth devoting a section to the more general question: is Priest himself a metaphysically realist dialetheist?<sup>29</sup> I will split the issue into two parts: (a) is he a realist dialetheist about *abstract* entities, such as sets and concepts? (b) Is he a realist dialetheist about concrete, spatio-temporally located and perceptually given entities?

As for point (a), Priest appears to show, first of all, a realist stance towards logical concepts in general. According to him, logical notions are not *established* by our best logical theories, but should be *captured* by them – and logical theories can grasp them, or fail to. For this reason, he rejects the idea that there can be no rivalry between different logics because each one forges its own logical vocabulary, on the basis of the Quinean motto “Change of logic, change of subject”:

At the root of this kind of answer is a simple confusion between a theory and what it is a theory of. [...] The theoretical object has to fit the real object; and how this behaves is not a matter of choice. [...] If it is to be applied, an account of negation must be considered not just as an abstract structure, but as a theory *of* something, just as a geometry is a theory of physical space. (Priest 2006, 76-7; see also 176ff. for a realist account of logical validity)

Getting closer to our point, it seems fair to say that Priest’s attitude towards sets and concepts, including inconsistent ones, may be reasonably labelled as a realist’s one. For instance, his (and Routley’s) dialethic set theories are justified *vis à vis* their consistent rivals, not only on the epistemic grounds of simplicity, explanatory power, etc., but as capturing the intuitive and naïve notion of set, i.e., what it is to *be* a set. As Woods said in his reconstruction of the “Dialethic Mini-History of Set Theory”,

[Dialetheism] offers prospect of a return to realism in logic and set theory. [...] There is a cost, of course. It is the cost of the ontological embrace of actual inconsistencies [...]. The realism that I am attributing to dialetheists need not be seen as Platonism. It need not even imply the necessity of dialethic truths. What it does imply is the rejection of any form of fictionalism in set theory, or semantics, or anywhere else save fiction itself. (Woods 2003, 156)

One could tell a similar story of the whole procedure for establishing the existence of inconsistent abstract objects at the “limits of thought”, formalized by the celebrated

---

<sup>29</sup> Mares 2004 claims that at least the early Priest was a “semantic paraconsistentist”, not a “metaphysically realist” dialetheist: in the paper where he presented the first version of his Logic of Paradox, Priest limited the possibility of true contradictions to circumstances in which we have semantic self-referentiality (see Priest 1979, 235). But Mares also considers Priest’s current and stabilized position as a clearly metaphysical, realist dialetheism, that is to say, a theory which holds “that there are things in the world that are actually inconsistent”; and “these inconsistencies do not arise merely because of our beliefs about the world or because of how we talk about things, but because of the way things really are *in themselves*” (Mares 2004, 265).

Inclosure Schema in *Beyond the Limits of Thought*; for “‘thought’ here should be understood in its objective, Fregean, sense” (Priest 1995, 3).

As for point (b), i.e., Priest’s realist dialetheism concerning concrete, empirical entities, this is a tricky issue but also in this case one could argue for a realist interpretation. Priest seems in general committed to a conjectural account of what we should believe about the world and, as Frederick Kroon has observed, “since Priest makes it abundantly clear that he is no relativist, it is clear that [...] he would resist the identification of truth with membership in some ‘best’ corpus of beliefs” (Kroon 2004, 251). In fact, Kroon’s paper also provides an interesting complete argument to the effect that Priest is a realist dialetheist. The argument relies on Priest’s discussion of *trivialism* (Priest 2000c and 2006, Ch. 3), i.e., of the repugnant view that everything (therefore, every contradiction) is true. Such a view can be easily rejected *a priori* by the orthodox-minded logician, since no contradiction, according to her, is true. However, what about the dialetheist? Once some true contradictions are admitted, where should we draw a line before admitting them all? We seem to need a consistency filter. According to Priest, we can exclude the truth of some contradictions, therefore, the triviality of the (empirical) world *a posteriori*, on the basis of a reliabilist conception of perception: “we know, then, that the world is not trivial, since we can see that this is so”. But *a posteriori* considerations based upon observation are, of course, defeasible, and “it is, in some sense, a priori possible that an inconsistent object appear in a consistent way” (Priest 2006, 62). How can we justify ourselves, then, in front of a convinced trivialist? Priest takes the challenge seriously, and this is exactly what a *realist* dialetheist is supposed to do: to be a realist in this respect entails that the actual world *might* be trivial even though we perceive, think, or are constrained to believe, that it is not.

Now, the only properly working line of reasoning, according to Priest, is a “transcendental argument” based on the phenomenology of choice: the trivialist’s behaviour, particularly her choice-based acting, shows that she does operate purposefully, accepting something and rejecting something else, and sees her activities as making a real difference in the empirical world. We can infer from this that she does not actually believe everything: “having to choose is something phenomenologically unavoidable”. However, transcendental arguments of this kind are usually taken as a commitment to some form of anti-realism or verificationism, for they aim at establishing categorical conclusions on the world from features of our consciousness. And this is *not* what Priest takes his argument to bring about:

This does not show that trivialism is untrue. As far as the above considerations go, it is quite possible that everything is the case; but not for me – or for any other persons. [...] The argument I have deployed provides a transcendental deduction from certain features of consciousness to the impossibility of being a trivialist. (Priest 2006, 70-1)

This is not a “transcendental argument” in the traditional sense: it does not take us from the impossibility of a trivial viewpoint to the *falsity* of trivialism – only to the much more modest result of the impossibility of *believing* it. According to Kroon, therefore, “Priest’s way of putting his own transcendental argument is likely to signify a commitment to some kind of realism”, and to a realist dialetheism:

The reason why even the compulsory rejection of trivialism is compatible with the truth of trivialism is that there is a conceptual gap between the content of our best beliefs and the way the world really is [...]. For all our clever attempts to insulate contradictions and provide reasons for thinking that the space of true contradictions is in fact very sparse, the world may yet fool us. [...] As a realist, Priest must grant that internal constraints on what we must believe

are not constraints on what the world is like independently of our best theories and beliefs.  
(Kroon 2004, 251-2)

### 6.3 Information Flow

We are then driven back to our original issue: is a realist dialetheism backed up by the dialetheist's interpretation of her favourite paraconsistent logics? In recent years, a plausible reading of the Routley-Meyer semantics for De Morgan negation and the relevant conditional has been proposed in terms of *information flow*, within the framework of situation semantics (See Barwise and Perry 1993; Mares 1996; Bremer and Cohnitz 2004). This approach is quite new but, informally, the basic idea is to model circuits hosting regulated flows of information – in Manuel Bremer's words:

Information flows in *distributed systems* (like a circuit that connects a switch with a bulb). These systems can be considered *channels* along which we reason. The laws and rules operative in such a system are *constraints* that define a proper working system and allow to reason along these constraints (like knowing that the switch was pressed if the bulb is burning). Talking in situation semantics language we can say that information about one *situation* is derived from another situation by some channel. The basic building blocks of information are *infons*, which resemble Russellian propositions. (Bremer 2005, 68-9)

In particular, within this framework we can have an intuitive characterization of the Routley star. Greg Restall has convincingly defended such a characterization, particularly in a 1999 essay whose subtitle deserves being quoted: “How I Stopped Worrying and Learned to Love the Routley Star”.

Two pieces of information or infons, Restall argues, can be compatible or incompatible with each other, thereby bringing in a certain partial compatibility or incompatibility between worlds containing them. We can accordingly introduce a binary compatibility relation  $C$  between worlds and characterize negation via such a relation:

$$(S\neg) \quad T_w(\neg\alpha) \Leftrightarrow \forall w_1 (C(w, w_1) \Rightarrow \text{Not } T_{w_1}(\alpha)),$$

that is to say:  $\neg\alpha$  is true in  $w$  if and only if  $\alpha$  is not true in all compatible worlds. We can read “ $C(w, w_1)$ ” as saying that nothing given by  $w$  is rejected by  $w_1$ , so the set of worlds compatible with  $w$  may represent what is not excluded by what is true in  $w$  (See Restall 1999, 61-2). And “this provides us with an *applied* semantics for negation” (*Ibid*, 61; my italics). In this context, and given some formal conditions on  $C$ , the “reverse twin” of  $w$ ,  $w^*$ , is the most informative, or comprehensive, among the worlds  $w_1$  for which  $C(w, w_1)$  holds: “the Routley star is a simplification of our compatibility clause for negation when we assume that  $C$  is symmetric, directed and convergent”; and “given that the compatibility semantics makes sense and is an applied semantics, it follows that its simple retelling, involving the Routley star, also makes sense, and it too is an applied semantics” (*Ibid*, 63; see also Bremer 2005, 72-3).

For the sake of the argument, let us suppose that the information flow representation is sufficiently transparent – which is doubted by some paraconsistent authors, e.g., Priest 2001, 198. Now the question is: what is an *infon*?<sup>30</sup> If worlds are considered as make-ups of

---

<sup>30</sup> A popular response is to follow Barwise and Perry: it is an ordered  $n$ -ple  $\langle R, o_1, \dots, o_n, b \rangle$ , where  $R$  is a  $n$ -ary relation,  $o_1, \dots, o_n$  are objects, and  $b$  is a *sign-bit*: 0 or 1.

infons (not simple sets, but something with more arrangement imposed on them, or even structures that act themselves as *conduits* for information in some way: this does not make a difference to the point); and if an infon somehow is information *on* something; then worlds are structures of information. In this case, worlds retain the right to be inconsistent and/or incomplete, since it is out of question that information can be inconsistent and/or incomplete. But again, we are dealing with theories, beliefs, etc., that is to say, representations.

Routley and Meyer, in fact, used to move back and forth on this point. In their 1972 paper the Routleys introduced worlds as “set-ups”, characterizing them quite syntactically as sets of *sentences* (see Routley and Routley 1972, 335); but in the Routley and Meyer 1973 paper, set-ups became what is *described by* a set of sentences (Routley and Meyer 1973, 200-1). Such slips obviously attract the charge of Russellian Fallacy – e.g., in John Woods’ words: “this is dialectic rhetoric *par excellence*. It takes us from doxastic inconsistency to ontic inconsistency, that is, from the fact that beliefs are sometimes inconsistent to the possibility that inconsistent beliefs are sometimes true, and so to the possibility that sometimes objects or states of affairs are inconsistent” (Woods 2003, 89-90).

On the other hand, the idea of an intuitive reading of relevant (im)possible worlds in terms of information was unwillingly provided by an aforementioned critic of relevant logics. We have seen how, according to Copeland, to grasp the meaning assigned to the logical vocabulary by a formal semantics we need an independent characterization of the evaluation function and of the ontological structure that provide a model for the calculus. However, this is not enough yet: the main task of the semantic apparatus is to represent a semantic property  $\Phi$ , such that understanding the condition under which a sentence containing one of the relevant logical symbols has that property is sufficient to understand the meaning of the symbol. Applied semantics is finally reached when we are able to read semantic clauses as “statement[s] of necessary and sufficient conditions for the possession, by sentences of the forms involved, of an antecedently understood property  $\Phi$ ” (Copeland 1986, 480). The case *par excellence* of such a semantic property  $\Phi$ , of course, is that of being *true*. Having an independent grasp of what “true” means in the context, we can understand what conjunction means, via the clause: “ $\alpha \wedge \beta$  is true if and only if...”, specifying a necessary and sufficient condition for a sentence containing it as its main connective to have that property. But if the worlds of the Routley-Meyer semantics are characterized as sets of sentences, beliefs, collections of hypotheses, *infons*, etc., then the property of being true in a world actually means that “the sentence  $A$  is a *member of* the set of sentences  $j$ , this set representing the beliefs of a given individual, or a given mathematical or physical theory” (*Ibid*, 486-7). Now, membership of a set of beliefs or a theory is a property quite different from *truth*. The predicate “is a member of belief set  $w$ ” does not in general satisfy the Tarskian schema; whereas many have considered the T-schema as a minimal constraint on a truth predicate (even independently of the issue whether the schema itself should be interpreted “deflationistically”, or in a robust sense). In Timothy Williamson’s words, even though “it is not claimed that a Tarskian theory tells the whole truth about truth”, nevertheless “it tells an essential part of the truth”. So “without a disquotational schema, it is doubtful that one has a truth predicate at all” (Williamson 1992, 268).

## 7. *Dialetheism as an Idealism*

The most plausible intuitive reading of the Routley-Meyer semantics, therefore, suggests that for relevance logics may hold what seems to hold for the various non-adjunctive

approaches: these paraconsistent theories are quite legitimate (actually, very interesting and useful) in order to model and manage inconsistencies in our cognitive luggage. As a matter of fact, several important applications of paraconsistent logics in the issues of cognitive management and belief revision are currently being developed (see e.g. Tanaka 1998, 2005). It may not be hazardous to predict that these applications will soon constitute a serious alternative to the standard Alchourrón-Gärdenfors-Makinson theories of belief change. But these perspectives, to begin with, seem to legitimate only a *weak* paraconsistent approach; not a strong or dialethic one, promising genuine counterexamples to the Law of Non-Contradiction. Contradictions are “true”; once an interpretation of the semantics has been produced, though, the intuitive meaning of “true” seems to have turned in our hands into something like “included in a theory or a system of beliefs”. Therefore, the dialetheist or supporter of strong paraconsistency faces the Dilemma. On the one side, if she claims to have exhibited true, i.e., real, contradictions, she is liable of committing the Russellian Fallacy. On the other side, if she bites the bullet and takes the shift from “true (of the world)” to “included in a theory” at face value, she appears to commit herself to some equivalence between *worlds* and *theories*, thereby taking the road of some constructivism or idealism: the view according to which the world is made of theories, or conceptual schemes.

It is noteworthy that Meyer and Martin seem to straightforwardly embrace the second horn of the Dilemma, in a 1986 paper in which they conduct a defence of the Australian Plan that has since become a classic. Contrasting the usual and, as we have seen, *epistemic* reading of the Belnap-Dunn American Plan, Meyer and Martin stress the “ontic” features of the inconsistencies admitted within the Routley-Meyer semantics (see Meyer and Martin 1986, 319). Nevertheless, at the same time they describe an inconsistent state of affairs as one in which “a given theory could both assert and deny  $A$ ” – and, in this case, “the obvious thing to say is that the theory is just mixed up about  $A$ ” (*Ibid*, 311). Just like in the Peircean perspective embraced by Rescher and Brandom, consistency and completeness are a regulative ideal – what our theories should be inclined to, in the long run. Hence comes the usual objection: there is a trade-off between worlds and corresponding long run theories, therefore, Fallacy ensues! It is the world, not our favourite theories, that confers truth and falsity upon sentences.

The reply is worth listening, for it consists exactly in embracing a form of idealism, and *retorting* it against the classical logician:

The World to which a given Logic is linked tends to be a lot like the Logic which is linked to it. When one investigates the techniques that are employed in proofs of completeness, one regularly runs into something not far from our Theory of Theories. [...] The semantic Universes in which our modelling goes on are *suspiciously like* formal theories themselves, endowed with what are supposed to be the right sort of properties. [...] This process of imbuing what one thought was syntactic with semantical significance can become quite bald, as it does in proofs of Classical first-order completeness in the style of Henkin 1949. To refute a given non-theorem  $A$ , one builds a “maximally consistent” theory  $T$  without  $A$ ; on a few further manipulation, the theory  $T$  becomes the model in which  $A$  is refuted. [...] To that extent, Classical logic is *self-justified*, not independently justified because it has been referred to the World. For the World to which it has been referred has been made in its own image. So one doesn’t really need worlds (or models, or whatever) to tell semantical stories. In the clearest sense, they are just imaginative copies of preferred theories. (*Ibid*, 324-6)

Talking about non-adjunctive approaches, I have claimed that metaphysical idealism may be considered as a quite respectable position – and by subscribing a consciously idealistic position dialetheists may avoid the risk of the Russellian Fallacy. Furthermore, once an

idealistic stance has been adopted, the very *distinction* between weak and strong paraconsistency seems to fade. The distinction was put forward by supporters of paraconsistency as a difference between (a) the claim that there can be inconsistent but interesting and non-trivial theories; and (b) the claim that there can be true contradictions, thus – given a minimally realist stance on “true” – inconsistent objects and/or states of affairs (those that makes the contradictions true). However, such a difference could not be easily maintained, when we somehow make the distinction between *theories* and *states of affairs* collapse. And this might be a welcome result for a strong paraconsistentist, supporting in a different, albeit circuitous, way Priest’s “slippery slope” from weak paraconsistency to dialetheism.\*

\* I am grateful to Achille Varzi, Luigi Perissinotto, and three anonymous referees, for their helpful comments.

## References

- Anderson, A.R. 1963, "Some Open Problems Concerning the System E of Entailment", *Acta Philosophica Fennica* 16, pp. 7-18.
- Anderson, A.R. and Belnap, N.D. 1975, *Entailment. The Logic of Relevance and Necessity*, vol. I, Princeton: Princeton University Press.
- Anderson, A.R., Belnap, N.D. and Dunn, J.M. 1982, *Entailment. The Logic of Relevance and Necessity*, vol. II, Princeton: Princeton University Press.
- Barwise, J. and Perry, J. 1983, *Situations and Attitudes*, Cambridge, Mass.: MIT Press.
- Batens, D. 1980, "Paraconsistent Extensional Propositional Logic", *Logique et analyse* 90, pp. 196-234.
- Batens, D., Mortensen, C., Priest, G. and van Bendegem, J.-P. (eds.) 2000, *Frontiers of Paraconsistent Logic*, Baldock: Research Studies Press.
- Beall, J.C. 2000, "On Truthmakers for Negative Truths", *Australasian Journal of Philosophy* 78, pp. 264-268.
- Beall, J.C. 2004a, "At the Intersection of Truth and Falsity", in: Priest, Beall, Armour-Garb 2004, pp. 1-19.
- Beall, J.C. 2004b, "True and False—As If", in: Priest, Beall, Armour-Garb 2004, pp. 197-216.
- Beall, J.C. and Colyvan, M. 2001, "Looking for Contradictions", *Australasian Journal of Philosophy* 79, pp. 564-569.
- Belnap, N.D. 1977, "A Useful Four-Valued Logic", in: Dunn, Epstein 1977, pp. 8-37.
- Benthem, J. van 1979, "What is Dialectical Logic?", *Erkenntnis* 14, pp. 333-347.
- Berto, F. 2005, *Che cos'è la dialettica hegeliana?*, Padova: Il Poligrafo.
- Berto, F. 2006a, "Meaning, Metaphysics, and Contradiction", *American Philosophical Quarterly* 45, pp. 283-297.
- Berto, F. 2006b, "Characterizing Negation to Face Dialetheism", *Logique et Analyse* 195, pp. 241-263.
- Berto, F. 2006c, *Teorie dell'assurdo. I rivali del Principio di Non-Contraddizione*, Roma: Carocci.
- Berto, F. 2007, "Hegel's Dialectics as a Semantic Theory", *European Journal of Philosophy*, forthcoming.
- Bobenrieth, A. 1998, "Five Philosophical Problems Related to Paraconsistent Logic", *Logique et Analyse* 161, pp. 21-30.
- Brady, R.T. 1996, "Relevant Implication and the Case for a Weaker Logic", *Journal of Philosophical Logic* 25, pp. 151-183.
- Brady, R.T. 2000, "Entailment, Negation and Paradox Resolution", in: Batens, Mortensen, Priest, van Bendegem 2000, pp. 113-136.
- Bremer, M. 2005, *An Introduction to Paraconsistent Logics*, Frankfurt a.M.: Peter Lang.
- Bremer, M. and Cohnitz, D. 2004, *Information and Information Flow. An Introduction*, Frankfurt a.M.: Peter Lang.
- Brown, B. 1999, "Yes, Virginia, there Really Are Paraconsistent Logics", *Journal of Philosophical Logic*, pp. 489-500.
- Brown, B. 2000, "Simple Natural Deduction for Weakly Aggregative Paraconsistent Logics", in: Batens, Mortensen, Priest and van Bendegem 2000, pp. 137-148.
- Brown, B. 2004, "Knowledge and Non-Contradiction", in: Priest, Beall and Armour-Garb 2004, pp. 127-155.
- Brown, B. and Priest, G. 2004, "Chunk and Permeate, a Paraconsistent Inference Strategy. Part I: The Infinitesimal Calculus", *Journal of Philosophical Logic* 33, pp. 379-388.
- Carnielli, W., Coniglio, M. and D'Ottaviano, I. (eds.) 2002, *Paraconsistency. The Logical Way to the Inconsistent*, New York-Basel.
- Copeland, B.J. 1979, "On When a Semantics is Not a Semantics: Some Reasons for Disliking the Routley-Meyer Semantics for Relevance Logic", *Journal of Philosophical Logic* 8, pp. 399-413.
- Copeland, B.J. 1986, "What is a Semantics for Classical Negation?", *Mind* 95, pp. 478-490.
- Da Costa, N.C.A. and Dubikajtis, L. 1968, "Sur la logique discursive de Jaskowski", *Bulletin of the Polish Academy of Sciences* 16, pp. 551-557.
- Dalen, D. van 1986, "Intuitionistic Logic", in: Gabbay, Guenther 1983-1989, vol. III, pp. 225-339.
- Dummett, M. 1973, "The Justification of Deduction", *Proceedings of the British Academy* 59, pp. 201-231.
- Dunn, J.M. 1976, "Intuitive Semantics for First-Degree Entailment and 'Coupled Trees'", *Philosophical Studies* 29, pp. 149-168.
- Dunn, J.M. 1986, "Relevance Logic and Entailment", in: Gabbay, Guenther (1983-1989), vol. III, pp. 117-224.
- Dunn, J.M. and Epstein, G. (eds.) 1977, *Modern Uses of Multiple-Valued Logics*, Dordrecht: Reidel.
- Evans, G. 1978, "Can There Be Vague Objects?", *Analysis* 38, p. 208.
- Fine, K. 1975, "Vagueness, Truth and Logic", *Synthese* 30, pp. 265-300.
- Foley, R. 1979, "Justified Inconsistent Beliefs", *American Philosophical Quarterly* 16, pp. 247-257.
- Fraassen, B. van 1966, "Singular Terms, Truth-Value Gaps, and Free Logic", *Journal of Philosophy* 63, pp. 481-495.
- Fraassen, B. van 1968, "Presuppositions, Implication and Self Reference", *Journal of Philosophy* 65, pp. 136-151.

- Gabbay, D. and Guenther, F. (eds.) 1983-1989, *Handbook of Philosophical Logic*, voll. I-IV, Dordrecht: Kluwer.
- Gabbay, D. and Wansing, H. (eds.) 1999, *What is negation?*, Dordrecht: Kluwer.
- Goodman, N. 1978, *Ways of Worldmaking*, Indianapolis: Hackett.
- Haack, S. 1978, *Philosophy of Logics*, Cambridge: Cambridge University Press.
- Hintikka, J. 1969, *Models for Modalities*, Dordrecht: Reidel.
- Hintikka, J. and Butts, R. (eds.) 1977, *Logic, Foundations of Mathematics and Computability Theory*, Dordrecht: Reidel.
- Hirsch, E. 1982, *The Concept of Identity*, Oxford: Oxford University Press.
- Hyde, D. 1997, "From Heaps and Gaps to Heaps of Gluts", *Mind* 106, pp. 640-660.
- Jaskowski, S. 1948, "Rachunek zdan dla systemow dedukcyjnych sprzecznych", *Studia Societatis Scientiarum Torunensis*, sectio A, I, 5, pp. 55-77.
- Keefe, R. 2000, *Theories of Vagueness*, Cambridge: Cambridge University Press.
- Keefe, R. and Smith, P. (eds.) 1996, *Vagueness. A reader*, Cambridge, Mass.: MIT Press.
- Kielkopf, C.F. 1975, "Adjunction and Paradoxical Derivations", *Analysis* 35, pp. 127-129.
- Kirkham, R.L. 1992, *Theories of Truth. A critical Introduction*, Cambridge, Mass.: MIT Press.
- Kirwan, C. 1978, *Logic and Argument*, London: Duckworth.
- Kotas, J. and da Costa, N.C.A. 1979, "A New Formulation of Discussive Logic", *Studia Logica* 38, pp. 429-444.
- Kripke, S. 1959, "A Completeness Theorem in Modal Logic", *Journal of Symbolic Logic* 24, pp. 1-14.
- Kripke, S. 1963a, "Semantical Analysis of Intuitionistic Logic I", in: J. Crossley, M. Dummett, eds., *Formal Systems and Recursive Functions*, Amsterdam: North-Holland, pp. 92-129.
- Kripke, S. 1963b, "Semantical Analysis of Modal Logic I: Normal Modal Propositional Calculi", *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* 9, pp. 67-96.
- Kripke, S. 1972, *Naming and Necessity*, Oxford: Basil Blackwell.
- Kripke, S. 1975, *Lectures on Truth*, unpublished MS, Princeton University.
- Kroon, F. 2004, "Realism and Dialetheism", in: Priest, Beall, Armour-Garb 2004, pp. 245-263.
- Kyburg, H.E. 1997, "The Rule of Adjunction and Reasonable Inference", *Journal of Philosophy* 94, pp. 109-125.
- Leblanc, H. (ed.) 1973, *Truth, Syntax and Semantics*, Amsterdam: North-Holland.
- Lewis, D. 1973, *Counterfactuals*, Cambridge, Mass.: Harvard University Press.
- Lewis, D. 1978, "Truth in Fiction", *American Philosophical Quarterly* 15, pp. 37-46.
- Lewis, D. 1982, "Logic for Equivocators", *Nous* 16, pp. 431-441, repr. in Id. 1998, *Papers in Philosophical Logic*, Cambridge: Cambridge University Press, pp. 97-110.
- Mares, E. 1996, "Relevant Logic and the Theory of Information", *Synthese* 109, pp. 345-360.
- Mares, E. 2004, "Semantic Dialetheism", in: Priest, Beall, Armour-Garb 2004, pp. 264-275.
- Meheus, J. (ed.) 2002, *Inconsistency in Science*, Dordrecht: Kluwer.
- Meyer, R.K. and Martin, E.P. 1986, "Logic on the Australian Plan", *Journal of Philosophical Logic* 15, pp. 305-332.
- Parsons, T. and Woodruff, P. 1995, "Worldly Indeterminacy of Identity", *Proceedings of the Aristotelian Society* 95, pp. 171-191.
- Pasniczek, J. 1998, "Beyond Consistent and Complete Possible Worlds", *Logique et Analyse* 161, pp. 121-134.
- Plantinga, A. 1974, *The Nature of Necessity*, Oxford: Oxford University Press.
- Priest, G. 1979, "The Logic of Paradox", *Journal of Philosophical Logic* 8, pp. 219-241.
- Priest, G. 1984, "Logic of Paradox Revisited", *Journal of Philosophical Logic* 13, pp. 153-179.
- Priest, G. 1987, *In Contradiction: A Study of the Transconsistent*, Dordrecht: Martinus Nijhoff.
- Priest, G. 1992, "What is a Non-Normal World?", *Logique et Analyse* 35, pp. 291-302.
- Priest, G. 1995, *Beyond the Limits of Thought*, Cambridge: Cambridge University Press.
- Priest, G. 1998, "What is so Bad about Contradictions?", *Journal of Philosophy* 8, pp. 410-426.
- Priest, G. 1999, "Perceiving Contradictions", *Australasian Journal of Philosophy* 77, pp. 439-446.
- Priest, G. 2000a, "Truth and Contradiction", *Philosophical Quarterly* 50, pp. 305-319.
- Priest, G. 2000b, "Motivations for Paraconsistency: the Slippery Slope from Classical Logic to Dialetheism", in: Batens, Mortensen, Priest, van Bendegem 2000.
- Priest, G. 2000c, "Could Everything be True?", *Australasian Journal of Philosophy* 78, pp. 189-195.
- Priest, G. 2001, *An Introduction to Non-Classical Logic*, Cambridge: Cambridge University Press.
- Priest, G. 2002, "Inconsistency and the Empirical Sciences", in: Meheus 2002, pp. 119-128.
- Priest, G. 2006, *Doubt Truth to be a Liar*, Oxford: Oxford University Press.
- Priest, G., Beall, J.C. and Armour-Garb, B. (eds.) 2004, *The Law of Non-Contradiction. New Philosophical Essays*, Oxford: Oxford University Press.
- Priest, G. and Routley, R. 1982, "Lessons from Pseudo-Scotus", *Philosophical Studies* 42, pp. 189-199.

- Priest G. and Routley, R. 1989, "Systems of Paraconsistent Logics", in: Priest, Routley, Norman 1989, pp. 151-186.
- Priest, G., Routley, R. and Norman, J. (eds.) 1989, *Paraconsistent Logic. Essays on the Inconsistent*, München: Philosophia Verlag.
- Quine, W.v.O. 1970, *Philosophy of Logic*, Englewood Cliffs, N.J.: Prentice Hall.
- Read, S. 1988, *Relevant Logic*, London: Basil Blackwell.
- Rescher, N. and Brandom, R. 1980, *The Logic of Inconsistency: a Study in Non-standard Possible Worlds Semantics and Ontology*, Oxford: Basil Blackwell.
- Restall, G. 1995, "Four-Valued Semantics for Relevant Logics (and Some of Their Rivals)", *Journal of Philosophical Logic* 24, pp. 139-160.
- Restall, G. 1999, "Negation in Relevant Logics (How I stopped worrying and learned to love the Routley Star)", in: Gabbay, Wansing 1999, pp. 53-76.
- Routley, R. 1979a, "Dialectical Logic, Semantics and Metamathematics", *Erkenntnis* 14, pp. 301-331.
- Routley, R. 1979b, *Exploring Meinong's Jungle and Beyond. An Investigation of Noneism and the Theory of Items*, Canberra: Dipartimental Monograph, Australian National University.
- Routley, R. and Meyer, R.K. 1973, "The Semantics of Entailment P", in: Leblanc 1973, pp. 194-243.
- Routley, R. and Meyer, R.K. 1976, "Dialectical Logic, Classical Logic and the Consistency of the World", *Studies in Soviet Thought* 16, pp. 1-25.
- Routley, R., Meyer, R.K., Plumwood, V. and Brady, R. (eds.) 1982, *Relevant Logics and Their Rivals I*, Atascadero, Ca: Ridgeview.
- Routley, R. and Priest, G. 1992, "Simplified Semantics for Basic Relevant Logics", *Journal of Philosophical Logic* 21, pp. 217-232.
- Routley, R. and Routley, V. 1972, "Semantics for First Degree Entailment", *Nous* 6, pp. 335-359
- Russell, B. 1923, "Vagueness", *Australasian Journal of Philosophy and Psychology* 1, pp. 84-92, now in: Keefe, Smith 1996, pp. 61-68.
- Sacks, M. 1989, *The World We Found. The Limits of Ontological Talk*, LaSalle, Ill.: Open Court.
- Schotch, P.K. and Jennings, R.E. 1989, "On Detonating", in: Priest, Routley, Norman 1989, pp. 306-327.
- Smiley, T. 1993, "Can Contradictions Be True? P", *Proceedings of the Aristotelian Society*, suppl. vol. 67, pp. 17-34.
- Sosa, E. 1999, "Existential Relativity", *Midwest Studies in Philosophy* 23, pp. 132-143.
- Talmy, L. 2000, *Toward a Cognitive Semantics*, Cambridge, Mass.: MIT Press.
- Tanaka, K. 1998, "What Does Paraconsistency Do? The Case of Belief Revision", in: T. Childers (ed.), *The Logica Yearbook 1997*, Prague.
- Tanaka, K. 2005, "The AGM Theory and Inconsistent Belief Change", *Logique et Analyse* 189-92, pp. 113-50.
- Tappenden, J. 1993, "The Liar and Sorites Paradox: Toward a Unified Treatment", *Journal of Philosophy* 90, pp. 551-577.
- Troelstra, A.S. and Dalen, D. van 1988, *Constructivism in Mathematics: An Introduction*, Amsterdam: North-Holland.
- Tye, M. 1990, "Vague Objects", *Mind* 99, pp. 535-557.
- Urchs, M. 1995, "Discursive Logic: Towards a Logic of Rational Discourse", *Studia logica* 54, pp. 231-249.
- Urchs, M. 2002, "On the Role of Adjunction in Para(in)consistent Logic", in: Carnielli, Coniglio, D'Ottaviano 2002, pp. 487-500.
- Varzi, A.C. 1997, "Inconsistency without Contradiction", *Notre Dame Journal of Formal Logic* 38, pp. 621-639.
- Varzi, A.C. 2000, "Supervaluationism and Paraconsistency", in: Batens, Mortensen, Priest, van Bendegem 2000, pp. 279-297.
- Varzi, A.C. 2004, "Conjunction and Contradiction", in: Priest, Beall, Armour-Garb 2004, pp. 93-110.
- Williamson, T. 1992, "Vagueness and Ignorance", *Proceedings of the Aristotelian Society* 66, pp. 145-62, now in Keefe, Smith 1996, pp. 265-280.
- Woods, J. 2003, *Paradox and Paraconsistency. Conflict Resolution in the Abstract Sciences*, Cambridge: Cambridge University Press.
- Yagisawa, T. 1988, "Beyond Possible Worlds", *Philosophical Studies* 53, pp. 175-204.