Cotes' Queries: Newton's Empiricism and Conceptions of Matter

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1 Introduction

The relation of Isaac Newton's natural philosophy to his method of inquiry is of central importance to Newtonian scholarship. In this paper, we investigate this relation as it concerns Newton's ideas about the nature and measure of matter. We argue that a conflict between two conceptions of "quantity of matter" employed in a corollary to proposition 6 of Book III of the *Principia* illustrates a deeper conflict between Newton's view of the nature of extended bodies and the concept of mass appropriate for the theoretical framework of the *Principia.* The conflict was first noted by the editor of the *Principia*'s second edition, Roger Cotes, and put to Newton by what we call the "two globes" objection. The objection makes it clear that two different measures of "quantity of matter" are at work in Newton's thinking, measures that are related to two competing views on the nature of matter. On what we call the "dynamical conception of matter"—the dominant conception in the *Principia*—quantity of matter is measured through a body's response to impressed force. On what we call the "geometrical conception of matter," quantity of matter is measured by the volume a body impenetrably fills. Newton's commitment to the geometrical conception comes out in his discussion with Cotes through the assumption that all atoms of matter have a uniform specific gravity; that is, that the inertia of completely filled bodies is proportional to their volume. On the dynamical conception of matter, there is no reason for this to be the case. A purely dynamical conception is consistent with the idea that the inertia of completely filled bodies is not in fixed proportion to their volumes, or even that a Boscovichian non-

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extended point particle can constitute a body. By analyzing the exchange with Cotes (as well as evidence from *De gravitatione*), we show that before Cotes' prodding in 1712, Newton held both conceptions of matter and apparently saw no conflict between them.

We trace Newton's failure to recognize the conflict between the dynamical and geometrical conceptions of matter to the fact that Newton allowed for the justification of natural philosophical claims by two types of *a posteriori*, empiricist methodologies, both of which turn away from the type of *a priori* speculation Newton found deplorable in Descartes. Although both proceed "from the phenomena," we argue that there are important differences between them. The first, which underlies the dynamical conception, is highly mathematical and relies on a nuanced interplay between specific phenomena and their theoretical description. Recent work by George Smith, Bill Harper, and Howard Stein (following the original characterization of the "Newtonian Style" by I.B. Cohen) has shown how this methodology was used in the *Principia* to justify the theoretical framework provided by the laws of motion. Drawing on their analyses, we briefly characterize this method, using Newton's reply to Cotes' better-known "invisible hand" objection as an illustration. The second empiricist method, which underlies the geometrical conception, also proceeds from the phenomena, but does not draw on the technical resources used in the first. Instead, its conclusions are intended to follow from general features of our experience (in conjunction with broad natural philosophical assumptions), in a way articulated most clearly in *De gravitatione* and through certain of Newton's examples in Rule III of the *Regulae philosophandi*. We argue that although both methods of inquiry are based on empirical considerations, the relationship of theory to evidence in each is different. Of course, centuries of debate attest to the difficulty in extracting from Newton's explicit methodological discussions a clear account of evidential warrant that fits the entirety of Newton's practice. We do not tackle this general question here. Instead, we merely highlight two different types of arguments from the phenomena endorsed by Newton and argue that he failed to clearly distinguish them. Consequently, he failed to recognize that one was not as secure as the other. In the *Principia* and *De gravitatione*, the two conceptions of matter are justified by these different types of arguments, yet prior to Cotes' "two globes" objection, Newton treated the two conceptions as if on equal footing, without recognizing their different sources of evidential warrant. Cotes' objection forced Newton to reconsider the status of the geometrical conception. Although he never drew general conclusions regarding the relation between his two methods of inquiry, he came to side with the method of inquiry that more elaborately tied empirical data to theoretical claims and, in the body of proposition III.6, framed the geometrical conception of matter only hypothetically. Given the deep-seated Cartesian and atomistic roots of the geometrical conception in Newton's thought, this was a profound shift.

We begin (in §2) by introducing the geometrical and dynamical conceptions of matter and the measures of quantity of matter associated with each. To do so, we review the empirical reasons that led Newton to abandon aether theories of gravitation and accept the existence of void spaces. Although the acceptance of void forced Newton to reject much of the Cartesian analysis of space and body, we show that the geometrical conception of matter present in *De gravitatione* (hereafter, *De grav*) and later work shows a lingering debt to Descartes, particularly in its geometrical method of quantifying matter. At the close of §2 we explicate the *a posteriori* method of inquiry that, for Newton, underlies the geometrical method of quantifying matter. In §3 turn to the *a posteriori* method that underlies the dynamical conception of matter. Drawing on these accounts of the contrasting methods, we then investigate the conflict between the two conceptions of matter in §4.

2 Newton's Conceptions of Matter

2.1 The Context: Newton's Empirical Case Against The Aether

In the years leading up to the *Principia*, Newton's natural philosophy underwent a "radical conversion" (to borrow R. S. Westfall's phrase); he abandoned the fundamental ideas of Cartesian natural philosophy and put in their place novel conceptions of space, body, and motion. This radical conversion was motivated in large part by Newton's rejection of the idea that the planets must be carried in their orbits by an aetherial vortex composed of subtle matter. In this section, we will focus on two crucial empirical reasons that led Newton to abandon the aetherial explanation of planetary motion and clarify how his *a posteriori* line of reasoning undermined the principles of Cartesian philosophy.

As early as 1664, Newton took the cause of terrestrial gravitation to be mechanical and formulated a mechanical aether theory akin to other contemporary theories. The central idea of these theories—that a body's weight could be explained in terms of a descending stream of aetherial fluid exerting pressure on the inner surfaces of a body—appears to have persisted in Newton's various versions of an aether hypothesis for gravity through the 1670s, even as the gravitational aether became more intricately tied to active principles inspired by his alchemical studies.[1]

[1] In response to Hooke's claim of priority in discovering the inverse square law, Newton referred Halley to a manuscript from 1675 ("Hypothesis Explaining the Properties of Light," sent to Oldenburg) that included an aether hypothesis which Newton claimed led naturally to an inverse square law (Newton (1977) II, p. 447; the manuscript sent to Oldenburg is reproduced in Newton (1977) I, pp. 364-6). Regardless of whether any of the various aether hypotheses lead to such quantitative results, from the mid 1660s onward they were a constant fixture in Newton's thoughts on gravitation. See "Of gravity and levity," in McGuire and Tamny (1983), pp. 362-5, 426-31, and Wilson (1976), pp. 192-5, regarding Newton's early views. Regarding the

The central role played by the gravitational aether in the 1670s makes its nearly complete (if temporary) disappearance from Newton's natural philosophy in the period preceding the composition of the *Principia* quite remarkable.[2] In the first edition of the *Principia* Newton gave two decisive empirical reasons for abandoning the aether. First, he became convinced that the planets and comets encounter negligible resistance to their motion. In first two theorems of the *De motu corporum in gyrum*, Newton derived Kepler's area law and the harmonic law for a central force *with no resistance*. The accuracy of Kepler's laws in describing planetary motions implied that there was no need to introduce a resisting force alongside the centripetal force holding the planets in their orbits.[3] Newton strengthened his case in later drafts and in the *Principia*. The persistence of planetary motion over thousands of years is incompatible with even very slight aetherial resistance (Herivel 1965, p. 302), since even slight resistance would lead to a steady decrease in quantity of motion. The motion of comets can also be ascribed to the same centripetal force as planetary motions, again without the need for a further force of resistance. But the motion of comets is particularly important because of their highly eccentric orbits and the existence of retrograde comets.[4] The negligible resistance encountered by the planets is compatible with an aetherial vortex in which the planets move with the aether, but it is much more difficult to reconcile the motion of comets, especially retrograde ones, with a vortex theory. Newton developed *De motu*'s argument that the celestial bodies move in non-resisting spaces into a more sustained attack on vortex theories in the *Principia*.

Second, Newton failed to detect aether resistance in a series of carefully designed pendulum experiments reported in Book II of the *Principia*. Based on the realization that a gravitational aether must penetrate to the inner surfaces of bodies—without such penetration, the aether's action could only depend on a body's surface area, not its "solidity" or bulk—Newton designed experiments to measure the *internal* resistance due to the aether.[5]

development of Newton's thought throughout the 1670s, and the many roles the aether played in Newton's natural philosophy, see Dobbs (1991), Chapter 4.

[2]See Dobbs (1988) for a detailed discussion of Newton's abandonment of the gravitational aether. For a review of the shifts in Newton's views regarding aether and void following the *Principia*'s first edition see McGuire (1966), pp. 103-110 *et passim*. Although Newton later considered an *immaterial* gravitational aether, he frequently repeated the two reasons discussed below for abandoning the hypothesis of a *material* aether.

[3]As Newton was well aware, even if there is no resistance Kepler's laws fail to hold *exactly* for universal gravity due to the perturbing effects of each planet on the other planets' orbits (Herivel 1965, p. 301). However, the departures from Keplerian motion due to these perturbations differ in character from the departures Newton expected for a resisting medium.

[4]Retrograde comets, such as Halley's comet, orbit the sun in the opposite direction to that of the planets. Newton also argued that the tails of comets show no sign of encountering resistance to a surrounding medium.

[5]See Newton (1999, General Scholium to Book II, Section 6, pp. 722-3). Note that *external* resistance may arise due to the air alone or the air and the aether conjointly, but the experiment is designed so that external resistance (as well as buoyancy of the air) is held constant. Cf. Kuhn (1970), pp. 106-8.

Newton constructed a pendulum consisting of a "round firwood box" suspended from a cord eleven feet long. He measured the oscillations of the empty box, and then filled the box with various heavy metals, adjusting the cord to the same length. The metal-filled box weighed 78 times as much as the empty, and so in the absence of internal resistance Newton believed the oscillations of the full pendulum bob (due to the bob's increased inertia) would take 78 times as long to decay. Newton initially assumed that filling the box would not change its *external* resistance. From the result that the decay only took 77 times as long, Newton concluded that the internal resistance must be over 5,000 times less than the external resistance. In the second and third edition of the *Principia*, Newton interpreted this to mean that *aether resistance* caused the damping. He wrote:

> This argument depends on the hypothesis that the greater resistance encountered by the full box does not arise from some other hidden cause but only from the action of some subtle fluid upon the enclosed metal (Newton 1999, p. 723).

However, in the first edition of the *Principia* (in a passage omitted in the second and third editions), Newton proposed a different cause: He wrote:

> But I suppose that the cause is very different [than the aether acting on the internal surfaces of the box]. For the times of the oscillation of the full box are less than those of the empty one, and therefore the resistance to the *external surface* of the full box is greater, by virtue of its velocity and the length of its oscillations, than to the empty box. From which it follows that the resistance [due] to the internal parts of the box is either zero or entirely insensible (translated in Kuhn 1970, pp. 106-7).

Newton's conclusion is phrased cautiously. He did not claim to settle the question of the aether's existence; instead, he inferred only that *if* there is an aether, then its *resistance* is either nil or negligible. However, he had also concluded on the basis of other experiments (reported in the same scholium) that the primary contribution to resistance is proportional to the *material density* of the fluid through which an object moves. Thus, the experiments gave Newton grounds to reject a *mechanical* aether, although with his usual care he did not make the stronger claim that they rule out an aether altogether.[6]

Although these considerations triggered Newton's radical conversion, they were not decisive for his contemporaries and successors. In Newton's later treatment in the *Principia*, fluid resistance arises primarily from the inertia of the fluid, and the dominant component of the force of resistance is proportional to ρv^2 (where ρ is the density of the fluid, and v is the relative velocity). Leibniz objected to this assumption in his correspondence with Clarke,

[6] See Smith (2001) for a thorough discussion of these experiments, Newton's treatment of resistance, and the changes made to this part of Book II between the first and second editions.

arguing that "it is not so much the quantity of matter as its difficulty in giving place that makes resistance" (Alexander 1956, Fifth letter, p. 65). Leibniz had earlier distinguished between two different sources of resistance, viscosity and density, and argued that the two make different contributions to the overall resistance for different types of fluids.[7] Drawing this distinction between different types of resistance opens up the possibility of an aetherial fluid that does not have resistance proportional to density—which would avoid Newton's arguments. In fact, as Smith (2001) has emphasized, the possibility is much easier to realize than Newton had anticipated. In 1752 D'Alembert showed that a fluid without viscosity has exactly zero resistance, completely undercutting Newton's proposal that the dominant contribution to fluid resistance arose from the inertia of the fluid. This error does not detract from Newton's genuine insight that a single force law was sufficient to account for planetary motions, but it does undermine Newton's empirical case against the aether. Of course, the problem was not solely with *Newton*'s arguments against the aether. Contemporary versions of aether theory were also based on misconceptions regarding fluids and the nature of resistance, and any aether theorist faced the daunting challenge of providing an account of how the aether produced gravitational effects without also causing appreciable resistance.[8]

2.2 The Geometrical and Dynamical Conceptions of Matter

Newton's rejection of a mechanical aether left him without a mechanical explanation of gravitation, along with an awareness of how difficult it would be to provide one. Here we focus on a fundamental consequence of this awareness for Newton's thought: he was forced to reconsider Descartes' doctrines regarding the nature of body and space and replace them with ones compatible with the existence of spaces nearly void of matter.

Newton's most sustained critical discussion of Descartes appears in *De grav.* The stated aim of the manuscript is the study of the gravitation and equilibrium of fluids, and it is written in the geometrical style, beginning with a series of definitions and closing with two theorems regarding inelastic fluids. Along the way, Newton clears the ground for his own definitions of space, body, and motion with a long philosophical discussion expressly devoted to undermining the corresponding Cartesian definitions—to "dispos[ing] of [Descartes'] *figmenta*", as it were. The main thrust of this long digression is that an adequate definition of motion requires an appropriate structure relating locations over time, assigning unique velocities to moving bodies, and suitably correlating motions to forces.[9] Descartes' plenum

[7] Leibniz was not unique in making such distinctions; Newton himself had also classified distinct types of fluids in which different mechanisms are the main source of resistance.

[8] See Aiton (1972) for an account of aether theories developed throughout the 18^{th} century.

[9] What is actually required for the dynamical theory of the *Principia* is the distinction between inertial and non-inertial motion; this only requires an affine connection (the structure needed to differentiate between straight and curved spacetime trajectories), and not the stronger structure that would be provided by

lacked the necessary structure, leaving Descartes with a definition of motion that failed to support distinctions he appealed to in developing his physical theory. Newton overcame this defect by introducing space as a distinct entity with a sufficiently strong structure, albeit an entity that did not fit neatly into traditional ontological categories.[10]

Before focusing on *De grav*'s positive theses, we should note that even in this overtly philosophical context Newton supported his arguments against Descartes with empirical evidence in favor of void space. On the basis of pendulum experiments that may have been the experiments discussed above or a precursor of them, Newton asserted that the resistance of the aether is "over ten or a hundred thousand times *less*" than the resistance of quicksilver (Newton 2004, p. 34).[11] Newton also took resistance to moving through a medium to be a consequence of the material nature of the medium's parts. As he put it, "if we set aside altogether every resistance to the passage of bodies, we must also set aside the corporeal nature [of the medium] utterly and completely" (Newton 2004, p. 34). This is because two bodies cannot simultaneously occupy the same region of space, and so, one body resists the passage of another body through the region it occupies. We noted above that this view was controversial, but if it is accepted then the failure to detect resistance is decisive evidence against the Cartesian plenum.

Rejecting the plenum posed a clear challenge to the Cartesian identification of extension as the principal attribute of body. In *De grav*, Newton singled out Descartes' so-called "elimination argument" (as presented in *Principles of Philosophy*, II.4 & II.11) as the main argument in favor of this thesis. According to Newton, Descartes argued that various sensory properties such as hardness, weight, and color can be abstracted from a body without endangering the status of that body *as a body*. Only the elimination of extension can destroy a body's corporeality, and so extension alone constitutes body's principal attribute,

identifying the "same position" over time; see Stein (1967). However, Newton seems not to have been entirely clear on this issue at the time of composition of the *De grav*; some of his criticisms of Descartes thus presume a stronger structure than necessary.

[10] For further analysis of Newton's criticisms of Descartes here and in the Scholium to the Definitions in the *Principia* see, for example, Stein (1967); Rynasiewicz (1995); DiSalle (2002).

[11] In *De grav* Newton briefly alluded to pendulum experiments but said almost nothing about them, and his stated conclusion is remarkably imprecise. B. J. T. Dobbs (1991, pp. 134-43 and 1988) argues that these pendulum experiments were performed between the composition of another manuscript, *De aere et aethere* (published in Newton 1962), and *De grav*. Westfall (1971), pp. 375-77, 341 reaches a similar conclusion, although he dates the sequence of events to 1679 rather than 1684. We find the ordering of events plausible (whether they occurred in 1679 or 1684), but there is insufficient textual evidence to make a compelling case. Jim Ruffner has also brought discussions of the aether in (undated) manuscripts regarding comets to our attention, and it is unclear whether these manuscripts are consistent with Dobbs' suggested dating.

De aere describes a probable precursor to these experiments, which fails to distinguish between air resistance and aether resistance since it fails to account for the buoyancy of the surrounding medium; *De grav* reports conclusions that might be related to the more sophisticated experiments. However, as Ruffner emphasized, the quoted result is both much more uncertain and numerically incompatible with Newton's later measurements of resistance.

or, as Newton put it "pertain to [body's] essence" (Newton 2004, p. 21).[12] To Descartes' argument, Newton countered that to be recognized as such, a body had to possess not only extension, but "faculties," in particular the ability to stimulate perceptions and to "transfer action" to other bodies.[13] The core of Newton's critique was the claim that "although philosophers do not define substance as an entity that can act upon things, yet everyone tacitly understands this" (Newton 2004, p. 21). Newton—*contra* Descartes—held that what we should primarily care about is not what a substance is, but what it does.

This difference of orientation is also evident in the stated aim of Newton's metaphysical speculation regarding body. Unlike Descartes' goal in the *Principles*, Newton's goal in *De grav* was the development of an account of body *sufficient* to serve as a basis for physical theory and *sufficient* to capture the phenomenal properties of bodies, the properties of "beings, in every way similar to bodies, whose creation we cannot fail to acknowledge to be within the power of God—and which thus we cannot certainly declare *not* to be bodies". Newton was clear, however, that he could not establish anything more than the sufficiency of his hypothetical account. In particular, he made no claims to reveal the essence or nature of body.[14]

But we must not stress only the *differences* between Newton and Descartes. Certainly, Newton's conception of body in *De grav* differed crucially from Descartes' both in its content and metaphysical pretensions. However, Newton's conception still possessed vestiges of Cartesianism. Although Newton defined body in terms of regions of space endowed with additional attributes—these additional attributes being foreign to Descartes' account of body—he still followed Descartes by treating bodies as regions of space, as extended geometrical structures, albeit not geometrical structures *simpliciter*. In *De grav*, the character of bodies is dependent to some extent on the character of space. Space, in turn, is an

[12] After imagining what a material body would be like if its hardness were removed and concluding that it would still be a body, Descartes wrote that "In the same way, it can be shown that weight, color, and all the other properties of this kind which are experienced in material substance, can be taken away; leaving that substance intact. From this it follows that the nature of matter does not depend on any such properties, but consists solely in the fact that it is a substance which has extension" (the last clause is present only in the French edition) (Descartes 1985, Vol. 1, p. 224) AT VIIA 42, CSM I 224.

[13] Newton was also familiar with the predecessor of the argument of *Principles* II.4 in the *Second Meditation*, but does not address it explicitly in *De grav*; see Harrison (1978, p. 132) and McGuire and Tamny (1983, p. 23). The criticism that something other than extension could not be eliminated was not unique to Newton; Leibniz, for example, also argued that an adequate concept of body must include a force of resistance or impenetrability in addition to extension.

[14] The epistemological status of Newton's account of space is quite different than that of his account of body. While Newton emphasized the tentative status of the account of body—it is merely one possible account of the structure of bodies compatible with our experiences—he did not treat the account of space as conjectural and tentative in the same sense, as Stein (2002) emphasizes. For this reason, when we speak of the 'nature' of body according to Newton, we do not mean to impute to him any form of essentialism or a conception of natural philosophy according to which the goal of philosophizing is to draw observable consequences from the the natures of the ontological primitives.

essentially geometrical structure—it is everywhere full of "all kinds of figures, everywhere spheres, cubes, triangles, straight lines, everywhere circular elliptical, parabolical, and all other kinds of figures, and those of all shapes and sizes, even though they are not disclosed to sight" (Newton 2004, p. 22, et seq.). Bodies, as regions of space, are consequently essentially geometrical, although they admit other essential properties as well.[15]

In *De grav*, Newton treated bodies as "*determined quantities of extension which omnipresent God endows with certain conditions*"(Newton 2004, p. 28, original emphasis). These conditions are:

> (1) that they be mobile; and therefore I did not say that they are numerical parts of space which are absolutely immobile, but only definite quantities which may be transferred from space to space;
> (2) that two of this kind cannot coincide anywhere; that is, that they may be impenetrable, and hence that oppositions obstruct their mutual motions and they are reflected in accord with certain laws;
> (3) that they can excite various perceptions of the senses and the imagination in created minds (Newton 2004, p. 28-29)

Central to Newton's account of body here is the notion that bodies are, first and foremost, "determined quantities of extension." The reliance on a determinate spatial substratum as a necessary precondition for the existence of bodies is one of the main features of his account. After providing the above definition of body, Newton immediately advertised one of its main anti-Aristotelian implications; namely, that is does away with the need to posit a substratum without properties as the metaphysical support for properties and forms and instead makes due with space itself:

> [F]or the existence of [bodies] it is not necessary that we suppose some unintelligible substance to exist in which as subject there may be an inherent substantial form; extension and an act of the divine will are enough. Extension takes the place of the substantial subject in which the form of the body is conserved by the divine will; and that product of the divine will is the form of formal reason of the body denoting every dimension of space in which the body is to be produced (Newton 2004, p. 29).

The analogy between Newton's account and hylomorphism makes clear that extension was as central to Newton's conception of body as the substantial subject was for the conception of body of his Aristotelian adversaries (as Newton understood them). On Newton's account, extension is necessary for the application of so-called "form" (in the guise of the three conditions listed above) and thus necessary for the existence of body. "Body," as Newton wrote in the official "Definitions" portion of *De grav*, "is that which fills space" (Newton

[15]See Stein (2002) for the relation in *De grav* of Newton's account of space to his account of bodies.

2004, p. 13). It is not necessarily that which gravitates, nor that which moves, nor that which is tangible and visible (although it may also be any of those)—it is that which fills space.

Although we cannot discuss the matter at length here, it seems to us that the conception of body as a filled-in quantity of extension takes precedence in *De grav* over the nascent conception of body as that which is governed by the laws of motion, a conception we will return to shortly. Although in *De grav* Newton certainly held that bodies must move "in accord to certain laws," the phrase does not acquire any special significance without the juxtaposition of *De grav* against the later dynamically-focused *Principia* and *De motu* drafts. Taken by itself, *De grav* seems to define body primarily as a region of filled-in extension and only secondarily as a region whose motion obeys further law-like constraints, although both elements are clearly present and clearly necessary. In fact, Newton often wrote in *De grav* as if bodies are, in the first instance, *impenetrable* regions of extension, and only in the second instance *mobile*, impenetrable regions of extension. For example, Newton motivated his "determined quantities of extension" definition with the following passage:

> If [God] should ... cause some space ... to be impervious to bodies and thus stop or reflect light and all impinging things, it seems impossible that we should not consider this space really to be a body from the evidence of our senses (which constitute our sole judges in this matter); for it ought to be regarded as tangible on account of its impenetrability, and visible, opaque, and colored on account of the reflection of light, and it will resonate when struck because the adjacent air will be moved by the blow (Newton 2004, pp. 27-28).

In other words, tangibility, visibility, and other traits that constitute the "corporeality" of matter according to our senses all depend, in the first instance, upon the impenetrability of geometrical regions of space. Motion has a secondary role in constituting that corporeality because motions only make manifest to our senses that impenetrable geometrical regions are in fact impenetrable, in accordance with a broadly mechanical conception of the operations of our sensory apparatus. In the train of reasoning of the above passage, motion is only introduced once regions of space are rendered impenetrable:

> [W]e may suppose that there are empty spaces scattered through the world, one of which, defined by certain [spatial] limits, happens by divine power to be impervious to bodies, and by hypothesis it is manifest that this would resist the motions of bodies and perhaps reflect them, and assume all the properties of a corporeal particle, except that it will be regarded as motionless. If we should suppose that that impenetrability is not always maintained in the same part of space but can be transferred here and there according to certain laws, yet so

> that the quantity and shape of that impenetrable space are not changed, there will be no property of body which it does not possess (Newton 2004, p. 28).

Of course, even if mobility has only a secondary status in this passage, it is clearly essential to Newton's account of body both here and in the "determined quantities of extension" definition. However, our point is merely that it is mobility *of impenetrable regions of extension* that is essential, not mobility taken by itself. The centrality in Newton's account of bodies of the impenetrability of the extensional substratum reveals Newton's residual Cartesianism: in *De grav* he considers bodies to be essentially extended geometrical structures—geometrical structures that are made real by a host of conditions, but geometrical structures nevertheless.

Newton's manner of quantifying body in *De grav* further illustrates his residual Cartesianism. When Newton provided a measure of a body's quantity of matter, he did so through a body's geometrical rather than dynamical properties. After defining the absolute quantity of force as a product of the force's intension ("the degree of its quality") and extension ("the amount of space or time in which it operates") Newton wrote:

> [M]otion is either more intense or more remiss, as the space traversed in the same time is greater or less, for which reason a body is usually said to move more swiftly or more slowly. Again, motion is more or less extended as the body moved is greater or less, or as it is diffused through a larger or smaller body. And the absolute quantity of motion is composed of both the velocity and the magnitude of the moving body (Newton 2004, p. 37).

In modern terminology, Newton equated momentum (which he calls the "force of motion") to the product of the velocity (the intension) and the "magnitude of the moving body" (the extension). Note, however, that the magnitude of the moving body is measured by the body's volume ("the amount of space in which [the force of motion] operates") rather than by the body's resistance to impressed forces (i.e., by its inertia). Since the force of motion is equated throughout Newton's writings (from the *Waste Book* onward) to the product of velocity and *quantity of matter*, in the above passage Newton estimated a body's quantity of matter through its quantity of extension (see Herivel 1965, p. 26). We call this method of quantification, along with Newton's account of the nature of bodies insofar as it relies on a substratum of determined quantities of extension, Newton's *geometrical conception of matter*.

Two caveats must be made regarding this geometrical conception. First, the conception is Cartesian in inspiration, but it is not wholly Cartesian. Newton did not attempt to reduce all of a body's properties to its geometrical properties, nor did he treat any other single property of body as its principal attribute. Newton's conception is less metaphysically minimalist and is presented as less certain than Descartes'. However, Newton did follow Descartes in considering extension both as essential to our understanding of body and as essential to the practice of physics vis-à-vis the measure of the quantity of matter associated with body. Second, although we've highlighted the geometrical conception's indebtedness to Newton's residual Cartesianism, the conception is also closely tied to Newton's atomism, particularly his belief in the uniformity of nature. That story is quite important for the development of Newton's thought, but it would take us far from our current course. We will, however, return to the uniformity of nature briefly in §4, when we consider Newton's allegiance to the geometrical conception in light of Cotes' 'two-globes' objection.

At any rate, in *De grav*, the geometrical measure of quantity of matter is not supplemented with a precise dynamical measure, as it is in the *De Motu* drafts leading to the *Principia* as well as in the *Principia* itself. According to what we call the *dynamical conception of matter* refined in these texts, a quantity of matter is measured by its response to impressed force, not by the volume of space which it impenetrably fills. As with the geometrical conception, the dynamical conception of matter also incorporates a view regarding the nature of bodies, one we will return to shortly. Newton introduced the dynamical measure of quantity of matter in Definitions I and III of the *Principia*, and it is the one most familiar to modern readers. In Definition III, Newton states that the internal force of a body (its *vis insita*) "is always proportional to the body and does not differ in any way from the inertia [*vis inertia*] of the mass except in the manner in which it is conceived (Newton 1999, p. 404)." We are to understand that *vis insita* is also proportional to a body's *quantity of matter* since Definition I states that:

> I mean this quantity whenever I use the term "body" or "mass" in the following pages (Newton 1999, p. 404).

Together with Law II, these two definitions establish a proportionality between a body's quantity of matter and the force responsible for the body's dynamical properties.[16]

Although Definition I also states that "Quantity of matter is a measure of matter that arises from its density and volume jointly," the quantification method implied by Definition III is used throughout the *Principia* almost exclusively. In fact, in Definition I itself Newton made explicit that quantity of matter "can always be known from a body's weight for—by making very accurate experiment with pendulums—I have found it to be proportional to the weight."

This is a far cry from the quantification method suggested in *De Grav*. Of course, Newton did define force in *De grav* as either "external"—"one that generates, destroys,

[16]For a discussion of the nature of *vis insita*, see McGuire (1994). For a more recent discussion of the connection between *vis insita* and *vis centrifuga* in orbital motion, see Meli (2006).

or otherwise changes impressed motion in some body"—or "internal"—"by which existing motion or rest is conserved in a body, and by which any body endeavors to continue in its state and opposes resistance" (Newton 2004, p. 36). But, unlike in the *Principia*, when it comes to *quantifying* body, the dynamical method is not used.[17] Lacking a clear statement of Law II, the dynamical measure of a quantity of matter remains rather vague in *De grav* and essentially intertwined with the conception of body as that which fills space. *De grav* adumbrates the dynamical conception of matter, but does not contain it fully and certainly does not contain its central element, the measurement of quantity of matter by a body's response to impressed force.[18] In the *Principia*, the two methods of quantifying matter co-exist, but the geometrical conception is relegated to the wings while the dynamical conception takes center stage.

What view regarding the nature of bodies goes along with the dynamical measure of matter in the *Principia*? In contrast to *De grav*, in the *Principia* Newton characterizes material bodies almost exclusively by their dynamical properties. Although the term "body" appears several times in the definitions and laws of motion, Newton does not offer a separate definition of body or an account of its possible nature like the one supplied in *De grav.* This suggests a transformation in Newton's view: the *Principia* provided clear formulations of the concept of force and the laws of motion, but bodies are defined only implicitly—as the entities subject to forces and for which the laws of motion hold.[19] The nature of body in the theoretical context of the *Principia* thus depends upon whatever constraints are implied by satisfaction of the laws.[20] Furthermore, the empirical support for this implicit account of the nature of bodies derives from the support for the laws of motion and the mechanical theory based on them.

Yet the dynamics of the *Principia* place surprisingly weak constraints on what could count as a body.[21] In particular, "bodies" satisfying all the Newtonian laws of motion need

[17]In the *De grav* Newton used the term *vis inertia* for the internal principle of motion (p. 36), in much the same sense as he used the term in an excised portion of Definition 1 in the Lucasian lectures (1685, Newton 1981, Vol. 5). This is by contrast with his usage of *vis insita* in the early *De Motu* drafts (see Herivel 1965, pp. 26-28). We thank George Smith for pointing this out to us. Yet despite the appearance of the term *vis inertia*, *De grav* is a transitional text which only hints at the concept of *vis inertia* developed in the *Principia*. This is not surprising, since in *De grav* Newton is working out the metaphysics of "natural power" on which *vis insita* ultimately depends; see Stein (1991).

[18]For example, Newton writes that "Bodies are denser when their inertia is more intense, and rarer when it is more remiss" (Newton 2004, p. 37), but is committed to conceiving of the absolute measure of force as the product of the intension of the force and the extension of the body in which the force is acting.

[19]Newton did define "body" in unpublished definitions intended for the third edition of the *Principia*, but these definition only make the reliance on dynamical properties explicit, see McGuire (1966). We will discuss these briefly in §4.

[20]Although we do not have the space here to discuss this view in detail, we are influenced by Katherine Brading's account of Newton's "methodological solution" of the problem of individuation of bodies; see her contribution to this volume.

[21]We thank Michael Friedman for a question that prompted the clarification in this paragraph. See also

not have any geometrical properties whatsoever. This may appear to conflict with Newton's various theorems regarding extended bodies, such as the famous proofs (*Principia* I.71-75) to the effect that a spherical body can be treated as if the mass were concentrated at a point. However, even these proofs only require that the force acting on or produced by the whole body is the sum over forces related to its constituent parts. They do not require the attribution of geometrical properties to the parts of the spherical bodies and are compatible with bodies treated as Boscovichian point-particles characterized by parameters such as "quantity of matter" that have no geometrical basis.[22] That the dynamical conception of matter is so austere reflects the limited mathematical framework of the *Principia*. The generalization of Newtonian theory to continuum mechanics (by the likes of Euler and Cauchy) leads to a much richer notion of body, which does have implications for the geometrical properties of bodies defined implicitly via the laws of motion. For example, Cauchy's generalization of Newton's first law involves contact forces defined over the boundary as well as the outward normal defined on the boundary; Boscovichian point particles do not satisfy these laws of motion as they lack boundaries and Cauchy's law does not apply.[23]

That said, the geometrical conception of matter developed in the *De grav* did not disappear from Newton's thought following the elaboration of the dynamical conception in the *Principia*. In drafts of corollaries to Proposition 6 of book III written in the 1690s, Newton assesses the connections between gravitational aethers, matter theory, and the existence of void.[24] In doing so, Newton assumes that the appropriate measure of quantity of matter is the volume of the basic particulate constituents of matter. Although we cannot discuss these manuscripts here, they indicate that Newton continued to take the geometrical conception of matter seriously after the publication of the *Principia*. Finally, Newton's reply to Cotes's "two globes" objection—which also concerns Proposition 6—clearly relies on the geometrical conception, and took place over 20 years later, in 1711. We will return to the two globes objection in §4 below. Now we turn to the two types of *a posteriori*, empiricist arguments we believe are associated with the two different conceptions of matter.

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the discussion of quantity of matter in Steinle (1992).

[22]That is not to say that it is straightforward to apply Newton's laws to "point-particles"; part of Euler's achievement in the *Mechanica* was to formulate the second law as "$\mathbf{F} = m\mathbf{a}$" and to show how to apply this law to the point-particles composing rigid bodies and fluids. We owe this point to George Smith, who also emphasized two further points: first, that the geometrical properties of bodies do play a limited role in the *Principia*, in particular in the study of resistance forces in Book II. Second, Newton had already begun to treat issues in continuum mechanics, in the treatment of wave propagation through fluids in Section 8 of Book II. It was only with later work that the implications of this approach for the geometrical properties of bodies were clarified.

[23]See Truesdell (1968) for a historical account of the later contributions, and Smith (2007) for a philosophically oriented discussion of the notion of body appropriate for classical continuum mechanics.

[24]For a thorough discussion of these manuscripts, see McGuire (1967).

2.3 The *A Posteriori* Character of the Geometrical Conception of Matter

In what sense did Newton establish, or believe he had established, his geometrical conception of body *a posteriori*? Two distinct *a posteriori* contributions can be discerned. First, the results of the pendulum experiments and the accuracy of Kepler's "laws" pushed Newton to reject the Cartesian identification of body with extension. In this sense, his path towards a new conception of body is similar to his path towards a new conception of light in his optical work in the 1670s.[25] In both cases, Newton took a rather narrow set of experimental results to be sufficiently crucial to warrant an overall revision of a fundamental concept of natural philosophy.

But there is an important difference: whereas the prism experiments, the crucial experiments in Newton's early optical work, were used to both refute the extant conception of light and suggest a new conception (i.e., that white light is not a natural kind but is composed of individually homogeneous light rays of differing refrangibilities), the pendulum experiments were used only to *refute* the Cartesian doctrine. On our account, Newton rejected an account of gravity based on the results of pendulum experiments along with his success in modeling planetary motions using a single force law without a resisting medium. This rejection then spilled over to the associated Cartesian accounts of body and space. However, the constructive element of Newton's geometrical conception of body was not secured by an *experimentum crucis*; rather it seems to have been secured by a different type of argument from the phenomena.

This argument proceeds from the experience of any body whatsoever. Newton attempted in *De grav* to provide an account of body that is sufficient for capturing the "evidence of our senses." The traits of body he aimed to save were all quite generic and are reflected in the overall character of our experience; for example, that body is visible, tangible, audible under certain conditions, etc. Newton's account of body as mobile, impenetrable, and sensible extension is only able to successfully save these traits because it is set against a background of natural philosophical presuppositions, but these presuppositions were not constituted by a set of first principles as they were for many mechanical philosophers. Rather, they were constituted by a set of natural philosophical explanations Newton considers plausible—e.g., that an object is visible because it reflects light or that it is audible because it is capable of pushing the adjacent air. However, given this larger natural philosophical framework, the evidential basis for Newton's conception of body includes any and all experiences of body. Moreover, success within Newton's natural philosophical framework does not rely on any quantitative notion of "strength of evidence" that can help arbitrate between Newton's

[25] For an account of this development see, for example, Shapiro (2004); Stein (ms).

and possible competing accounts—where by "strength of evidence" we mean any measure of the fit between a given theory and its evidential basis that allows discrimination among competing theories according to degree of evidential warrant. Strikingly, the relationship between Newton's account of body and the evidence on which it is based does not involve a sophisticated notion of evidential warrant of the type found in the argument for universal gravitation. Rather, it involves a notion much closer to the one employed by traditional mechanical philosophers in order to justify their mechanical models, but one that does not appeal to first principles or privileged modes of explanation. We will return to both types of arguments in the following section, but note Newton's explicit reference to the significant underdetermination of *De grav*'s account of body:

> [I]t is hardly given to us to know... whether matter could be created in one way only, or whether there are several ways by which different beings similar to bodies could be produced... [H]ence I am reluctant to say positively what the nature of bodies is, but I would rather describe a certain kind of being similar in every way to bodies, and whose creation we cannot deny to be within the power of God, so that we can hardly say that it is not body (Newton 2004, p. 27).

Newton is explicitly open to the possibility that another hypothesis regarding the nature of body can save the phenomena equally well.

This *a posteriori* method of arriving at claims regarding the nature of matter resembles in certain respects the one offered in Rule III of the *Regulae Philosophandi* and its drafts.[26] In Rule III, Newton claimed that certain qualities of bodies are "universal," qualities that can be attributed to any body whatsoever and so constitute the core of our understanding of body, the "foundation of all natural philosophy" (Newton 1999, p. 796). Often, Newton referred to such claims of universality as being "deduced from phenomena" (e.g., (Newton 1999, p. 943)). The list of universal qualities consists of extension, hardness, impenetrability, mobility, and inertia. However, Newton's evidence for their universality and thus the philosophical import of Rule III is by no means homogeneous. One of the theses of this essay is that deducing or gathering propositions "from phenomena" does not have a univocal meaning for Newton, and so the resemblance of Rule III to *De grav* concerns only the first three qualities—more will be said about the rest in §3.3. First, Rule III, like *De grav*, appeals to our general experience of bodies as the evidential basis from which claims regarding the extension, hardness, and impenetrability of bodies ought to be drawn. For example, Newton wrote in language echoing that of his earlier treatise that:

[26] See McGuire (1970, 1968). Although we will not discuss the drafts here, their implicit methodology is even closer to that of *De grav* than that of Rule III, both in the evidential basis they recommend for natural philosophical claims and the way in which the relationship between that basis and the resulting claims is conceived.

> The extension of bodies is known to us only through our senses. . . [and] because extension is found in all sensible bodies, it is ascribed to all bodies universally. We know by experience that some bodies are hard. . . [and] justly infer from this not only the hardness of the undivided particles of bodies that are accessible to our senses, but also of all other bodies. That all bodies are impenetrable we gather not by reason but by our senses. We find those bodies that we handle to impenetrable, and hence we conclude that impenetrability is a property of all bodies universally.(Newton 1999, p. 795, emphasis added).

In each case, our experience of bodies, broadly conceived, forms the evidential basis of the generalization. Of course, according to Rule III *only* those qualities that pass the intension and remission criterion and that are found in "all bodies on which experiments can be made" can be "taken as qualities of all bodies universally" (Newton 1999, p. 795). Consequently, the evidential basis recommended by Rule III is more restrictive than the one used in *De grav*. Only *some* features of our experience of bodies remain relevant to generalization about the nature of body; visibility and audibility, for example, are ruled out. However, the remaining features are those that are truly general—they are part of *all* our experiences of body. In fact, achieving this generality is precisely the point of Newton's application of the intention and remission criterion. Any quality that is not *always* part of our experience of bodies—i.e., one that can be remitted to zero and thus disappear, or one that is not present in some bodies—is ruled out.

Second, regarding the first four qualities mentioned, Rule III, like *De grav*, does not utilize a notion evidential warrant is similar in complexity to the one used throughout the *Principia*. This is because while the rule's condition of application—the intention and remission criterion—can be made precise, it is unclear when in the course of empirical investigation we can be content that it is sufficiently satisfied for "all bodies on which experiments can be made" to warrant judgments of universality. Newton's examples do not help. Newton argued that the extension of bodies is made manifest in *all sensible bodies*. However, he also held that we know by experience that hardness is only found in *some* bodies while impenetrability is only found in "those bodies that we *handle*"—presumably a smaller class than "all *sensible* bodies". Is the judgement of universality regarding one of these better off than the others? Newton hinted at a notion of strength of evidence, but did not make it explicit. He wrote that "the argument from phenomena will be *even stronger* for universal gravity than for the impenetrability of bodies, for which. . . we have not a single experiment, and not even an observation, in the case of the heavenly bodies" (Newton 1999, p. 796). It seems that something like simple enumerative strength is at work here: the more instances of a quality we have, the stronger the judgement of its universality. However, this is still a far cry from the sophisticated and more robust relation between theory and evidence implicit in the *Principia*.

The lack of a robust notion of evidential warrant or "strength of evidence" would not be bothersome by itself, but we will argue in §4 that Newton, on at least one occasion, overemphasized the evidence in favor of the geometrical conception of matter. The reason, we will argue, is that Newton failed to distinguish the type of argument given in *De Grav* for the geometrical conception of matter from the type of argument used in the *Principia*. We must first clarify the latter type of argument. To do so, we'll use Cotes' invisible hand objection.

3 The Invisible Hand

As the editing of the *Principia*'s second edition neared completion in 1713, Roger Cotes took on the task of writing a preface contrasting Newton's "experimental philosophy" with the approach of the Cartesians and Aristotelians. To exemplify Newton's method he intended to present a "short deduction of the Principle of Gravity from the Phænomena of Nature, in a popular way" (Newton 1977, V, Doc. 985, p. 391). However, he encountered a difficulty in presenting the argument given in the *Principia*.

Cotes accepted the first two steps of Newton's argument for universal gravitation, to the effect that (1) the planets are held in their orbits by an inverse square centripetal force directed towards the sun, and (2) that this force can be identified with terrestrial gravity, via the moon test. What gave him pause was the next step, first mentioned in Cor. 1 of Proposition 5 of Book III and discussed again in Proposition 7. In this third step, Newton applied the third law to the centripetal force holding planets in their orbits, and concluded that a given planet also attracts the sun.[27] In other words, Newton argued that gravity is a *mutual interaction* between the sun and planet. Cotes's invisible hand objection is meant to illustrate that this third step requires further hypotheses about the nature of gravitation. Cotes wrote:

> ...ye Force by which they [the planets] are continually diverted from the Tangents of their Orbits is directed & tends towards their Central Bodies. Which Force (from what cause whatever it proceeds) may therefore not improperly be call'd Centripetal in respect of ye revolving Body & Attractive in respect of the Central. [...] But in the first Corollary of the 5th [proposition of Book III] I meet with a difficulty, it lyes in these words *Et cum attractio omnis mutua sit.* I am persuaded they are then true when the Attraction may properly be so call'd, otherwise they may be false. You will understand my meaning by an Example.

[27] The third law reads as follows in all editions: "To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction" (Newton 1999, p. 417).

> Suppose two Globes A & B placed at a distance from each other upon a Table, & that whilst A remains at rest B is moved towards it by an invisible Hand. A by-stander who observes this motion but not the cause of it, will say that B does certainly tend to the centre of A, & thereupon he may call the force of the invisible Hand the Centripetal force of B & the Attraction of A since ye effect appeares the same as if it did truly proceed from a proper & real Attraction of A. But then I think he cannot by virtue of this Axiom [*Attractio omni mutua est*] conclude contrary to his Sense & Observation that the Globe A does also move towards the Globe B & will meet it at the common centre of Gravity of both Bodies. This is what stops me in the train of reasoning by which I would make out as I said in a popular way the 7th Prop. Lib. III. I shall be glad to have Your resolution of the difficulty, for such I take it to be. [...] For 'till this objection be cleared I would not undertake to answer one who should assert that You do *Hypothesim fingere*. I think You seem tacitly to make this Supposition that the Attractive force resides in the Central Body (Newton 1977, V, Doc. 985, p. 392).

There are two ways of reading Cotes. On a first, literal reading, there is a stark *empirical* contrast between the "invisible hand" scenario and Newton's account of gravitation. According to Cotes, the invisible hand imparts motion to Globe B without imparting motion to Globe A—Globe A *does not move*. According to Newton, however, interactions are truly mutual, and so the central body of a gravitational system (represented by Globe A) is predicted to move, however slightly. The mismatch between prediction and observed motion is Cotes' problem: "I think [an observer] cannot by virtue of this Axiom [*Attractio omni mutua est*] conclude *contrary to his Sense & Observation* that the Globe A does also move towards the Globe B" (Newton 1977, V, Doc. p. 985, p. 392, emphasis added). Cotes is presuming that in a gravitational system, like in the invisible hand case, "Sense & Observation" will always show that the central body does not move. This is a false empirical presumption. Determining the motion of a central body in a real-world case is by no means a simple task, but it is possible. In fact, for truly mutual interactions between two bodies (without external forces that effect the two bodies differently), there is a "two-body" correction to Kepler's third law (see, e.g., Smith 2002b, p. 44). Although Newton or Cotes could not have made an empirical case in favor of this correction term on the basis of the data available to them, the question was open to empirical resolution. Cotes jumped the gun by presupposing that observation will show that the central body does not move.

Yet this was a perfectly natural response on Cotes' part. Neither Newton nor Cotes could have made an empirical case for the motion of the central body of the solar system, and Cotes was right to press Newton on that point. But Newton did not respond directly to this challenge and instead subtly shifted the terms of the debate. To account for the subsequent exchanges we will focus on a second, slightly modified, reading of Cotes. On this reading, we take Cotes to suggest that there are two invisible hands acting in concert to move both globes in a way that is identical with the predictions of Newton's theory. This new scenario does not amount to an empirically distinguishable alternative to Newton's description of planetary motions; instead, it reveals that Newton's argument for the extension of the third law to the central body rested upon an unacknowledged, substantive assumption about the nature of gravity.[28] Although Cotes clearly claimed that Globe A does *not* move, Newton seems to have responded to Cotes as if he had posed this more telling objection to Newton's methodology.

Newton's replies to Cotes focused on two points, both of which were reflected in changes to the text of the *Principia*. First, Newton defended the third law as a crucial feature of his conception of force by showing that it is essential to extending the first law to systems of interacting bodies. We will argue that this reply missed the point (at least of the second, stronger reading of Cotes); Cotes did not challenge the third law itself, but rather the identification of the bodies involved in the application of the third law. Nonetheless, Newton's discussion of the third law reveals its importance in going from a mathematical characterization of force based on the first two laws to a physical characterization of forces treated as mutual interactions among bodies. Second, Newton responded to the charge of feigning hypotheses; he clarified the nature of hypotheses in his method and argued that objections of a particular kind, exemplified by Cotes' invisible hand, should simply be set aside. We will follow Newton's lead in using this as an occasion to consider the status of the laws of motion (and to make it clear why we do not read this as Newton evading the question). We will briefly contrast the limited sense in which the laws are "hypothetical" on Newton's account with the role of hypotheses for his contemporaries and the a posteriori character of the geometrical conception of matter, and by doing this describe the sophisticated *a posteriori* reasoning made available by the mathematical framework of the *Principia*, the type of reasoning on which the dynamical conception of matter is based.

3.1 Applying the Third Law

The invisible hand highlights the ambiguity in Newton's application of the third law in the third step of the AUG. Suppose that invisible hands and an attractive force produce indistinguishable motions of an orbiting and a central body. The third law implies that there is an equal and opposite force corresponding to the force holding the body in its orbit,

[28]Kant re-discovered the problem, apparently independently of Cotes, and characterized Newton as being "at variance with himself" for denying that gravitational attraction is an essential property of matter; see Remark 2 to Proposition 7 of the *Dynamics* (4:515) and the discussion in Friedman (1992)[pp. 149-159]. Several more recent commentators have also discussed the invisible hand objection: Densmore (1996); Koyré (1965); Harper (2002); Stein (1991); see also Harper's contribution to this volume.

but it does not specify the nature and location of this force. Should it be a reaction force acting on the central body, or a force pushing against the invisible hand?

Assume that there is no empirical way to distinguish the two options. Newton's argument depends on one of two assumptions. Either, first, gravity is a force of attraction causally residing in the interacting bodies alone, an attractive force "properly so call'd" as Cotes put it; or, second, whatever underlying mechanism is responsible for gravitation must itself produce a reaction force on the central body. Suppose, for example, that an aether mediates the gravitational interaction. Newton's application of the third law would be appropriate only if there is no net momentum transfer from the two gravitating bodies to the aether. In this case, even though the third law *properly applied* yields a reaction force on the aether pressing against the planet, the aether interacts with the planet and sun in precisely the right way to produce a reaction force on the sun. Either option conflicts with Newton's claim that the validity of the AUG does not depend upon "hypotheses" regarding the nature of gravity.[29]

Newton responded to Cotes by defending the validity of third law of motion itself. He asked Cotes to consider two bodies A and B acted on by no net external forces, such that the forces *between* A and B do not satisfy the third law. Say, for example, that A exerts a greater force on B than vice versa. Newton emphasized that the resulting imbalance of forces would cause the bodies to accelerate off to infinity, a result that conflicts both with experience and the first law of motion (Newton 1977, V, Doc. 988, p. 397). Newton added text to the *Principia* to the same effect. In the scholium following the Laws of Motion, Newton asked the reader to imagine sections of the Earth cut off by parallel planes equidistant from the center.[30] As before, an imbalance of the gravitational forces felt by

[29]A second issue regarding the third law apparently did not concern Cotes: granting that the reaction force inheres in the central body, are the motions cited in the AUG sufficient to establish that the action and reaction force are equal in magnitude? In modern terms: The celestial bodies interact via acceleration fields that are fixed by the product of the gravitational constant G and the mass M of the body producing the field. Purely gravitational interactions do not provide any basis for distinguishing the contributions of G and M. Consider, for example, two bodies with masses M_A, M_B interacting gravitationally (see Harper (2002) for a thorough discussion of this point, which we draw on here). The motion of these two bodies is compatible with assigning acceleration fields with magnitudes GM_A, GM_B *or* instead assigning different masses, $G_A M'_A, G_B M'_B$, provided that $GM_A = G_A M'_A$ and $GM_B = G_B M'_B$. In other words, the masses can be assigned freely if the value of G is allowed to vary for acceleration fields associated with different bodies. However, changing the values of the masses to M'_A, M'_B also changes the motive forces between the two bodies, spoiling the equality in magnitude required by the third law. The upshot of this second objection is that Newton had to also assume, quite plausibly, that the value of G is a characteristic of this type of force and not a characteristic of the body producing the acceleration field. Unless G is a truly "universal" constant (having the same value for all bodies), the motion in response to an acceleration field produced by body A cannot be used to measure M_A. The assumption that G is a universal constant amounts to treating the various acceleration fields produced by the celestial bodies as instances of the same type of force, and Cotes did not challenge this assumption.

[30]Newton had used a similar example in the first edition of the *Principia*; in the earlier example, A and B are separated by an obstacle, so that the application of the third law to the pressure exerted by bodies A

these two parts of the Earth would lead to the Earth accelerating off to infinity with no net external force.[31] These examples reveal the intimate connection between the third and first laws. In order for the first law to hold for the center of mass of a closed system of interacting bodies, the third law must hold for the interactions among the bodies, although Newton's examples only involve *contiguous* bodies pressing against one another.[32]

This line of response highlights the importance of the "mutuality" of force, the crucial novelty introduced in Newton's formulation of the laws of motion. As Stein (2002) has emphasized, speaking of *separate* forces acting on two bodies, which happen to come in an action-reaction pair, is misleading. In Newton's usage, the "force" corresponds to an interaction between bodies that is *not* broken down into separate "actions" and "reactions," except in our descriptions of it. Newton's own "popular" version of the third book, the *System of the World*, included a clear statement to this effect:

> It is true that we may consider one body as attracting, another as attracted; but this distinction is more mathematical than natural. The attraction resides in each body towards the other, and is therefore of the same kind in both. [...] In this sense it is that we are to conceive one single action to be exerted between two planets arising from the conspiring natures of both. (§20, in Newton (1966), p. 568)

This conception of force as a single interaction manifested by equal and opposite impressed forces on separate bodies is built into the Laws of Motion. It also plays a crucial role in distinguishing apparent forces from real forces, in the following sense. Given a body in motion, the first two laws allow one to infer the existence of a force producing the motion

and B on the obstacle can be extended to the attractions between them.

[31] Applying this line of reasoning to orbital motion is less straightforward. Two mutually interacting bodies with $|f_{AB}| = |f_{BA}|$ describe orbits around a common central point c. The bodies move such that their distances from c are constant, with the respective distances fixed by the relative strength of the acceleration fields of the two bodies. If the third law holds, the point c is also the center of mass c_m for the two bodies; at c the weights of the two bodies (that is, their motive forces towards each other) are precisely balanced. As Harper (2002) shows, if the third law fails ($|f_{AB}| \neq |f_{BA}|$) then c_m no longer coincides with c; instead, the center of mass c_m moves in uniform circular motion around c. Unlike the first two cases, this does not lead to an obvious violation of the first law such as an isolated body running off to infinity. However, Newton makes it clear that such motion of c_m is incompatible with the laws of motion in Corollary 4 to the Laws of Motion (Newton 1999, pp. 421-423). Harper (2002) further argues that it is unclear whether this motion of the center of mass violates the first law, due to an ambiguity in Newton's discussion of the first law. The brief explanatory paragraph accompanying the first law gives various examples of motion that would continue unabated if not for resistance, including the spinning motion of a hoop (Newton 1999, p. 416). Harper concludes from this that Newton may understand the first law to apply to uniform circular motion (Harper 2002, p. 92). However, we read the expository paragraph as answering an obvious empirical objection to the first law – that bodies do not remain in motion in everyday experience – rather than as a further elaboration of the "states of motion" to which the first law applies, especially since the law explicitly applies to motion "at rest or ... moving uniformly straight forward." In any case, Newton is quite explicit in the statement and discussion of Corollary 4.

[32] For the importance of the notion of a closed system, see Van Dyck's paper in this volume.

that may be well-defined quantitatively (given a definite magnitude and direction), without considering the question of what produces the force. But the third law further requires that the force results from an interaction between the body in motion and some other body. The Coriolis force—the apparent force exemplified by the deflection of bodies from an inertial trajectory when viewed from a rotating reference frame—illustrates this distinction: the force is well-defined quantitatively and could be inferred from observing a body in motion using the first two laws of motion and results from Book I, but there is no "interacting body" to be found. The first two laws figure primarily in treating forces from a mathematical point of view whereas the introduction of the third law marks an important physical constraint. Although Newton famously abstained from requiring a full account of the "physical cause or reason" of a force as a precondition for establishing its existence, any further account of the physical nature of the force would have to satisfy the constraint imposed by the third law.[33] We do not wish to imply that satisfying the third law is the *only* further constraint Newton imposes on a physically real force; Smith (2002a, p. 150) argues that Newton imposes five conditions that must be satisfied for a component of a mathematically characterized force to qualify as physical in his discussion of gravity in Book III and resistance forces in Book II. See also Janiak (2007, 2008), for a careful broader assessment of the distinction between Newton's mathematical and physical characterizations of force.

But even granted this conception of force as mutual interaction, Cotes' query prompted justification for Newton's identification of the central body in a gravitational interaction as the second "conspiring" body. The first two steps of the AUG established that the force producing the orbits of the celestial bodies is closely related to the respective central bodies: it is directed toward the central body, and it varies as the inverse square of the distance from the central body. These features make plausible Newton's identification of the central body as the second body whose "conspiring nature" produces the interaction. Furthermore, if the list of candidates is limited to other *known* bodies, there are few, if any, plausible choices other than the central body. Cotes accepted the first two steps of the AUG, and so accepted those features of the force law apparently related to the central body. However, Cotes was correct to insist that this plausibility argument falls short of a proof. Newton's rivals pursuing a vortex theory of planetary motion aimed to recover both these aspects of the force without introducing a truly mutual interaction with the central body. They did so by introducing an analogue of the "invisible hand"; namely, an aether that was unobservable except for its hypothesized gravitational effects.

In sum, although this part of Newton's response emphasizes the viability of the third

[33] There is much more to be said regarding the subtle distinction Newton draws between a mathematical and physical characterization of force, its use in defending his account of gravitation, and its relation to earlier debates regarding the status of physics, mathematics, and physico-mathematics.

law, this clarification is not sufficient to answer Cotes without some further stipulations regarding the bodies referred to by the law. However plausible these further stipulations may seem, they beg the question as a reply to Newton's contemporary critics. And so, one may charge that these stipulations are inconsistent with Newton's own method, that they are feigned hypotheses. Newton responded to this charge directly.

3.2 The Status of the Laws of Motion and The *A Posteriori* Character of the Dynamical Conception of Matter

Since Cotes concluded that Newton did indeed "feign hypotheses" in the AUG, Newton offered two clarifications of the meaning of hypothesis in his method. First, Newton reprimanded Cotes for applying the term hypothesis too broadly:

> ... as in Geometry the word Hypothesis is not taken in so large a sense as to include the Axioms & Postulates, so in experimental Philosophy it is not to be taken in so large a sense as to include the first Principles or Axiomes wch I call the laws of motion. These Principles are deduced from Phænomena & made general by Induction: wch is the highest evidence that a Proposition can have in this philosophy (Newton 1977, V, Doc. 988, pp. 396–97).

To make this clear in the *Principia*, Newton directed Cotes to add the widely cited passage immediately following "hypotheses non fingo" to the General Scholium:

> For whatever is not deduced from the phenomena must be called a hypothesis; and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction. The impenetrability, mobility, and impetus of bodies, and the laws of motion and the law of gravity have been found by this method. (Newton 1999, p. 943).

Thus, apart from defending his application of the third law, Newton further argued that the laws of motion (and other claims) in the *Principia* had a quite distinctive status.

How to characterize this status more precisely is a delicate question, but the exchange with Cotes and a comparison of Newton with his contemporaries can shed some light on Newton's position. Newton clearly regarded the laws of motion as having a much more secure status than the hypothetical models pursued by "mechanical philosophers" such as Huygens. Huygens characterized the aim of physics as the construction of mechanical models that rendered various phenomena intelligible. Confidence in a hypothetical model was based on the quality of explanations it offered, along with its ability to predict novel phenomena and other virtues such as simplicity. Yet this approach was open to a well-known objection: the possibility of alternative models that offered differing, yet equally satisfactory, explanations.[34] Mechanical philosophers typically responded by insisting that their models satisfied further constraints, such as the compatibility of the models with certain privileged first principles, particularly with the ontology of matter in motion. These constraints, however, did not solve the underdetermination problem, they merely limited its scope.

Newton was well aware of the underdetermined nature of hypothetical reasoning, but he did *not* conclude that certainty would forever exceed our grasp in reasoning about natural phenomena. His criticisms of mechanical philosophers were combined with the assertion that his own method could establish results with as much certainty "as the nature of things admit." This is because Newton insisted on a criterion of evidential warrant distinct from that of the mechanical philosophers (cf. Harper and Smith 1995). Propositions that met this more stringent demand qualified as "deduced from the phenomena" or "proved by experiments," and Newton claimed that they were not subject to the same objections as the method of hypotheses. In particular (as we also saw in §2.3) Newton's reasoning about natural phenomena did not depend on the conformity of such reasoning with first principles regarding fundamental natures.

But there is more to "deduction from phenomena" or "proof by experiment" than a disregard for first principles, particularly in the context of the *Principia*. Specifying this difference is no mean feat, but we are able to draw on recent work by Howard Stein, Bill Harper, and George Smith that advances the understanding of the implicit methodology of the *Principia* by carefully reconstructing its argumentative structure.[35] Despite disagreements on several finer points, this line of work highlights two general contrasts between Newton and the earlier mechanical philosophers. First, Newton's predecessors—take Galileo, for example—did not deal with the complexity of actual motions in the same way as Newton. Although the consequences of Galileo's theory of uniformly accelerated motion were not taken to apply exactly to *actual* motions, a rough conformity between actual and theoretically described motions was taken as evidence in favor of the theory. Yet proper judgments of conformity require an assessment of factors such as the effects of air resistance and measurement imprecision, and these are problematic precisely because such factors are not treated in the theory.[36] By way of contrast, Newton had an elegant way of handling the

[34]Newton emphasized this point in response to criticisms of his optical theories: "For if the possibility of hypothesis is to be the test of the truth and reality of things, I see not how certainty can be obtained in any science; since numerous hypotheses may be devised, which shall seem to overcome new difficulties" (Newton 1978, p. 106).

[35]See, in particular, Harper (1990, 2002); Harper and Smith (1995); Smith (2001, 2002a,b); Stein (1991, ms), and the contributions of Harper and Smith to this volume.

[36]Many mechanical philosophers expected that such effects could not be incorporated into theories of motion; Galileo, for example, doubted whether air resistance could ever be handled theoretically. See McMullin (1985) for a discussion of the various ways in which Galileo defended the applicability of mathematical

complexity of actual motions. He took the care to prove theorems that could underwrite "robust inferences," that is, inferences whose conclusions (usually claims regarding forces) hold approximately if their antecedents (usually observational claims) hold approximately (Smith 2002b). For example, Newton's use of the precession theorem in the first step of the AUG makes it possible to infer properties of the gravitational force law from actual motions even if they only *approximately* satisfy a simple mathematical description, such as Kepler's laws. An initial theoretical description of some phenomena is thus not blocked by the complexity of actual motions. The argumentative structure of the *Principia* further illustrates that Newton approached the full complexity of actual motions piecemeal, building up from the initial theoretical description to more complicated descriptions in what Cohen called the "Newtonian Style" (Cohen 1980). As we shall see immediately below, this style is also crucial for establishing the epistemological warrant of Newtonian mechanics.

Second, the Newtonian laws of motion are remote from directly observed motions. That is, the laws of motion do not by themselves entail specific predictions about, for example, falling bodies, but must be supplemented with assumptions regarding the forces in play to yield predictions. Consequently, the laws are not *directly* "deduced from phenomena" on the basis of successful predictions. This claim apparently runs counter to Newton's defense of the laws in the scholium following the Laws and Corollaries (Newton 1999, pp. 424-430). There, Newton discussed experiments which could plausibly be taken as the basis for a "deduction" of each of the laws from phenomena such as the ballistic pendulum (intended to demonstrate the applicability of the third law to impacts). Newton also presented a defense of the third law as a natural extension of the static treatment of forces to cases where the mutually balanced forces apply to different bodies. We do not deny that the successful treatment of these and other experiments provides evidence in favor of the laws of motion, but we insist that this is not a case of simple predictive success. The treatment of pendulums, falling bodies, and simple machines all require further assumptions regarding the forces at play.[37] We take the scholium to establish quite convincingly that a variety of phenomena are compatible with the laws of motion when the motion of bodies is characterized dynamically using Newton's definitions. The challenge in giving an account of the status of the laws—and the status of Newtonian mechanics more generally—is to clarify the sense in which indirect empirical support is accrued to the laws and with them the dynamical conception of matter that is nonetheless stronger than that offered by mere predictive success of hypothetical models.

idealizations. Descartes similarly believed that a mathematical treatment of air resistance in free fall is intractable, see Gabbey 1993 and Garber 2000.

[37]We thank George Smith for pressing us to clarify this point, and for helpful discussions.

We focus on Bill Harper's and George Smith's reconstruction of the status of the laws. Harper analyzes Newton's criteria of empirical success as the requirement that observed motions provide multiple agreeing measurements of the parameters of the force law used to describe those motions. This approach shifts the focus from the predictive success of a single model to the stability of parameter values across a set of theoretical descriptions of observed motions. Smith emphasizes the importance of approaching actual motions by a series of approximations (Smith 2002b). An initial inference only establishes the approximate validity of the gravitational force law as applied to actual motions, but one can further calculate trajectories on the assumption that the gravitational force holds *exactly* in a precisely specified situation—such as two point-masses interacting solely via the gravitational force. Discrepancies between this initial theoretical account and the actual motions may indicate that some idealizing assumptions do not hold, and the next step in the series of approximations is obtained by dropping assumptions—for example, by considering the gravitational effect of a third point-mass—and obtaining a more elaborate theoretical description. On Smith's account, the laws of motion (and, for our purposes, the dynamical conception based on them) accrue empirical support with each success in accounting for discrepancies between actual motions and the n^{th} stage in a series of approximations. These successes involve showing that the discrepancies can be handled by relaxing idealizing assumptions in a way that is consistent with the entire mathematical framework, and in a way that identifies further physical details of the system being studied.

Although this is far too brief a sketch of these ideas, we have said enough to contrast the methods of inquiry associated with the dynamical and geometrical conception of matter. Clearly, both Harper's and Smith's accounts of Newton's method depend crucially on the exactness made available by the mathematical framework of Book I. But Newton claimed to have established "from the phenomena" not only facts about the gravity and lawful motion of bodies, but also claims about their impenetrability and extension (see §2.2-2.3). Can similar accounts be given for these claims?

Certainly, Newton does *not* introduce parameters characterizing impenetrability and show how various phenomena give agreeing measurements of them. Nor does he give controlled idealizations that can be utilized as first approximations in order to derive the properties of impenetrability and extension from observed motions, and then proceed to develop successively more detailed approximations. The evidential warrant for such inferences "from the phenomena" relies on a less sophisticated chain of reasoning than does the evidential warrant provided for the laws of motion. We will return to this issue shortly, when we consider a case in which a deduction from the phenomena that utilizes a precise mathematical framework is pitted against a deduction from the phenomena that does not.

Before doing so, however, we must note a feature of Newton's *a posteriori* method of inquiry that is *shared* by both the sophisticated and less sophisticated types of inferences "from the phenomena".

Newton's further comments to Cotes warn against overstating the certainty of any *a posteriori* deductions:

> ... Experimental philosophy proceeds only upon Phenomena & deduces general Propositions from them only by induction. And such is the proof of mutual attraction. And the arguments for ye impenetrability, mobility & force of all bodies & for the laws of motion are no better. And he that in experimental Philosophy would except against any of these must draw his objection from some experiment or phænomena & not from a mere Hypothesis, if the Induction be of any force. (Newton 1977, V, Doc. 989, p. 400)

Newton acknowledged that the laws of motion were "hypothetical" in the limited sense of being open to revision based on subsequent discoveries, but carefully delimited what would count as a valid objection. Although Newton and Cotes continued to use the term "hypothetical" in their correspondence, in modern terminology "provisional" or "corrigible" would more aptly describe the status Newton ascribed to the laws. According to Newton, the laws of motion are not provisional due to the threat of underdetermination and alternative "hypothetical models". Rather, they are provisional because in establishing them one must generalize from a limited set of phenomena, and this necessarily inductive step may be overturned by new empirical evidence. In an unsent draft of the letter to Cotes quoted above, Newton discussed the idea at greater length:

> One may suppose that God can create a penetrable body & so reject the impenetrability of matter. But to admitt of such Hypotheses in opposition to rational Propositions founded upon Phænomena by Induction is to destroy all arguments taken from Phænomena by Induction & all Principles founded upon such arguments. And therefore as I regard not Hypotheses in explaining the Phenomena of nature so I regard them not in opposition to arguments founded upon Phcænomena by Induction or to Principles setled upon such arguments. In arguing for any Principle or Proposition from Phænomena by Induction, Hypotheses are not to be considered. The Argument holds good till some Phænomenon can be produced against it. This Argument holds good by the third Rule of philosophizing. And if we break that Rule, we cannot affirm anyone general law of nature: we cannot so much as affirm that all matter is impenetrable. . . .
>
> It is not enough to object that a contrary phænomenon may happen but to make a legitimate objection, a contrary phenomenon must be actually produced. (Newton 1977, V, p. 398)

By the time of this exchange, however, the earlier portions of Book III of the second edition had already been printed, so new material could not be added to them. Yet in the third edition, Newton added a claim to much the same effect, now treated as an independent fourth rule of philosophizing rather than a consequence of Rule III:

> In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions.
>
> This rule should be followed so that arguments based on induction may not be nullified by hypotheses (Newton 1999, p. 796).

Rule IV makes it clear that the uncertainty Newton associated with deductions from the phenomena was quite different than that associated with mechanical models. Taking the results of such a deduction to apply generally introduced uncertainty, the uncertainty of any inductive generalization. Newton *further* acknowledged the possibility that the original results of the deduction from phenomena may only be an approximation to further, "more exact" theoretical descriptions. But in both cases, Newton held that the best way to handle the associated uncertainty was to pursue comparisons of observations and their theoretical descriptions, with the hope of turning up contrary phenomena indicating error. Pursuing "hypotheses" in the sense of the mechanical philosophy had no part in this effort.

3.3 Gravity as an Essential Property

How did Cotes respond to Newton's elaboration of his method? Cotes was tempted to bite the bullet and assert that the matter of the central body actively produces the gravitational force felt by the orbiting body, that is, that it is the physical seat of the force of gravitation.[38] The third law applies in this instance because the central body, rather than some intermediary, is *directly* responsible for the force felt by the orbiting body. However, this suggests an intimate connection between matter and gravitation, and so a question arises about how to characterize this connection. In writing the preface to the second edition Cotes initially called gravitation an essential property of matter—by which he meant a property "without which no others belonging to the same substance can exist" (Newton 1977, V, Doc 1001)—but was reprimanded by Clarke. In response, Cotes substituted "primary" for "essential," but still treated gravitation as on par with impenetrability, extension, and mobility; it has, he wrote, "as fair a claim to that title" as the other properties.[39]

[38]Here we have in mind a distinction between two senses in which gravitation can be ascribed to matter (see also McMullin 1978, pp. 59-61). First, gravity causes deviations from inertial motion in accordance with the second law, and in this sense matter plays only a *passive* role, by responding to the impressed force of gravity. But in addition, a body must also produce the impressed force felt by other bodies, and this second, *active* sense is more problematic for Newton. For the third law to apply to an attractive force between two bodies, without anything else mediating the interaction, each body must respond to and also produce the force.

[39]See Edleston (1969), p. 158, and Newton (1999), p. 392.

Cotes did not elaborate further in the correspondence, but it seems plausible that he would have defended himself as follows. Inertia is taken to be essential to matter and material bodies precisely because the laws of motion—the laws detailing the relations between inertia, impressed force, and motion in bodies—require it. To be a body subject to the laws of motion means to be necessarily a body with inertial properties. Likewise, gravity has a "fair claim" to the title of an essential property because the understanding of attractive forces at work in the *Principia* requires it. The *Principia* demonstrates that all bodies attract one another according to a single force law, and so, taking this force law as his guide in determining the essential properties of matter, and having no indication that this force law could be explained by some deeper mechanism, Cotes is ready to claim that gravity is essential to material bodies. For Cotes, physical theory itself is the guide to determining essential properties, and physical theory is best established by the detailed apparatus of the *Principia*. As promised in §2.3, we can now also see why the list of qualities generalized by Rule III of the *Regulae Philosophandi* is heterogenous. The force of inertia, for example, is essential for the Newtonian theoretical description of actual motions. But extension, hardness, and impenetrability are not.[40] Moreover, gravity and inertia are established by the complex method outlined in the previous section, but extension, hardness, and impenetrability are not. As we shall see in §4, Cotes clearly recognized that the qualities treated by Rule III are not on an equal footing and thus that not all "deductions from phenomena" generalized by Rule III are equally meritorious.

For Newton, however, responding to the objection by taking gravity as an essential property was a step in the wrong direction. The physical characterization of gravity as a real rather than merely apparent force requires at least that it is a mutual interaction satisfying the third law. This is an important constraint on the nature of the force and it runs deeper than might be expected, but Newton does not follow Cotes in taking this to have direct implications for the ultimate cause of gravitation or the essential properties of matter. Newton's original reprimand of Cotes—that he applied the term "Hypothesis" too broadly—is instructive in this regard. For Newton, the application of the third law to the orbiting and central body is crucial part of making the step from a mathematical characterization of a force, as a well-defined quantity inferred from observed motions, to a characterization of the physical causes, species, and proportions of real forces. But making this step emphatically does not require determining the cause of gravity or the relation of gravity to the essential properties of matter.[41] The application of the third law has a "hypo-

[40]Mobility has a curious status as an object of the intension/remission criterion, so we leave it aside here.

[41]Cotes' treatment of essential properties collapses the distinction between these two; for a general discussion of these issues, see Schliesser (2009b).

thetical" or provisional character, in the limited sense in which the overall framework of the laws of motion is "hypothetical". However, this sense is not analogous to the hypothetical character of mechanical models. The true nature of the gravitational force—i.e., whether or not it acts *immediately* as a force of interaction between the orbiting and central bodies—is a separate question, not directly related to the status of laws of motion, and Newton reserved judgment regarding it.[42] To speculate, as Cotes did, that the application of the third law is inconsistent with the true, yet unknown, cause of gravitation is to repeat a common mistake of the mechanical philosophers, namely to judge an experimentally established proposition on the basis of its compatibility with claims regarding the fundamental nature of bodies. Given his skepticism regarding such claims, Newton rejected the need for such a compatibility check, and this was one of the most distinctive aspects of his method.

In sum, in our opinion Newton's answers to Cotes only *seem* to fail to recognize the question of whether gravity is mutual *per se* because Newton *purposely* rejected any discussion of what gravity is, *per se*. Newton's reference to the conspiring nature of both orbiting and central bodies should not be taken to mean that gravitational attraction resides essentially in either. Had Newton explicated his own methodological tenets with enough clarity, he could have made it clearer to Cotes that he chose to remain agnostic about the implications of his own theory regarding the essential natures of bodies. However, his lack of explicitness on this occasion, the fact that he often entertained deeper explanations (albeit with sufficient caveats), *and* the fact that he was the sole natural philosopher endorsing this approach, all contributed to Cotes' confusion and willingness to consider such implications. The same pattern of misunderstanding recurs in Cotes' query about the proportionalities that hold between weight, inertia and quantity of matter discussed in the next section. There, however, Cotes proves Newton to be deeply mistaken about the claims warranted by his own natural philosophical method.

[42] Newton famously denied that his characterization of gravitational force implied that brute matter could act directly at a distance; for example, his correspondence with Richard Bentley included the following, oft-quoted vehement denial:

> It is inconceivable that inanimate brute matter should, without the mediation of something else which is not material, operate upon and affect other matter without mutual contact, as it must do if gravitation, in the sense of Epicurus, be essential and inherent in it. And this is one reason why I desired you would not ascribe innate gravity to me. That gravity should be innate, inherent, and essential to matter so that one body may act upon another at a distance through a vacuum without the mediation of any thing else by and through which their action or force may be conveyed from one to another is to me so great an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it (Newton 1977, III, pp. 240-244)).

See Janiak (2007) and Schliesser (2009a) for a recent contrasting interpretations of this passage.

4 Proportionalities

In Proposition 6 of Book III, Newton demonstrated that:

> All bodies gravitate toward each of the planets, and at a given distance from the center of any one planet the weight of any body whatever toward that planet is proportional to the quantity of matter which the body contains (Newton 1999, p. 806).

It follows that the weight of a body does not depend on properties, such as form or texture, other than its quantity of matter, as Newton noted in the first Corollary. This result distinguishes the force of gravity from other forces such as magnetism and also sets Newton's view apart from other contemporary accounts of gravitation.[43] In the text of the proposition, Newton described a pendulum experiment meant to establish that near the surface of the Earth the *weight* of a body is proportional to its quantity of matter, and further that the weight of Jupiter's moons is proportional to their quantities of matter.[44]

The pendulum experiment was first mentioned in two manuscripts which follow the initial drafts of *De Motu* and is remarkably simple. Newton constructed two pendulums, each with a wooden box for the bob and equal lengths of 11 feet, and then filled the wooden boxes with equal weights of gold, silver, lead, glass, sand, common salt, wood, water, and wheat. For each pair of materials, he measured the periods of oscillation of the pendulums. The connection between the periods of oscillation and the proportionality between weight and quantity of matter depends on one of the *Principia*'s earlier propositions, proposition 24 of Book II. That proposition establishes the general result that the mass of a pendulum bob is proportional to the product of its weight and the square of the pendulum's period of oscillation, $m \propto w \cdot p^2$. (As Newton notes in Corollary 5 to the proposition, this result holds as well with the "relative weight" (or buoyant weight) of the pendulum bob in place of w, because for a body immersed in a medium the motive force is the relative weight.) The proof of the proposition begins with two basic proportionalities. First, $f_m \propto \frac{m \triangle v}{\triangle t}$, where f_m is the motive force, v is the velocity and t the time. The first proportionality holds that motive force measures the change in quantity of motion generated in a given time due to an impressed force. This quantity is proportional to the product of quantity of matter and velocity. This proportionality merely re-states the definition of motive force

[43]Seventeenth century treatments of gravitation left open the possibility that gravity could depend upon a wide variety of a body's properties. Newton considered a long list of such possibilities in the 60s in the *Waste Book*. Likewise, the analogy between gravitation and magnetism invoked by thinkers such as Kepler and Hooke suggested that gravity may act partially, not at all, or even repulsively on some bodies, depending upon their composition; see Westfall (1967, pp. 246-51), Koyré (1965), pp. 173-179, 185-187, and references therein.

[44]See also Harper's contribution to this volume for a discussion of the second part of the argument of Proposition 6 regarding Jupiter's moons.

as it is given in Definition 8 of the *Principia*. Clearly, the quantity of matter here—the m appearing on the right hand side—represents a body's *inertial resistance* to change in motion produced by impressed force. The second proportionality used is $f_m \propto w$; that is, that motive force is proportional to the weight of the pendulum bob. The proof proceeds by replacing terms in the (slightly rearranged) expression $m \propto \frac{f_m \triangle t}{\triangle v}$, to derive first $m \propto \frac{w \triangle t}{\triangle v}$, and then $m \propto w \cdot p^2$. The body of the proof consists of showing that $p^2 \propto \frac{\triangle t}{\triangle v}$. For a simple pendulum near the earth's surface, the period depends upon both the length of the pendulum and the acceleration due to gravity. Since Newton used two pendulums of equal length, the periods would have differed only if the gravitational acceleration of the bobs differed. Put differently, if the gravitational acceleration differed for pendulum bobs composed of different materials, the difference would be reflected in the pendulums' periods.

Newton reported that the periods of two pendulums containing different types of matter (gold and wood, for example) were in fact the same, to within an accuracy of $\frac{1}{1000}$. From this experiment, he concluded that:

> Accordingly, the amount of the matter in the gold (by II.24, corollaries I and VI) was to the amount of matter in the wood as the action of the motive force upon all the gold[,] to the action of the motive force upon all the wood–that is, as the weight of one to the weight of the other. And it was so for the rest of the materials (Newton 1999, p. 807).

In other words, using the same notation as above, $m \propto f_m \propto w$ for all the materials tested in the experiment. Newton generalized this to include all bodies, including those composed of materials not tested in the experiment. He wrote: "This is the quality of all bodies within reach of our experiments, and therefore (by Rule 3) is to be affirmed of all bodies whatsoever." The Corollaries of Proposition 6 treat the implications of this proposition for matter theory.

Of particular importance to us is Corollary III. In Corollary III, Newton argued that a vacuum must exist. If a plenum existed, he argued, then all bodies would have the same specific gravity and no body would rise or descend through air. The Corollary reads as follows in the first edition (Newton 1999, p. 810):

> And thus a vacuum is necessary. For if all spaces were full, the specific gravity of the fluid with which the region of the air would be filled, because of the extreme density of its matter, would not be less than the specific gravity of quicksilver or gold or of any other body with the greatest density, and therefore neither gold nor any other body could descend in air. For bodies do not ever descend in fluids unless they have a greater specific gravity.

Cotes' objected that this argument implicitly assumes that completely filled regions of space possess identical specific gravities, which, in turn, can be the case if and only if those regions contain identical quantities of matter. He illustrated the objection with a thought-experiment:[45]

> Let us suppose two globes A & B of equal magnitudes to be perfectly fill'd with matter without any interstices of void Space; I would ask the question whether it be impossible that God should give different *vires inertia* to these Globes. I think it cannot be said that they must necessarily have the same or an equal *Vis Inertia*. Now You do all along in Your Philosophy, & I think very rightly, estimate the quantity of matter by the *Vis Inertia* & particularly in this VIth Proposition in which no more is strictly proved than that the Gravitys of all Bodys are proportionable to their *Vires Inertia*. Tis possible then, that ye equal spaces possess'd by ye Globes A & B may be both perfectly fill'd with matter, so no void interstices remain, & *yet* that the quantity of matter in each space shall not be the same. Therefore when You define or assume the quantity of Matter to be proportionable to its *Vis Inertia*, You must not at the same time define or assume it to be proportionable to ye space which it may perfectly fill without any void interstices; unless you hold it impossible for the 2 Globes A & B to have different *Vires Inertia*. Now in the 3rd Corollary I think You do in effect assume both these things at once (Newton 1977, V, Doc. 893, p. 228).

Cotes pointed out, in other words, that the two ways of quantifying matter—based on its response to impressed force (*vis inertiæ*) and based on the volume it impenetrably fills—need not agree. The third corollary presupposes that these two measures of quantity of matter do, in fact, agree. However, if they do not, one can account for differences in specific gravity without recourse to a vacuum. The implications for Newton's anti-Cartesian, anti-plenum arguments are clear.[46]

[45] Cotes' Cambridge contemporary Robert Greene lodged essentially the same objection to Newton's argument for the vacuum in Chapter VI of Greene (1712), albeit not nearly as perspicaciously as Cotes. It appeared in a series of rebuttals to arguments in favor of a vacuum, one part of an overall critique of Newtonian natural philosophy and defense of the treatment of matter as active rather than passive or inert. We do not know what impact, if any, Greene's published discussion of the point made here by Cotes had at the time.

[46] As with the invisible hand objection, Kant also criticized Newton on precisely this point. Proposition XII of the pre-Critical *Physical Monadology* states that "The specific difference of the density of bodies, which are able to be observed in the world, cannot be fully explained without reference to the specific difference in the inertia of their elements." Kant explicitly replies to Newton's argument discussed above, and comments that trying to explain variation in density starting with homogeneous atoms with the same density would be to "indulge in an exaggerated passion for conjecture" (Kant 2003, p. 64). A passage from the *Critique* more closely parallels Cotes' argument (we thank Kent Baldner for bringing this passage to our attention):

> Nearly all natural philosophers (Naturleher), since they perceive a great difference in the quantity of matter of different sorts in the same volumes (partly through the moment gravity, or weight, partly through the moment of resistance against other, moved matter) unanimously infer from this that this volume (extensive magnitude of appearance) must be empty in all matter, although to be sure in different amounts. But who among these for the most part mathematical and mechanical students of nature ever realized that their inference rested solely

But Cotes' objection also has broader implications, implications that tie together the various threads in Newton's thought we have discussed so far. As we have seen, the geometrical and dynamical definitions of "quantity of matter" both played a significant role in Newton's thought. Cotes' objection shows that he recognized the possibility of measuring "quantity of matter" in two distinct ways and that he saw a possible conflict between them. Assuming that both methods are correct, i.e. that both *vis insita* and extension are proportional to quantity of matter, it should follow that both are proportional to one another. It turns out, however, that a proportionality between the dynamical and geometrical quantification methods can be given neither *a priori* nor empirical justification. First, there is nothing in the concepts of spatial impenetrability or force of inertia that necessitates a determinate proportionality between the two. Second, although the pendulum experiments are intended to prove that gravitation depends upon the *quantity of matter*, and not on the form or texture of matter, as Cotes indicated to Newton, "no more is strictly proved [in these experiments] than that the Gravities of all Bodys are proportionable to the *Vires Inertiae*."[47] Whether the gravities of bodies are further proportional to their *quantities of matter* depends on how one defines 'quantity of matter'. If one defines it to be proportional to the inertia of a body, then the experiments support the desired conclusion. But if one defines it to be proportional to the extension a body impenetrably fills, the experiments undermine the desired conclusion. What Cotes' objection thus reveals, although Cotes does not point to it directly, is that the choice of definition threatens the very validity of the argument for Universal Gravitation. To see this, assume that the proportionality of quantity of matter, defined geometrically, to *vis inertiæ* can vary, as in Cotes' two globes. We can replace *vis inertiae* with weight in the conclusion to Proposition 6, since the pendulum

> on a metaphysical presupposition, which they make so much pretense of avoiding? - for they assume that the *real* in space (I cannot call it here impenetrability or weight, since these are empirical concepts), is *everywhere one and the same*, and can be differentiated only according to its extensive magnitude, i.e., amount. Against this presupposition, for which they can have no ground in experience and which is therefore merely metaphysical, I oppose a transcendental proof, which, to be sure, will not explain the variation in the filling of space, but which still will entirely obviate the alleged necessity of the presupposition that the difference in question cannot be explained except by the assumption of empty spaces... For there we see that, although equal spaces can be completely filled with different matters in such a way that in neither of them is there a point in which the presence of matter is not to be encountered, nevertheless everything real has for the same quality its degree (of resistance or of weight) which, without diminution of the extensive magnitude or amount, can become infinitely smaller until it is transformed into emptiness and disappears (Kant 1998, (A173/B215–A174/B216)).

Kant's most thorough discussion of quantity of matter is in the *Metaphysical Foundations of Natural Science*, although we do not have space to pursue the issue further here.

[47]Here a modern reader might object that Newton and Cotes are guilty of conflating inertial and gravitational mass. While there is truth to this (see Densmore (1996), pp. 313-330 for a detailed assessment of the pendulum experiments taking this into account), the problem is distinct from the objection under discussion.

experiments show that they are proportional at a fixed distance from the earth; thus, quantity of matter geometrically defined is not proportional to weight (at a fixed distance). If quantity of matter is defined to be proportional to quantity of extension, it further follows that, even at a fixed distance from the Earth, *the quantity of matter of a body can vary in relation to its weight*. Thus, Cotes' objection undermines not just the third Corollary to Proposition 6, but Proposition 6 itself, and thus a crucial step of the argument for universal gravitation. If Newton wants to maintain that quantity of matter can be defined by *either* quantity of extension or quantity of inertia, he must assume that the two are determinately proportional, a claim for which he can offer no justification. This is precisely Cotes' point.

In his responses, Newton attempted to rebut Cotes' objection by claiming that matter has inertial properties that are proportional to its quantity, that matter is impenetrable and thus has geometrical properties, and that these two facts suffice to entail the determinate proportionality of inertia to extension. Of course, this is still missing Cotes' point. The point is simply that these two facts, which Cotes does not dispute, do *not* entail the proportionality of inertia to extension.[48] Newton's second response to Cotes (after Cotes reiterates his reasoning) illustrates his misunderstanding and his continuing commitment to *both* the dynamical and geometrical conceptions of matter and their *a posteriori* character. Newton wrote:

> I have reconsidered the third Corollary of the VIth Proposition. And for preventing the cavils of those who are ready to put two or more sorts of matter you may add these word[s] to the end of the Corollary: [1] From pendulum experiments it is established that the force of inertia is proportional to the gravity of a body. [2] The force of inertia arises from the quantity of matter in a body and so is proportional to its massiness [*massa*]. [3] A body is condensed by the contraction of the pores in it, and when it has no more pores (because of the impenetrability of matter) it can be condensed no more; and so in [completely] full spaces [the force of inertia] is as the size of the space. Granted these three principles the corollary is valid (Newton 1977, V, Doc. 898, p. 240).

Since Newton and Cotes explicitly agreed on [1], the source of their disagreement lies in [2] and/or [3]. In [2], Newton implicitly defined quantity of matter to be proportional to the force of inertia. Since Cotes had already written to Newton that "all along in your Philosophy, & I think very rightly, you estimate the quantity of matter by the *Vis Inertiae*," the source of conflict must be [3]. In [3], Newton deduced from [2] and the impenetrability of matter that the inertia of matter is proportional to the extension it solidly fills. Clearly

[48] Cotes' position shifted slightly during this exchange of letters: whereas initially he objected to the implicit assumption of the proportionality of inertia to quantity of extension ("You *must not* at the same time define ..."), he later allowed that the proportionality could be invoked as an unproved assumption. In either case, his objection is that Newton's explicit commitments do not entail that the proportionality holds.

Newton took this to be a valid inference. According to Cotes, however, Newton's reasoning is circular: he implicitly assumed that the force of inertia is determinately proportional to the extension solidly filled by matter in order to deduce that, after condensation, the force of inertia would be determinately proportional to the extension filled by matter. As he wrote in his subsequent response to Newton:

> I am not yet satisfied as to the difficulty unless You will be pleased to add, That it is true upon this concession, that the Primigenial particles ... have all the same *Vis Inertiae* in respect to their magnitude or extension in *Spatio pleno*. I call this a concession because I cannot see how it may be certainly proved either a Priori by bare abstracted reasoning; or be inferr'd from Experiments (Newton (1977) V, Doc. 899, p. 242).

Cotes took Newton to be putting a uniformity constraint on the fundamental, "Primigenial" particles of matter, particles that are not directly accessible to experimental investigation. The uniformity constraint amounts to the claim that all fundamental particles have identical specific gravities; or, what amounts to the same thing, that their quantity of matter is directly proportional to their extension. Newton had appealed to the uniformity constraint from his earliest philosophical writing in the *Certain Philosophical Questions*, to his draft and final revisions to Hypothesis III of the *Principia*'s first edition, and to, finally here, his considered arguments against the vacuum. It is a fixture of Newton's thought that had gone unchallenged until this exchange with Cotes, although Newton appears to have justified the constraint by subtly different means at different points in his career.[49] At the beginning of this exchange with Cotes, however, Newton seems to believe that the constraint could be justified *a posteriori*. His initial responses make it clear that, by his own lights, the uniformity of primigenial particles followed from observable facts regarding the extension, impenetrability, and inertia of matter. Cotes' objection pointed to the conflict between the geometrical and dynamical measures of matter, both of which, according to Newton, were derived *a posteriori*.

However, as we have suggested, the geometrical definition (and the conception of matter underlying it) is derived by a different sort of *a posteriori* argument than the dynamical definition (and the conception of matter underlying it). The geometrical definition is derived from the claim that, as Newton puts it in Rule III and as he articulated more elaborately in *De grav*, "extension is found in all sensible bodies". This derivation is in some sense immediate—it rests on no mathematical chain of reasoning, no process of approximation, and no fixing of causal parameters. Within Newton's broadly mechanical account of per-

[49]McGuire (1970) traces Newton's changes of attitude towards the uniformity assumption, as well as the assumptions' sources in ancient atomism and its impact on Newton's transmutation hypothesis and inferences regarding the nature of micro-matter.

ception, it simply follows from the our experience of any body whatsoever. The dynamical definition, on the other hand, is *a crucial part of Newton's mathematical account of force, developed and used to account for a variety of motions in the body of the* Principia.

Ultimately, Newton backed down. In Corollary 4 to Proposition 6 of the Second Edition, he rephrased the anti-vacuum argument in the form of a conditional, acknowledging the assumption Cotes insisted on:

> If all the solid particles of all bodies have the same density and cannot be rarefied without pores, there must be a vacuum. I say particles have the same density when their respective forces of inertia [or masses] are as their sizes.

That is, *if* the fundamental particles have a fixed ratio between inertia and volume, *then* a vacuum must be granted. Yet the interchange with Cotes makes clear that Newton's initial inclination was to maintain that all primigenial particles are similarly extended in proportion to their quantities of matter, despite the fact that his own pendulum experiments and the mathematical structure in which they were embedded recommended no such steadfastness. In fact, the dynamical conception supported by the results of the *Principia* is compatible with treating matter as constituted by Boscovichian point-particles, with the quantity of matter appearing solely as a parameter of these point-particles. The geometrical properties of matter simply play no role in physical explanations in this schema, since such explanations depends solely on the laws of motion and the further specification of inter-particle forces.

Newton's initial failure to see this point reflects, on our view, a failure to clearly distinguish the distinctive *a posteriori* methods described above. As we saw in the treatment of Rule III, Newton conceived of properties like extension and impenetrability as having the same status as inertia, despite the fact that they were supported by a distinctive line of argument that was not intertwined with the argument for universal gravitation or the deduction of the laws of motion from phenomena. Cotes, to his credit, was quite clear that Newtonian mechanics does not support a geometrical conception of matter. While writing the preface to the second edition, he pointed out the shaky status of extension to Clarke: "I understand by Essential propertys such propertys without which no others belonging to the same substance can exist: and I would not undertake to prove that it were impossible for any of the other Properties of Bodies to exist without even Extension" (Newton 1977, V, Doc. 1001). Cotes here comes face to face with the possibility that Boscovichian non-extended point-particles can constitute bodies and that our *experience* of bodies—even our experience of those qualities that seem immutable and invariably present—is no guide in questions of essentiality. For Cotes, to repeat a point made in §3.3 regarding gravity, physical theory itself is the guide to determining essential properties. It just so happens that within Newtonian mechanics inertia plays a central role in giving an account of observed motions whereas extension does not. Insofar as Cotes is concerned, so much the worse for extension.

Newton was not far behind. After Cotes' objection pointed out the incongruity between his two conceptions of matter, Newton came to withdraw his support from his long-held geometrical conception of matter and the remainder of his views developed accordingly. The change of mind for an astute and tenacious figure such as Newton is significant: Newton did not back down in response to the invisible hand objection because he was certain of his correctness. In response to the two globes objection, however, Newton modified his views appropriately. In a series of draft definitions intended for Book III of the third edition of the *Principia* (dated by McGuire (1966) to 1716),[50] Newton explicitly addressed his now-changed conception of body. He wrote:

> *Definition II* Body I call everything which can be moved and touched, in which there is resistance to tangible things, and its resistance, if it is great enough, can be perceived (McGuire 1966, p. 115).

Lacking from this definition is any mention of the extension of bodies. The *only* definitional property of body here is its inertial resistance. A far cry indeed from *De grav*'s definition:

> Definition 4. Body is that which fills space (Newton 2004, p. 13).[51]

5 Conclusion

We have stressed two main themes of Newton's thought in this paper. The first is an account of Newton's empiricist method, and the two approaches he took to justifying claims in natural philosophy. The approach exemplified by the argument for universal gravitation in the *Principia* contrasts sharply with the method of the mechanical philosophers. Unlike the mechanical philosophers, Newton did not allow for the satisfaction of intelligibility constraints (e.g., that only contact action in comprehensible) to serve as justification, even if partial, for a particular physical theory or model; the justificatory support for the laws of motion and universal gravitation derives entirely from their ability to serve successfully as a framework for describing motions. In fact, Newton took the results of such deductions from the phenomena to be more secure than any claimed first principles regarding the

[50] See McGuire (1966). McGuire sees Newton's battles with Leibniz as the main impetus for these definition. We take this essay to suggest that Newton also had independent reasons for formulating his new definitions, ones motivates by his exchange with Cotes.

[51] We take the development of Newton's views on body to show that *De grav* cannot be automatically taken to reflect Newton's mature metaphysical views. Rather, it is best taken as Newton's relatively early attempt to explicate the philosophical infrastructure in which his physics is embedded, but by no means the last word. This is often an under-appreciated fact in Newtonian scholarship.

nature of matter and motion. Newton's response to Cotes' invisible hand objection reflected this methodological stance: Cotes objected that Newton had inappropriately assumed that gravitational force must be produced by the orbiting and central bodies despite his professed agnosticism regarding the underlying cause of gravity; Newton responded by clarifying that his characterization of gravity as a force obeying the three laws was hypothetical in the same limited sense that the laws of motion are hypothetical—namely, all were fallible—and did not entail further assumptions regarding the essential properties of matter or the underlying cause of gravity. The second approach to establishing results *a posteriori* is exemplified by the account of body in *De grav* and some of Newton's statements in Rule III, and it involves a much more direct argument, essentially reading off the universal properties of matter from the general experience of bodies. It does not draw on a precise mathematical framework like that of the *Principia*, and so the ways of clarifying evidential warrant in relation to the first approach do not carry over. This leaves it much less clear how to assess the strength of the conclusions derived from this type of reasoning, and Newton says very little to help in fleshing out the account.

Second, there is an uncomfortable union in Newton's thought between two competing conceptions of matter. The geometrical conception of matter—the dominant conception at the time—reflects Newton's Cartesian roots and was linked to the possibility of an aether explanation of gravitation. Although Newton decisively rejected several aspects of Cartesian thought in *De grav*, he retained an account of bodies which took their geometrical properties to be fundamental. At the same time, Newton developed the distinctive dynamical conception of matter incorporated in the *Principia*, which measures quantity of matter by a body's response to impressed forces. Newton never explicitly drew this distinction, and he apparently treated the two quantitative measures of matter as simply different aspects of an underlying, coherent account of matter. Cotes' second objection brought out the tension between these two conceptions; Cotes argued that Newton's claims would not be sustained without an explicit assumption regarding the fundamental constituents of matter, betraying Newton's professed agnosticism on such matters. Newton's response to Cotes reflect his failure to clearly distinguish the two approaches to *a posteriori* reasoning characterized above.

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