The Epistemic Significance of Valid Inference – A Model-Theoretic Approach*

Constantin C. Brîncuş

1. Introduction

The problem analysed in this paper is whether we can gain knowledge by using valid inferences, and how we can explain this process from a model-theoretic perspective. According to the *paradox of inference* (Cohen & Nagel 1936/1998, 173), it is logically impossible for an inference to be both valid and its conclusion to possess novelty with respect to the premises. I argue in this paper that valid inference has an epistemic significance, i.e., it can be used by an agent to enlarge his knowledge, and this significance can be accounted in model-theoretic terms. I will argue first that the paradox is based on an equivocation, namely, it arises because logical containment, i.e., logical implication, is identified with epistemological containment, i.e., the knowledge of the premises entails the knowledge of the conclusion. Second, I will argue that a truth-conditional theory of meaning has the necessary resources to explain the epistemic significance of valid inferences. I will explain this epistemic significance starting from Carnap's semantic theory of meaning and Tarski's notion of satisfaction. In this way I will counter (Prawitz 2012b)'s claim that a truth-conditional theory of meaning is not able to account the legitimacy of valid inferences, i.e., their epistemic significance.

The paper has five sections. I will start by presenting the paradox of inference, according to which a valid inference has no epistemic usefulness, and I will argue that we can dismiss it once we realize that it is based on an equivocation. In the second section I will show why we can gain knowledge

* ACKNOWLEDGEMENT: This paper is supported by the Sectoral Operational Programme Human Resources Development (SOP HRD), financed from the European Social Fund and by the Romanian Government under the contract number POSDRU/159/1.5/S/133675.

by performing valid inferences, and I will argue following (Prawitz 2012a) that we should make a distinction between stating an argument and performing an inference. In the third section, I will present Prawitz's constructivist account to the legitimacy of valid inference, i.e., to their epistemic significance, and his arguments for the idea that a truth-conditional theory of meaning does not have the necessary resources to account this epistemic significance. Starting from Carnap's semantic views on meaning, I will show in the fourth section how we can explain the epistemic significance of valid inference from a model-theoretic perspective, and more precisely, by using a truth-conditional theory of meaning. In the fifth section, by introducing Tarski's notion of satisfaction, I will briefly indicate why the account is also sound for the inferences involving quantifiers. Finally, I will conclude by stating the conditions under which an agent can gain knowledge by using valid inferences, and by arguing that, in order to acquire the piece of knowledge expressed by the conclusion of an inference, an agent should know that the inference performed by him is valid.

2. The *Paradox of Inference*

An explicit presentation of the so-called *paradox of inference* can be found in M. Cohen & E. Nagel's book *An Introduction to Logic and Scientific Method*, namely:

> If in an inference the conclusion is not contained in the premise, it cannot be valid; and if the conclusion is not different from the premises, it is useless; but the conclusion cannot be contained in the premises and also possess novelty; hence inferences cannot be both valid and useful (Cohen & Nagel 1936/1998, 173).

If we translate the conclusion of this paradox in modal terms, what it tells us is that it is *logically impossible* for an inference to be valid and useful in the same time. We may obtain a clear-cut representation of this idea if we reconstruct the argument in a slightly different way, as follows:

P1. If an inference is valid then the conclusion is *contained* in the premises.
P2. If an inference is useful then the conclusion is *different* from the premises.
P3. The conclusion cannot be *contained* in the premises and also *different* from them.
C. Inferences cannot be both valid and useful.

The paradox is meant to be a criticism to the value of formal logic, but, as Cohen and Nagel mention, it is based upon several confusions[1]. What does it mean to say that the conclusion "is contained" in the premises, or that it possesses "novelty" with respect to the premises? Clearly, as the authors emphasize, for a certain person, "a conclusion may be surprising or unexpected even though it is correctly implied by the premises" (Idem, 174). For instance, a theorem in Euclidean geometry may have no novelty for a teacher who has proved it several times before, but, certainly, a student who approaches the subject for the first time, by proving it, may encounter a psychological novelty, i.e., "a feeling of novelty" (Idem, 176). Nevertheless, this feature has nothing to do with the validity of an inference. The theorem in question necessarily follows from the axioms independently from the persons who inquire Euclidean geometry. This fact shows us that psychological novelty and logical novelty, i.e., logical independence among propositions[2], should be distinguished. Moreover, the term "containment" is taken by the authors, correctly I think, to denote the relation of logical implication, and, since in a valid argument the conclusion is implied by the premises, the conclusion cannot possess logical novelty with respect to the premises, i.e., it is not logically independent from them. In addition, since the relation of implication is an objective relation that is established among propositions, it follows that implications can only be discovered. As a

[1] For this disscusion you may also read the section "Deduction and Novelty" from (Cohen 1944/1965, 25-28).
[2] Two propositions are logically independent if, and only if, the truth-value of one of them in no way determines or limits the truth-value of the other. (see also Cohen & Nagel 1936/1998, 52-57).

consequence, logical novelty, in contradistinction to psychological novelty, is also an objective feature of a proposition with respect to others. Cohen and Nagel consider that psychological novelty, the mark of usefulness, arises because the *conventional*[3] meaning of a proposition, implied by a set of propositions, may not be present to the reasoner's mind, although, from an objective point of view the propositions are logically connected. Consequently, the conclusion of the paradox may be dismissed.

Although I agree that the terms "containment", "novelty", and "usefulness" are used in a quite vague manner in the original formulation of the paradox, given the fact that valid inferences are essentially used in epistemic contexts, I think that we should take the terms "novelty" and "usefulness" as aiming to epistemic novelty and epistemic usefulness. In addition, since an inferred theorem may be new not just for a particular person but for the entire scientific community, the concept of novelty must be thought of as an epistemological concept. Moreover, and most importantly, by making an inference we are expecting to get a *justification* for the inferred proposition, which, no doubt, is an essential feature of epistemic contexts. Therefore, the main point of the argument is that the conclusion of a valid inference cannot possess epistemic novelty with respect to its premises, and, consequently, inferences do not have any epistemic usefulness.

In order to have a clear-cut understanding of the so-called paradox of inference, I consider that it is necessary to offer explicit definitions of the central terms involved in its formulation. Namely, we should clearly distinguish between logical containment and epistemological containment, and between logical novelty and epistemological novelty. In order to make explicit these concepts I propose the following four definitions:

> Definition 1 $_{\text{Logical Containment}}$: A proposition P is logically contained in a set of propositions Γ if and only if Γ logically implies P.

[3] The authors distinguish between the conventional meaning of a proposition and the propositions it implies. The conventional meaning is defined as "that minimum of meaning which is required if a group of inquirers can be said to address themselves to the *same* proposition" (Idem, 176). The conventional meaning of a proposition enlarges when we discover a new logical consequence of this proposition. Initially, the meaning of this new proposition was not part of the conventional meaning of the initial proposition. Given my interpretation of the paradox, I think that this conventional meaning should be identified with the *known associated meaning* with a sentence, at a certain moment of time.

Definition 2 $_{\text{Logical Novelty}}$: A proposition P has logical novelty with respect to a set of propositions Γ if and only if Γ and P are logically independent.

Definition 3 $_{\text{Epistemological Containment}}$: A proposition P is epistemologically contained[4] in a set of propositions Γ, relative to a person A if and only if by knowing Γ then the person, *ipso facto*, knows P.

Definition 4 $_{\text{Epistemological Novelty}}$: A proposition P has epistemological novelty for a person A relative to a set of propositions Γ, if, and only if, A knows Γ without knowing, *ipso facto,* P.

These definitions allow us to understand more clearly why an inference can be both valid and useful or, in other words, to see why logical containment and epistemological novelty are compatible. The compatibility becomes evident once we realize that the paradox, as I strongly believe, is based on an equivocation, namely, logical containment is identified with epistemological containment. This compatibility can be easily made explicit, in several steps of reasoning, in the following manner: if logical containment implies epistemological containment, then a person who knows a proposition also knows all its consequences. However, it is a fact that a real person may know a proposition without knowing all its consequences. Hence, logical containment does not imply epistemological containment. Moreover, epistemological containment is equivalent to epistemological non-novelty. This means that logical containment does not imply epistemological non-novelty. Therefore, logical containment and epistemological novelty are logically compatible.

If we want, for precision, we may represent more explicitly this reasoning as follows:

(A) If logical containment implies epistemological containment, then a person who knows a proposition also knows all its consequences. (Def.1 and Def.4)

---

[4] If we want, we may label the concept of epistemological containment with the term of "epistemological consequence", i.e., a proposition is an epistemological consequence of a set of propositions, relative to an agent state of knowledge, if it is epistemologically contained in them. Of course, according to definition 3, Γ may logically imply P, or it may not imply it.

(B) A person may know a proposition without knowing all its consequences. (Description of a fact.)
(C) Logical containment does not imply epistemological containment. (MT –A, B)
(D) Epistemological containment is equivalent to epistemological non-novelty. (Def.3 and Def.4)
(E) Logical containment does not imply epistemological non-novelty. (C, D)
(F) Logical containment and epistemological novelty are compatible. (E)

The distinction between logical novelty and epistemological novelty is also important. The generally accepted idea is that we can obtain a new piece of knowledge only from experience. However, this is not necessarily the case. Of course, if a proposition has empirical content and is logically independent from what we already know, we cannot infer it from what we know. In this case we may obtain a justification for it only by appealing to experience. Nevertheless, we may also gain new knowledge by way of deductive thinking, as I will argue below. Therefore, not all that is epistemologically novel is logically novel.

With these distinctions in mind we can now defuse the so-called *paradox of inference*: if an inference is valid then the conclusion is *logically contained* in the premises. If an inference is useful then the conclusion must have *epistemological novelty* with respect to the premises. But, the conclusion can be *logically contained* in the premises and also possess *epistemological novelty*. Consequently, inferences can be both valid and useful, i.e., it is logically possible for an inference to be both valid and useful.

The distinction formulated above between logical containment, i.e., logical implication, and epistemological containment, i.e., epistemological consequence, can be found in a slightly different way in (Fine 2007, 47-48). K. Fine distinguishes between classical consequence and manifest consequence. The point is that although a proposition may be a classical consequence of a set of propositions it is not necessarily a manifest consequence of them. According to Fine's definition, a proposition q is a manifest consequence of other propositions $p_1, p_2, ,..., p_n$ if it is a classical consequence of them and if, in addition, it would be manifest to any ideal cognizer who knew the propositions $p_1, p_2, ,..., p_n$, that q was indeed a classical consequence of those propositions.

The fact that these two concepts do not overlap is indicated by K. Fine by way of a very simple example. We can imagine an ideal cognitive agent who is perfectly competent in drawing consequences from what he knows, and we may ask ourselves if he will know every classical consequence of what he already knows. More precisely, let us consider the agent A who knows that Paderewski is a brilliant pianist (having heard him at a concert), and who also knows that he is a charismatic statesman (having heard him at a political rally), but who does not realize that is the same person who is both. Therefore, the agent A knows that *p is P*, and he also knows, from another context, that *p is S*, but the agent A is not in the position to know that p is both P and S. In other words, although *(∃x)(Px & Sx)* is a classical consequence of *Pp* and *Sp*, the agent knows *Pp and Sp* without knowing *(∃x)(Px & Sx)*. If we use the epistemic operator `$K_A$_`, i.e., A knows _, and let the sign "$\vdash$" denote the relation of manifest consequence, we may write:

$$K_A\,Pp,\ K_A\,Sp \nvdash K_A(\exists x)(Px \,\&\, Sx)$$

Having in mind this example, we may believe that, in order to know the conclusion, the agent must have some extra, empirical, knowledge. However, this is not necessarily the case. For instance, what the agent may need is simply coordinating his thoughts in order to realize that Paderewski the pianist is the same person with Paderewski the statesman. Of course, this example may raise further questions, but they lie beyond the scope of the present paper. We can make a clear-cut distinction between logical containment and epistemological containment without entering in mental considerations, as we will see below.

3. Knowledge Through Inference

The idea that logical containment, i.e., logical implication, and epistemological containment do not overlap can be illustrated by more simple and frequent situations. For instance, let us consider the following argument for which we assume that the premises are in fact true:

> If the safe was opened, it must have been opened by Smith, with the assistance of Brown or Robinson. None of these

> three could have been involved unless he was absent from the meeting. But we know that either Smith or Brown was present at the meeting. So since the safe was opened, it must have been Robinson who helped open it. (Forbes 1994, 86)

No doubt, this argument is logically valid, i.e., the premises logically imply the conclusion, but, certainly, an *untrained detective* may know that each premise in part is true without recognizing, *ipso facto,* the truth of the conclusion. Speaking more generally, when there is an epistemic gap, sufficiently wide, between the premises and the conclusion of a valid argument, the person in question must make an *epistemic effort* – which remains to be explained – in order to be able to assert the conclusion. Probably the best example which enforces the distinction between logical containment and epistemological containment is that of an argument which has as premises the axioms of a theory and as conclusion a certain theorem of that theory[5]. In this case, although the conclusion follows from the axioms, certainly we may not be able to infer it. If this would not be so, then a person who knows the axioms of a theory will also know, *ipso facto,* each theorem of that theory. But certainly, this is not the case. Logical omniscience is not a feature of the actual human minds.

If we introduce again the epistemic operator 'K_A_' we may consider the following two propositions (Rescher 2005, 14-15):

(A) If K_A p and K_A (p ├ q), then K_A q.
(B) If K_A p and p ├ q, then K_A q, or, equivalently, if p ├ q, then K_A p ├ K_A q,

I think that we all agree that although the proposition (A) is true, proposition (B) needs a supplementary condition in the antecedent in order to be true, i.e.: (B') If K_A p and p ├ q, and … , then K_A q. I think that the additional condition that should be introduced is the following: agent A infers[6] q from p. In this case, if we reconsider the argument stated above, in

[5] This example is also used by Dag Prawitz (2012a, 890; 2012b, 11) in order to illustrate that the Tarskian semantic notion of validity is insufficient for explaining the epistemic significance of valid inferences.

[6] By "agent A infers q from p" I mean "agent A correctly (or validly) infers q from p" since, as far as I understand these terms, an agent who does not correctly infer a proposition from a set of propositions, basically, he does not infer it.

order to get in possession of the piece of knowledge expressed by the conclusion, an agent must infer the conclusion from the premises. In other words, he must supplement the premises with the following steps of inference (I_1, I_4):

P_1. If the safe was opened, it must have been opened by Smith, with the assistance of Brown or Robinson.
P_2. None of these three could have been involved unless he was absent from the meeting.
P_3. Either Smith or Brown was present at the meeting.
P_4. The safe was opened.
I_1. Smith opened the safe and either Brown or Robinson helped. (from P_4 and P_1)
I_2. Smith was absent from the meeting. (from I_1 and P_2)
I_3. Brown was present at the meeting. (from I_2 and P_3)
I_4. Brown did not help to open the safe. (from I_3 and P_2)
C. Robinson helped open the safe. (from I_4 and I_1). (see also Forbes 1994, 87)

In sum, the agent gets a justification for asserting the proposition expressed by the conclusion *only if* he performs the inferences I_1-I_4. By simply asserting P_1 –P_4 and then C, the agent does not have any real justification in order to assert the proposition expressed by the conclusion C. Generalizing from this individual case, we can say that although a proposition is logically implied by a set of true propositions, i.e., it is logically contained in them, an agent does not know the proposition expressed by the conclusion if he is not able to infer it from the premises. We can say, following (Prawitz 2012a), that stating an argument and making an inference are two quite different things. To make an inference means to assert the premises and then to infer the conclusion from the premises. To state an argument means just to assert the premises and the conclusion, and claiming the existence of a certain relation between them (A, B. Hence C.).

Returning to our initial problem, i.e., whether we can gain knowledge through valid inferences, now we can definitely give an affirmative answer. As we saw, a proposition may be logically implied by a set of propositions without entailing, by itself, that an agent who knows that set of propositions, *ipso facto,* knows each proposition logically implied by them, i.e., in our initial terms, logical containment does not entail epistemological containment. In order to entail epistemological containment, logical containment must be supplemented with the condition imposed on a real agent, namely, to realize the necessary acts of inference. The epistemic effort that we have mentioned above consists precisely in performing these acts.

In the following sections it remains to explain why the relation of logical implication in addition with the acts of inference may entail epistemological containment. Dag Prawitz (2012a) proposed an explanation of the epistemic significance of valid inferences from a constructivist point of view, and he argued (in Prawitz 2012b) that a truth-conditional theory of meaning does not have the necessary resources to explain the legitimacy of inferences, i.e., their epistemic significance. The main aim of the next sections is to account for the epistemic significance of valid inferences from a model-theoretic point of view, and more precisely, by using a truth-conditional theory of meaning.

4. The Legitimacy of Inferences

The problem of accounting for the epistemic significance of valid inferences presupposes an explanation of the fact that an agent who knows[7] the true propositions expressed by the premises of a valid argument, and performs a chain of inferences from the premises that leads him to the conclusion, will, *ipso facto,* know the proposition expressed by the conclusion. A necessary condition for grasping the piece of knowledge expressed by the conclusion is, of course, that the agent understands the meanings of the sentences involved in the stated argument, i.e., the propositions that they express. Consequently, for explaining the proposed problem we must state an adequate theory of meaning which allows us to explain how an agent may gain sometimes new knowledge by using valid inferences. This fact indicates us the real importance of a theory of meaning and its central place in philosophical approaches.

[7] The concept of knowledge engaged in this approach is such that an agent knows a proposition if that proposition is true and the agent has a justification that guarantees the truth of that proposition.

One of the most important philosophers who vividly recognized the importance of the theory of meaning in analysing philosophical issues was Rudolf Carnap. Although he initially reduced philosophy to the logic of science, and more precisely to the syntax of the language of science (Carnap 1937, 277-280), he soon realized that

> many philosophers and scientists interested in the logical analysis of science have become aware that we need, in addition to a purely formal analysis of language, an analysis of the signifying function of language – in other words, a theory of meaning and interpretation. (Carnap 1942, v).

Consequently, he redefined the task of philosophy as consisting in semiotic analysis. i.e., syntactic, *semantic* and pragmatic analyses. (Carnap 1942, 250). The theory of meaning developed by Carnap, as we will see below, is *a truth-conditional theory of meaning*.

Michael Dummett is another important philosopher who recognized the importance of a theory of meaning in philosophy. He considered that a meaning-theoretical investigation is the only way that we can clarify, roughly speaking, philosophical problems. In fact, he turned this idea into a general philosophical program meant to settle the debate between Realism and Anti-realism (see also Prawitz 2012b). However, the theory of meaning developed by Dummett, in relative opposition to the truth-conditional theory of meaning as I will try to argue below, is *a proof-conditional theory of meaning*.

The main assumption of Dummett's meaning-theoretical approach is that an adequate theory of meaning should account for all the features of the use of expressions that depend on knowing their meaning. Of course, as we indicated in the first part of this article, a main use of sentences which depends on knowing their meaning is their use in valid inferences and arguments. Therefore, an adequate theory of meaning must be able to account for this important use of sentences. In addition, since an important, and probably the most important, use of valid inferences is in expanding the knowledge of an agent or of a community, an adequate theory of meaning must explain this epistemic significance of valid inferences – in Prawitz's words, their legitimacy.

Dag Prawitz considers that a truth-conditional theory of meaning is not able to account for the legitimacy of inferences, and, consequently, it is not an adequate theory of meaning. His general argument may be represented in the following manner:

As. A theory of meaning should account for all features of the use of expressions that depend on knowing their meaning.
P_P1. A deductive inference is legitimate if it can be used to obtain knowledge, i.e., to get a conclusive ground for an assertion.
P_P2. The legitimacy of a deductive inference is part of our use of language and depends on knowing the meaning of the sentences involved.
P_P3. A meaning theory must give an account of the legitimacy of deductive inference. (from As., P_P2.)
P_P4. Legitimacy is not explained by a truth-conditional theory of meaning (T.-C.T.M.)
P_C. A T.-C.T.M. is an inadequate theory of meaning. (Prawitz 2012)

Prawitz's main arguments against a truth-conditional theory of meaning are stated in support for the fourth premise. However, before exposing these specific arguments let us briefly analyse his explanation for the legitimacy of inferences. The main concept in his theory of meaning, used to explain the legitimacy of arguments, appears in the general argument presented above in the first premise, namely, the concept of ground. Shortly, an inference is legitimate if it offers to the agent who has grounds for the premises, and performs that inference, a ground for the conclusion.

Essentially, in this new theory of meaning,

> the sense of a sentence is given in terms of how it is established as true, in other words, in terms of what is required to be justified in asserting the sentence *or to have a ground for the assertion*. (Prawitz, 2012b, 12).

The concept of ground is defined as "something that one gets in possession of by doing certain things". (When he speaks about grounds, Prawitz refers only to conclusive grounds, i.e., grounds that guarantee truth

(Prawitz 2012a, 890)). The real content of the term "ground" is given relative to the type of sentences that we analyse. A ground for an empirical sentence is obtained by doing an empirical action, i.e., an adequate observation. A ground for a non-empirical sentence is obtained by performing a mental action, namely, for a mathematical sentence a ground is obtained by performing a relevant calculation, and for a logically compound sentence a ground is obtained by operating on grounds for asserting its constituents. (Prawitz 2012a, 893). These grounds are "abstract entities that can be constructed in the mind" or

> we may think of a ground for a judgement as just a representation of the state of our mind when we have justified a judgement (Prawitz 2009, 195).

By using the notion of ground, and the primitive notion of grounding operation, Prawitz redefines an inference as a quadruple containing premises, grounds for premises, a grounding operation, and conclusion. To make an inference means exactly to apply the grounding operation to the grounds for the premises. By applying the grounding operation to the grounds for the premises an agent obtains, *ipso facto,* a new ground for the conclusion and thereby is justified in asserting the conclusion. As a general remark, this theory follows Gentzen's idea that the introduction inferences – or canonical inferences, in Prawitz's terms – determine the meaning of the logical constants. The main difference is that the introduction inferences are seen as having attached grounding operations. These grounding operations operate on the grounds for the premises to which they are applied and transform them in a ground, a new ground, for the conclusion.

To take a simple example, if a person understands the meaning of conjunction and has grounds for each of its conjuncts, then, in virtue of the meaning of conjunction, she will also have, *ipso facto*, a ground for the compound conjunctive sentence. The meaning of conjunction is explained as being determined by what counts as a ground for it. Of course, having a ground for each of its conjuncts is a necessary and sufficient condition for having a ground for the conjunctive sentence. More specifically, this new ground for the conjunctive sentence -&G(α,β)- is obtained by applying the conjunctive grounding operation -&G- to the grounds -α and β- for each conjunct. *Mutatis mutandis* for the other sentences formed by Gentzen's introduction rules.

Now we may return to analyse Prawitz's arguments for P_P4, i.e., "legitimacy is not explained by the T.-C.T.M". I think that these arguments are essentially two, one regarding the Tarskian model-theoretic notion of validity, and the other regarding the idea of determining the meaning of a sentence by its truth-conditions.

The first argument runs as follows: the model-theoretic notion of validity is defined as truth-preservation under all assignments of meaning to the non-logical terms from the sentences involved in an inference. However, an inference which has as premises the axioms of a theory and as conclusion an arbitrary theorem of the theory, although it is valid, if there is a sufficiently wide epistemic gap between the premises and the conclusion, then it does not offer to the agent that performs it the knowledge expressed by the conclusion. Therefore, validity is not a sufficient condition for legitimacy, and, consequently, legitimacy is not explained by a T.-C.T.M.

The second argument emphasizes the idea that

> truth-conditions contain too little information to allow us to infer that a person who knows the meaning of a sentence also knows what counts as ground for asserting the sentence (Prawitz 2012a, 12).

More specifically, the argument may also be represented as a *modus tollens*, namely: an adequate theory of meaning must show how an agent who (1) knows the meaning of the sentences involved in an inference, (2) is justified in asserting its premises, and (3) performs the inference is, *ipso facto*, justified in asserting the conclusion. Nevertheless, if the meaning of the sentences is given by truth-conditions then the criteria (1), (2), and (3) are not satisfied because an agent will not know that the proposition expressed by the conclusion is true without making additional inferences. Consequently, legitimacy is not explained by a T.-C.T.M.

The first argument, I think, could be easily resisted. Certainly, we must agree that validity by itself is not a sufficient condition for legitimacy. In fact, this idea was implicit in our analysis of the paradox of inference when we distinguished logical containment from epistemological containment. Without supplementing the idea of logical implication with the condition of inferring the conclusion of a valid argument from its premises, an agent could not get a justification for asserting the conclusion. However, this does not imply that a T.-C.T.M. cannot explain this entire process. What remains to be done is to explain in truth-conditional terms why an agent who understands truth-conditionally the meanings of the sentences of an argument, knows that the premises are true, and infers the conclusion from them will get a justification for asserting the conclusion. Basically, we must explain in terms of truth-conditions why an agent who infers a conclusion from certain premises already known obtains a justification for the conclusion. This will be explained after I will introduce some insightful ideas from Carnap's truth-conditional theory of meaning. In this way I think that we will also be able to resist Prawitz's second argument according to which a truth-conditional theory of meaning is not a good candidate for explaining the legitimacy of inferences.

5. Inference and Meaning *via* Truth-Conditions

The idea that the meaning of a sentence is determined by its truth-conditions has a long standing philosophical tradition. Gottlob Frege was the first philosopher who systematically defined the meaning of a sentence in this way. Of course, this definition of meaning presupposes a primary understanding of the concept of truth. Nevertheless, after Alfred Tarski had succeeded in offering a materially adequate and formally correct definition of truth for a certain formal language, the truth-conditional definition of meaning received a powerful foundation.

Tarski was sceptical regarding the extension of his theory of truth to natural languages, but Donald Davidson emphasized that the T-clauses from the theory of truth for a certain language could also serve as definitions for the meaning of the sentences from that language. In particular,

> the definition works by giving necessary and sufficient conditions for the truth of every sentence, and to give truth conditions is a way of giving the meaning of a sentence. To know the semantic concept of truth for a language is to know what it is for a sentence – any sentence – to be true, and this amounts, in one good sense we can give to the phrase, to understand the language (Davidson 1967, 310).

However, long before Davidson, Rudolf Carnap, strongly influenced by Tarski's work in semantics, had recognized the importance of Tarski's definition of the semantic concept of truth for the theory of meaning. In his 1942 book, *Introduction to Semantics*, Carnap makes the following assertions:

> By a semantical system (or interpreted system) we understand a system of rules, formulated in a metalanguage and referring to an object language, of such a kind that the rules determine a truth-condition for every sentence of the object language, i.e. a sufficient and necessary condition for its truth. In this way the sentences are *interpreted* by the rules, i.e., made understandable, because to understand a sentence, to know what is asserted by it, is the same as to know under what conditions it would be true. To formulate it in still another way: the rules determine the *meaning* or *sense* of the sentences. Truth and falsity are called truth-values of sentences. To know the truth-conditions of a sentence is (in most cases) much less than to know its truth-value, but it is a necessary starting point for finding out its truth-value (Carnap 1942, 22).

In this passage we find again the basic idea of a truth-conditional theory of meaning, i.e., meaning is given by truth-conditions, and we also discover, as we should, that the knowledge of the truth-conditions for a sentence, i.e., of its meaning, is (in most cases) only a necessary condition for determining its truth-value. Moreover, Carnap details this general description by way of a very simple and useful example:

> Suppose that Pierre says: 'Mon crayon est noir' (S). Then, if we know French, we understand the sentence S, although we may not know its truth value. Our understanding of S consists in our knowledge of its truth-condition; we know that S is true if and only if a certain object, Pierre's pencil, has a certain color, black. This knowledge of the truth-condition for S tells us what we must do in order to determine the truth-value of S, i.e. to find out whether S is true or false, what we must do in this case is to observe the color of Pierre's pencil (Idem, 22-23).

Although we may not be justified in generalizing from this particular example, what is interesting is that Carnap believes that the knowledge of the meaning of a sentence tells us what *we must do* in order to determine the truth-value of that sentence. It is interesting because this idea allows us to make an analogy to Prawitz's definition of ground, i.e., "something that one gets in possession *of by doing* certain things". This would mean that by knowing the truth-conditions for a sentence, i.e., its meaning, a person will also know what she must do in order to obtain a ground or justification for that sentence, or for the assertion of the proposition expressed by that sentence. Of course, being an empirical sentence, "Mon crayon est noir", what a person must do is to perform an adequate observation – as Prawitz also emphasizes when he speaks about grounds for empirical sentences. Nevertheless, I do not think that by knowing the truth-conditions for a sentence we will also know what counts as a ground for it. We may understand very well a sentence without knowing exactly what would constitute a ground for asserting it. For instance, we understand Goldbach's conjecture but we do not know what *specifically* we must do in order to obtain a ground for it. As a consequence, we may have doubts regarding Prawitz's idea that to understand a sentence means to know what counts as a ground for asserting it.

Going further, what do the truth-conditions for a logically compound sentence tell us? To answer this question, it is helpful to briefly analyse Carnap's semantic view for the logical operators. In this respect, we can take into account a short description from Carnap's 1958 book, *Introduction to Symbolic Logic and Its Applications,* namely:

> What the truth table of a connective gives is primarily a necessary and sufficient condition for the truth of a compound so connected, in terms of the truth-values of its members. Suppose that a person knows the sense of the sentences 'A' and 'B', where perhaps 'A' says that it is (now, in Paris) snowing and 'B' says that it is raining; and suppose that no translation of 'v' has been given him, but only the truth-table. Can the person comprehend the meaning of the sentence 'A v B' so that (a) he knows when it is permissible to assert this compound on the basis of his factual information; and (b) he can extract from a communication

> having the form of this compound the factual information being communicated? The answer is: he can. Perceiving from the NTT that the compound holds in the first three cases but not in the last, our subject knows precisely the conditions under which the compound sentence may be asserted and he knows precisely what information it conveys as a communication. […] He knows that the compound may not be asserted if indications are it is neither snowing nor raining. […] The remarks support a general statement: a knowledge of the truth-conditions of a sentence is identical with an understanding of its meaning (Carnap 1958, 14-15).

The problem to which I think that Prawitz hints when he says that the "truth-conditions contain too little information to allow us to infer that a person who knows the meaning of a sentence also knows what counts as ground for asserting the sentence" is the problem of justification. A person who understands the premises of an inference, knows that they are true, and correctly infers a certain conclusion from them, will know that the conclusion is also true – logical consequence being necessarily truth-preserving. But, is the person justified in asserting the truth of the proposition expressed by the conclusion?

Prawitz's proposal – which I find very interesting – was to reconsider and to enlarge the definition of an inference by adding to its structure two new elements, namely, grounds and a grounding operation *corresponding* to Gentzen's introduction inferences (Prawitz 2009, 195). Nevertheless, do we really need these new elements in order to explain the preservation of justification when passing correctly from the premises to the conclusion of an inference? I think that they are not necessary, and that a truth-conditional theory of meaning handles the situation. Since inferences are valid in virtue of the meanings of the logical constants from their structure, I think that *there must be a connection between the meaning of the logical constants and justification,* and this is why the ideas expressed by Carnap are very useful regarding this issue.

Essentially, one main idea here is that the meaning of the logical operators is what makes an inference valid[8]. An inference is necessarily

[8] The converse relation, that the rules of inference -or certain rules of inference- determine the meanings of the logical operators, i.e., the inferentialist thesis, is not engaged in this approach.

truth-preserving in virtue of the invariant meanings of the logical operators in all the valuations of the non-logical expressions of that inference, i.e., the meaning of the logical operators indicates us a necessary and sufficient condition for the truth-value of the compound sentences formed with their help, and this is an essential feature for explaining the preservation of truth. For instance, let us consider the inferences from Gentzen's natural deduction system for propositional logic, which is sound and complete. Gentzen considers the introduction inferences as determining the meaning of the logical operators. However, if we do not conceive the introduction inferences as evidently valid, and ask why the introduction inferences are valid, the intuitive answer that we may give is that these rules are valid in virtue of the meanings of the logical operators that they introduce. In general, a formal system of logic – or a logical calculus – is meant to formalize, or to represent, a system of logic, i.e. a semantic system. This is precisely why a full formalization of logic consists in constructing a formal system which formalizes, in addition to logical truth and logical consequence, all the logical properties of the logical operators (Carnap 1943, 96).

Since the meaning of a propositional logical operator indicates -as Carnap emphasizes- a necessary and sufficient condition for the truth-value of the compound sentences formed with its help, this means that a person who knows the truth-values of the simple sentences and understands the meaning of the logical operators, will also know the truth-value of a compound sentence inferred by using the rules of inference that introduce its operators. Of course, in order to know the truth-values of the simple sentences from which a compound sentence is inferred, a person must possess a justification that guarantees the truth-value of those simple sentences. But, again, is a person who knows the premises of a valid argument and infers a conclusion from them, justified in asserting that conclusion?

I think that she is justified. One main feature of the truth-conditional theory of meaning that is also implicitly present in Carnap's description is compositionality, i.e., the meaning of a compound sentence is given by the meanings of its constituents (together with the structure of the sentence). More specifically, if an agent knows the truth-conditions for the simple sentences that form a compound sentence and understands the meaning of the logical operators – as they are given by the normal truth-tables (NTT) – then he will also know the truth-conditions for the compound sentence formed by those operators. As a consequence, and this is the main idea, by knowing the truth conditions for the logically compound sentence, the agent will *know in which conditions it is permissible to assert* the compound sentence, i.e., he will know when he *is justified* in asserting the proposition expressed by the compound sentence. Hence, the knowledge of the truth of the premises, i.e., of the fulfilment of their truth conditions, together with the knowledge of the meaning of the logical constants are necessary and sufficient conditions for an agent to be *justified* in asserting the proposition in question. This is what I think that Carnap refers to when he says that the agent "knows when it is permissible to assert" the compound sentence on the basis of the factual information from which he/she infers it.

The idea that there is a strong connection between truth-conditions and justification was implicitly present also in Frege's approach to inferences. Frege believed that the transitions between the premises and conclusion in derivations must have three properties, namely: (a) the transitions must be truth-preserving, (b) *the premises must justify the conclusion*, and (c) each transition must be immediately evident (Peacocke 1992, 799-800). Frege did not develop a detailed philosophical account to the relation of justification but Peacocke considers that in the Fregean conception the following account seems natural:

> one formula of *Grundgesetze* justifies a second if the truth-conditions for first, as determined by Frege's stipulations, guarantees fulfilment of the truth conditions for the second, as determined by stipulations (Peacocke 1992, 799-800).

To sum up, and to answer the proposed question, by knowing the truth-conditions for the logically compound sentences, an agent will know in which conditions it is justified to assert the proposition expressed by that compound sentence. In addition, if the agent has justifications which guarantee the truth of the propositions expressed by the sentences from which the compound sentence is inferred[9], then he will also have a justification for asserting the proposition expressed by the compound

[9] It is important to emphasize that an inference is not necessarily a proof-theoretical instrument. To perform an inference means to transform some propositions into another proposition according to certain rules of inference.

sentence. More specifically, if a person (1) knows truth-conditionally the meaning of the sentences involved in an inference, (2) is justified in asserting its premises, and (3) performs the inference then he/she is, *ipso facto*, justified in asserting the conclusion. The validity of this argument will be explicitly explained in the last section.

6. Inferences Involving Quantifiers

The idea that an act of inference may have an epistemic function, which can be explained in truth-conditional terms, could also be instantiated on the inferences involving quantifiers. What we must show is that the meaning of a sentence involving quantifiers is compositional, and that by inferring quantified sentences from sentences for which we have justifications we get in possession of a justification for the inferred sentence. The central notion that allows us to see why these two conditions are fulfilled is the notion of *satisfaction*. This is the central notion of the Tarskian semantics for the predicate language. Since we cannot directly assign a truth-value to the atomic sentences containing free variables from the predicate logic, a quantified sentence is not a truth function of the truth-values of its components. Nevertheless, with the help of the notion of satisfaction we can define the truth-value for this sentences. Without entering in details, roughly, an open sentence with n-free variables is true if and only if it is satisfied by a sequence of objects. This propositional function is satisfied by a sequence of objects if and only if the first n-objects from the sequence instantiate the property expressed by the predicate[10].

Having in mind these features, we may now analyse the two quantified sentences inferred from atomic sentences with the help of two basic introduction inferences from Gentzen's natural deduction system for predicate calculus. If we know the satisfaction conditions for the premises and we understand the meaning of the logical quantifiers, then we also know the satisfaction conditions for the quantified sentences. Furthermore, in addition to the understanding of the meaning of the atomic sentences and quantifiers, if we know that the propositional function is in fact satisfied,

[10] For some basic insight of the Tarskian semantics the reading of (Taylor 1998, 113-145) could be useful.

then we will also know that the quantified sentence inferred from it is satisfied. In essence, if we understand the meaning of the quantifiers and we are justified in asserting the propositional functions to which they apply, then we will also be justified in asserting the quantified sentences formed by applying the quantifiers to the propositional functions. The knowledge of the meanings of the quantifiers tells us in which conditions we are justified to assert a quantified sentence. We can simply describe the situation as follows:

a) $\dfrac{A(t)}{(\forall x)A(x)}$ b) $\dfrac{A(t)}{(\exists x)A(x)}$

a) If we know the satisfaction-conditions (S-C) for the propositional function $A(x_1)$, and we know that the propositional function is satisfied, then, in virtue of the meaning of '$\forall$', we will also know that the proposition $(\forall x)A(x_1)$ is satisfied. The propositional function $A(x_1)$ is satisfied (SAT) by a sequence of objects (Σ) $<a_1, a_2, ..., a_n>$ iff a_1 satisfies x_1.

SAT [Σ, $A(x_1)$] iff a_1 has the property A.

SAT [Σ, $(\forall x)A(x)$] iff (for any sequence $\Sigma^* \sim_i \Sigma$)[11] (SAT [Σ^*, $A(x_1)$])

b) If we know the satisfaction-conditions (S-C) for the propositional function $A(x_1)$ and we know that the propositional function is satisfied, then, in virtue of the meaning of '$\exists$', we will also know the S-C for $(\exists x)A(x_1)$. The propositional function $A(x_1)$ is satisfied by a sequence of objects $< a_1, a_2, ..., a_n>$ iff a_1 satisfies x_1.

SAT [Σ, $A(x_1)$] iff a_1 has the property A.

SAT [Σ, $(\exists x)A(x_1)$] iff (there is a sequence $\Sigma^* \sim_i \Sigma$)(SAT [Σ^*, $A(x_1)$])

The idea that these inferences are legitimate, i.e., they have an epistemic function, is explained by D. Prawitz in the same manner as for the propositional inferences, with the help of the notions of ground and grounding operations, and, in addition, as for the inferences involving assumptions, by operating the distinction between saturated and unsaturated grounds. You may find below a short exemplification but for more details see (Prawitz 2009, 193-194).

a) $\dfrac{A(t)}{(\forall x)A(x)}$ b) $\dfrac{A(t)}{(\exists x)A(x)}$

[11] For any sequence of objects Σ^* differing from Σ in at most the i-th place.

a) If we have a ground for the propositional function $A(x_{1,} x_2, ..., x_n)$, then, in virtue of the meaning of '∀', we will also have a ground for $(\forall x_{1,} x_2, ..., x_n) A(x_{1,} x_{2,} ..., x_n)$. A ground for propositional function $A(x_{1,} x_{2,} ..., x_n)$ is an unsaturated ground $\alpha(x_{1,} x_{2,} ..., x_n)$ such that for individuals $a_{1,} a_{2,} ..., a_n$ in the domain in question $\alpha(a_{1,} a_{2,} ..., a_n)$ is a ground for the assertion $A(t_{1,} t_{2,} ..., t_n)$, where t_i denotes a_i. The ground for $(\forall x)A(x)$ is formed by applying the grounding operation ∀G to the ground $\alpha(x)$.

b) If we have a ground for the propositional function $A(x_1)$, then, in virtue of the meaning of '∃', we will also have a ground for $(\exists x_1)A(x_1)$. A ground for propositional function $A(x_1)$ is an unsaturated ground $\alpha(x_1)$ such that for individuals $a_{1,} a_{2,} ..., a_n$ in the domain in question $\alpha(a_{i,})$ is a ground for the assertion $A(t_{1,} t_{2,} ..., t_n)$, where t_i denotes a_i. The ground for $(\exists x)A(x)$ is formed by applying the grounding operation ∃G to the ground $\alpha(x)$.

7. Final Remarks

I have argued that valid inferences have an epistemic function, and that this function can be explained in model-theoretic terms, and more precisely, in truth-conditional terms. As we have seen before, by performing a valid inference from premises for which an agent already has justifications which guarantee their truth, the agent may obtain a justification for the truth of the proposition expressed by the conclusion of the performed inference. The argument which endorses the epistemic significance of valid inferences can be expressed as follows:

ES_P1: There is a valid inference from P1, P2,…, Pn to C.
ES_P2: The agent A has justifications for the premises P1, P2,…, Pn.
ES_P3: The agent performs the inference, and knows that it is valid.
ES_C: The agent obtains a justification for asserting the proposition expressed by C.

The explication of the validity of this argument may run as follows: since C is logically implied by P1, P2,…, Pn, i.e., is a logical consequence of them, the truth of the premises – by definition – is a necessary and sufficient condition for the truth of the conclusion, i.e., it guarantees the truth of C. In addition, by inferring C from P1, P2,…, Pn by means of valid inferences, known by the agent to be valid, the agent knows that C is a logical consequence of P1, P2,…, Pn. Consequently, he will know that if P1, P2,…, Pn are true then C is also true, i.e., the truth of the premises guarantees the truth of the conclusion. Of course, the agent may know that the truth of P1, P2,…, Pn guarantees the truth of C without having actually any justification for P1, P2,…, Pn. Nevertheless, if P1, P2,…, Pn are true then so is C. As a consequence, if the agent obtains a justification which guarantees the truth of the premises, then the knowledge of the truth of the premises and of the meanings of the logical constants from the inference performed, will justify the agent in asserting the truth of the conclusion.

The second conjunct of the premise 3, i.e., the agent knows that the inference is valid, is declined by Dag Prawitz as a necessary condition for explaining the legitimacy of valid inferences because "we do not normally establish the validity of an inference before we use it" (Prawitz 2009, 184). If this condition would be necessary, Prawitz argues, then a regress would result, namely, in order to establish the validity of the inference we would need an argument which establishes its validity, and then another argument for the validity of the previous argument and so on. However, I think that we can stop this regress because, as we have seen in section four, we know the validity of inferences in virtue of the meaning of the logical operators from their structure.

According to Prawitz, an agent gets a ground for the conclusion of an inference by performing it – in the sense defined in section 3 –, and "if the inference she has made is valid, then she is in fact in possession of a ground for her judgement, and this is exactly what is needed […] to know that the affirmed proposition is true" (Prawitz 2009, 199). In addition – Prawitz continues – although this is not necessary, "reflecting on the inference she has made the agent can prove that the inference is valid" (Idem). I think that if the agent did not know that the inference is valid, then we could not say that he really has a ground for the conclusion. For instance, someone may possess a ground for a certain proposition without being aware of the fact that it is indeed a ground for that proposition. (Think of a person who has made certain observations in the chemistry laboratory, but does not know for what propositions these observations may serve as grounds). *Mutatis mutandis*, a mathematician who infers a proposition by means of some inferences whose validity is unknown to him, cannot be considered justified in asserting the truth of that proposition, even though the proposition is in fact true. This is why I think that the knowledge of the validity of the inference is a necessary condition for obtaining a justification for its conclusion. Therefore, if the stated conditions are satisfied, then valid inferences may provide us with valuable knowledge.[12]

References

Carnap, Rudolf. 1942. *Introduction to Semantics*. Cambridge: Harvard University Press.

Carnap, Rudolf. 1943. *Formalization of Logic*. Cambridge: Harvard University Press.

Carnap, Rudolf. 1958. *Introduction to Symbolic Logic and Its Applications*. New York: Dover Publications.

Cohen, Morris. 1944/1945. *A Preface to Logic.* Cleveland and New York: Meridian Books, The World Publishing Company.

Cohen, Morris and Ernest Nagel. 1936/1998. *An Introduction to Logic and Scientific Method*. New Delhi: Allied Publishers Limited.

Davidson, Donald. 1967. "Truth and Meaning". *Synthese* 17(1): 304-323.

Forbes, Graeme. 1994. *Modern Logic. A Text in Elementary Symbolic Logic*. New York: Oxford University Press.

Fine, Kit. 2007. *Semantic Relationism*. Malden: Blackwell Publishing.

Helman, Glen. 1992. "Proofs and Epistemic Structure". In *Proof, Logic, and Formalization*, ed. Michael Detlefsen. London: Routledge.

Martin-Lof, Per. 1985/1996. "On the Meanings of the Logical Constants and the Justifications of the Logical Laws". *Nordic Journal of Philosophical Logic* 1, 1: 11-60.

[12] This paper was presented at the Special Workshop *Meaning and Truth*, on January 17, 2015, Bucharest, at the CELFIS Seminar on January 21, 2015, Faculty of Philosophy, University of Bucharest, and on August 7, 2015, Helsinki, at the 15th Congress on Logic, Methodology, and Philosophy of Science (CLMPS) and Logic Colloquium 2015. I want to thank Alexandru Dragomir, Mircea Dumitru, Jose Martinez-Fernandez, Henry Galinon, Virgil Iordache, Ilie Pârvu, Cristi Stoica, Vladimir Svoboda and John Woods for their comments and remarks. I also want to thank Oana Culache, Mihai Rusu and Iulian D. Toader for useful observations and comments regarding the ideas and the English language of the paper.

Peacocke, Christopher. 1992. "Sense and Justification". *Mind*, New Series 101, 404: 793-816.

Prawitz, Dag. 2006. "Meaning Approached via Proofs". *Synthese* 148: 507-524.

Prawitz, Dag. 2009. "Inference and knowledge". In *The logica yearbook 2008*, ed. M. Pelis, 183-200. London: College Publications, King's College London.

Prawitz, Dag. 2012a. "The Epistemic Significance of Valid Inference". *Synthese* 187: 887-898.

Prawitz, Dag. 2012b. "Truth as an Epistemic Notion". *Topoi* 31: 9-16.

Rescher, Nicholas. 2005. *Epistemic Logic. A Survey of the Logic of Knowledge*. University of Pittsburgh.

Taylor, Kenneth. 1998. *Truth and Meaning. An Introduction to the Philosophy of Language.* Malden: Blackwell Publishing.