Categoricity and Negation. A Note on Kripke's Affirmativism

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Abstract: The idea that an adequate language for science needs a negation operator was recently dismissed by Kripke as "yet another dogma of empiricism". That a scientist could, and even should, drop negation implies at least three points: 1. negativist theories, i.e., theories formulated in languages that include negation, are conservative extensions of their affirmativist versions; 2. negativist theories have no serious advantages over their affirmativist versions; 3. negativist theories are dispensable and should better be replaced by their affirmativist versions. We argue that all three points are problematic.

Keywords: negation, categoricity, idealization

1 Introduction

Kripke (2015) argues that an affirmativist language is adequate for science: "In strictly scientific discourse, or serious discourse generally, limning the true and ultimate nature of reality, restriction to affirmativist terminology is the way to go" (p. 384). So if you're a scientist you could, and even should, drop negation. This implies that, in science, negativist theories are conservative extensions of their affirmativist versions, have no serious advantages over their affirmativist versions, and are actually dispensable and so better replaced by their their affirmativist versions. But we think that all these three points are problematic. We will take them in turn.[1]

We point out, first, that negativist theories can be conservative extensions of their affirmativist version provided that one takes the notion of logical complement as an affirmativist notion. Secondly, we describe what Kripke

[1]We should note that, in an *Addendum* to his paper, Kripke confessed that it is rather a "parody" of some of Quine's own arguments, but also that he cannot deny that his argument is sound, since he would thereby be lapsing into a negativistic idiom. Parody or not, suppose that we take Kripke's paper seriously. Then our discussion below entails that one should either reject his argument or dismiss this supposition.

took the practical advantages of affirmativist theories to be, and then consider the view that an epistemic advantage of negativist theories is that they are simpler than their affirmativist versions: the simplifying effects of classical negation are analogous to those of imaginary numbers in mathematics. But we emphasize that while in mathematics such effects might be explained in terms of categoricity, classical logic is non-categorical, for it admits of non-normal interpretations. We also prove that eliminating negation does not help one get rid of such interpretations: positive (and affirmativist) logic is non-categorical as well. Finally, we argue that affirmativist restrictions on science are at odds with the typical understanding of scientific idealization.

2 Is T_- a conservative extension of T_+?

Let's start with Kripke's argument for the claim that negativist theories are conservative extensions of their affirmativist versions. Consider the affirmativist first order language, L_+, of an affirmativist theory, T_+. L_+ contains a finite list of primitive predicates, conjunction and disjunction as primitive connectives, and the universal and existential quantifiers. Is L_+ adequate for science?

Let us assume that L_+ is adequate only if negation is added. By De Morgan Laws, every sentence in the augmented language, L_-, is logically equivalent to a sentence with negation applied only to atomic formulae. Let us eliminate negation and extend L_+ by adding to each predicate P_i a predicate P_i^* for its complement, and likewise for any atomic sentence. Thus, everything expressible in L_- is expressible in the affirmativist language thus supplemented, L_+^*.

Kripke admits that the notion of complement is negativistic, but claims that this is not a problem since the argument just given is directed at negativists, not affirmativists. This argumentative move is adopted by Kripke from Quine (1960), where mentalist terms are employed in explaining to mentalists the physicalist view about the mind. Using negativist notions in Kripke's affirmativist argument directed at negativists is analogous to using mentalist notions in Quine's physicalist argument directed at mentalists. Be that as it may, if the notion of complement is negativistic, then how can L_+^* be counted as an affirmativist language? Kripke's argument for the idea that negativistic theories are conservative extensions of their affirmativist versions goes through only if one assumes that the notion of complement is an affirmativist one. But in many logical systems, especially algebraic logical systems, the complement is precisely the notion that expresses negation.[2]

We should further note that, unlike classical positive logic, Kripke's affirmativist logic does not have a classical semantics: it eliminates not only the negation operator, but also the notion of "falsity". A conjunction $\ulcorner A \wedge B \urcorner$ is true if and only if A is true and B is true, and a disjunction $\ulcorner A \vee B \urcorner$ is true if and only if A is true or B is true. However, if the normal truth tables (NTTs) for conjunction and disjunction contain only the first line[3], then we cannot really distinguish conjunction from disjunction, since only the other three lines of the NTTs allow us to make a distinction between conjunction and disjunction. Why would "and" and "or" mean different things for the affirmativist?

But even if the affirmativist could distinguish between "and" and "or", the class of logical rules of inference and of logical truths of affirmativist logic is drastically affected. For instance, $\ulcorner A \rightarrow (B \rightarrow A) \urcorner$ could no longer be treated as a logical truth, since the material conditional is defined only on the first line of the NTT and thus, without knowing the value of B, we may be reluctant to accept $\ulcorner B \rightarrow A \urcorner$, and thus $\ulcorner A \rightarrow (B \rightarrow A) \urcorner$.

3 Negation as an ideal element

What might be the advantages of affirmativist theories? Kripke claims that affirmativism is more advantageous than negativism, because it improves the civility of our debates, for example. If you thought that by eliminating negation, you would not be able to say anymore that the negation of a true statement is false, Kripke would say that you should never state "Your opinion is false!" in the first place. Instead, what you should say is "I am reluctant to accept your view". He thinks that this may lead, in the long run, to world peace. No argument is given for this claim, other than the observation that many conflicts have been preceded by negativistic characterizations of one's opponents' assertions as "false". Kripke also thinks that affirmativism offers a solution to paradoxes, since he thinks that all of them are easily seen

[2]As a matter of logical fact, the negation operation could not be defined with the help of other sentential operations. This is why classical positive logic, i.e., classical logic without the negation operator but with classical semantics, is incomplete – logical truths such as Peirce's Law cannot be derived without negation.

[3]From Kripke's remark on this matter, it is natural to suppose so. He explicitly states that we do not need the third extra lines from the truth-table for conjunction, the first line is sufficient. This would suggest that the lines that contain the sign for falsity should be dismissed. If Kripke introduced a sign for the attitude of reluctance to replace the sign for falsity, then, of course, the propositional operators could be easily defined and distinguished one from another.

to invoke a negativistic notion. More precisely, without negation, paradoxes would just "disappear". However, it is not clear that this is the case. As Skolem (1952) proved, Dedekind's naive set theory still leads to inconsistency via the comprehension axiom, even if the logic used in the background is positive predicate calculus (i.e., first-order logic without negation; see also Hilbert & Bernays, 1934/1968).

Furthermore, imposing affirmativist restrictions in science assumes that the alleged practical advantages of affirmativism (e.g., less conflicts, peace, etc.) outweigh any other advantages negativist theories might have. But there is a long tradition in the philosophy of logic, stemming from Hilbert's school, which considers negation as an idealization: "Negation plays the role of an ideal element whose introduction aims at rounding off the logical system to a totality with a simpler structure, just as the system of real numbers is extended to a more perspicuous totality by the introduction of imaginary numbers." (Bernays, 1927). Negativist theories, it is claimed here, are simpler than their affirmativist versions. For example, the simplifying effects of classical negation are comparable to the simplifying effects of the imaginaries, and so eliminating negation is comparable to the elimination of the imaginaries from mathematics. Extending a logical system by adding connectives, like negation, leads to a totality with a simpler structure. As Hilbert also emphasized, negation makes possible the logical closure and completeness of a system (see, e.g., Hilbert, 1931).

This view raises some important questions. Let's assume that extending a number system by adding new objects leads to a more perspicuous totality. In particular, adding imaginary numbers to the reals forms the algebraic closure of the real numbers, but what makes an algebraic closure a more perspicuous totality? One answer to this question might be given in semantic terms like categoricity: the algebraic closure of the real number field is unique up to isomorphism (Steinitz, 1910). But what makes a closed and complete logical system a totality with a simpler structure? Could a similar answer be given in terms of categoricity?

4 The non-categoricity of classical logic

Let a semantic property of an expression be fully formalized by a calculus if and only if the expression possesses that property in every interpretation for which the calculus is sound. As is well known, however, classical logic allows for non-standard models, i.e., interpretations for which the standard calculi remain sound and complete, but in which the logical constants have different meanings than the standard ones (Carnap, 1943). The existence of these interpretations shows that the standard propositional and quantificational calculi do not fully formalize all the semantic properties of the logical terms and, thus, fail in uniquely determining their meanings. The rules for negation, disjunction, material implication, and the quantifiers, in contrast to the rules for conjunction, do not determine all the properties of these operators as defined by their normal truth tables (NTT) and the standard semantics for quantifiers.

In propositional logic, non-normal interpretations are possible because the usual formalizations of classical logic state conditions only for logical derivability (C-implicate, in Carnap's terms) and logical theoremhood (C-truth). Thus, they can formalize only those semantic properties definable on the basis of logical consequence (L-implicate) and logical truth (L-truth). However, the semantic properties of L-exclusive and L-disjunct are not definable on this basis, thus, they are not formalized by the usual systems. Two sentences are L-exclusive if and only if they are not both true (thus, at least one is false), and two sentences are L-disjunct if and only if at least one of them is true (thus, they are not both false). Since L-exclusive and L-disjunct are not fully formalized, the principles of non-contradiction and excluded middle are not represented in the usual formalizations of classical logic.

There are two mutually exclusive types of non-normal interpretations for classical propositional operators: (I) all sentences are true, and (II) at least one sentence is false. Thus, there are non-normal interpretations in which a sentence and its negation are both true, and non-normal interpretations in which they are both false (and so, their disjunction is true and their implication is false). In type (I) interpretations, the principle of non-contradiction is violated; in type (II), the principle of excluded middle is violated. Thus, these two principles are not fully formalized, since they hold in some interpretations (i.e., in the normal interpretations), but they do not hold in others (i.e., in the non-normal interpretations). In addition, even if all propositional operators had only normal interpretations, there would still exist non-normal interpretations of the quantifiers (as shown in Carnap 1943, chapter F). In particular, there are sound interpretations of quantificational logic in which "$(\forall x)Fx$" could be interpreted as "every individual is F, and b is G", where "b" is an individual constant. Likewise, "$(\exists x)Fx$" could be interpreted as "at least one individual is F, or b is G". The possibility of these non-normal interpretations arises because, in the standard formalizations of first-order logic, a universal sentence is not deductively equivalent (C-equivalent, in

Carnap's terms) to the conjunction of all the instances of the operand, and an existential sentence is not C-equivalent with the disjunction of all the instances of the operand. The existence of such interpretations shows that the logical calculus fails in uniquely determining the meaning of logical terms. Unlike the rules for conjunction, those for negation (as well as those for disjunction, implication, and quantifiers) do not determine all the properties of these operators as defined by NTTs and standard semantics.

Kripke took Carnap (1943) as an illustration of a negativistic theory, as it includes not only rules of theoremhood, but also rules of rejection, and he thought that discussing it in more detail would be "superfluous". But we think that it is important to note that Carnap proved that if negation has a standard interpretation, then all other operators have a standard interpretation. Thus, the categoricity of negativist theories like classical logic would require the elimination of non-standard interpretations of negation. The affirmativist could point out that if one eliminates negation, its non-standard interpretations are thereby eliminated as well. This seems natural enough since the principles of non-contradiction and excluded middle do not belong to positive logic. This would suggest a shorter, if more radical, route to categoricity.

However, this assumes that a positive version of classical logic does not allow for non-normal interpretations. A closer investigation, however, shows that the elimination of negation does not immediately entail that positive logic admits of no non-normal interpretations. The normality of negation constrains disjunction (and, thus, all the other operators) to be normal only if negation is, of course, part of the system. If negation has a standard interpretation, then disjunction has a standard interpretation. This is due to the Disjunctive Syllogism (DS): $A \vee B, \neg A \vdash B$. If A and B would be false, and negation is standard (thus, $\ulcorner \neg A \urcorner$ is true), then $\ulcorner A \vee B \urcorner$ cannot be true, otherwise the rule would be unsound. However, we do not have the DS in positive logic. In the absence of negation, what happens to disjunction? As we shall see in a moment, in positive logic, A and B can be false, but $\ulcorner A \vee B \urcorner$ true.

5 The non-categoricity of positive logic

The semantic property of disjunction displayed on line D4 of its NTT is not determined by the deduction rules for disjunction. Let us analyze what happens when D4 is violated, that is, what kind of non-normal interpretation would result from this violation, and then let us check whether this interpretation is non-empty. Let us assume that D4 is violated with respect to the propositional constants A and B. On this assumption, we can reason as follows:

a) A is false, B is false, and $\ulcorner A \vee B \urcorner$ is true. (by assumption)
b) A is different from B (and conversely). Proof: If A were B, then A, which is derivable from $\ulcorner A \vee A \urcorner$, would be derivable from $\ulcorner A \vee B \urcorner$ and, thus, true. But A is false. Thus, A is different from B.
c) Any sentence derivable both from A and from B is true. Proof: If a sentence is derivable both from A and from B, then it is derivable from $\ulcorner A \vee B \urcorner$, and, thus, true.
d) A does not follow from B, nor B from A. Proof: If A were derivable from B, since it is derivable from itself, it would be derivable from $A \vee B$ and, thus, true. But A is false.
e) $\ulcorner A \rightarrow B \urcorner$ is false. Proof: $A \vee B \vdash ((A \rightarrow B) \rightarrow B)$. Thus $\ulcorner (A \rightarrow B) \rightarrow B \urcorner$ is true. Since B is false, then according to line I4 in the NTT, $\ulcorner A \rightarrow B \urcorner$ has to be false. But since A is false as well, then I4 is violated with respect to A and B.

Thus, we can see that the violation of D4 would lead to a non-normal interpretation of the positive calculus in which the truth value of $\ulcorner A \rightarrow B \urcorner$ on line I4 in the NTT is not determined by the rules of material implication, since this operator is non-extensional, i.e., it behaves normally in its main occurrence in $\ulcorner (A \rightarrow B) \rightarrow B \urcorner$, but non-normally in $\ulcorner A \rightarrow B \urcorner$. What we have to examine now is whether the non-normal interpretation with the features just sketched is non-empty. That, indeed, it is non-empty will be shown by the construction of an example, namely, the construction of an interpretation, V_+, as follows:

1. if p is a theorem of the positive calculus, then $V_+(p)$ is true.
2. if p is not a theorem of the positive calculus, then $V_+(p)$ is false, but in the following two cases:
   (a) $V_+(A \vee B)$ is true.
   (b) For every C, if $A \vee B \vdash C$, then $V_+(C)$ is true.

This interpretation assigns 'truth' to every theorem of positive calculus and 'false' to every non-theorem of the positive calculus, except in the two cases described: namely, it will assign 'truth' to $\ulcorner A \vee B \urcorner$ and to all formulas derivable from it. Thus, since it assigns a determinate truth value to all formulas of positive logic, V_+ is a full interpretation of the positive calculus.

What we have to show now is that the positive calculus remains sound under this interpretation.

Proof: Let Γ be a set of premises and σ an arbitrary sentence in the language of positive calculus, and let us further suppose that $\Gamma \vdash \sigma$. There are three cases to be considered:

1. if in Γ we have only theorems, then σ will be a theorem and, thus, true in V_+.
2. if in Γ we have a non-theorem different from $\ulcorner A \vee B \urcorner$ and from any C derivable from $\ulcorner A \vee B \urcorner$, then the sequent $\Gamma \vdash \sigma$ will be valid even if σ is false.
3. if Γ contains a set of theorems (Δ) and, in addition, it contains a non-theorem which is either $\ulcorner A \vee B \urcorner$, or any C derivable from $\ulcorner A \vee B \urcorner$, then σ either follows from Δ, and thus it is true, or it follows from $\ulcorner A \vee B \urcorner$, and thus it is also true.

Therefore, the calculus of positive logic remains sound in the interpretation V_+. However, as constructed, this interpretation is non-normal, since it makes a disjunction true, although both of its disjuncts are false. What this shows is that the existence of non-normal interpretations of a logical calculus does not require the non-normality of logical negation. Even without negation, the rules for disjunction do not completely determine the semantic properties of disjunction as defined by its NTT. In addition, since positive quantificational logic is obtained by adding the standard rules for the existential and universal quantifiers to the positive fragment of classical logic, it is in no different position than classical quantificational logic with respect to non-normality: the non-normal interpretations of the quantifiers in classical logic are also present in the case of positive quantificational logic.

Furthermore, things are similar in affirmativist logic, because the elimination rule for disjunction involves no negation, and it is responsible for the non-standard models for disjunction. More exactly, in affirmativist logic, one may take a disjunction to be true, although one may be reluctant to accept its disjuncts. Therefore, the elimination of negation and falsity does not entail that there are no non-standard interpretations.

To take stock, we have argued that negativist theories may be conservative extensions of their affirmativist versions, but only if the notion of logical complement is accepted as an affirmativist notion, and that these affirmativist versions may have important practical advantages in that they may be less conflictual and more civil, but that one should not overlook some important epistemic advantages of negativist theories, e.g., simplicity. We have also pointed out that negativist theories do not obstruct purported semantic advantages of their affirmativist versions: since positive and affirmativist logics are non-categorical, one cannot maintain that categoricity is lost due to the addition of the negation operator. Thus, the affirmativist's elimination of negation cannot be justified on such semantic grounds.

6 Affirmativism and idealization

Let's further suppose that one has strong enough reasons to prefer practical to epistemic advantages, and so let's suppose that one accepts the affirmativist restrictions on science. Consider again imaginary numbers and their simplifying effects. The latter could also be explained in terms of linear factorization, i.e., via the fact that the only irreducible polynomials are those of degree one. This is stated by the Fundamental Theorem of Algebra: "every equation of degree n has n roots". The theorem is, of course, false in the real number system, but true in its algebraic closure, i.e., in the complex number system. According to the affirmativist, however, one should not say that the theorem is false in the real number system, since falsity has been eliminated, but only that one is reluctant to accept the theorem in the real number system. Analogously, one should not say that, for instance, the ideal gas law is false for real gases, but only that one is reluctant to accept it. More generally, one should not say that an idealized statement is false for non-idealized systems, but only that one is reluctant to accept that statement. However, this is entirely missing the point of scientific idealization. In science, idealized statements are typically rather unreluctantly accepted, for even though they are false for non-idealized systems, they are thought to have great explanatory power (Toader, 2015).

In conclusion, *pace* Kripke, we argued that an affirmativist first order language, L_+, cannot be adequate for science. We believe that if Kripke's affirmativist view is taken seriously, then our argument entails that his view should be rejected. More precisely, our argument entails that L_+ cannot be taken as the object language of a genuine scientific theory. But this leaves other questions open. Could L_+ be adequate as a metalanguage for science, i.e., as the language of the metatheory of a scientific theory? In response to this, we would like just to note here a recent effort to do model theory in the framework of positive logic: "a non first order analogue of classical model theory where compactness is kept at the expense of negation" (Ben-Yaacov, 2003). Such an approach has been taken, for instance, to the formal semantics of quantum mechanics (see, e.g., Zilber, 2016). But we are fine, for the time being, with a negationless metatheory.

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