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IMPOSSIBLE WORLDS AND PROPOSITIONS: AGAINST THE PARITY THESIS

By Francesco Berto

Accounts of propositions as sets of possible worlds have been criticized for conflating distinct impossible propositions. In response to this problem, some have proposed to introduce impossible worlds to represent distinct impossibilities, endorsing the thesis that impossible worlds must be of the same kind; this has been called the parity thesis. I show that this thesis faces problems, and propose a hybrid account which rejects it: possible worlds are taken as concrete Lewisian worlds, and impossibilities are represented as set-theoretic constructions out of them. This hybrid account (1) distinguishes many intuitively distinct impossible propositions; (2) identifies impossible propositions with extensional constructions; (3) avoids resorting to primitive modality, at least so far as Lewisian modal realism does.

> I could construct excellent ersatz worlds in ever so many ways, drawing on the genuine worlds for raw material.
>
> David Lewis, *On The Plurality of Worlds* (Oxford: Blackwell, 1986)

I. THE GRANULARITY PROBLEM

I begin by rehearsing some well known facts. Within possible-worlds model-theoretic semantics, propositions can be defined in terms of worlds, as functions from worlds to truth-values, or as sets of worlds. In some classic accounts, this is taken as a direct ontological reduction. A proposition *is* a set-theoretic construction out of worlds: it is the set of worlds at which it is true. If this is what propositions are, then we should believe in propositions, provided we believe in worlds and in set-theoretic constructions out of things we believe in.

This provides a criterion of identity for propositions. If A is a given sentence, let |A| be the proposition expressed by A. Then |A| and |B| are the same proposition iff |A| and |B| are true at the same possible worlds iff |A| and |B| are the same set of possible worlds.

Obviously, the criterion is thoroughly extensional, thus solving Quinean perplexities about propositions as intensional entities (where one can define

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entities of some kind or other as intensional iff (1) such entities are associated with extensions at the actual world, and (2) the associated extensions are insufficient to discriminate between intuitively distinct entities of that kind), or 'creatures of darkness', only to the extent that the notions adopted in the characterization – specifically, the notion *possible world* – are extensional. The point can be phrased conceptually (that is, in terms of the definition or elucidation of modal and intensional concepts), or ontologically (that is, in terms of identification of intensional entities with constructs out of worlds). Speaking conceptually, if it turns out that our notion *possible world* is characterized in terms of some modal concept, then we may have an illuminating reduction of the concept *proposition* in terms of another modal concept; but the reduction does not count as fully extensional. In terms of ontological reduction, if it turns out that possible worlds are themselves constructions out of intensional entities of some kind K, then we have just reduced a kind of intensional entities, namely, propositions, to the more basic kind of intensional entities K.

Notoriously, Lewis' modal realism promises such a fully extensional ontological reduction and conceptual explanation as its main theoretical benefit (a gain in ideology paid for in the coin of ontology, to speak in the Quinean way). Lewisian worlds are existing, non-actual (in Lewis' indexical sense of 'actual'), concrete maximal mereological sums of spatiotemporally interrelated individuals. Propositions are set-theoretic constructs out of worlds; individuals and sets are extensional; therefore propositions are reduced to extensional entities.

But the possible-worlds framework, whether phrased in terms of modal realism or not, has an equally notorious problem with impossible propositions: intuitively distinct impossible propositions – say, that swans are blue and it is not the case that swans are blue, that two is odd, that Charles is a married bachelor – hold at the same possible worlds, namely, none. If propositions are sets of worlds, believing or asserting one of these should be the same as believing or asserting the other, which it is not (ditto for necessary propositions, with the ensuing shortcomings concerning logical omniscience, etc.). At least some propositions appear to be hyperintensional: intensional equivalence is insufficient for identity. Barwise calls this the 'granularity problem', and I shall adopt the terminology.[1]

The granularity problem is a major shortcoming of possible-worlds accounts of propositions as compared with their main rival, the structured-propositions accounts, in which, roughly, propositions are built up from the semantic values of the sentences expressing them, and display a structure

[1] J. Barwise, 'Information and Impossibilities', *Notre Dame Journal of Formal Logic*, 38 (1997), pp. 488–515.

that basically mirrors the syntactic structure of the sentences themselves.[2] Treating propositions as set-theoretic constructions out of worlds leads to a very coarse individuation of propositions, and because of this it has been subject to apparently devastating attacks, for instance, by Scott Soames.[3]

II. IMPOSSIBLE WORLDS TO THE RESCUE?

In response to the granularity problem, several authors have proposed that worlds semantics should be extended so as to include *impossible* worlds.[4]

What are impossible worlds? One can find four main definitions in the literature. Ordered from the more to the less general, they are the following.

First, dual to the identification of possible worlds with ways things could have been comes the identification of impossible worlds with ways things could not have been. Not everything is possible, that is, some things just cannot happen. Anything that just cannot happen must be an impossibility; and these ways the world could not be are impossible worlds.[5]

Next, another definition has it that impossible worlds are worlds where the laws of logic are different. This is logic-relative: given some logic *L*, an impossible world is one in which the set of truths is not one that holds in any acceptable interpretation of *L*.

A third, more restrictive, definition claims that impossible worlds are worlds where the set of things that hold is not the set of things that hold in any *classical* interpretation (classical logicians, it is said, can consider a world where the law of excluded middle fails as a logically impossible world, since they take classical logic as the correct logic).[6]

A still more specific definition has it that an impossible world is a world where sentences of the form A and ¬A hold, against the law of non-contradiction.[7]

[2] See J. King, 'Structured Propositions', in E.N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/propositions-structured.

[3] See S. Soames, 'Lost Innocence', *Linguistics and Philosophy*, 8 (1985), pp. 59–71, and 'Direct Reference, Propositional Attitudes, and Semantic Content', *Philosophical Topics*, 15 (1987), pp. 47–87.

[4] See V. Rantala, 'Impossible World Semantics and Logical Omniscience', *Acta Philosophica Fennica*, 35 (1982), pp. 106–15; T. Yagisawa, 'Beyond Possible Worlds', *Philosophical Studies*, 53 (1988), pp. 175–204; G. Priest, 'What is a Non-Normal World?', *Logique et Analyse*, 35 (1992), pp. 291–302; W. Lycan, *Modality and Meaning* (Dordrecht: Kluwer, 1994); D. Nolan, 'Impossible Worlds: a Modest Approach', *Notre Dame Journal of Formal Logic*, 38 (1997), pp. 535–72; J. Pasniczek, 'Beyond Consistent and Complete Possible Worlds', *Logique et Analyse* , 161 (1998), pp. 121–34.

[5] See J. Beall and B. van Fraassen, *Possibilities and Paradox* (Oxford UP, 2003); G. Restall, 'Ways Things Can't Be', *Notre Dame Journal of Formal Logic*, 39 (1997), pp. 583–96.

[6] See Priest, 'Editor's Introduction', *Notre Dame Journal of Formal Logic*, 38 (1997), pp. 481–7.

[7] See W. Lycan, *Modality and Meaning*, p. ✱✱.

Impossible worlds are called for if we claim that we are capable of considering logically impossible situations, and of making discriminations about what goes on at them.[8] Worlds semantics for minimal logic includes non-normal worlds in which excluded middle and *ex falso quodlibet* (that is, the law according to which a contradiction entails everything) fail. The former also fails in standard Kripke semantics for intuitionist logic. It seems that we refer to these worlds when we evaluate such counterlogical conditionals as 'If intuitionist logic were the correct logic, then excluded middle would fail' (true); and 'If intuitionist logic were the correct logic, then *ex falso quodlibet* would fail' (false). Anyone who understands intuitionism, or minimal logic, or quantum logic, etc., knows how things would be if one of these logics were correct (assuming they are not).

The same goes for counterpossible mathematical reasoning if one supposes that mathematical truths hold at all possible worlds: if the axiom of choice were false (assuming it is true), then it would follow that the cardinals are not linearly ordered, but presumably it would not follow that $7 + 5 = 13$. Discourse on ways things could not be has its own logic in a broad sense: some inferences in it are correct, some are not.

Impossible worlds are supposed to help with impossible propositions and the granularity problem. One can have an impossible world w_A at which some swan is blue and not blue, a distinct impossible world w_B at which Fermat's last theorem is false, and yet another distinct impossible world w_C at which bachelors are married but swans and Diophantine equations behave wisely. According to impossible-worlds theorists, these are three ways the world could not have been. This intuitively suggests that the kingdom of the absurd is not like Hegel's night, in which all cows are black. Impossible worlds allegedly allow fine-grained distinctions unavailable in standard possible-worlds semantics. That |A| is an impossible proposition now does not mean that |A| is an empty set of worlds, but rather that |A| includes only impossible worlds.

III. THE PARITY THESIS

This line of argumentation by itself does not establish the ontological status of such entities of recalcitrant bent as impossible worlds. Famously, the two main options in the metaphysics of modality are Lewis' modal realism, and ersatzism (or actualism, or abstractionism) in its various forms. Are impossible worlds Lewisian or ersatz worlds? Most if not all supporters of

[8] For this line of argumentation on behalf of impossible worlds, see F. Berto, *How to Sell a Contradiction* (London: King's College Publications, 2007).

ways things could *not* be maintain neutrality on this issue. We find early impossible-worlds theorists such as Rescher and Brandom advancing what they call the 'parity thesis':

> We are not concerned to argue that possible worlds should be considered as part of the furniture of the universe, only that there is nothing to choose between standard and non-standard possible worlds [i.e., impossible worlds] in this regard.... Recall that we do not want to argue that we should treat any possible world as real, only that the considerations which can be advanced in favour of so treating standard possible worlds apply equally to non-standard possible worlds.[9]

Here is Graham Priest:

> As far as I can see, any of the main theories concerning the nature of possible worlds can be applied equally to impossible worlds: they are existent non-actual entities; they are non-existent objects; they are constructions out of properties and other universals; they are just certain sets of sentences.... There is, as far as I can see, absolutely no cogent (in particular, non-question-begging) reason to suppose that there is an *ontological* difference between merely possible and impossible worlds.[10]

However, each option on the metaphysical status of impossible worlds which sticks to the parity thesis has rather unpalatable consequences. After listing such consequences in the following section, I shall explore in §V an alternative approach, its advantages and its potential limits. The approach looks *prima facie* very interesting, especially for those who aim at (a) refining the possible-worlds apparatus in order to deal with impossibilities more satisfactorily, while at the same time (b) retaining the alleged capacity of modal realism to provide a fully reductionist account of modalities. (However, as an anonymous referee has pointed out to me, the strategy will sound less appealing to those who have no particular problems with primitive modalities and non-reductive accounts of intensional phenomena. More on this in what follows.) As I shall show, the strategy entails rejection of the parity thesis: possible and impossible worlds, metaphysically speaking, are *not* of a kind.

IV. THE DILEMMA

The dilemma, quickly put, is the following.

(a) If impossible worlds are taken as concrete entities of the same kind as Lewisian possible worlds, then we face two drawbacks: first, we seemingly have to resort to conceptually primitive modality in order to delimit

[9] N. Rescher and R.B. Brandom, *The Logic of Inconsistency* (Oxford: Blackwell, 1980), pp. 64–5.
[10] Priest, 'Sylvan's Box', *Notre Dame Journal of Formal Logic*, 38 (1997), pp. 573–81, at pp. 580–1.

impossible from possible worlds, thereby losing the alleged main advantage of modal realism over rival accounts of modality. Secondly, we have to admit that impossible propositions can be true at the *actual* world – which is quite hard to swallow.

(b) Impossible worlds may then be taken as ersatz worlds. Since ersatzism comes in various forms, this horn of the dilemma embeds some sub-dilemmas. Each theory of impossible propositions inherits the limits of ordinary accounts of propositions in terms of ersatz possible worlds. Each of these theories has to resort to intensional entities in its explanation of what ersatz worlds are, or to primitive modal notions (most often, to both). So the ontological reduction of propositions to more basic entities is still not extensional; or the definition of propositions as sets of worlds is conceptually modal (most often, both).

The main proponent of option (a) is Yagisawa, with his extended modal realism (EMR). Yagisawa claims that one of the main advantages of (EMR) is that it preserves the capacity of Lewis' modal realism to provide a purely extensional ontological identification of propositions with sets of worlds, while at the same time making the fine-grained distinctions whose unavailability within the ordinary possible-worlds framework produces the granularity problem. However, (EMR) with its concrete impossible worlds faces the two aforementioned drawbacks, which can be explicated as follows.

First, the alleged main advantage of Lewisian modal realism, namely its provision of an extensional, non-modal account of modal concepts, is put at stake by the admission of concrete impossible worlds. Once an impossible world enters the stage, the standard modal clause for possibility

P. It is possible that A iff there is a world w such that, at w, A

becomes false from right to left. So (EMR) needs a principle which restricts the quantification in the right-hand side of the biconditional to possible worlds; how to do that without making use of modal notions is an unsolved problem.[11]

The second objection comes from Lewis himself. If one is a modal realist, 'at world w' works as a restricting modifier: its main task consists in restricting the quantifiers within its scope to parts of w. If so, then it should make no difference with respect to the connectives, and should distribute through them. This means in particular that

$$\text{At } w\text{: } (A \wedge \neg A)$$

entails

$$\text{At } w\text{: } A \wedge \neg(\text{At } w\text{: } A).$$

[11] On this point, see J. Divers, *Possible Worlds* (London: Routledge, 2002), p. 69.

Thus within (EMR) any inconsistency at some impossible world automatically spills over into an inconsistency at the actual world. If a way the world might not be is a way some concrete impossible world is, impossibilities are out there: there exist *impossibilia* that instantiate them.

Yagisawa bites the bullet and replies that in order to speak the truth about contradictory things, one has to contradict oneself. But although the claim that there are things about which we speak truly in contradiction, that is, dialetheism,[12] has nowadays gained a certain respectability, this will not do for any non-dialetheist modal metaphysician, that is, for the large majority of them.

Suppose, then, one turns to option (b), ersatzism. Ersatz impossible worlds are endorsed, e.g., by Mares and Vander Laan,[13] and all hands agree that such worlds come at no great ontological or theoretical cost, once one has accepted ersatz possible worlds. After all, ersatz worlds are abstract: they account for impossibilities not by instantiating them, but by representing them in some way or other.

It is unlikely that a single objection can be raised against all ersatz accounts of impossible worlds. But there is a general danger for ersatz-impossible-worlds theorists, namely, a problem of *modal circularity*. This is hinted at by Stalnaker, who admits that the notion *ersatz impossible world* as such is unproblematic enough, but has qualms about its overall utility.[14] Some of the main ersatz accounts of possible worlds take them as maximal consistent sets of propositions.[15] Given this approach, it is natural to have impossible worlds as sets of propositions which are locally inconsistent and/or incomplete (as suggested by Lycan, and developed by Vander Laan). But now, Stalnaker points out, the machinery obviously cannot perform any explanatory work with respect to *propositions*: if one analyses worlds as sets of propositions, one cannot then analyse (possible and impossible) propositions as sets of worlds. So, Stalnaker concludes, the notion *impossible world* cannot do any interesting philosophical job in this respect.

Things get better with ersatz accounts of worlds which avoid characterizing them directly in terms of propositions. But the danger is still there,

[12] See F. Berto and G. Priest, 'Dialetheism', in Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/dialetheism; Berto, 'Meaning, Metaphysics, and Contradiction', *American Philosophical Quarterly*, 43 (2006), pp. 283–97, and '*Adynaton* and Material Exclusion', *Australasian Journal of Philosophy*, 86 (2008), pp. 165–90.

[13] E. Mares, 'Who's Afraid of Impossible Worlds?', *Notre Dame Journal of Formal Logic*, 38 (1997), pp. 516–25; D. Vander Laan, 'The Ontology of Impossible Worlds', *Notre Dame Journal of Formal Logic*, 38 (1997), pp. 597–619.

[14] See R. Stalnaker, 'Impossibilities', *Philosophical Topics*, 24 (1996), pp. 193–204.

[15] See R.M. Adams, 'Theories of Actuality', *Noûs*, 8 (1974), pp. 211–31, repr. in M.J. Loux (ed.), *The Possible and the Actual* (Cornell UP, 1979), pp. 190–209; W. Lycan, 'The Trouble with Possible Worlds', also in *The Possible and the Actual*, pp. 274–316.

and it is a well known one, for it dogs such theories independently of their admitting ersatz impossible worlds besides possible ones.

Plantinga takes worlds to be particular states of affairs.[16] Ontologically speaking, states of affairs are suspiciously similar to propositions, and in any case, 'it is unlikely that one could have the concept of some particular state of affairs (say, that of Jimmy Carter's being president) without having the concept of the corresponding proposition (here, the proposition that Jimmy Carter is president)'.[17] Besides, possible worlds are defined within Plantingan ersatzism as both maximal and possible states of affairs; that a state of affairs which is a world *w* is maximal means that for any state of affairs *s*, either *w* includes *s* or *w* precludes it, where 'includes' means that it is not possible that *w* obtains without *s* obtaining, and 'precludes' means that it is not possible that *w* obtains together with *s*. Overall, we still have possibility as a primitive notion, and one which is heavily exploited in the definitions.

So-called nature ersatzism, or Stalnakerian ersatzism, takes worlds, as total ways things might be, to be world-natures, or particular structured properties, or complete properties of maximal individuals.[18] This may improve things with respect to the circularity issue, but it has long been recognized that the result is not comparable with the capacity of Lewisian realism to provide fully extensional ontological identifications: world-natures or world-properties, just like obtainable states of affairs, are intensional *par excellence*.[19] These species of ersatzism can offer, at best, 'intra-intensional ontological identifications'[20] – in particular, a reduction of such intensional entities as propositions to allegedly more primitive intensional entities. So propositions are reduced, at best, to more basic creatures of darkness.

Linguistic ersatzism fares better. Worlds are taken as world-books, maximal-complete stories, that is, sets of sentences of a 'worldmaking' language (Carnap's state-descriptions, Jeffrey's complete consistent novels, etc.).[21] It is easy to admit impossible worlds of the same kind, that is, world-books which are occasionally inconsistent (and, say, incomplete).

Troublemakers could point out that sentence types, as abstract entities, dangerously resemble propositions too. This fear may be dispelled, as Lewis himself suggested (*On the Plurality of Worlds*, p. 143) by taking the words of the worldmaking language as the sets of their concrete tokens (particular

[16] A. Plantinga, *The Nature of Necessity* (Oxford: Clarendon, 1974), and 'Two Concepts of Modality', *Philosophical Perspectives*, 1 (1987), pp. 189–231.

[17] Loux (ed.), *The Possible and the Actual*, p. 50.

[18] See Stalnaker, 'Possible Worlds', *Noûs*, 10 (1976), pp. 65–75, repr. in Loux (ed.), *The Possible and the Actual*, pp. 225–34.

[19] Loux (ed.), *The Possible and the Actual*, p. 49.

[20] Divers, *Possible Worlds*, p. 197.

[21] See R. Carnap, *Meaning and Necessity* (Chicago UP, 1947); R. Jeffrey, *The Logic of Decision* (Chicago: McGraw-Hill, 1965).

inscriptions, for instance). Token words are concrete individuals, so world-books are set-theoretic constructions out of these concrete individuals. Reducing possible and impossible propositions to these (consistent and inconsistent) set-theoretic constructions seems to be reduction to a safe and sane extensional ontology.

But the main objection raised by Lewis against possible worlds taken as world-books is also well known: linguistic ersatzism must resort to conceptually primitive modality. The same applies, of course, to an expanded linguistic ersatzism including inconsistent-impossible world-books. For are not possible world-books, unlike impossible ones, just sets of sentences which *could* all be true together?

Lewis concedes that one may distinguish possible from impossible world-books, when the impossibilities at issue are of a narrow logical character, by purely syntactic means, that is, by deeming impossible any world-book which includes both A and ¬A for any A. But not all absolute impossibilities are narrowly logical in this sense: that some subatomic particle *p* is both positively and negatively charged, $Pp \wedge Np$, or that Charles is a married bachelor, $Mc \wedge Bc$, for instance, may count as absolute impossibilities. So we have syntactically consistent impossible world-books including both *Pp* and *Np*, or both *Mc* and *Bc*. One might reply that these are metaphysical, or (in Charles' case) so-called analytic, impossibilities, and that metaphysical and analytic impossibilities are not absolute impossibilities: they are narrower than purely logical impossibility. The only absolute impossibilities are the syntactically specifiable logical ones. To this, Lewis replies (p. 153) that the answer would just 'falsify the facts of modality', and many agree.

If, on the other hand, we resort to non-logical axioms working as 'meaning postulates' or 'metaphysical postulates' to rule out such sentences (thus we add to our theory such things as $\neg\exists x(Mx \wedge Bx)$, or $\neg\exists x(Px \wedge Ax)$), according to Lewis we have to resort to primitive modality again. Lewis' main argument goes thus. The worldmaking language is supposed to be rich enough to include both micro-descriptions in terms, say, of fundamental logical particles (Lewis calls them 'local descriptions'), and macro-descriptions in terms of donkeys and bachelors ('global descriptions'). It may be the case that some non-logical impossibilities lurk in the form of incompatibilities between local and global descriptions. We may have a maximal set of sentences including local descriptions of the basic properties and spatiotemporal arrangement of all the basic particles of the world, and 'lo, we have implied [at the micro-level] that there is a talking donkey, or we have fallen into inconsistency if we also say explicitly [at the macro-level] that there is not' (p. 155). To rule out these impossibilities we need additional bridge axioms connecting local and global descriptions, 'conditionals to the

effect that *if* – here follows a very long, perhaps infinitary, description of the arrangement and properties of the point particles – *then* there is a talking donkey' (pp. 155–6). Since the required axioms would be infinitely many, or in any case impossible to specify in practice, linguistic ersatzers can never completely state their theory without resorting to primitive modality.

To generalize: the introduction of ersatz impossible worlds makes apparent for any species of ersatzism the first problem faced by Yagisawa's extended modal realism, that is, the problem of how to qualify the standard possibility clause

P. It is possible that A iff there is a world *w* such that, at *w*, A

without resorting to primitive modality. The point is that each of the species of ersatz modal realism affords the resources to produce ersatz impossible worlds, so to speak, out of the same stuff ersatz as possible worlds are made of; ersatz impossible worlds are *of the same kind as* their possible mates.

In the end, the supporter of ersatz impossible worlds may bite the bullet. After all, if the above arguments are right, in order to resolve the granularity problem we have to resort to primitive modality in any case – even if we are extended modal realists. Specifically, concerning propositions, we can keep identifying them with sets of worlds; by admitting impossible worlds, we can solve the granularity problem and differentiate impossible propositions. To be sure, worlds are themselves characterized in terms of some other intensional entity, or in such terms that primitive modality is conceptually unavoidable. At least we have reduced propositions to more basic modal notions, and that is about the best we can do. As an impossible-worlds theorist has it,

> Not all possible-worlds accounts purport to provide an analysis of modality. An ontology of possible states of affairs, for example, might make no attempt to explain what a possible state of affairs is without use of the notions of possibility, necessity, or some other modal term. In fact, it is rather commonly thought that any such attempt would be futile because possibility, necessity, and their ilk, form, as it is said, a tight circle of interrelated modal notions, none of which can be properly analysed without recourse to some element of the circle.[22]

Linguistic ersatzism does fare better than other forms of ersatzism anyway, for it is committed (provided Lewis' argumentation succeeds) to primitive *conceptual* modality, by having to resort to primitive modal notions in order to differentiate possible and impossible worlds. But it is not committed to primitive intensional *entities*, for the basic ontological kinds admitted by linguistic ersatzism can be just sets and individuals.

[22] Vander Laan, 'The Ontology of Impossible Worlds', p. 608.

V. HYBRID MODAL REALISM

But suppose one still wants to retain the advantages of both worlds, ersatz and genuine, when it comes to impossibilities and impossible propositions. Suppose, that is, (a) one wants to employ a modal framework including both possible and impossible worlds to account for the granularity problem; (b) one wants to have the capacity to discriminate between distinct impossibilities and impossible propositions, without issues of circularity, and better, with a reduction of propositions to fully extensional entities (*contra* ersatzism); but also (c) one wants to avoid the unwelcome consequences of concrete impossible worlds instantiating impossibilities, such as having true contradictions invade the actual world, *contra* (EMR). One could then try the following hybrid solution: (1) go realist when it is about *possible* worlds, and (2) exploit the set-theoretic machinery of modal realism to represent different *impossible* worlds and impossible propositions as distinct *ersatz* abstract constructions.

I shall call this position *hybrid modal realism* (HMR). As announced above, whereas ersatzists can live with their ersatz impossible worlds and cheerfully swallow a non-reductive account of modalities and intensional phenomena, (HMR) is especially palatable for those who desire a reductive modal theory that also effectively addresses the granularity problem. Moreover, (HMR) entails rejection of the parity thesis – a thesis which, as I have said, has been endorsed by most if not all impossible-worlds theorists.

How is this approach to be pursued in detail? By rejecting concrete, genuine impossible worlds and *impossibilia* inhabiting them, (HMR) cannot represent distinct impossibilities directly, by instantiation, as it does with distinct possibilities. However, it seems that (HMR) can account at least for *some* distinct impossible situations and impossible propositions via distinct ersatz-abstract constructions. Its ontology has the resources to take ersatz impossible worlds and propositions as set-theoretic constructions out of genuine, concrete possible worlds.[23] The account may go as follows.

Genuine, concrete possible worlds are the basic stuff. Basic, atomic propositions, such as that swans are blue or that Charles is married, are sets of possible worlds. Distinct impossible situations cannot be represented by genuine impossible worlds and their inhabitants, for there are no such things. But distinct impossible situations can be represented by distinct world-books or world-stories, taken as sets constructed out of atomic

[23] The idea comes from a suggestion found in Divers, *Possible Worlds*, p. 313, fn. 19.

propositions. If book-stories are sets of atomic propositions, then they are sets of sets of genuine possible worlds.[24]

So we can have distinct inconsistent world-books, taken as sets of mutually inconsistent atomic propositions, representing intuitively distinct impossibilities. For instance, take two distinct contradictions, $A \wedge \neg A$ and $B \wedge \neg B$, where $|A|$ and $|B|$ are ordinary, contingent propositions – say, that swans are blue and that Charles is married. Consider a simplified model $\mathbb{M} = \{w_1, w_2, w_3, w_4, w_5\}$ as the set of genuine, concrete possible worlds. Say that $|A| = \{w_1, w_2\}$, so $|\neg A| = \{w_3, w_4, w_5\}$, i.e., the set-theoretic complement of $|A|$ in $\mathbb{M}$. Say that $|B| = \{w_2, w_3, w_4\}$, so $|\neg B| = \{w_1, w_5\}$. The impossible proposition that A and $\neg A$ just is the inconsistent set $\{|A|, |\neg A|\} = \{\{w_1, w_2\}, \{w_3, w_4, w_5\}\}$, and the impossible proposition that B and $\neg B$ just is the inconsistent set $\{|B|, |\neg B|\} = \{\{w_2, w_3, w_4\}, \{w_1, w_5\}\}$. So we have that $\{\{w_1, w_2\}, \{w_3, w_4, w_5\}\} \neq \{\{w_2, w_3, w_4\}, \{w_1, w_5\}\}$, and so these are distinct inconsistent sets of sets of worlds – they are sets of *mutually disjoint* sets of genuine worlds: what makes them inconsistent is that their subsets have no common element. That no genuine world appears in each of them shows that the propositions such subsets consist in can be jointly true in no possible world.

Next, take three distinct atomic contingent propositions, $|C|$ = that Nassau Street runs east–west, $|D|$ = that the railroad nearby runs north–south, and $|E|$ = that Nassau Street is parallel to the railroad nearby.[25]

Say that $|C| = \{w_1, w_5\}$, $|D| = \{w_2, w_3, w_5\}$, and $|E| = \{w_1, w_2, w_3, w_4\}$. At some possible world, namely w_5, Nassau Street runs east–west and the railroad nearby runs north–south; at some possible world, namely w_1, Nassau Street runs east–west and is parallel to the railroad nearby; and at some possible worlds, namely w_2 and w_3, the railroad runs north–south and is parallel to Nassau Street. But the three propositions are jointly inconsistent, so $C \wedge D \wedge E$ cannot be true in any possible world. Such an impossible proposition can be represented by the set $\{|C|, |D|, |E|\} = \{\{w_1, w_5\}, \{w_2, w_3, w_5\}, \{w_1, w_2, w_3, w_4\}\}$. What makes this set inconsistent is again the fact that no genuine world is a member of each of its subsets.

Of course, some such set-theoretic constructions will represent also *possible* worlds. Take another simplified model $\mathbb{N} = \{w_1, w_2, @\}$ as the set of genuine, possible worlds. This has a distinguished element @ = the actual world. Then the atomic contingent propositions available with respect to this model are $|F| = \{w_1\}$, $|G| = \{w_2\}$, $|H| = \{@\}$, $|I| = \{w_1, w_2\}$, $|L| = \{w_1, @\}$, $|M| = \{w_2, @\}$. We can build a set $\{|G|, |I|, |M|\} = \{\{w_2\}, \{w_1, w_2\}, \{w_2, @\}\}$. With respect to our simplified model $\mathbb{N}$, this is a representation of possible world w_2, including all and only the atomic propositions true at that world. Now take $\{|H|, |L|, |M|\} = \{\{@\}, \{w_1, @\}, \{w_2, @\}\}$; that this is a representation of the actual world is shown by its having @ included in each of its subsets – it is the set of all and only the atomic propositions true at @. Since $|M| = \{w_2, @\}$, $|\neg M| = |F| = \{w_1\}$ since this is the set-theoretic complement of $|M|$ with respect to N. So take the set $\{|H|, |L|, |M|, |\neg M|\} = \{\{@\}, \{w_1, @\}, \{w_2, @\}, \{w_1\}\}$. This is another inconsistent set, representing an impossible situation which is exactly like the actual world, except that in it both M and $\neg M$ hold. And so on.

How are we to regard world-books that represent possible worlds? These world-books, being abstract set-theoretic constructions, are not (genuine, concrete) possible worlds. They are not even competitors with concrete worlds, but are delivered free, so to speak, once we believe in (genuine, concrete) possible worlds and set-theoretic constructions out of them.[26]

If more than one possible world appears in each subset of such set-theoretic constructions, then we have an incomplete description – a description which is true at more than one possible world. If exactly one possible world appears in each subset, we have a consistent, complete description, that is, the description of a possible world. If the world at issue is @, this is a description of the actual world. If no possible world at all is a member of each subset, then we have an inconsistent set-theoretic construction.[27]

VI. EVALUATING (HMR)

Prima facie, (HMR) has some theoretical virtues. First, there is no circularity in the order of explanation. We begin with genuine, concrete Lewisian

[24] In 'Ways Things Can't Be', Greg Restall presents an account which is in some ways close to the one endorsed here, except that it takes impossible worlds directly as *sets of* possible worlds, not as sets of sets of them. The approach is formally well developed and presents a formal definition of 'truth at a set of possible worlds' as an extension of the standard possible worlds semantics. But Restall's account is forced to endorse a paraconsistent logic (specifically, Priest's three-valued LP), just because it is read off the semantics, and the logical connectives need to be (re-)defined accordingly. A possible advantage of the account sketched here is that, on the contrary, by itself it requires no logical revisionism.

[25] The example comes from Lewis, 'Logic for Equivocators', *Noûs*, 16 (1982), pp. 431–41.

[26] As an anonymous referee has pointed out to me, this would be in accordance with the spirit of Lewis' idea that modal ontology is not a matter of finding which objects really are the worlds, but of identifying which objects are suited to play the right theoretical roles.

[27] One may notice a certain trade-off between 'impossible worlds' and 'impossible propositions'. This is not accidental. What counts in the theory as a single 'impossible proposition' (such as that A and $\neg A$), and what as an impossible world-book, is *relative* to the 'model', taken as the set of concrete possible worlds we start with. For instance, with respect to the sample model $\mathbb{N} = \{w_1, w_2, @\}$ introduced above, the set $\{|H|, |L|, |M|, |\neg M|\} = \{\{@\}, \{w_1, @\}, \{w_2, @\}, \{w_1\}\}$ can count as an impossible world-book; but with respect to a different model including more basic possible worlds, this can be regarded as an impossible conjunctive proposition – the proposition that $H \wedge L \wedge M \wedge \neg M$ – which is not an impossible world-book.

worlds (and they are automatically possible, given that they exist). Next, we claim that propositions, possible and impossible, are abstract set-theoretic constructions out of genuine worlds. Impossible worlds are taken as ersatz worlds, in the sense that they themselves are abstract set-theoretic constructions out of genuine worlds (they are sets of sets of such worlds).

Secondly, the ontological reduction is fully extensional. What we believe in, at the base level, are concrete Lewisian worlds, that is, maximal mereological sums of spatiotemporally related individuals, and sets. Next, we believe in set-theoretic constructions that represent impossible propositions (such as $\{|A|, |\neg A|\} = \{\{w_1, w_2\}, \{w_3, w_4, w_5\}\}$) and impossible worlds (such as $\{\{@\}, \{w_1, @\}, \{w_2, @\}, \{w_1\}\}$, which counts as an impossible world-book with respect to the model $\mathbb{N} = \{w_1, w_2, @\}$), for we believe in set-theoretic constructions out of things we already believe in.

Thirdly, the basic clause for possibility

P. It is possible that A iff there is a world w such that, at w, A

works without resorting to primitive modality, if we take the quantification in the right-hand side to range only over concrete, Lewisian genuine worlds. That is, we rule out from the domain of quantification *abstract* objects of any kind – including set-theoretic constructions out of concrete worlds.

Fourthly, (HMR) has no problem with the Lewisian objection to impossible worlds: inconsistencies at impossible worlds do not automatically spill over into the actual world, nor into any possible, concrete, non-actual Lewisian world. For impossible worlds are world-books – abstract set-theoretic constructions. So 'at impossible world w_A' means 'according to the world-book w_A'. From the fact that according to w_A: (A and it is not the case that A), it does not follow that according to w_A: A and it is not the case that according to w_A: A.

To what extent does (HMR) solve the granularity problem by representing intuitively distinct impossibilities as distinct? This is an open problem. Everything works fine in so far as one represents impossibilities which are adequately phrased as explicit contradictions or, as it were, structured and purely logically specifiable impossibilities of the form $A \wedge \neg A$. Things are less obvious when it comes to representing distinct impossibilities which, despite being structured in the sense that they are encoded by molecular propositions, do not appear to be strictly logical, such as Charles being married and a bachelor, or some subatomic particles being both positively and negatively charged. Could one try to reduce these impossibilities to strictly logical ones by means of bridge principles, that is, by adding as an axiom that if something is a bachelor then it is unmarried, or that if something has positive charge then it cannot have negative charge? Things

get more complicated when we consider that some impossible propositions are just *atomic* ones – that two is odd, for instance – which is why in the examples above I have been speaking of *contingent* propositions to begin with. There *may* be a way of analysing these atomic impossibilities into set-theoretic constructions out of worlds after translating them into structured impossibilities; but at present I have no idea of how it could be done systematically. In any case, this would probably take us quite far from the 'surface grammar' of the sentences expressing such impossibilities.

One of the well known advantages of the structured account of propositions over the possible-worlds account is that the former keeps track of the semantic values of subsentential expressions, and so it can treat propositions expressed by atomic sentences as structured entities built up from the semantic values of the respective subsentential constituents. It is clear that sets of worlds and set-theoretic constructions out of worlds cannot in general achieve the same level of fine-grainedness. However, one can also exploit the resources of Lewisian ontology to provide an extensional ontological individuation of subsententially structured atomic propositions: we just need more complex set-theoretic constructions out of *possibilia*, that is, out of parts of Lewisian concrete possible worlds.

According to some mainstream accounts, structured propositions are constructions out of individuals, properties and relations. As is well known, modal realism aims at providing a fully extensional account also for properties and relations: they are identified with sets of their instances, this- and other-worldly individuals. This helps to discriminate between accidentally coincident properties; having kidneys and having a heart are distinct, for there are possible worlds having parts which have a heart but no kidneys (and *vice versa*). This suggests that one could mimic within the modal realist framework the account of structured propositions which identifies them as constructions out of individuals, properties and relations. Such an identification, in the modal realist framework, would still be an extensional reduction.

But what about necessarily coincident but intuitively distinct properties, such as being trilateral and being triangular, or being a theorem of first-order logic and being a logical truth? If properties are sets of individuals, in order to differentiate such properties while retaining extensionality we may need ersatz impossible individuals – something which is triangular but not trilateral, etc. These may be taken as sets of (structured) properties. We can build structured properties out of unstructured basic properties and relations. Lewis' example concerns triangularity and trilaterality. We start with the two binary relations A, being an angle of, and S, being a side of. Next, we consider the relation R which holds between a basic property X and a basic relation Y iff X is the property of being something to which

exactly three things bear the relation Y. Triangularity is the unique property which bears relation R to A, and trilaterality is the unique property which bears relation R to S. So one can take triangularity as <R, A> and trilaterality as <R, S>. These are necessarily co-extensive but structurally distinct properties.

Hybrid modal realism, so far, is in any case just the sketch of a theory. But those who endorse a Lewis-style reductionism may find it appealing; its (partial) success in distinguishing at least some intuitively distinct impossibilities, while at the same time avoiding ontological intensionality and conceptually primitive modality (at least, to the extent that modal realism avoids them), shows that it is an option worth being pursued by Lewisians. This option entails rejection of the parity thesis: possible and impossible worlds, after all, may not be of a kind.[28]

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