WHITEHEAD'S PRINCIPLE

BEN BLUMSON AND MANIKARAN SINGH

Abstract. According to *Whitehead's rectified principle*, two individuals are connected just in case there is something self-connected which overlaps both of them, and every part of which overlaps one of them. Roberto Casati and Achille Varzi have offered a counterexample to the principle, consisting of an individual which has no self-connected parts. But since atoms are self-connected, Casati and Varzi's counterexample presupposes the possibility of gunk or, in other words, things which have no atoms as parts. So one may still wonder whether Whitehead's rectified principle follows from the assumption of atomism. This paper presents an atomic countermodel to show the answer is no.

1. Introduction

Some things are connected, and others are not – right now I am connected to my chair, but when I stand up, my chair and I will be disconnected. The oceans are connected to the Earth, since they lie on its surface, but the Moon is not, since it orbits in space. The Northern hemisphere is connected to the Southern hemisphere, since they meet at the equator. In contrast, the North temperate zone is not connected to the South temperate zone, since they are separated by the tropics. Many more specific examples are easy to find.

But when *in general* are two things connected? An intuitive answer to this question is *Whitehead's principle*, according to which two things are connected if and only if there is something which overlaps both of them, and

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which is such that every part of it overlaps one of them.[1] So according to Whitehead's principle, Europe and Asia, for example, are connected because Russia overlaps both Europe and Asia, and because every part of Russia overlaps Europe or overlaps Asia.

Intuitive as it is, Whitehead's principle is incompatible with the possibility of scattered objects or, in other words, things which are not self-connected.[2] New Zealand, for example, is divided into two disconnected parts – the North island and the South island. Nevertheless, there is something which overlaps both the North island and the South island, and every part of which overlaps either the North island or the South island – namely, New Zealand itself. So Whitehead's principle falsely implies that the North and South islands are connected after all.

This suggests that what seemed intuitive about Whitehead's principle might instead be captured by *Whitehead's rectified principle*, according to which a pair of individuals is connected if and only if there is something *self-connected* which overlaps both of them, and every part of which overlaps one of them. So according to Whitehead's rectified principle, Europe and Asia, for example, are connected because mainland Russia is self-connected, mainland Russia overlaps both Europe and Asia, and every part of mainland Russia overlaps either Europe or Asia.

Roberto Casati and Achille Varzi have presented a counterexample to Whitehead's rectified principle inspired by the Cantor set, the *Cantor bar*, which consists of an individual which has no self-connected parts, or is entirely scattered (Casati and Varzi 1999, p. 61). As we explain in detail

[1]See Varzi 1996, p. 267 and Casati and Varzi 1999, p. 11. Whitehead's application of the principle was to "junctions" of events, rather than connection of individuals. See Whitehead 1919, p. 102 and Whitehead 1920, p. 76.

[2]See Casati and Varzi 1999, pp. 12-13. For scattered objects see Cartwright 1975, pp. 157-9 and Chisholm 1987. Biro 2017 denies there are scattered objects.

in section (7), the Cantor bar presupposes not only the possibility of scattered objects, but also the possibility of gunk, or individuals which are not composed of any atoms.

The complexity of the example naturally raises the question of whether Whitehead's rectified principle is provable in the context of atomism. This paper presents a very simple, albeit counterintuitive, atomistic countermodel to show that it is not. We conclude that the truth of Whitehead's rectified principle is logically independent from the question of atomism – both atomistic and atomless mereologies are consistent with the acceptance or rejection of Whitehead's rectified principle.

2. Classical Extensional Mereology

The difficulties for Whitehead's principle are starkest in *classical extensional mereology*. Roughly speaking, classical extensional mereology is the combination of two theses – *extensionalism*, according to which no things compose more than one thing, and *universalism*, according to which all things compose at least one thing.[3] In particular, universalism is inconsonant with Whitehead's principle because, as we shall see, they jointly entail that everything is connected to everything else, thus trivialising the relation of connection.

To be more precise, we follow Casati and Varzi in axiomatising classical extensional mereology with a single primitive relation P, where Pxy is interpreted as meaning that x is an improper part of y.[4] It's assumed that parthood is reflexive, antisymmetric, and transitive:

(1) Pxx — Reflexivity

(2) $Pxy \wedge Pyx \rightarrow x = y$ — Antisymmetry

(3) $Pxy \wedge Pyz \rightarrow Pxz$ — Transitivity

[3] For this characterisation see, for example, Lewis 1991, pp. 72-3 and Lando 2018.

[4] See Varzi 1996, pp. 260-5 and Casati and Varzi 1999, pp. 29-47.

Then, a pair of individuals are defined as overlapping if and only if there is something which is part of both of them:

(4) $Oxy =_{def} (\exists z)(Pzx \wedge Pzy)$ Overlap

Egypt overlaps Asia, for example, because Sinai is part of Egypt and part of Asia.

With the definition of overlap in hand we can state the axiom of strong supplementation, according to which if something is not part of another, some part of the latter does not overlap the former:

(5) $\neg Pyx \rightarrow (\exists z)(Pzx \wedge \neg Ozy)$ Strong Supplementation

Since Egypt is not part of Africa, for example, there is some part of Egypt, viz. Sinai, which does not overlap Africa. Together with antisymmetry, strong supplementation captures the extensionalist aspect of classical extensional mereology.[5]

The universalist aspect of classical extensional mereology is captured by the axiom schema of fusion, according to which for any satisfied predicate, there is something which overlaps all and only the things which satisfy that predicate:

(6) $(\exists x)\psi(x) \rightarrow (\exists x)(\forall y)(Oyx \leftrightarrow (\exists z)(\psi(z) \wedge Oyz))$ Fusion

Since there is an ocean, for example, there is something which overlaps all and only things which overlap an ocean.

Then the fusion or general sum of things which satisfy a predicate (which is unique in the context of extensionalism), is defined as the thing which overlaps all and only things which satisfy that predicate:

(7) $(\sigma x)\psi(x) =_{def} (\imath x)(\forall y)(Oyx \leftrightarrow (\exists z)(\psi(z) \wedge Oyz))$ General Sum

[5]Some axiomatisations use weak instead of strong supplementation. See Simons 1987, p. 41; Varzi 1996, p. 265 and Casati and Varzi 1999, pp. 45-7. However, this fails to capture extensionalism, except in the presence of a stronger version of the fusion axiom. See Pontow 2004; Hovda 2009; Varzi 2009 and Varzi 2019. The relationship between antisymmetry and extensionalism is emphasised in Cotnoir 2010 and Cotnoir 2016.

The Ocean, for example, is the thing which overlaps all and only things which overlap an ocean. (Further definitions of classical extensional mereology will be introduced as we need them).

3. Whitehead's Principle

With these definition in hand, Whitehead's original principle, according to which a pair of individuals is connected if and only if there is something which overlaps both of them, and every part of which overlaps one of them, can be stated (using Cxy to mean x is connected to y) as:

(8) $Cxy \leftrightarrow (\exists z)(Ozx \wedge Ozy \wedge (\forall w)(Pwz \rightarrow (Owx \vee Owy)))$

According to Whitehead's principle, Asia is connected to Africa, for example, because Egypt overlaps Africa, Egypt overlaps Asia, and every part of Egypt overlaps Africa or Asia.[6]

Whitehead's principle has the advantage that if it were true, it would provide a reductive analysis of connection in purely mereological terms.[7] In particular, Whitehead's principle offers an analysis of connection in terms of just parthood and overlap, which is itself defined in terms of parthood. So if Whitehead's principle were true, there would be no mystery about when two things are connected – or at least, any mysteries about when two things are connected would reduce to questions about when one thing is part of another. This is a considerable theoretical advantage.

However, in the context of classical extensional mereology, Whitehead's principle is completely unacceptable, since it entails that everything is connected to everything else. To see why, define the sum of two individuals as

[6]See Varzi 1996, p. 267 and Casati and Varzi 1999, p. 51

[7]In some systems, parthood itself is analysed in terms of connection (see Clarke 1981, Varzi 1996, pp. 276-7 and Casati and Varzi 1999, pp. 63-4). In this case, Whitehead's principle would provide a reciprocal rather than reductive analysis of connection, where connection and parthood are both analysed in terms of each other.

the individual which overlaps all and only individuals which overlap the first or overlap the second:

(9) $x + y =_{def} (\iota z)(\forall w)(Owz \leftrightarrow (Owx \vee Owy))$ Sum

The sum of Europe and Asia, for example, is Eurasia – the individual which overlaps all and only things which overlap Europe or overlap Asia.

In classical extensional mereology, it follows by substituting $z = x \vee z = y$ for $\psi(z)$ in the fusion axiom schema that every pair of individuals has a sum (which is unique in the context of extensionalism):

(10) $(\exists z)(\forall w)(Owz \leftrightarrow (Owx \vee Owy))$ Sum Closure

Since Europe, for example, is identical to Europe or identical to Asia, it follows from fusion that there is something, viz. Eurasia, which overlaps all and only things which overlap Europe or overlap Asia.

Then Whitehead's principle is trivial in classical extensional mereology because its right hand side is always satisfied – the sum of $x + y$ overlaps x and overlaps y, and every part of $x+y$ overlaps x or overlaps y. Consider, for example, India and Australia. According to classical extensional mereology, India and Australia have a sum, Indo-Australia. Indo-Australia overlaps India, and Indo-Australia overlaps Australia. Moreover, every part of Indo-Australia overlaps India or overlaps Australia. So, according to Whitehead's principle, it follows that India is connected to Australia.

To put the problem another way, we noted in the introduction that Whitehead's principle is incompatible with the existence of scattered objects. But classical extensional mereology exacerbates this problem, because it entails that whenever two individuals are not connected, their sum is a scattered object. So unless all individuals are connected, classical extensional mereology entails that there are scattered objects. So unless all individuals are connected, classical extensional mereology is incompatible with Whitehead's principle.

4. Classical Extensional Mereotopology

The problem just raised with Whitehead's principle is that whereas Russia and Egypt are self-connected, Indo-Australia is scattered, so the fact that Indo-Australia overlaps both India and Australia does nothing to connect them. But what is the difference between individuals which are self-connected, like mainland Australia, and individuals which are scattered, like the Indonesian Archipelago? This question cannot be answered in mereological terms alone, but can be answered by introducing connection as an additional primitive.[8]

In particular, *classical extensional mereotopology* is the theory which extends classical extensional mereology with an additional primitive relation C, where Cxy is interpreted as meaning x is connected to y, and is governed by the following axioms:

(11) Cxx Reflexivity

(12) $Cxy \rightarrow Cyx$ Symmetry

(13) $Pxy \rightarrow (\forall z)(Czx \rightarrow Czy)$ Monotonicity

Here, monotonicity is responsible for capturing the relationship between parthood and connection.[9]

With connection adopted as an additional primitive, self-connection can be defined in terms of connection and summation. According to this definition, an individual is self-connected if and only if for every way of partitioning it into a sum, the summands are connected:

(14) $SCz =_{def} (\forall x)(\forall y)(z = x + y \rightarrow Cxy)$ Self-Connection

New Zealand, for example, is not self-connected according to this definition, because New Zealand is the sum of the South Island and the North Island, but the South Island and the North Island are not connected.[10]

[8] See Varzi 1996, p. 268; Casati and Varzi 1999, pp. 12-13.

[9] For classical extensional mereotopology see Grzegorczyk 1960; Smith 1993 and Smith 1996. For this axiomatisation see Varzi 1996, p. 271 and Casati and Varzi 1999, p. 58.

[10] For this definition see Varzi 1996, p. 271 and Casati and Varzi 1999, p. 58.

5. Whitehead's Rectified Principle

With the distinction between self-connected and scattered individuals in hand, it appears that Whitehead's principle can be corrected. In particular, according to *Whitehead's rectified principle*, a pair of individuals is connected if and only if there is something *self-connected* which overlaps both of them, and such that every part of it overlaps one of them:

(15) $Cxy \leftrightarrow (\exists z)(SCz \land Ozx \land Ozy \land (\forall w)(Pwz \rightarrow (Owx \lor Owy)))$

So Europe is connected to Asia, for example, not only because mainland Russia overlaps Europe, mainland Russia overlaps Asia, and every part of mainland Russia overlaps Europe or Asia, but also because mainland Russia is self-connected.[11]

Whitehead's rectified principle, unlike its predecessor, cannot serve as a reductive analysis of connection in purely mereological terms, since it analyses connection partly in terms of self-connection, which is not purely mereological, since it is itself defined in terms of connection. Nevertheless, if Whitehead's rectified principle were true, it could still provide a reciprocal analysis of connection, where connection and self-connection are both analysed in terms of each other. So it could still provide partial illumination on the question of when two things are connected.

Moreover, the right to left direction of Whitehead's rectified principle is a theorem of classical extensional mereotopology. To see why, define the product of two overlapping individuals as the individual all and only parts of which are part of both of them:

(16) $x \times y =_{def} (\iota z)(\forall w)(Pwz \leftrightarrow (Pwx \land Pwy))$ Product

Sinai, for example, is the product of Egypt and Asia, since something is part of Sinai if and only if it is part of Egypt and Part of Asia.

In classical extensional mereology, every pair of overlapping individuals has a product (which is unique in the context of extensionalism):

[11] See Varzi 1996, p. 271 and Casati and Varzi 1999, p. 61.

(17) $Oxy \to (\exists z)(\forall w)(Pwz \leftrightarrow (Pwx \wedge Pwy))$ Product Closure

Since Malaysia overlaps Borneo, for example, there is something – namely, East Malaysia – all and only parts of which are part of Malaysia and part of Borneo.

Moreover, in classical extensional mereology, if an individual overlaps two others, then its product distributes over their sum:

(18) $(Oxy \wedge Oxz) \to x \times (y + z) = (x \times y) + (x \times z)$ Distributivity

Since Turkey overlaps Europe and Asia, for example, the product of Turkey with Eurasia is the sum of the product of Turkey with Europe and the product of Turkey with Asia.[12]

Finally, the right to left direction of Whitehead's rectified principle follows from the definition of self-connection, distributivity and monotonicity:

(19) $(\exists z)(SCz \wedge Ozx \wedge Ozy \wedge (\forall w)(Pwz \to (Owx \vee Owy))) \to Cxy$

Proof. Suppose z satisfies the right hand side of Whitehead's rectified principle. Since every part of z is part of x or part of y, z is part of $x + y$, so $z = z \times (x + y)$. And since z overlaps x and z overlaps y, it follows from distributivity that $z = z \times (x + y) = (z \times x) + (z \times y)$. Then since z is self-connected it follows from the definition of self-connection that $z \times x$ is connected to $z \times y$. But $z \times x$ is part of x and $z \times y$ is part of y, so it follows from the monotonicity of connection that x is connected to y.[13] $\square$

However, as we will see, the left to right direction of Whitehead's rectified principle is *not* provable, even in a theory stronger than classical extensional mereotopology.[14]

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[12]For a detailed proof of distributivity see Pietruszczak 2018, pp. 102-4

[13]For this theorem and its proof see Casati and Varzi 1999, p. 61.

[14]Varzi 1996, p. 271 mistakenly suggests that the left to right direction of Whitehead's rectified principle is provable in closed extensional mereotopology, a weaker theory in which fusion is replaced with sum and product closure.

6. CLOSURE CONDITIONS

Casati and Varzi favour a stronger theory they call *classical extensional mereotopology with closure conditions* (abbreviated $GEMTC$), which adds to classical extensional mereotopology axioms inspired by the Kuratowski closure axioms of mathematical topology.[15] In order to state these conditions, we say that something is an internal part of another if and only if the former is a part of the latter and everything connected to the former overlaps the latter:

(20) $IPxy =_{def} Pxy \wedge (\forall z)(Czx \rightarrow Ozy)$ Internal Part

Bolivia, for example, is an internal part of South America, since Bolivia is part of South America and everywhere connected to Bolivia overlaps South America. Brazil, on the other hand, is not an internal part of South America, since the Pacific is connected to Brazil but does not overlap South America.[16]

In terms of internal parthood, the interior of an individual can be defined as the general sum of its internal parts:

(21) $i(x) =_{def} (\sigma z)IPzx$ Interior

Supposing that the countries of South America are its atomic parts, for example, then the interior of South America is the sum of Bolivia and Paraguay.

With these definitions in hand, we can define $GEMTC$ as the theory which extends classical extensional mereology with the following axioms:

(22) $P(ix)x$ Inclusion

(23) $i(ix) = i(x)$ Idempotence

(24) $i(x \times y) = i(x) \times i(y)$ Product

[15]See Varzi 1996, p. 273 and Casati and Varzi 1999, p. 59.

[16]See Varzi 1996, p. 268 and Casati and Varzi 1999, p. 55.

GEMTC, according to Casati and Varzi, "may be considered the archetype of a mereological theory" (Casati and Varzi 1999, p. 59).[17] Nevertheless, as we shall see in the next two sections, the axioms of *GEMTC* are still too weak to entail Whitehead's principle.

7. The Cantor Bar

In order to show that the left to right direction of Whitehead's rectified principle is not a theorem of *GEMTC*, Casati and Varzi give as a counterexample the *Cantor Bar*, which they describe as follows:

> ... the creation of the Cantor Bar. The first step is to remove the middle third of a self-connected bar. The next step is to remove the middle of each of the remaining bars. Repeating this over and over again creates a scattered object with no self-connected parts (Casati and Varzi 1999, p. 61).

Casati and Varzi's idea is that after countably many steps *every* self-connected bar will have been divided into two, and so the remainder will have *no* self-connected parts.[18]

Since it has no self-connected parts, the Cantor Bar is a counterexample to the left to right direction of Whitehead's rectified principle. As Casati and Varzi write:

> ... if x has no self-connected parts, then there may be things to which x is connected (e. g. x itself) without there being

[17] See also Varzi 1996, p. 273. Note that axiom (22), inclusion, is redundant, since it's provable in classical extensional mereotopology (Casati and Varzi 1999, p. 59).

[18] Casati and Varzi's example is inspired by the *Cantor Set*, which is constructed as follows. Let $E_0 = [0, 1]$, each $E_n = \frac{E_{n-1}}{3} \cup (\frac{2}{3} + \frac{E_{n-1}}{3})$, and the Cantor set $C = \bigcap_{i=1}^{\infty} E_i$. The Cantor set with the subspace topology inherited from the usual topology on the real line is a Hausdorff space. And the collection of nonempty regular open subsets of a Hausdorff space is a model of classical extensional mereotopology (Grzegorczyk 1960; Gerla 1995, p. 1021; Varzi 1996, p. 272). So the collection of nonempty regular open subsets of the Cantor set is a model of classical extensional mereotopology.

> any self-connected z doing the job required by Whitehead's principle (Casati and Varzi 1999, p. 61).

The problem is that although the Cantor Bar is connected to itself, the only things which overlap the Cantor Bar, and which all parts of which overlap it, are parts of it – but none of these are self-connected.

Notice that in order to be an individual with no self-connected parts, the Cantor Bar must be composed of "gunk" or, in other words, must have no atoms as parts.[19] To see this, define an atom as an individual which has no parts other than itself:

(25) $Ax =_{def} (\forall y)(Pyx \rightarrow x = y)$ Atoms

Fundamental physical particles, for example, are atoms under this definition, since fundamental physical particles have no parts other than themselves.[20]

Then in classical extensional mereotopology, it follows that all atoms are self-connected (Masolo and Vieu 1999, p. 243):

(26) $Ax \rightarrow SCx$

Proof. Suppose x is an atom. To show x is self-connected, we have to show that for all y and z, if $x = y + z$ then y is connected to z. So suppose $x = y + z$. It follows from the definition of a sum that everything which overlaps y overlaps z, and so it follows from strong supplementation that y is part of x. But then since x is an atom, it follows from the definition of an atom that y is x. By the same reasoning, it follows that z is x. So it follows from the reflexivity of connection that y is connected to z. □

So in order to be a counterexample to Whitehead's rectified principle, no atoms can be part of the Cantor Bar.[21]

[19] For a summary of the debate over the possibility of gunk see Hudson 2007, pp. 297-9.

[20] For this definition see Masolo and Vieu 1999, p. 240. See also Simons 1987, p. 41.

[21] Note that in the model of classical extensional mereotopology provided by the collection of nonempty regular open subsets of the Cantor set is atomless, since although $\{\frac{1}{4}\}$, for example, is a subset of the Cantor set, it is not a regular open subset.

8. An atomic countermodel

So the example of the Cantor bar raises the question of whether the left to right direction of Whitehead's rectified principle is provable from an additional axiom, according to which everything is composed of atoms:

(27) $(\forall x)(\exists y)(Ay \wedge Pyx)$ Atomism

This axiom would rule out the Cantor Bar, for example, since no part of the Cantor Bar is an atom.[22]

But there is a very simple countermodel to show that even under the assumption of atomism, classical extensional mereotopology does not entail the left to right direction of Whitehead's rectified principle. Suppose everything is composed of just four atoms a, b, c and d, none of which are connected to each other. And suppose that $a + d$ is connected to $b + c$, and that no other individuals are connected without overlapping. In this model, everything has its atoms amongst its interior parts, so since everything is the general sum of its atoms, everything is its own interior. So the axioms of $GEMTC$ are all satisfied.

However, in this model nothing is self-connected except the four atoms, since because the atoms aren't connected to anything except themselves, everything else may be partitioned into a sum in which one summand is an atom to which the other summand is not connected. And none of the four atoms overlap both $a+d$ and $b+c$, so there is nothing self-connected to satisfy the right hand side of Whitehead's rectified principle. Hence, Whitehead's rectified principle is not a theorem of classical extensional mereotopology with closure conditions, even under the assumption of atomism.

To make the countermodel more intuitive, imagine, for example, that a and c are protons, whereas b and d are electrons, so that $a + b$ and $c + d$ are hydrogen atoms (in the chemical sense) which connect to form a H_2

[22]For this characterisation of atomism see Masolo and Vieu 1999, p. 240. See also Simons 1987, p. 42; Varzi 1996, p. 265 and Casati and Varzi 1999, p. 48.

molecule (where we think of connection in terms of chemical bonding, rather than contact). Then although the two hydrogen atoms are connected, there is nothing self-connected which overlaps both of them, and so, according to the example, Whitehead's rectified principle is false.

9. Additional Axioms

A striking feature of the atomic countermodel is that $a + d$ is connected to $b + c$, without being connected to b or connected to c (and likewise, $b + c$ is connected to $a + d$, without being connected to a or connected to d). So one might consider adding to mereotopology an axiom according to which if an individual is connected to a complex, then it is connected to a proper part of that complex, viz.:

(28) $\neg Ax \to (\forall y)(Cyx \to (\exists z)(Pzx \wedge z \neq x \wedge Cyz))$

To be connected to Australia, for example, a region must be connected to an Australian state or territory.

If the number of individuals were *finite*, then this axiom would entail the left to right direction of Whitehead's rectified principle, since for any two complex individuals which are connected, we could keep applying the axiom until we reach two atoms which are connected to each other. The sum of these atoms would be self-connected, and every part of it would overlap one of the original two individuals, and so it would satisfy the right side of the rectified principle.

However, there are countermodels in which the number of individuals is infinite. Suppose, for example, that there are countably many atoms, and so continuum many individuals in total. And suppose two individuals are connected if and only if either they overlap or else they are composed of infinitely many atoms. In this model, only atoms are self-connected, since any other individual may be partitioned into a sum in which one summand is an atom to which the other summand is not connected. So although (28) is satisfied, the left to right direction of Whitehead's principle is not, since disjoint individuals composed of infinitely many atoms are connected, but not overlapped by anything self-connected.

A stronger axiom would require that if two individuals are connected, then they have atomic parts which are themselves connected:

(29) $Cxy \rightarrow (\exists z)(Az \wedge Pzx \wedge (\exists v)(Av \wedge Pvy \wedge Czv))$

This axiom would entail the right to left direction of Whitehead's rectified principle. For suppose z is connected to y. It follows there exists atoms z and v such that z is part of x, v is part of y and z and v are connected. Then $z + v$ satisfies the right hand side of Whitehead's rectified principle.

Notice that axiom (29) is very strong, since it entails atomism. For letting x and y be the same, the consequent of (29) follows from the reflexivity of connection, and so it follows that in order to be connected to itself, every individual must have an atom as a part, as atomism requires. Thus, axiom (29) rules out not only the example of the Cantor bar, as it is required to do in order to entail the left to right direction of Whitehead's rectified principle, but rules out the possibility of gunk altogether.

Moreover, even under the assumption of atomism, (29) is stronger than it needs to be in order to entail the left to right direction of Whitehead's rectified principle. It entails, for example, that the closed interval $[0, 1]$ cannot be connected to the open interval $(1, 2)$, since no point in $[0, 1]$ is connected to any point in $(1, 2)$. But the connection of $[0, 1]$ to $(1, 2)$ is not a counterexample to the left to right direction of Whitehead's rectified principle since, for example, it is satisfied by the interval $(.9, 1.1)$.

10. Whiteheadian Mereotopology

Is there an axiom which can be added to classical extensional mereotopology that entails the left to right direction of Whitehead's rectified principle, but which does not entail anything which the left to right direction of Whitehead's rectified principle does not entail? The most obvious answer is the left to right direction of the principle itself:

(30) $Cxy \rightarrow (\exists z)(SCz \wedge Ozx \wedge Ozy \wedge (\forall w)(Pwz \rightarrow (Owx \vee Owy)))$

Casati and Varzi call the mereotopology reached by adding axiom (30) to a mereotopology its "Whiteheadian extension" (Casati and Varzi 1999, p. 61).

As it must in order to exclude the countermodel of the Cantor bar, the principle entails that there is no scattered gunk or, in other words, that everything has a self-connected part:

(31) $(\forall x)(\exists z)(SCz \wedge Pzx)$

Proof. Letting x and y be the same, it follows from the reflexivity of connection and the left to right direction of Whitehead's rectified principle that there is z such that $SCz \wedge Ozx \wedge (\forall w)(Pwz \rightarrow Owx)$. Then suppose for reductio that z is not part of x. It follows from (5) strong supplementation that there exists v which is part of z and does not overlap x. But every part of z overlaps x, so this is a contradiction. It follows that $SCz \wedge Pzx$, thus concluding the proof.[23] □

The atomic countermodel above has already shown that the converse is not the case, and the left to right direction of Whitehead's rectified principle does not follow from (31), the inexistence of scattered gunk. But since that countermodel is atomic, one naturally wonders whether (31) does entail the left to right direction of Whitehead's rectified principle under the assumption of atomlessness, according to which everything has a part other than itself:

(32) $(\forall x)(\exists y)(Pyx \wedge x \neq y)$ Atomlessness

This axiom would rule out the atomic countermodel, since the atoms in that countermodel do not have any parts other than themselves.[24]

For an atomless countermodel to the entailment from (31) to the left to right direction of Whitehead's rectified principle, imagine there are just four

[23]Casati and Varzi say that "The thesis that everything has at least one self-connected part ... would then be a theorem of any such [Whiteheadian] theory" (Casati and Varzi 1999, p. 61). But we cannot reconstruct the proof without using strong supplementation.

[24]For the characterisation of atomlessness see Simons 1987, p. 42; Varzi 1996, p. 266; Casati and Varzi 1999, p. 48 and Masolo and Vieu 1999, p. 240.

pizzas, every slice of which may be sliced in half, but only along the radii of the pizzas, so that every slice is connected to all the other slices of the same pizza at that pizza's center. Atomlessness is satisfied because every slice of pizza may itself be sliced in half. But (31) is also satisfied, since every slice of pizza is self-connected, and everything has a slice of pizza as a part.

Now in order to complete the countermodel imagine that the four pizzas play the role of the four atoms in the simple atomic countermodel. In other words, imagine that none of the pizzas are connected to each other, but – contrary to (28) – that the sum of the first two pizzas is connected to the sum of the second two pizzas. Then although the two pizza sums are connected, nothing which overlaps both of them is self-connected, and so there is nothing to satisfy the left to right direction of Whitehead's rectified principle.[25]

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