Reply to Rosanna Keefe's 'Modelling higher-order vagueness: columns, borderlines and boundaries'

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The following is an expanded written version of my reply to Rosanna Keefe's paper 'Modelling higher-order vagueness: columns, borderlines and boundaries' (Keefe 2015), which in turn is a reply to my paper 'Columnar higher-order vagueness, or Vagueness is higher-order vagueness' (Bobzien 2015). Both papers were presented at the Joint Session of the the *Aristotelian* Society and the *Mind* Association in July, 2015. At the Joint Session meeting, there was insufficient time to present all of my points in response to Keefe's paper. In addition, the audio of the session, which is available online[1] becomes inaudible at the beginning of my reply to Keefe's comments due to a technical defect. The following is a full version of my remarks.

First let me repeat that I am thankful to Rosanna Keefe for her set of comments, which made me see more clearly various points of the theory I propose. Second, the reader should be aware that Keefe explicitly and deliberately chose not to engage with the core element of my paper.[2] The paper's core element is the normal modal logic that one obtains by adding the axiom FINAX to the fixed-domain first-order modal system QKT4M+BF (or QS4M+BF). Other than in my paper, the logic QKT4M+BF+FINAX has so far not attained any philosophical application, and the logic KT4M has only been used to illustrate a rather obscure notion of time. Keefe says nothing at all specifically about the logic QKT4M+BF+FINAX and how I link it to vagueness and higher-order vagueness. She does also not say anything at all specifically about the logic KT4M (S4M). Yet, it is this *specific* logic QKT4M+BF+FINAX by which columnar higher-order vagueness is defined, and it is this *specific* logic that gives the proposed theory, that vagueness is higher-order vagueness, its character and its strengths. Thus, in order to learn what

[1] The audio version contains audible summary versions of my paper and of Keefe's reply. It can be found here https://lecturecapture.warwick.ac.uk/ess/echo/presentation/15cf5dd2-f22b-49d6-a393-0a613a9caf5d?ec=true

[2] 'In Bobzien (2015), one of Bobzien's main aims is to take the propositional modal logic she has defended elsewhere and develop and explore an extension to first-order logic. I will focus, however, on more general questions about the suitability of the general framework for capturing vagueness.' (Keefe 2015, p 92). Keefe rigorously avoids talking about the main body of my paper.

columnar higher-order vagueness is, and what its strengths are, the reader will have to revert to my original paper, or at least to its audio summary. The reader would also need to devote at least a minimal effort to studying the basic features of the logic QKT4M+BF+FINAX.

Instead of focusing on the core of my paper, Keefe focuses on some more general questions regarding the overall framework of my theory. These are questions that I have discussed either in previously published papers or in forthcoming papers.[3] I note that many of Keefe's points of criticism hold equally against epistemicist theories of vagueness, and indeed against the majority of currently predominant theories of vagueness. Thus in many respects, Keefe's paper at least implicitly registers discontent with most prevailing approaches to vagueness.

In the following, I first take up some of Keefe's more major points in each of her five sections and then supplement this with a list of smaller items, page by page, mainly noting instances where Keefe appears to have misread or misunderstood my paper.

Keefe's main criticism in her Section I: clear borderline cases

Keefe's main point of criticism in her Section 1 (pp. 90–94) concerns the existence of clear borderline cases. It is a main element of my theory of higher-order vagueness that there are no clear borderline cases that are relevant to a modal representation of higher-order vagueness. Simplified, I interpret borderlineness of *a* with regard to *F* as the inability of qualified individuals to determine whether F*a*. Here then is Keefe's argument:

> <Bobzien> claims that in presenting *a* as a clear borderline case of *F*,
> [Y]ou must be able to distinguish *a* from the non-borderline F cases and the non-borderline ¬*F* cases. But this means, I maintain, that you have, perhaps inadvertently and at least temporarily, shifted to the above-described in-between borderlineness, or still another kind of borderlineness, and that you equivocate on 'borderline'. (Bobzien 2015, p.80)
> Why think this? Why not think that you can determine that you can't determine that *a* is *F* (i.e. invoking <Bobzien's> undecidability conception)? (Keefe 2015, p. 94)

First, I note, that the passage Keefe quotes is not in the part of the paper in which I set out my view, but rather in its last section, in which I reply to certain common objections, and that what she quotes is part (and part only!) of my reply to *Objection 1*, that there is – assumed to be – evidence

[3] Bobzien (forthcoming).

that there are clear borderline cases (= *Objection 1a*). (The distinction between in-between borderlineness and undecidability borderlineness that Keefe mentions is set out summarily on pp. 77–80 of Bobzien 2015.) Now, Keefe's objection to my reply to *Objection 1a* is in fact identical with my own *Objection 1b* from the paragraph directly following her quote. Here is the objection:

> *Objection 1b*: I can tell that I can't tell whether this object is *F*. So this object is clearly borderline *F*. (p. 80)

And my reply to *Objection 1b* is this:

> '*Reply*: Given the interpretation of operators C and B (§§i and ii above <i.e. pp 63–8>, if you *actually* can tell that you can't tell (as opposed to mistakenly believing you can), there are two possibilities: (i) you use a notion of tellability that includes lack of qualification of individuals and thus is not the one suggested. ('I can tell that *I* can't tell whether *Fa*, but don't rule out that someone better qualified might be able to tell that *Fa*'); (ii) you use the notion of tellability introduced above. That notion abstracts from all individual human handicaps. As a result, if you can tell that you can't tell, there must be something *in a with regard to the predicate F* that you pick up on and that allows you to distinguish *a* from those cases that are non-borderline *F* and non-borderline not-*F*. In that case, again, I maintain that you have, perhaps inadvertently and at least temporarily, shifted to using 'borderline' for in-between borderlineness, and not as it is used in this paper, that is, for undecidability borderlineness, and that you may be equivocating on 'borderline'.' (p. 81)

So, in the very paper on which Keefe comments, in the very paragraph that follows the one that triggers her objection, I provide (i) the objection she puts forward and (ii) a reply to that objection. Keefe seems to have overlooked this fact.

Keefe's main criticism in her Section II: borderline clear cases

Keefe's main criticism in her Section II (pp. 95–96) concerns borderline-clearly-*F* cases and borderline-clearly-not-*F* cases, where for the first it holds that $\neg CCFa \wedge \neg C\neg CFa$ and for the second it holds that $\neg C\neg C\neg Fa \wedge \neg CC\neg Fa$ (p. 95).[4] Keefe claims:

1. 'On the standard hierarchical conception of higher-order vagueness, there are <such cases>'. (p. 95)

[4] There is a typo in the second formula which I have corrected.

2. 'Even if we can't produce a definitive clear first-order borderline case, we can typically distinguish between borderline clear *F*s and borderline clear not-*F*s.' (p. 96) Bobzien's theory does not keep those types of second-order borderline cases apart, and, by inference, is 'counterintuitive'. (p. 96)
3. Keefe claims that the first cases are likely *clearly* not clearly not *F*, the last are likely *clearly* not clearly *F*. Thus we obtain the two cases $\neg CCFa \wedge \neg C \neg CFa \wedge C \neg C \neg Fa$ and $\neg C \neg C \neg Fa \wedge \neg CC \neg Fa \wedge C \neg CFa$. Again, Keefe claims, Bobzien's theory does not allow for this distinction. (p. 95–96)
4. 'Any individual's categorisation through the sorites series will distinguish between these cases'. (p. 96)
5. 'There will typically be widespread agreement that something a well-placed observer is inclined to call borderline clearly red should definitely/clearly not also count as clearly orange.' (p. 96)
6. 'A plausible view of higher-order vagueness would require a theory to keep those types of second-order borderline cases apart. (p. 96)

Let me first get a few small points out of the way.

Keefe's ***Point 1*** is straightforwardly false. In the standard hierarchical conception of higher-order vagueness there are no borderline-clearly-*F* cases and no borderline-clearly-not-*F* cases. The standard hierarchical conception, or in any case the one I define and discuss in the paper, is the one presented e.g. by Sainsbury 1991, pp. 168–9, and in Shapiro 2005 and 2006, and was introduced in my Section I, entitled *Columnar Higher-order Vagueness and Hierarchical Higher-order Vagueness*, based on these and similar publications. ***Point 4*** is also straightforwardly false. There are many individuals, philosophers and others, who do not distinguish these cases. (Keefe does not back up her universal claim with evidence.) ***Point 5*** conflates the red/not-red distinction with the red/orange distinction. Only the first leads to a Sorites paradox. Still, even if we replace 'orange' with 'not-red' in her sentence, the truth of her claim would depend on what notion of borderlineness the well-placed observers are using. If it is Shapiro's and Sainsbury's, they will not have the inclination Keefe mentions. The truth of ***Point 6*** could reasonably be doubted, given what I said about points 1 and 4. In any event, my own theory does allow one to keep apart the two types of second-order borderline cases Keefe mentions, see my response to *Points 2* and *3* below.

Keefe's ***Point 2*** is also false, since my theory can keep Keefe's two types of second-order borderline cases apart. This is easily shown. First we distinguish between the logic I introduce and my theory of vagueness. My *logic* has as its purpose to pinpoint the structural features of vague expressions that are responsible for (i) there being a grey area, or *penumbra*, in any Sorites series, and (ii) our being unable to say where this grey area starts or ends; and moreover (iii) to forestall any higher-order vagueness paradoxes in the course of this. My logic is not meant to reflect all reasonable natural language uses of 'borderline', not even all those uses people may come up with when confronted with a Sorites paradox. So the *logic* does not provide for the two cases Keefe introduces, because in my view distinguishing such cases does nothing to further its purpose, nor does it anything to further a solution of the Sorites. My *theory of vagueness*, on the other hand, accommodates those two kinds of borderline clear cases just fine. There is a natural language use of 'borderline' that I call o*nly-just borderlineness*. An object *a* is borderline *F* in this sense if and only if it is *F*, albeit only just. Using the term 'borderline' in this way, borderline-clearly-red cases are those that are clearly red, but only just. Equally, borderline-clearly-not-red cases are those that are clearly not-red, but only just. So my theory does satisfy the condition Keefe sets out in her *Point 2*. It does also satisfy the condition she sets out in her ***Point 3***. With *only-just borderlineness*, borderline-clearly-red cases are *clearly* not clearly not-red and borderline-clearly-not-red cases are *clearly* not clearly red. For, evidently, anything that is clearly red, even if only just, is not orange and also not not-red. And evidently, anything that is clearly not-red, even if only just, is not red. My theory thus explains easily why people distinguish between borderline-clearly-*F* and borderline-clearly-not-F things (*Point 2*), and does so in a way that makes perfect sense. And my theory also satisfies the condition Keefe set out in *Point 3*.

Perhaps Keefe means that a theory of vagueness should, with *one and the same* notion of borderlineness (a notion with which what is borderline *F* is neither clearly *F* nor clearly not-*F*, and with which what is clearly *F* is such that we can tell that it is *F*) be able to explain why philosophers and other people distinguish, in a Sorites series, borderline-clearly-*F* cases, borderline-clearly-not-*F* cases and clearly-borderline-*F* cases. In my view, such a requirement is a non-starter. Still, let us consider this requirement. Keefe's formulations in her Section II make it clear that in her view *we can tell* of the borderline-clearly-*F* things that they are borderline-clearly-*F*, of the borderline-clearly-not-*F* things that they are borderline-clearly-not-*F*, and of the clearly-borderline-*F* things that they are clearly-borderline-*F*. The result is a sequence of the following third-order modalities, strung up along a Sorites series from *F* to ¬*F*:

1 CCC*F* CBC*F* CCB*F* CBC¬*F* CCCB¬*F*.

Here we have four sharp borders, instead of the original *one* between the *F* and the not-*F* cases. So *prima facie*, this suggestion is no improvement. What would Keefe suggest at this point? Here are her options.

(1) Fill in borderline cases between the clear cases, just as this was done with the sharp border between *F* and not-*F* by introducing clear cases

and borderline cases. Consider first *the outer cases*. We could fill in borderline cases in one of the following two ways (in bold):

CCC*F* **BBC*F*** CBC*F* CCB*F* CBC¬*F* **BBC¬*F*** CCCB¬*F*

CCC*F* **BCC*F*** CBC*F* CCB*F* CBC¬*F* **BCC¬*F*** CCCB¬*F*

Neither makes much sense. What if we fill in the two middle cases? The two options are (in bold):

CCC*F* CBC*F* **BCB*F*** CCB*F* **BCB¬*F*** CBC¬*F* CCCB¬*F*

CCC*F* CBC*F* **BBC*F*** CCB*F* **BBC¬*F*** CBC¬*F* CCCB¬*F*

Again, neither makes much sense. Additionally, this approach is well-known to lead to higher-order vagueness paradoxes. So Keefe's first option is doomed.

(2) Keefe could opt for what I have called 'in-between borderlineness', and give up the idea of representing clarity and borderlineness with a modal logic that preserves compositionality of the borderline-operator. But this seems not to be what she has in mind.

(3) Keefe could opt for saying that the reason why there are not four sharp borders in ***1*** is that, at some higher order, the cases at issue are borderline, and are so for all subsequent higher orders. Then we have something like:

2 B^nC^mCCF B^nC^mBCF B^nC^mCBF $B^nC^mBC\neg F$ $B^nC^mCCB\neg F$

with *m* a finite number, and for any *n*.[5] The problem with this suggestion is that Keefe's argument then crumbles. Both for the existence of clearly-borderline-*F* and for the existence of borderline-clearly-*F* and borderline-clearly-not-*F* her argument relies on the assumption that individuals take it to be obvious that there are such cases. But if (at some higher level) those individuals cannot tell whether a case is clearly-borderline-*F*, etc., then there is nothing obvious about these cases anymore. In fact, if one can tell that C*A* but cannot tell whether one can tell that C*A*, one is epistemically in exactly the same position one is in with my own theory, where precisely this is also true. So, to sum up, whichever way one turns, Keefe's main criticism in her Section II is unsuccessful.

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[5] This is pretty much the road Williamson goes down, although, *pace* Keefe (p.95), Williamson never says that his theory can accommodate clear borderline cases in a Sorites series, and for good reason.

Keefe's main criticism in her Section III: If I accept axiom 4 I must accept axiom 5

Keefe's main point of criticism in her Section III (pp. 96–99) is this. She claims that, given my interpretation of clarity in (Bobzien 2010), if I accept axiom 4 (C*A*→CC*A*) I have to accept axiom 5 (¬C*A*→C¬C*A*). She supports this claim with the further claim that a competent speaker of the kind I introduce in my 2010, a *CRISP* that is, 'will or at least should – doubt her competence' (98), and that this would commit me to axiom 5.

Keefe's objection fails. First of all, it is criticism based on the concept of a *CRISP* (a **c**ompetent, **r**ational, **i**nformed **sp**eaker, and the interpretation of the C-operator from (Bobzien 2010). As such it is no criticism of the paper to which she replies (Bobzien 2015), which does not rely on (Bobzien 2010). In particular, my present view has evolved beyond the one I presented in (Bobzien 2010). Second, Keefe disregards my 2010 *definition* of the C-operator in terms of *CRISP*s. In her argument, she ignores the fact that the competence of *CRISP*s (their *CRISP*ness, for short) is indexed to the bracket of the expression with regard to which they are competent. But it *is* so indexed. As a result, my claim is that those *CRISP*s who are competent with regard to *both* *CRISP*ness *and Fa* can tell that they can tell that *Fa*. And since *CRISP*ness is transparent, those *CRISP*s (the ones that are competent with regard to their *CRISP*ness regarding *Fa*) have no reason to doubt their competence. Third, in her argument, Keefe also ignores the fact that the ability to tell that is part of the interpretation of the C-operator is defined not by a *CRISP*'s competence, but modally quantifies over all *CRISP*s indexed to the expression with regard to which they are competent. 'It is clear that *A*' is thus short for 'In all possible situations of evaluating all competent speakers can tell that *A*'. Given these three points, Keefe's main criticism in her Section III falls flat. With or without my 2010, my acceptance of axiom 4 does in no way commit me to accept axiom 5. (Keefe's objection is somewhat similar to saying that if a theory implies the semi-decidability of a set of questions Σ, then it implies the decidability of Σ.)

Keefe's main criticism in her Section V: introducing a technical term in philosophy is bad

Since what I have to say about Keefe's main objection in her Section IV depends on what I have to say about her main objection in Section V, I start with Section V (pp. 102–106). Keefe's main criticism here is that my use of the expression 'borderline' is technical (Section V, *passim*). More specifically, Keefe claims that 'regarding <the operator> C <and hence the operator B> as a technical term undermines the attempt to uncover the structure of vagueness and borderlineness' (p.103), and that, if my use of the expression 'borderline' is technical, this 'leaves open the question what the real structure of borderline cases is and *since vagueness is so closely tied to borderline cases*, we have no reason to think that the theory

<Bobzien suggests> illuminates vagueness at all' (ibid., my italics). Frankly, I find what Keefe says about my use of technical terms puzzling, for a variety of reasons.

First, the vast majority of uses of 'borderline case' and 'borderline borderline' are technical: almost all occurrences are philosophical, medical or legal terminology. The idea that there is a natural use of 'borderline' that technical uses try to capture is empirically unfounded. (Where I use 'technical term' in (Bobzien 2015), I assume a simple dictionary definition: a word that has a specific meaning within a specific field of expertise.)

Second, there are at least five structurally different and mutually incompatible – potential – natural uses of 'borderline'. Hence when Keefe talks about '*the* natural notion' or '*the* natural language <operator>'[6] (italics mine) this is unhelpful. In addition to the natural-language uses of borderline distinguished in (Bobzien 2015) and (Bobzien 2013), there are the above-mentioned only-just use, the not-quite use (*a* is borderline *F* if it is almost *F*, but not quite), and the quite different case of borderline-borderlineness in which one predicate is a borderline case of another predicate that is a borderline case of still another predicate (e.g. 'is pudding' of 'is solid' and 'is custard' of 'is pudding').[7]

Third, philosophers frequently confuse different presumed natural uses of 'borderline', to detrimental effect for their theories, which suggests this may not be such a good route to take. (This is the main topic of (Bobzien 2013)).

Fourth, Keefe's claim that 'vagueness is so closely tied to borderline cases <in a unique non-technical sense of "borderline case">' is simply false. There is no *prima facie* connection between ordinary language uses of 'borderline' and the Sorites. Over the last few decades, philosophers have unsuccessfully grappled with the Sorites paradox by using the expression 'borderline case' in various, often incompatible, ways. However, the Sorites has been discussed for over 2300 years.

Fifth, what vagueness *is closely tied to* is some sort of hedging behaviour and possible disagreement and fickleness people display when asked to say of some non-polar cases of a sorites series whether they are *F*. That is, *all there is, is a certain set of empirical data of speaker behaviour which can quite easily be described without ever using the term 'borderline case'*. This point cannot be made too emphatically. Philosophers resort to the use of the

[6] Cf. 'the natural language notion of 'clearly' (*sic*)' (Keefe 2015, p.104), 'a Definitely operator resembling the natural language one (*sic*)' (ibid. p.103).

[7] And of course these can be combined, so for example Sorensen 2010 mixes a number of these together.

expression 'borderline' in order to describe certain phenomena like hedging behaviour, or undetectable switches in truth-evaluations, and the like. Generally, they do *not* try and device a theory that captures the natural language use of 'borderline'. This too makes it clear that Keefe's claim that "vagueness is so closely tied to borderline cases (in a unique non-technical sense of 'borderline case')" (p. 103) is simply false.

Sixth, I have made clear in my paper (p. 65, p. 66) that my interest is in certain philosophical problems that are directly related to the Sorites, not in the investigation of the semantics of natural language expressions. If it was illegitimate to try to uncover the logical structure that underlies a philosophical problem if there was no natural language expression in existence that would capture it, progress in philosophy would be severely hampered. The fact that for millennia philosophers have introduced new terms into their language in order to express their solutions to philosophical problems, and are still very busy doing just this, should speak for itself.

Seventh, as I say explicitly in the paper to which Keefe responds (Bobzien 2015, p.65, p. 66), I have not used the natural language expressions 'clear' and 'borderline' in order to capture any intuitions that may come with some common use.[8] The reason I use the expression 'borderline' for the analogue to the contingency operator in QKT4M+BF+FINAX is that it shares a large number of structural features with the analogues to the contingency operators in *other modal logics* that have been used in attempts by *other* philosophers to capture what is characteristic for vagueness, and which these *other* philosophers call 'borderline cases'. Like my own, these other modal theories that have been suggested (i) generally aim at capturing the distinction between *what's inside and what's outside the grey area*. (ii) They use the structural equivalents of *a necessity and a contingency operator*. (iii) They assume that their analogue to the necessity operator is *factive or veridical*, and accordingly include axiom T. (iv) They call the cases picked out by the analogue to the contingency operator *borderline cases*. (v) Their borderline cases are *defined by the axioms and rules* of their modal system. As such, each of these theories uses 'borderline case' as a technical term. (vi) A final shared feature is *the syntactic definition of higher-order vagueness*. Most theories that employ modal logic maintain that there is higher-order vagueness *in the sense that* there are borderline-borderline cases, borderline-borderline-borderline cases, etc. And these theories use the term higher-order vagueness as follows, with minor variations in terminology: 'Object *a* is a first-order borderline case of *F* in modal system S iff B*Fa* in S. And *a* is an n+1th-order borderline case in S iff $B^{n}BFa$ in S.' This is a standard way of defining higher-order vagueness

[8] 'I have no interest in providing a semantics for the natural language expression 'can tell'.' (Bobzien 2015, 65), 'I am not interested in the semantics of the natural language expression 'it is clear'.' (ibid. 66).

qua borderline-borderlineness, and I use it in exactly the same way. So modally my theory is expressed with a borderlineness operator, and with higher-order vagueness each defined syntactically in exactly the same way other theories do that define vagueness modally. The difference lies in which modal system is chosen.

So, Keefe's objection to my use of 'borderline' as a technical term and her claim that this 'undermines the attempt to uncover the structure of vagueness and borderlineness' (p. 103) seem to me entirely unfounded.

A second major criticism in Keefe's *Section V* is her point that some kind of sharp boundaries are part of my theory. More precisely, Keefe presents the fact that my theory defines vagueness in such a way that to the left and right of the borderline zone there are a last clearly-F case and a first clearly-not-F case as if this were an extremely rare and very terrible thing (Keefe 2015, Section V *passim*). So, let me say this. First of all, my theory has sharp boundaries *only* in the sense that there is a last clearly-F case and that there is a first clearly-not-F case. With allowances for terminology ('clear', 'definite', 'determinate', etc.) such sharp boundaries are present in the vast majority of theories of vagueness: epistemicism, degree theories, glut theories, gap theories, various contextualist theories and supervaluationism, including Keefe's own brand of supervaluationism. In other words, this kind of sharp boundaries are the norm, not the exception. It is unclear to me why Keefe spends so much energy on criticising a feature of my theory that it shares with her own.

Keefe's main criticism in her Section IV: Columnar higher-order vagueness contains an assumption that cannot be verified

Keefe's main point of criticism in her Section 4 (pp. 100–102) is this. She claims (a) that it is problematic that 'the theory <of columnar higher-order vagueness> is committed to "not clearly X", where X is one of its own basic assumptions' (p. 100). And she claims (b) that the theory 'declares a key element of itself <i.e. X> merely of borderline status', which she thinks 'is, at best, odd' (p.102). I take these two points in turn. First, the objection needs some unpacking. What Keefe claims to be my assumption X is

3 $\neg C \neg \exists x B F x$

and what she claims to be the problematic statements I am committed to are

4 $\neg C \neg C \neg \exists x B F x$.

and

5 B¬C¬∃*x*B*Fx*.[9]

My actual assumption is

6 For any Sorites series a_1 to a_n of a predicate φ, ¬C¬∃*x*Bφ*x*.[10]

Accordingly, Keefe's objection should run more precisely 'the theory <of columnar higher-order vagueness> is committed to "not clearly *X*" and "borderline *X*" where *the consequent X* of one of its own basic second-order-universal assumptions is such that one cannot tell whether *X*.'

For now I disregard the point that the assumptions and the two commitments are each relative to there being a Sorites series with *F*, and, for the purpose of argument, assume Keefe's two points to be correct, i.e., that my theory is committed to the combination of ***3*** and ***4*** and to the combination of ***3*** and ***5***.

*The combination of **3** and **4***: Keefe herself notes (i) that a claim that a basic assumption *A* of one's theory is unclear (so that one cannot tell whether *A*) is very problematic for any theory that, like her own supervaluationism, holds that borderline cases have a third semantic value but (ii) that such a claim 'looks a lot less problematic' 'if we move to an epistemic reading of C' as "can tell that", since it is *not* 'in general … an unacceptable consequent of a theory that by its own lights we do not and cannot know <one of its basic assumptions> to be true' (p. 101). Now, in the paper to which she responds, I only present the bivalent version of columnar higher-order vagueness, and say nothing about what kind of non-bivalent version I envisage. Let me here simply say that I do not envisage it as having a third semantic value, beyond truth and falsehood. (The reader will have to wait for my paper on modal semantics for columnar higher-order vagueness for details.)

*The combination of **3** and **5***: This leaves us with Keefe's claim that the fact that columnar higher-order vagueness 'declares a key element of itself merely of borderline status' (p. 102), 'is, at best, odd'. Keefe backs up her criticism with the fact that my clarity-operator C is defined in terms of borderlineness and not the other way about (p.101). Now, although it is true that I understand the C-operator in terms of the B-operator, i.e. as non-borderlineness, what matters is not this fact, but that, in turn, I define the B-operator in terms of 'can tell', as set out in the paper and above. If we account for the definition of the B-operator in terms of 'tell-ability', what remains of Keefe's criticism boils down to the

[9] She does not say how she gets to *5*.

[10] Bobzien 2015 p. 72 (*4.3*). Or, with my interpretation of the C-operator: 'For any Sorites sequence a_1 to a_n of a predicate *φ*, one cannot rule out (i.e. cannot tell that it is not the case) that it contains a borderline case of *φ*.'

following. Since the structure of tell-ability (as defined by the box-operator of the logic QKT4M+BF+FINAX) is designed specifically to capture the epistemic constraint of inability-to-tell which is to explain the hedging phenomenon in Sorites series (rather than any old inability to tell), it is objectionable or odd that

> a basic assumption *A* of the theory of columnar higher-order vagueness is itself such that one cannot tell whether *A*, where 'cannot tell whether *A*' is designed to structurally approximate the phenomenon of hedging behaviour in Sorites series.

If we now add the relativity to Sorites series from **6** above, we see that accurately formulated the criticism would be: it is objectionable or odd that

> the basic assumption of the theory of columnar higher-order vagueness, that for any Sorites series with *F*, $\exists x\mathrm{B}Fx$ (or $\neg\mathrm{C}\neg\exists x\mathrm{B}Fx$), is such that one cannot tell whether $\exists x\mathrm{B}Fx$ (or $\neg\mathrm{C}\neg\exists x\mathrm{B}Fx$), where 'cannot tell whether' is designed to structurally approximate the phenomenon of hedging behaviour in Sorites series (by the modal logic QKT4M+BF+FINAX, of course).

Thus the real question is not whether this *sounds odd*, but whether a logic that, for Sorites series with *F,* has both $\neg\mathrm{C}\neg\exists x\mathrm{B}Fx$ and $\mathrm{B}\neg\mathrm{C}\neg\exists x\mathrm{B}Fx$ can be the right logic for vagueness. As I said above, the main point of (Bobzien 2015) is precisely that this *is* the right logic for vagueness. As far as I can see, it is the only *normal modal logic* that can in one go (i) explain the hedging phenomenon in the sense that it reasonably approximates the area where people hedge and (ii) avoid introducing two or more sharp boundaries in place of one[11] and (iii) avoid all known higher-order vagueness paradoxes. (It can also be given a plausible philosophical interpretation, based on the viewpoint relativity of our assessments in the grey area. I explore this point in a different paper, which provides an interpretation of the modal semantics of QKT4M+BF+FINAX.)

Now, if one thinks that the purpose of the modal operator for borderlineness is to define extensions for various kinds of borderline and non-borderline cases (clearly borderline, borderline borderline, borderline clear, clearly borderline not, etc.) along a Sorites series, one is bound to miss the purpose the logic QKT4M+BF+FINAX. The purpose is not to define extensions (if it were, there would be no

[11] It does so by precluding a decision as to whether there is one or there are two such boundaries.

difference to S5). The purpose is to introduce the logical structure of a very specific kind of epistemic constraint which we encounter, for example, when being marched through a Sorites. And which is characteristic for vague predicates. This structure is a structure of semi-decidability – as I illustrated it in the summary representation of the paper at the Joint Session.[12]

* * *

I conclude with some points where Keefe seems to have either misunderstood my paper or drawn inaccurate conclusions from what I say.

On p. 91, Keefe writes: 'Bobzien models her structure with the modal logic S4M.' Strictly speaking, this is inaccurate. I distinguish between a general logic of columnar higher-order vagueness and a normal modal logic of columnar higher-order vagueness. The latter is quantified. So, neither logic is S4M.

On p. 92 Keefe writes: '*Since* all borderline cases are also borderline borderline cases, Bobzien *must deny* that there are clear borderline cases' (Italics mine). Here Keefe seems to misunderstand my theory, reversing *explanans* and *explanandum*. It is my firm view that there are no clear borderline cases that are expressible modally as set out on p.66 in (2.1) and that are relevant to the Sorites paradox. And it is *for this reason* that I introduce theorem V, that if something is borderline, it is borderline borderline – not the other way about. (This is explained in detail in my 2010, which Keefe cites.)

On p. 93 Keefe writes: 'Bobzien criticizes what she calls 'in-between borderlineness … . Instead she endorses "undecidability borderlineness".' Keefe here seems to have misunderstood my view on the two conceptions of borderlineness. I do not criticize in-between borderlineness at all. I fully endorse it. What I criticize is that philosophers conflate the two uses of 'borderline' that I mention and that this confusion gives rise to the so-called higher-order vagueness paradoxes. (I set this out in detail in my 2013, which Keefe cites, and also on pp.77-80 in very paper to which Keefe replies, i.e. Bobzien 2015.)

On p. 94 Keefe writes: 'the two conceptions <of borderlineness> may be neither exhaustive nor accurate'. This implies that she thinks it is my view that the two conceptions of borderlineness are exhaustive. However this is not my view. As I mentioned above, it is my view that

12 https://lecturecapture.warwick.ac.uk/ess/echo/presentation/15cf5dd2-f22b-49d6-a393-0a613a9caf5d?ec=true

there are at least five structurally different natural language uses of ‘borderline’. Of these I discuss the two that I believe are frequently confounded in discussions of vagueness. In fact, Keefe herself quotes a sentence of my paper in which I imply that there are more than two uses of ‘borderline’ (see above, on her Section I: ‘or still another kind of borderlineness’).

On p. 95 Keefe writes: ‘[A]n advocate of Williamson’s epistemic view can accommodate clear borderline cases’. I doubt that Williamson’s theory of vagueness as a whole can accommodate clear borderline cases. Williamson never says he can. But assume for the sake of argument that he can. One can then easily show that the only clear borderline cases Williamson can have are those that are borderline clear at some higher order. That is, if *a* is borderline *F* and you can tell that *a* is borderline *F*, then from some higher order upwards, you cannot tell whether you can tell whether/that you can tell … whether/that you can tell that *a* is borderline *F*. No argument of the kind Keefe adduces (p.94–95), that it is obvious that there are clear borderline cases, has any relevance to Williamson’s theory.

On p. 96, with footnote 5, Keefe writes: ‘One way to show that <in her theory the borderline clearly *F* and the borderline clearly not-*F* cases> coincide is to appeal to the result cited in Bobzien (2013), that formulae starting with $\neg C$ and having an odd number of negation signs (e.g. $\neg CCFa$, the first conjunct of the first type of case) are equivalent to $\neg CFa$ and those starting with $\neg C$ and having an even number of negation signs (e.g. the second conjunct, $\neg C\neg CFa$) are equivalent to $\neg C\neg Fa$. Both types of second-order borderline case are thus provably equivalent to $\neg CFa \wedge \neg C\neg Fa$, that is, a first-order borderline case.’

This argument is fallacious. (1) Whether Keefe’s borderline clearly *F* and borderline clearly not-*F* are provably equivalent to a first-order borderline case depends on whether they are *clearly* borderline clearly *F*/not-*F* or *not clearly* borderline clearly *F*/not-*F*. And since for Keefe these cases are the ones which we can typically pick out, or in any case well-placed observers can (p. 96), and thus *one can tell that* they are borderline clearly *F*/not-*F*, the right way to formalize them would be as $CBCFa$ and $CBC\neg Fa$. Thus we obtain $C[\neg CCFa\wedge\neg C\neg CFa]$ and by $\wedge$-distribution $C\neg CCFa\wedge C\neg C\neg CFa$ which according to the source on which she relies (Bobzien 2013) is equivalent with the contradiction $C\neg Fa\wedge CFa$. The same holds for $C[\neg CC\neg Fa\wedge\neg C\neg C\neg Fa]$ which by $\wedge$-distribution provides $C\neg CC\neg Fa\wedge C\neg C\neg C\neg Fa$ and by the equivalence rule she relies on the contradictory $CFa\wedge C\neg Fa$. This is exactly as it should be according to my theory, which does not have, and is not meant to have, clear borderline cases with the borderlineness relevant to the Sorites. (2) If Keefe suggested that one *cannot* tell of any BCF/not-F

case that it is BC*F*/not-*F*, then she is in the same situation with regard to her claim that we can tell that the borderline cases on the left are BC*F* and those on the right are BCnot-*F* as if she adopted *my* theory. (Williamson is in this situation. For him all borderline clearly *F* cases, i.e. all cases governed by his equivalent for BC*F*, are such that we can't know that they are BC*F* – or else his solution to the Sorites fails, see Bobzien 2012, pp. 205-10.) (3) If Keefe were to suggest that one can tell of some but not necessarily all BC*F*/not-*F* cases that they are BCF/not-F, this does not improve things. Since by her own lights, these are the cases right next to the clearly clear F/not-F cases, we can tell of the left-most and right-most cases that they are BCF/not-F respectively, and we have a sharp border between the CC and the CB cases at both ends. Of course she may say that, where that border is, we have BCC and BCB cases (like Williamson would). But as long as we can tell of *some* cases that they are BC*F*/not-*F*, we can infer that those to their left (for CBC*F*) or right (for CBCnot-*F*) are *also* CBC*F*/not-*F* (by Keefe's own assumption), except if we assume that they are BCBC*F*/not-*F*, which leads us back to the – unsuccessful options (1) and (2).

In sum, either Keefe's borderline clear cases are not clearly so, which is incompatible with Keefe's own assumption, or they are clear borderline clear cases, which leads to (at least) two sharp borders of the kind those who introduce a modal operator to express borderlineness aim to avoid: sharp borders between $C\varphi a_n$ and $C\neg\varphi a_{n+1}$. Columnar higher-order vagueness avoids such sharp borders.

On p. 97 Keefe writes: 'In *this* paper <i.e. Bobzien 2015>, she maintains that C is a technical term, so she may no longer be committed to such interpretations <i.e. as in her 2010>' (italics Keefe's). In fact, I very clearly present my use of 'borderline' as technical in my 2010 (esp. pp. 8-18).

On p. 99 Keefe claims (referring to Bobzien 2012, pp. 194-195) that if the pair of sentences 'it's clear that *A*' and 'it's unclear whether it's clear that *A*' are judged to contradict by many people, then the pair of sentences 'it's unclear whether *A*' and 'it's unclear whether it's unclear whether *A*' must be so judged by these people – where 'unclear', would express the 'doubt characteristic of borderline cases'. Keefe provides no reasons for this claim, and I believe it to be false.

On p. 101 Keefe writes about the assumption she discusses in her Section IV: 'The assumption, note, is much weaker than the claim that there are borderline cases: it is merely the claim that it is not clear that there are *not*.' This is correct. The implication that it is not part of my theory that there are borderline cases (and that this is a negative feature of my theory) however, is not correct. It is easy to see that in S4+BF+FINAX my assumption entails that there are borderline cases.[13]

On p. 102 Keefe writes about the motivation for going beyond first-order vagueness: '[t]he need to avoid replacing the two-fold categories with three (equally sharp) ones is a typical part of the explanation of the need to accommodate higher-order vagueness' and that columnar higher-order vagueness is unsuitable as a theory of higher-order vagueness, since its 'three columns or categories of cases' commit it 'to sharp boundaries'.

This shows that Keefe misses the point of my using S4+BF+FINAX for higher-order vagueness, and simply treats it as if it was S5. To see the point of S4+BF+FINAX, one has to be willing to think out of the box. As I say in the paper (p.68), it is the combination of the meta-principles (2.12) $\mathrm{C}A \leftrightarrow \mathrm{C}^{n}A$ for any *n*, and (2.13) $\mathrm{B}A \leftrightarrow \mathrm{B}^{n}A$ for any *n*, that ensure that borderlineness and radical higher-order borderlineness are co-extensive and that clarity and radical higher-order clarity are co-extensive. And as is generally known, the motivation for introducing radical higher-order vagueness is that it will avoid a sharp border between the non-borderline or clear and the borderline cases. And it does so irrespective of whether it is co-extensive with first-order vagueness. In this way, the theory retains the feature of radical higher-order vagueness without leading to any known higher-order vagueness paradox. (Readers not familiar with this discussion are invited to study the literature, including Sainsbury 1991, Shapiro 2005, and especially Wright 1992 and Zardini 2013, in connection with Section VI of Bobzien 2015.)

On p. 104 Keefe writes: 'On Bobzien's picture, the appeal to a technical term to do the work of modelling borderlineness does not build on a background theory of the semantics of vague predicates.' This is inaccurate. My theory does build on a semantic theory: a Kripke semantics for first-order logic, which I specify as reflexive, transitive and final (Bobzien 2015 pp. 61, 68, 70). I also provide a detailed philosophical interpretation, or 'background theory', based on the viewpoint sensitivity of assessments of sentences with vague predicates in contexts. I do this in a different paper, since it is not the topic of 2015.

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13 Proof:

(1)	$\neg\mathrm{C}\neg\exists x\mathrm{B}Fx$	assumption
(2)	$\neg\mathrm{C}\exists x\mathrm{B}Fx$	theorem of S4+BF+FINAX*
(3)	$\neg\mathrm{C}\neg\exists x\mathrm{B}Fx \wedge \neg\mathrm{C}\neg\exists x\mathrm{B}Fx$	1, 2 ∧-introduction
(4)	$\mathrm{B}\exists x\mathrm{B}Fx$	3 def B
(5)	$\exists x\mathrm{B}Fx \leftrightarrow \mathrm{B}\exists x\mathrm{B}Fx$	theorem of S4+BF+FINAX
(6)	$\mathrm{B}\exists x\mathrm{B}Fx \rightarrow \exists x\mathrm{B}Fx$	5 PC
(7)	$\exists x\mathrm{B}Fx$	5, 6 MP

* A proof of (2) can be found in Appendix II (i) of (Bobzien 2015).

(The topic of Bobzien 2015 is not the philosophical interpretation of the semantics, but the element of my theory by which vagueness and higher-order vagueness coincide and that is responsible for the fact that it avoids all known higher-order vagueness paradoxes, both topics about which Keefe says nothing.)

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