

Review of Joe Salerno's (ed) *New Essays on the  
Knowability Paradox*\*

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There are many truths about the world that seem beyond our epistemic reach. Even if a fly lost its right wing at a certain spot in Australia's Great Sandy Desert on April 4, 1444, it is dubious whether anyone will ever know it. On the face of it, however, we should not expect this fact about our non-omniscience to commit us to the much more dubious claim that there is a truth that could not *possibly* be known by anyone at any time. But Fitch has shown us otherwise: if there are unknown truths, there are in principle unknowable truths. Joe Salerno has collected an excellent stack of essays on Fitch's *knowability paradox*.

Salerno (essay 3) teaches us that it was Church who first established the paradox and anonymously conveyed it to Fitch in a pair of referee-reports (essay 1) on an early Fitch paper on value concepts. The anonymous referee is credited in Fitch's well-cited 1963 paper "A Logical Analysis of Some Value Concepts" (essay 2)—the only previously published material in the volume—that sparked the huge interest in

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the knowability paradox. Salerno’s essay also gives a critical evaluation of the debate in 1945 between Church and Fitch and offers an explanation of why Fitch ultimately included the proof of the knowability paradox in his 1963 paper.

In light of this, the more suitably named *Church-Fitch paradox* shows that (Unknown) and (Knowability) entail (Omniscience), where  $\diamond$  and  $K$  abbreviate “It is metaphysically possible that” and “It is known by someone at some time that” respectively:

- (Unknown)      $\exists p(p \wedge \neg Kp)$
- (Knowability)    $\forall p(p \rightarrow \diamond Kp)$
- (Omniscience)    $\forall p(p \rightarrow Kp)$

(Unknown) states that there is a truth that is unknown by anyone at any time. (Knowability) captures the spirit of certain anti-realistic views—such as verificationism and mathematical intuitionism—that impose an epistemic constraint on truth: if  $p$  is true, then  $p$  is knowable in principle. (Omniscience) says that any true proposition is known by someone at some time. Regardless of whether (Omniscience) is true for divine creatures, it seems clearly false even for idealized copies of cognizers like us.

(Unknown) and (Knowability) entail (Omniscience) in conjunction with the following principles (I omit reference to quantifiers):

- (Factive)      $Kp \rightarrow p$
- (Distribution)    $K(p \wedge q) \rightarrow (Kp \wedge Kq)$
- (Necessitation)   For any theorem  $p$ , infer  $\Box p$

(Factive) is uncontroversial. It captures the truism that known propositions are true. (Distribution) is accepted by most, and it says that subjects that know a conjunction

also know each of its conjuncts. (Necessitation) holds in all normal modal logics, and it says that theorems are necessarily true.

The crucial part of the Church-Fitch argument shows that it is impossible to know the so-called *Fitch-conjunction* ( $p \wedge \neg Kp$ ):

1.  $K(p \wedge \neg Kp)$       (Assumption)
2.  $Kp \wedge K\neg Kp$       1 (Distribution)
3.  $Kp \wedge \neg Kp$       2 (Factive)
4.  $\neg K(p \wedge \neg Kp)$     1, 3 (Reductio)
5.  $\Box\neg K(p \wedge \neg Kp)$    4 (Necessitation)
6.  $\neg\Diamond K(p \wedge \neg Kp)$    5 (Duality of  $\Box$  and  $\Diamond$ )

Suppose now that (Unknown) is true—that is, suppose there is a  $p$  such that  $(p \wedge \neg Kp)$ . We can then apply (Knowability) and infer  $\Diamond K(p \wedge \neg Kp)$ . But that contradicts the claim in step 6 that  $\neg\Diamond K(p \wedge \neg Kp)$ . To avoid this, we can either deny (Knowability) or (Unknown). In the latter case, we can infer  $\neg(p \wedge \neg Kp)$  and universally generalize to  $\forall p\neg(p \wedge \neg Kp)$ , which is classically equivalent to  $\forall p(p \rightarrow Kp)$  and hence (Omniscience).

So, if there are unknown truths, the claim that all truths are in principle knowable entails that all truths are known. Insofar as (Knowability) is the least plausible principle used in the argument, we must give up (Knowability) to avoid (Omniscience).

Adherents of (Knowability) must reply to the Church-Fitch argument. Three main types of replies have emerged: weaken the logic, restrict (Knowability), or reformulate (Knowability). Postponing until last the four essays that investigate the Church-Fitch argument from a broader philosophical perspective, I will briefly survey the remaining essays according to how they fit with these different types of replies.

## Non-classical replies

Dummett (essay 4) uses intuitionistic logic to avoid the Church-Fitch argument. Intuitionistically, the step from  $\neg(p \wedge \neg Kp)$  to  $(p \rightarrow Kp)$  is invalid because it presupposes an inference from  $\neg\neg Kp$  to  $Kp$ . However, the inference from  $\neg(p \wedge \neg Kp)$  to  $(p \rightarrow \neg\neg Kp)$  remains intuitionistically valid, and  $(p \rightarrow \neg\neg Kp)$  seems almost as bad as (Omniscience). But Dummett claims that when  $(p \rightarrow \neg\neg Kp)$  is read intuitionistically—roughly as “If  $p$ , there is an obstacle in principle to our being able to deny that  $p$  will ever be known”—it is innocuous. In fact, he claims that  $(p \rightarrow \neg\neg Kp)$  is the best characterization of anti-realism when the logical constants are understood intuitionistically.

Rasmussen (essay 5) and Bermúdez (essay 6) continue the intuitionistic thread. Rasmussen uses the intensional interpretation of intuitionistic negation to argue that (Knowability) is inessential to anti-realism. He considers alternative formulations of the knowability thesis and seems most optimistic about a (time-indexed) formulation along the Dummettian lines above. Bermúdez aims to provide further motivation for the intuitionistic reply by applying Dummett’s notion of indefinite extensibility to propositions.

Priest (essay 7) uses a dialetheist system that contains an absurd world where all propositions are both true and false to invalidate the reductio rule that we use to infer  $\neg K(p \wedge \neg Kp)$  in step 4. By endorsing the dialetheist idea that contradictions can be true, we can have (Knowability) and (Unknown) without (Omniscience).

Beall (essay 8) offers both paracomplete and paraconsistent replies to the Church-Fitch argument, neither of which is committed to dialetheism. Beall’s first paracomplete system utilizes truth-value gaps to allow for possibly—but not actually—true contradictions, whereas the second utilizes epistemically abnormal worlds at which

principles such as (Distribution) fail. Beall’s third system combines the previous two in a paraconsistent, non-dialetheist system.

### Restriction replies

Tennant (essay 14) modifies and defends the *Cartesian* restriction strategy. The basic idea behind this strategy is to restrict the scope of (Knowability) to *Cartesian propositions*: propositions knowledge of which does not lead to absurdity. Since  $(p \wedge \neg Kp)$  leads to an inconsistency,  $(p \wedge \neg Kp)$  is not Cartesian. So we cannot use (Knowability) to infer  $\diamond K(p \wedge \neg Kp)$ , and the Church-Fitch argument is blocked. Tennant shows that certain modifications to the basic idea are needed to defend it against specific derivations of the knowability paradox that exploit the KK-principle:  $(Kp \rightarrow KKp)$ .

Williamson (essay 12) criticizes Tennant’s (unmodified) Cartesian restriction strategy. In particular, he argues that it is still committed to  $\neg(p \wedge \neg Kp)$ , which is intuitionistically equivalent to  $(\neg Kp \rightarrow \neg p)$ . Effectively,  $(\neg Kp \rightarrow \neg p)$  says that what is never known is not true. And arguably, this is not much better than (Omniscience).

Kvanvig (essay 13) argues that the restriction reply is a red herring. For it does not give an explanation of the lost logical distinction between merely possible knowledge and actual knowledge:  $\forall p(p \rightarrow \diamond Kp) \Leftrightarrow \forall p(p \rightarrow Kp)$ . If this is what a proper solution to the knowability paradox requires, the restriction strategy is unsuccessful because it merely blocks rather than diagnoses a fault in the Church-Fitch argument.

Jenkins (essay 18) criticizes Kvanvig by arguing that there remains a firm logical distinction between  $\diamond Kp$  and  $Kp$ . According to Jenkins, the Church-Fitch argument teaches us that what we thought expressed anti-realism properly—namely (Knowability)—actually does not. In place of (Knowability), she proposes an unmodalized principle that she holds can capture the core anti-realist relation between

truth and knowledge.

### Reformulation replies

Linsky (essay 11) uses type theory to block the Church-Fitch argument. Since  $K$  gets typed differently when other epistemic operators fall within its scope, the two knowledge operators in step 1— $K(p \wedge \neg Kp)$ —should be typed as  $K^{(2)}$  and  $K^{(1)}$  respectively. In turn, step 3 is typed as  $(K^{(2)}p^1 \wedge \neg K^{(1)}p^1)$ , and there is no longer a contradiction. Hart (essay 19) complains that the type theory solution still entails that all true propositions are actually known at *some* type level.

Hand (essay 17) argues that the Fitch-conjunction  $(p \wedge \neg Kp)$  reflects the pragmatic phenomenon of *self-defeat* that occurs with sentences such as “I am not speaking”: it can be true but never asserted truthfully. Similarly,  $(p \wedge \neg Kp)$  is self-defeating with respect to knowledge: it can be true but never known to be true. Hand claims that the interesting debate between the realist and the anti-realist is a semantic debate. Since the problems with the Fitch-conjunction are pragmatic, they are not central to the debate between realism and anti-realism. As such, an anti-realist need not endorse (Knowability) as a core feature of her view.

Kelp and Pritchard (essay 20) replace (Knowability) with a weaker principle (JD): for all true propositions  $p$ , it is possible to justifiably believe  $p$ . They argue that (JD) can do the main work that (Knowability) is intended to do, namely to rule out evidence-transcendent truths. Because justification is not factive, the Church-Fitch argument fails. At least, they argue, it fails when we also reject the seemingly plausible principle that if we justifiably believe that we do not justifiably believe  $p$ , then we do not justifiably believe  $p$ .

Restall (essay 21) replaces (Knowability) with a weaker principle (CK): every truth is conjunctively knowable. Roughly, a proposition  $q$  is *conjunctively knowable*

just in case  $q$  is equivalent to a conjunction, each of whose conjuncts is knowable. Since each conjunct in  $(p \wedge \neg Kp)$  is knowable, (CK) allows that a sentence logically equivalent to  $(p \wedge \neg Kp)$  is conjunctively knowable. Although (CK) is compatible with (Unknown), Restall shows that the alethic accessibility relation cannot be transitive on pain of trivializing (CK).

### **Knowability from a broader philosophical perspective**

Van Benthem (essay 9) looks at the Church-Fitch argument in the setting of dynamic epistemic logic. He argues that propositions such as the Fitch-conjunction have a *self-afflicting* property: they change their own truth-value when they are announced. Van Benthem shows that similar epistemic phenomena often occur with information update in social dynamic contexts. And he expresses hope that we can broaden our understanding of Fitch-style issues in a richer dynamic-epistemic framework.

Burgess (essay 10) investigates a translation of (Knowability) into temporal modal logic: if  $p$  is true now, then  $p$  will be known at some point in the [open-ended] future. Burgess argues that various restrictions on this principle must be invoked to deal with the fact that propositions are tensed in temporal logic. The problems of finding such restrictions, he claims, can help us shape our understanding of the knowability paradox.

Brogaard (essay 15) uses a Church-Fitch style reasoning against *strong modal fictionalism*, which holds that talk of possible worlds is a useful fiction that is literally false. She argues that fictionalism must acknowledge certain principles that in turn trigger a Church-Fitch style argument showing that fictionalism is committed to the existence of possible worlds. Brogaard speculates that a restriction strategy may be the best way for the fictionalist to avoid this unacceptable commitment.

Bueno (essay 16) uses a Church-Fitch style argument to evaluate different views

in the philosophy of mathematics. He argues that full-blooded Platonism—roughly the view that all logically possible mathematical objects actually exist—and mathematical fictionalism—roughly the view that mathematical objects do not exist—are particularly vulnerable to Church-Fitch style problems. Although a similar fate need not befall other forms of platonism and fictionalism, Bueno argues that such views cannot easily explain how knowledge of mathematical objects is possible.

Let me end by pointing readers looking for a more elaborate overview of the collection to Salerno's very readable introduction (pp. 1-10). But more than anything, those who are interested in epistemology, logic, or metaphysics should dig into the essays themselves. For they are well worth a read.