A Never-Ending Story

Abstract: Take a strip of paper with ‘once upon a time there’ written on one side and ‘was a story that began’ on the other. Twisting the paper and joining the ends produces John Barth’s story Frame-Tale, which prefixes ‘once upon a time there was a story that began’ to itself. In this paper, I argue that the possibility of understanding Frame-Tale cannot be explained by tacit knowledge of a recursive theory of truth in English.

Keywords: understanding, recursion, metafiction.

Take a strip of paper with ‘once upon a time there’ written on one side and ‘was a story that began’ on the other. Twisting the paper and fastening the ends produces John Barth’s (1968: 1-2) Frame-Tale, which prefixes a token of ‘once upon a time there was a story that began’ to itself. Frame-Tale is understood: it’s because I understand it that I’m able to judge, for example, that Frame-Tale is true if it’s a story which began. But explaining this raises a puzzle.

Traditionally, the ability to understand an infinite number of English sentences is explained via tacit knowledge of a recursive theory of truth, with base clauses which state the truth-conditions of atomic sentences directly, and recursive clauses which state the truth-conditions of complex sentences in terms of the truth-conditions of their simpler constituents. (In a complete theory, these clauses would themselves be theorems derived from further base and recursive axioms.)

There are two problems with extending this strategy to explaining the ability to understand Frame-Tale. The first problem is that a statement of Frame-Tale’s truth-conditions cannot be derived from a recursive theory of truth in a finite number of steps, so tacit knowledge of such a theory cannot explain the ability of someone with finite intellectual capacities to understand it. I will argue that this problem cannot be solved, even by adopting a non-recursive theory of truth.

The second problem is that neither ‘once upon a time’ nor ‘there was a story that began’ is associated with an appropriate base clause from which to begin a derivation of Frame-Tale’s truth-conditions. I will argue that this problem can be avoided by a theory of truth which associates a non-recursive clause with ‘there was a
story that began’. But this theory of truth still cannot explain the ability to understand *Frame-Tale*, because the first problem remains unsolved.

It’s helpful to compare *Frame-Tale* with a simpler example. Take a strip of paper with ‘it rains and then’ written on one side. Fastening the ends produces *Fan-Tale*, an inferior fan fiction inspired by *Frame-Tale*, which prefixes a token of ‘it rains and then’ to itself. *Fan-Tale* differs from *Frame-Tale* because ‘it rains’ is associated with an appropriate base clause. But a statement of *Fan-Tale*’s, like *Frame-Tale*’s, truth-conditions cannot be derived in a finite number of steps.

*Frame-Tale* is important for two reasons. First, one important motivation for attributing tacit knowledge of recursive theories of truth to natural language speakers is to explain the fact that natural language speakers have the ability to understand an infinite number of sentences, even with only finite intellectual capacities. So if natural language speakers have the ability to understand never-ending sentences like *Frame-Tale*, tacit knowledge of a theory of truth should explain this too.

Second, *Frame-Tale* is an intriguing story. If the ability to understand it cannot be explained in the usual way via tacit knowledge of a theory of truth, this reveals something about why it is intriguing. So although this paper is primarily intended as a contribution to the philosophy of language, it’s also intended as a contribution to literary criticism. *Frame-Tale* illustrates that philosophy of language is interesting not only for its own sake, but also for understanding literature.

Three clarifications. First, it’s sometimes objected that *Frame-Tale* is not a grammatical sentence of English, so that the possibility of understanding it need not be explained. But *Frame-Tale* raises similar problems for a syntactic theory of English grammar as it does for a semantic theory of truth in English. Just as *Frame-Tale* should be accommodated by a semantic theory of truth in English, it should also be accommodated by a syntactic theory of English grammar.

It’s sometimes objected, for example, that *Frame-Tale* is not a sentence – and so the possibility of understanding it need not be explained – because it cannot be properly punctuated. It cannot contain, for example, an initial capital letter, a final full stop, or rightmost quotation marks. But the correct conclusion to draw from this is not that *Frame-Tale* is not a sentence, but simply that not every sentence ends with a full stop, and that not every quotation requires quotation marks.

Second, some linguists argue that just as there is no finite limit to the length of sentences in natural languages, there is no infinite limit either (see Langendoen and
Postal (1984) as well as Collins (2010) for recent discussion). These linguists concede that just as we can’t understand very but finitely long sentences, we can’t understand infinitely long sentences either. It’s just, they argue, that our inability to understand infinitely long sentences doesn’t preclude them from being sentences.

The problem raised by Frame-Tale is not just that a recursive theory of truth does not entail a statement of its truth-conditions. This is a problem, but it is the same as the problem raised by some linguists about the inability of recursive grammars for natural language to accommodate infinitely long sentences. Frame-Tale also raises an additional problem, because whereas very long, including infinitely long, sentences can only be understood in principle, Frame-Tale is understood in practice.

Third, some philosophers who emphasise the role of recursive theories of truth in the explanation of the ability to understand infinitely many sentences are extremely cagey about whether such theories are tacitly known (see, for example, Foster (1976: 2)). Other philosophers say explicitly that a recursive theory of truth is tacitly known in a substantive and psychologically real sense (see especially Evans (1981) as well as Davies (1981: 52-86) and Larson and Segal (1996: 56-62)).

According to this substantive sense of tacit knowledge, for a theory of truth to be tacitly known, the causal structure of the interpreter’s judgements must mirror the derivational structure of the theory (Davies 1981: 79). In particular, if the derivation of a particular theorem of the theory of truth is infinitely long, the causal explanation of the interpreter’s judgement that that theorem is true should also be infinitely long. It is this version of the thesis that Frame-Tale is a counterexample to.

Frame-Tale is understood: it’s because I understand it that I can judge, for example, that it is true if it is a story which began. Likewise, Fan-Tale is understood: it is because I understand it that I can judge, for example, that it’s true if and only if it rains eternally. Traditionally, the ability to understand and judge the truth-conditions of Frame-Tale and Fan-Tale would be explained by tacit knowledge of a theory of truth for a language which entails these judgements.

A theory of truth for a language is a theory which entails for each sentence in the language a theorem of the form:

\[ s \text{ is true if and only if } p \]
where ‘s’ is replaced by a structural description of that sentence and ‘p’ by that sentence or its translation (Davidson 1967: 23). So a theory of truth in English should entail, for example, that ‘snow is white’ is true if and only if snow is white.

To explain the ability to understand Frame-Tale, a theory of truth in English should entail:

(1) ‘... once upon a time there was a story that began once upon a time there was a story that began ...’ is true if and only if ... once upon a time there was a story that began once upon a time there was a story that began ...

where the ellipses signify that the sentence continues infinitely in both directions.

It might be objected that instead of an infinitely long theorem such as (1), a theory of truth in English need only entail a finitely long theorem such as:

(2) A token of ‘once upon a time there was a story that began’ prefixed to itself is true if and only if a token of ‘once upon a time there was a story that began’ is prefixed to itself

which appears preferable to (1) because it is only finitely long.

Since Frame-Tale is a token of ‘once upon a time there was a story that began’ prefixed to itself, (2) entails, whereas (1) does not, that Frame-Tale is true. But this is undesirable, because someone who understands Frame-Tale may doubt that it is true, since they may doubt whether it is a story, or whether it begins. Likewise, (1) entails, but (2) doesn’t, that Frame-Tale is true if every token of ‘once upon a time there was a story that began’ immediately succeeds and precedes another on an infinite strip.

But Fan-Tale illustrates that these differences are inessential. To explain the ability to understand Fan-Tale, a theory of truth should entail either:

(3) ‘... it rains and then it rains and then it rains ...’ is true if and only if ... it rains and then it rains and then it rains ...

where the ellipses signify that the sentence continues infinitely in both directions, or:

(4) A token of ‘it rains and then’ prefixed to itself is true if and only if it rains eternally

which appears preferable to (3) because it is only finitely long.

Although (1) and (2) disagree about the truth-conditions of Frame-Tale, (3) and (4) agree about the truth-conditions of Fan-Tale, because both (3) and (4) agree that Fan-Tale is true if and only if it rains eternally. This suggests that the differences between (1) and (2) are inessential, and that it’s possible in principle to formulate a
variation of (2) which is finitely long but in agreement with (1). But even though (2) and (4) are finitely long, (1) and (3), I will argue, are still preferable.

It’s possible to understand *Frame-Tale* and *Fan-Tale* without previously having encountered tokens prefixed to themselves, so axioms which suffice to explain the ability to understand ordinary sentences should suffice to explain the ability to understand *Frame-Tale* and *Fan-Tale*. But axioms which don’t explicitly mention tokens prefixed to themselves suffice to explain the ability to understand ordinary sentences. So axioms which don’t explicitly mention tokens prefixed to themselves should suffice to explain the ability to understand *Frame-Tale* and *Fan-Tale*.

But since (2) and (4) do mention tokens prefixed to themselves explicitly, axioms which don’t explicitly mention tokens prefixed to themselves would not suffice to entail them. So tacit knowledge of a theory of truth which entails (2) and (4) would be unlikely to explain the possibility of understanding *Frame-Tale* and *Fan-Tale* without previously having encountered tokens prefixed to themselves. Although (2) and (4) are finitely long, the ability to understand *Frame-Tale* and *Fan-Tale* must be explained by a theory of truth entailing (1) and (3).

The ability to understand an infinite number of sentences is traditionally explained by tacit knowledge of a recursive theory of truth, with base clauses which directly state the truth-conditions of atomic sentences and recursive clauses which state the truth-conditions of complex sentences in terms of the truth-conditions of the sentences which compose them. There are two problems for explaining the ability to understand *Frame-Tale* via tacit knowledge of a recursive theory of truth.

The first problem is that although statements of the truth-conditions of an infinite number of sentences can be derived from the base and recursive clauses of a theory of truth, infinitely long statements of the truth-conditions of sentences such as *Fan-Tale* or *Frame-Tale* cannot be derived. So although tacit knowledge of a theory of truth can explain the ability to understand infinitely many sentences, it cannot explain the ability to understand *Fan-Tale* or *Frame-Tale*.

Although, for example, statements of the truth-conditions of an infinite number of sentences can be derived from a base clause stating:

\[(5) \text{‘it rains’ is true if and only if it rains}\]

and a recursive clause stating:
(6) \( \varphi \) and then \( \gamma \) is true if and only if \( \varphi \) is true and then \( \gamma \) is true

A statement of Fan-Tale's truth-conditions cannot be derived.

A statement of the truth-conditions of an infinite number of sentences can be derived since substituting 'it rained', for example, for \( \varphi \) and \( \gamma \) in (6) obtains: 'it rains and then it rains' is true if and only if 'it rains' is true and then 'it rains' is true, which together with (5) entails:

(7) 'it rains and then it rains' is true if and only if it rains and then it rains

as desired.

Likewise, substituting 'it rains and then it rains' for \( \varphi \) and 'it rains' for \( \gamma \) in (6) obtains in combination with (7) and (5) a statement of the truth-conditions of 'it rains and then it rains'. In general, once a statement of the truth-conditions of a sentence has been derived, substituting that sentence for \( \varphi \) and 'it rains' for \( \gamma \) in (6) obtains in combination with that statement and (5) a statement of the truth-conditions of a longer sentence.

This procedure can be repeated any finite number of times to obtain a statement of the truth-condition of a sentence of any finite length composed of 'it rains' and 'and then': the number of repetitions equals the number of occurrences of 'it rains' in the sentence whose truth-condition is stated. But if the number of repetitions and occurrences of 'it rains' are equal, a finite number of repetitions can't obtain an infinitely long statement of truth-conditions such as (1) or (3).

So jointly, the base clause for 'it rains' and the recursive clause for 'and then' entail a theorem stating the truth-condition of sentences of any finite length composed of 'it rains' and 'and then'. But the base clause for 'it rains' and recursive clause for 'and then' don't entail theorems stating the truth-condition of any unending sentences, since only theorems of finite length are entailed by the base clause for 'it rains' and the recursive clause for 'and then'.

It might be thought desirable to stipulate that by performing this procedure an infinite number of times one would perform an infinitely long derivation, the result of which is the limit of the results of performing the procedure one, two, three, … or any finite number of times. Since (3) is the limit of (5), (7), … and so on, this stipulation would allow (5) and (6) to entail (3), via an infinite number of steps (this stipulation is not trivial, since standardly (5) and (6) do not entail (3) at all).
So if infinitely long derivations were permitted, then (3) may be obtained from (5) and (6) via performing the procedure an infinite number of times. But since Fan-Tale can be understood with merely finite intellectual capacities, the derivation which explains this should be finitely long. Likewise, since Frame-Tale can be understood with finite intellectual capacities, the derivation which explains how it is understood should also be merely finitely long.

The second problem for explaining the ability to understand Frame-Tale via tacit knowledge of a recursive theory of truth is that since no constituent of Frame-Tale, unlike Fan-Tale, is an atomic sentence, there is no appropriate base clause from which to begin deriving (1), regardless of the derivation’s length. I shall argue that this problem can be solved by introducing a non-recursive clause for ‘there was a story that began’, but that the resulting derivation is still infinitely long.

Since ‘once upon a time’, for example, is an expression that takes a sentence to make a sentence, its meaning should be given by a recursive clause like:

\[
\text{(8) } \text{once upon a time } \phi \text{ is true if and only if once upon a time } \phi \text{ was true.}
\]

‘Once upon a time it was a dark and stormy night’, for example, is true if and only if once upon a time ‘it was a dark and stormy night’ was true or, in other words, if and only if once upon a time it was a dark and stormy night.

But if ‘there was a story that began’ is also an expression that takes a sentence to make a sentence, then its meaning should also be given by a recursive clause like:

\[
\text{(9) there was a story that began } \phi \text{ is true if and only if there was a story that began } \phi \text{ is true.}
\]

According to (9), ‘there was a story that began it was a dark and stormy night’ is true if and only if there was a story that began ‘it was a dark and stormy night’ is true.

It’s obvious that (9) is incorrect, since ‘there was a story that began’ prefixed to a sentence may be true even if there is no story which begins by saying that that sentence is true. There is a story that began it was a dark and stormy night, for example, but no story that began ‘it was a dark and stormy night’ is true. So, contrary to (9), it’s false that ‘there was a story that began it was a dark and stormy night’ is true if and only if there was a story that began ‘it was a dark and stormy night’ is true.

But even if (9) were correct, tacit knowledge of (8) and (9) couldn’t explain the ability to understand Frame-Tale: since both (8) and (9) state the truth-conditions
of sentences containing ‘once upon a time’ and ‘there was a story that began’ in terms of the truth-conditions of other sentences, they don’t entail a direct statement of the truth-condition of any sentence. To do that, they would have to be combined with a base clause like (5), which directly states the truth-conditions of an atomic sentence.

But since no atomic sentence is a constituent of Frame-Tale, there’s no appropriate base clause from which to begin deriving its truth-conditions. So if ‘once upon a time’ and ‘there was a story that began’ are both associated with recursive clauses which state the truth-conditions of sentences containing them in terms of the truth-conditions of other sentences, there’s no derivation of Frame-Tale’s truth-conditions from these clauses which could explain the ability to understand it.

The lack of an appropriate base clause suggests that the possibility of understanding Frame-Tale should be explained by a non-recursive theory of truth. An exceptionally simple derivation is offered by a theory whose only axiom is:

\[
(10) \quad (\forall \phi) \quad \text{true English if and only if } \phi.
\]

Substituting ‘... once upon a time there was a story that began once upon a time there was a story that began ...’ for \( \phi \) in (10) straightforwardly obtains (1).

But notoriously, knowing this axiom is insufficient for understanding English, since although I know that all its instances are true, substituting \( \phi \) for a sentence I don’t understand produces a theorem I’m equally unable to understand, and therefore unable to know (Lepore and Loewer 1983). Since I’m unable to understand the first line of *Finnegan’s Wake* (Joyce 1939: 1), for example, I’m also unable to understand the result of substituting its first line for \( \phi \) in (10).

This problem can be avoided by a more limited employment of substitutional quantification in the following non-recursive clause for ‘there was a story that began’:

\[
(11) \quad (\forall \phi) \text{there was a story that began } \phi \text{ is true if and only if there was a story that began } \phi.
\]

Substituting a sentence I don’t understand for \( \phi \) in (11) produces a theorem I am able to understand, and therefore able to know in virtue of knowing (11).

Although I don’t understand the first line of *Finnegan’s Wake*, for example, I do understand: ‘there was a story that began riverrun, past Eve and Adam’s, from swerve of shore to bend of bay, brings us by a commodius vicus of recirculation back to Howth Castle and Environs’ is true if and only if there was a story that began
riverrun, past Eve and Adam’s, from swerve of shore to bend of bay, brings us by a commodius vicus of recirculation back to Howth Castle and Environs.

But substituting the right hand side of (1) for $\varphi$ in (11) obtains:

\[
\begin{align*}
(12) & \quad \text{there was a story that began ... once upon a time there was a story that began once upon a time there was a story that began ... is true if and only if there was a story that began ... once upon a time there was a story that began ...} \\
& \quad \text{which is not equivalent to (1).}
\end{align*}
\]

According to (12), for example, *Frame-Tale* begins ‘there was a story that began’ rather than ‘once upon a time’, whereas according to (1) it begins with neither. And according to (1) ‘once upon a time’ immediately succeeds every occurrence of ‘there was a story that began’, whereas according to (13) no occurrence of ‘once upon a time’ nor ‘there was a story that began’ immediately succeeds the first occurrence of ‘there was a story that began’.

A better suggestion is to substitute ‘once upon a time there was a story that began ...’ (with an ellipses only on the right hand side) for $\varphi$ in (11) to obtain:

\[
\begin{align*}
(13) & \quad \text{there was a story that began once upon a time there was a story that began... is true if and only if there was a story that began once upon a time there was a story that began ...} \\
& \quad \text{in which ‘once upon a time’ does immediately succeed ‘there was a story that began’.}
\end{align*}
\]

According to (13), *Frame-Tale* begins ‘there was a story that began’, whereas according to (1) it does not. But in combination with (8), (13) entails:

\[
\begin{align*}
(14) & \quad \text{once upon a time there was a story that began once upon a time there was a story that began ... is true if and only if once upon a time there was a story that began once upon a time there was a story that began ...} \\
& \quad \text{which begins with ‘once upon a time’ instead of ‘there was a story that began’.}
\end{align*}
\]

If infinitely long derivations are permitted, (1) may be obtained by substituting the sentence mentioned in (14) for $\varphi$ in (11) followed by repeating the step from (13) to (14) an infinite number of times. But since *Frame-Tale* can be understood with finite intellectual capacities in a finite amount of time, the derivation which explains this ability should be finitely long. So the ability to understand *Frame-Tale* cannot be explained even by a non-recursive theory of truth.
It might be suggested that Frame-Tale is ambiguous between the sentence mentioned in (13) and the sentence mentioned in (14). This suggestion has the advantage that it’s not determinately true that Frame-Tale begins with ‘there was a story that began’ nor determinately true that it begins with ‘once upon a time’. Moreover, since both these disambiguations of Frame-Tale have a beginning, it’s determinately true on this suggestion that Frame-Tale does begin, just as it says.

But although (13) and (14) were both derived in a finite number of steps, this suggestion does not address the underlying problem, since no corresponding suggestion will eliminate the infinitely long derivation required to understand Fan-Tale. Even if Fan-Tale were paraphrased ‘it rains and then it rains and then …’ with an ellipses only on the right, deriving its truth-conditions would still require an infinite number of steps, since it could not draw on an analogue of (11).

The ability to understand Frame-Tale cannot be explained by tacit knowledge of a recursive theory of truth, since a statement of it’s truth-conditions cannot be derived in a finite number of steps and there’s no appropriate base clause from which such a derivation could begin. But the ability to understand Frame-Tale cannot be explained by tacit knowledge of a non-recursive theory either, because a statement of it’s truth-conditions still can’t be derived in a finite number of steps. Frame-Tale is understood, but how it’s understood isn’t understood.¹

References


¹ I am grateful for comments from Cliff Kerr, Weng-Hong Tang and Peter Shiu-Hwa Tsu, as well as audiences at the University of Sydney, especially Daniel Nolan and Lionel Shapiro, the National University of Singapore, the Australasian Association of Philosophy New Zealand conference at Massey University, the Joint Session of Mind and the Aristotelian Society at the University of Sussex, and the Turing 2012 conference at De La Salle University Manila.


