Abstract

Deductionism assimilates nature to conceptual artifacts (models, equations), and tacitly holds that real physical systems are such artifacts. Some physical concepts represent properties of deductive systems rather than of nature. Properties of mathematical or deductive systems can thereby sometimes falsely be ascribed to natural systems.

1. Introduction

Science can be described as a system of concepts that support technical recipes, which tell us how to manipulate physical processes and materials to advantage. It is convenient in many ways if such a system of concepts can be formally expressed, either mathematically or otherwise as a formal deductive system. Deductionism is here defined as the faith that all of physical reality can be mapped by formal constructs, especially by deductive systems or mathematical models. It is thus the premise that nature is ultimately rational, insofar as it is reducible to formal terms. Such a belief, often tacit, is an example of what Gerald Holton calls thematic content within the practice of science. It began with the ancient Greeks, who held that one should be able to deduce natural details strictly from first principles, in the way that theorems of geometry are proven from axioms. In its extreme, deductionism holds that mathematical models work because there is virtually no distinction between the model and the natural reality it models. Indeed, in practice, models are often the actual objects of scientific study. They are usually defined by equations, and the natural systems chosen for study are re-defined in such a way that they can be described by (preferably simple) equations. Deductionism thus blurs the very distinction between mathematics and the physical world [Tegmark 2007].

However, one key difference is that physics depends on physical evidence, while mathematics depends only on reason and definitions. When a physical system has been re-defined as a mathematical model, however, the correspondence between mathematics and physics is then actually a correspondence of one deductive system to another. The correspondence with reality remains a separate question or else an article of faith. But that is often a self-confirming faith, since phenomena that can be treated with existing mathematics are the ones generally selected for study. Given that a deductive system is conceptually equivalent to a machine, the deductive approach is compatible with the philosophy of mechanism. It serves technological advance, since mathematical models can often be engineered into literal machines or devices. These include the apparatus involved in experiments, which materially embody the theory behind them. Yet, seeing the world through deductive eyes can limit what is perceived as significant for investigation. Experiment is “theory-laden” and also affirms the categories of theory in a self-reinforcing cycle.
2. The unreasonable effectiveness of reason

Leibniz took note of the apparent correspondence between mathematics and physical reality, which he took to be an act of God, a divinely “pre-established harmony.” Physicist Eugene Wigner [1960] would later famously call this harmony “the unreasonable effectiveness of mathematics.” Let us grant that logic and mathematics in fact reflect and generalize aspects of natural reality. One reason for the correspondence, then, is that these abstractions from nature are then projected back upon nature as underlying its features, and are also reflected in the ways experiments are constructed. The expectation that nature will behave mathematically is then something of a tautology. It is no coincidence that fundamental laws of nature are mathematical; for, mathematical laws maximally compress empirically observed patterns [Chaitin 2008]. It is no surprise that equations, which idealize natural properties and relationships, happen to describe idealized experimental setups. For, to be described mathematically at all, a natural system must first be redefined in idealized terms. Thus, another reason for the correspondence is that there is a selection effect involved, so that reality appears intrinsically mathematical simply because we focus on those aspects of it that can be most easily treated mathematically, or which correspond to the constrained behavior produced in experiments. Idealizations are chosen that do correspond closely enough to reality.

While laws of nature succinctly capture results of observation and experiment, following Popper such inductive generalizations cannot be proven, only disproven. Theorems of mathematics, on the other hand, can be proven because they are propositions within formal systems, not empirical assertions; the very concept of proof is integral to logic and formalism. Whatever power of necessity natural laws appear to possess derives from translating inductive generalizations into theorems of a formal system. If we marvel at the effectiveness of simple mathematical expressions to portray the real world, we should bear in mind that far more complicated expressions for the same relationships can be found. We are pleased by simple equations because they are succinct and solvable—a preference dating at least to the Pythagoreans, for whom “rational” meant tidy proportions, as in rational numbers. Often in the name of esthetics, we seek simplified aspects of nature because these are what we can deal with mathematically and technologically. A further benefit is that conceptual devices can lead to the construction of real devices, such as computers, automobiles, and radio telescopes.

While the pre-established harmony between mathematics and physics is amazing, perhaps the true marvel is the general ability of thought to model the external world at all: the pre-established harmony between world and brain. Einstein (following Hume) considered the capacity of reason to grasp reality to be little short of miraculous. Since his generation, understanding of the relationship between reason and reality has been naturalized by evolutionary epistemology: logic is a form of cognition, and the relationship between cognition and world is established through natural selection. Since that relation is historical and not a priori, logic itself must be an evolutionary product. An evolutionary general theory of intelligence would help to account for the astonishing effectiveness of mathematics to describe the world in simple terms, which is but one development of a broader capacity to model, abstract, and generalize [Baum 2007].
Moreover, it is psychologically reassuring to believe that physical reality is rational and can generally be redefined in human terms. Successful mathematical treatment of nature, reflected in technology, richly confirms that faith.

Abstract creations lead to tangible creations. Scientists understandably focus on conceptual artifacts because these are just the things they can most readily understand and make use of. One sees this materially reflected in the popularity at one time of mechanical models of the solar system and the atom, and in the current use of molecular models in chemistry. But scientific constructs also include a diversity of subtler abstractions. Philosopher Nancy Cartwright generically calls such conceptual artifacts nomological machines [Cartwright 1983; 1989]. Deductionism is the belief that such artifacts can exhaustively map all of physical reality; the universe may even be a literal machine such as a quantum computer [Lloyd 2013].

A controlled experiment is a nomological machine [Mets 2012], which often consists of literal machinery. Select factors are isolated for study, with other influences deliberately excluded—which is hardly the situation in nature, where the real system generally cannot be controlled to isolate specific factors of interest, and the number of factors involved is indefinite. In an experiment, ideally a single factor is isolated for study. Idealization is justified on the ground that the gain of treating the situation mathematically outweighs the loss from simplification. The isolation of causes is justified on the ground that in nature some causal factors are overwhelmingly more significant than others. It is argued that science would simply not be possible if this were not so [Wigner 1960]. Yet, “significance” is relative, and is evaluated within the terms of the theory that is to be tested.

“Twiddling a knob” is a theorist’s figure of speech [Davies 1995] that treats initial conditions, natural constants, and even the laws of nature themselves as variables of a nomological machine that can be adjusted by the theorist at whim. The world is then taken to be a mechanism whose (mathematical) settings can be controlled to make a different universe.

Closed reversible systems are a staple of classical physics because they can be well treated mathematically. The fundamental laws of physics are generally “time-reversal invariant,” even though many physical processes at the macroscopic scale seem irreversible, as expressed in the second law of thermodynamics. However, this reversibility of laws is not a property of nature but a mathematical property of equations (−t can be substituted for t). One calculates the behavior, backward or forward in time, of the model, which is reversible by definition. Though time plays a key role within such a framework, deductive systems themselves are timeless by definition. In the real universe, there is a direction of time. Irreversibility predominates because no part of the real world is actually a deductive system, even though it may be convenient to treat it as ruled by (reversible) equations. Irreversibility is fundamental even at the quantum level, as demonstrated by the “collapse of the wave function.”

While physical symmetry is ever only empirically approximate, mathematical symmetry has the precision of definition. The success of mathematical symmetry arguments at predicting discoveries in high-energy physics suggests that symmetry (of

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1 Cf. Vico’s “maker’s knowledge”.
laws, at least) is an objective property of nature. Does this mean that nature itself must be profoundly symmetrical at deep levels simply because the mathematical theory of the day uses symmetry principles? “Symmetry breaking” describes the departure of reality from an ideal. When symmetry is considered a natural ground state, an event is sought to account for any departure from it. (This concept may be compared to Aristotle’s notion of a “natural” tendency or state, which can be perturbed by an external efficient cause.) In modern physics, symmetry breaking is reified as a causal agent; whereas symmetry and asymmetry are but ideas imposed by the theorist upon natural situations, which are generally neither static nor perfectly symmetric.

Most real numbers are non-computable, which is sometimes an inconvenience for a science based on the continuum (differential equations). However, while physics could be reformulated in terms of a discrete mathematics, this would reduce it to a deductive system, with all that does not fit into its Procrustean bed left undefined. One might expect to find integers or rational numbers at the base of a digital world, but instead we find numbers such as e and π [Tipler 2005]. That in itself is evidence that the world is not a deductive system, computer program, or simulation.

3. Models and deductive systems

The great advance represented by formal thinking is to define things precisely and unambiguously [Cassirer 1980]. Ordinary words are ambiguous, if only because they are tied to references in the complexly interrelated social world. Ideally, scientific terms are precisely defined and free from intuitive secondary meanings or associations. In principle, formal scientific concepts mean only what they are explicitly held to mean, so that one always knows what one is talking about. (A disadvantage, however, is that it is difficult then to talk of anything else.) Deductionism is the belief that physical processes correspond to such artificially constructed meanings. In the extreme, it is the premise that nature itself is reducible to definitions and mathematical models, which are often the actual objects of scientific study.

Models are deductive systems, usually defined by equations, representing select aspects of the real world. Equations are distilled expressions of empirical data, which are records of specific interactions between observer and world [Smolin 2013]. A model and its equation(s) express the same idealization of an inductive generalization. However, neither model nor equation should be assumed to be isomorphic to the real process or system modeled [Giere 1999]. Even if a deductive system is complete in the mathematical sense, it cannot represent reality exhaustively. For, model, equations, and deductive systems are finite products of human definition, whereas natural reality is not. Scientific modeling resembles other cognitive processes that are highly selective, serve biological needs, and are limited by the intentions behind them and the material processes that support them. Mathematical prediction, after all, presupposes an agent with reasons to predict the future and values that inform those reasons [Turchin 1990].

The correspondence of mathematics with physical reality parallels the correspondence of our ordinary cognition with the external world. In both cases, the

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2 One should bear in mind that “new” particles (whether or not predicted through symmetry arguments) are not naturally occurring entities but artificial products of high-energy experiments.
correspondence is a matter of utility, not identity or truth. In the case of perception, cognitive modeling serves the creature and is endorsed by its evolutionary success. However, mathematical modeling, though underpinning our present civilization, has not been around long enough to prove its long-term evolutionary value to the species. While our perceptual models are so ingrained that we take them for the world itself, taking mathematics for the world itself does not enjoy the same warrant.

The world is made intelligible through a synergy of its real properties with successful strategies employed to model and understand it. For example, the extreme velocity of light rendered it feasible to ignore the very small relativistic effects occurring at speeds that could be studied during the first two centuries of classical physics. Similarly, the very small size of atoms, electrons, and energies of visible light made it possible to ignore quantum effects occurring among classical objects. Such circumstances rendered the development of physics possible in the first place, but also required its eventual revision. On the other hand, if it had been the case that light travels with infinite speed, as was early supposed, life would not have been shielded from the simultaneous arrival of all the radiation and other distant influences that might occur at a given moment. Similarly, but for being quantized, matter would not be stable, and neither chemistry nor chemists would not be possible.

If life itself has depended on relative isolation and separation of effects on different scales, so has science [Hartle 1997]. Without such localism, the notion of the isolated system would not have been feasible and physical laws could not have taken a simple and practically computable form. This was also facilitated by the apparent continuity of the macroscopic world, for which differential equations were effective [Barrow 1991, p50]. The huge disparity in energy between macroscopic objects and photons allows one to neglect the physical effects of observation in everyday circumstances. Consequently, one can postulate the existence of real objects independent of observers and of observers independent of objects. Without this effect of scale—which might be but an incidental fact of nature—there could be no clear distinction even between subject and object. To that degree, philosophy is at the mercy of physics; for, such categories cannot be taken as a priori if they depend on contingent physical facts.

The fundamentals of deductive systems are true by definition or fiat. A deductive system is self-contained and coheres purely by logical necessity. There is no reference outside itself until it is “interpreted” as a mapping of some portion of the real world—just as plane geometry, for example, can be applied to the physical properties of the earth’s surface. One can know with certainty what a deductive system contains because it has been defined to contain just those things. As with a computer program, one knows in advance that there is a specified output to any input. In order to infer what a natural system contains, by contrast, one must either take it apart or else observe its outputs in relation to inputs. Imagining from the outset that the system is a deductive system lends support to the notion of determinism and the assumption that systems are well defined.

One proposes a model to assimilate observed correlations in nature to a rational scheme. But logical relations between elements in the model do not necessarily reflect causal relations in nature. All we can be certain of is that correlations exist among data; there can always be alternative models to account for them, and there can always be undiscovered correlations. While relationships within deductive systems are logically necessary, only by analogy are relationships within nature “necessary.” Since, causality
involves an inference from actual observed patterns and measurements, the notion of *causal necessity* simply transfers, for psychological reasons, the model’s internal *logical necessity* to the physical system concerned. Causality is an empirical issue, while determinism is a property of models.

4. Determinism

The only truly deterministic systems are deductive systems, which are logically self-contained and coherent by definition. Nature and natural systems, in contrast, are open and ambiguous, always eluding definition. It is not nature that is deterministic, but human thought systems projected upon it.\(^3\)

The precision associated with determinism can be of two very different types: either the precision of definition or that of large statistical runs. All empirical knowledge is statistical in essence; it pertains to data before making claims about the world. Even in the classical realm, one simply disregards the spread of error in order to imagine a precise and necessary link between cause and effect. It might seem that the classical realm is distinguished from the quantum realm by the nature of its entities, but the essential difference is epistemological rather than ontological, a function of the type of statistics.

While nature cannot be reduced to a deductive system, it does often seem to mimic one. Many classical systems, such as the solar system, conform to their mathematical descriptions to great accuracy. Quantum entities appear to manifest the precise and simple integrity characterizing products of definition. One can accurately predict the future of classical systems (such as planetary motions) because such real systems coincide, for all practical purposes, with deductive systems. This is because, in such massive systems deviations on a micro scale cancel out to yield a statistically precise averaged macroscopic pattern that can be codified as a law. This sort of predictability is an emergent effect of large numbers. If the motion of a planet-sized object could be made to depend, through amplification, on individual micro events (as in the case of Schrödinger’s cat), predicting its path would be impossible [Ellis 2006].\(^4\) On the other hand, a very large *ensemble* of such hypothetical unpredictable planets should average out to approach classical expectations!

Determinism gives the comforting impression that the laws of physics are magical incantations that can be invoked to predict the future. This power seems guaranteed within a deterministic system, where future is logically linked to past by known algorithms. This may even give the illusion that a deterministic system *aims* toward a pre-given end.\(^5\) However, there is no guarantee that the world actually is such a system,

\(^3\) Conversely, however, if it could be proven that nature *is* deterministic, this would imply that the cosmos itself is a deductive system—perhaps a simulation or an alien engineering project, or a divine creation as originally supposed. In other words, nature is deterministic only if it is not natural!

\(^4\) Such amplification would require great entropic resources, as in George Ellis’ example of a massive object powered by rocket engines that are triggered by quantum events.

\(^5\) At least in systems presumed to be deterministic, descriptions by means of action principles are no more than equivalent alternatives to dynamical descriptions. This was noted by many commentators since Maupertuis, who were anxious to preserve the independence of science from
or—if it is—that the appropriate formulae can be known. Knowledge that enables one to predict the future is based on extrapolating from patterns of the past. While these patterns may be expressed mathematically, no equation has causal power to set the future.

Determinism and chance are complementary concepts that are alike nebulous. What does ‘chance’ (or accident) mean other than that which does not occur by intention? And what does ‘determine’ mean if not (by an act of free will) either to ascertain what is so or else to intentionally make it so (e.g., in an experimental set-up)? Does causality then even have a meaning apart from conscious agents? Confusion surrounding these concepts leads to the impression that laws of nature “govern,” or that mathematical formulae somehow carry a power to fix the behavior of matter in advance of actual events.

Finally, what does it mean for nature to be undetermined, random? Indeed, what does random mean, other than that no explanatory precedent or ordered pattern can be found? Finding such causes or patterns (or failing to find them) involves an act of searching by a conscious agent (the “observer”). The observer is not merely a passive witness to what occurs, but an active participant. There is always an entangling interaction of object with subject. Randomness is therefore not a property of a system, process, collection of events that does not include the observer. On the contrary, it is a relation to the observer. Yet, it is a natural mental habit to view the world as self-existent and independent of the observer. This psychological fact leads to paradoxical notions such as involved in the measurement problem. The very concepts of determinism and indeterminism are reified as observer-independent properties of a system. In truth, however, there is always someone who determines or cannot determine. Indeterminism means only that a given agent (and perhaps no agent at all) can find out what the state of the individual system actually is. The only other coherent sense of ‘determine’ is to fix, as in setting experimental conditions. One may interpret the quantum wave function to imply that a system exists in a “mixed state.” But that interpretation simply reifies the uncertainty of the observer as though it were a property of the system itself; whereas knowledge, certainty, and uncertainty are properties of the relationship of the observer to the system.

5. The deductionist program

Galileo famously called mathematics the language in which nature is “written.” But natural things are not literal numbers and nature is not literally a text. Mathematics is rather the language of science—or its grammar, at least. Scientific explanation is communication, whether in a natural or in a formal (mathematical) language. Scientific theories consist of propositions, communicated by one or more scientists primarily to other scientists. Yet, mathematics can no more capture all aspects of natural reality than ordinary language can capture all of human experience. Of course, this has not daunted the inclination toward deductionism nor the general mathematization of science.
The deductionist program in science was first fully expressed in Newton’s *Principia*, presented as geometric proofs in the style of Euclid. It had been a major theme of the ancients and is inherent in the later thought of Einstein, whose confidence in mathematical formalism was inspired by his success with general relativity [Barrow 1991, p244]. It is encouraged by textbooks, which teach physics in terms of logical rather than historical development—a revisionist approach that makes the laws of nature seem falsely simple and inevitable [Barrow, p156]. It also creates the impression that science, if not nature itself, can be axiomatized, in a final story that has erased its conceptual tracks and all traces of historical process. After all, a deductive system is timeless, eliminating dependency on the messy particulars of the real world. However, the reality of nature consists in the particular, not the general.

Mathematical treatment restricts the operative factors involved to the few that can be treated mathematically. While useful “for all practical purposes,” this creates the misleading impression that natural reality involves only these defined factors. It may be true that science as we know it could not have developed without such simplification; but this does not make it a literally true picture of the world. Nor should the history of scientific practices constrain its future. We cannot know what purposes in future may be deemed “practical.”

Deductive systems are effective just to the extent that simplification and idealization are useful toward specific aims. One can gloss over discrepancies between real and ideal, even defying common sense, by redefining the target system as a deductive system, an idealization tailored to make sense. Mathematical equations define toy systems that model selected aspects of the real world. But neither equation nor model is strictly isomorphic to the real process or system it represents. The faith that nature is rationally comprehensible amounts to a program to assimilate it to such models.

6. Conclusion

The only well-defined systems are deductive systems. The only “necessity” is the force of logic. The only absolutely certain knowledge is deductive-axiomatic. All physical knowledge is necessarily statistical, involving some uncertainty. However perfectly differential equations may describe idealized systems, they correspond only imperfectly to real systems.

The concepts of isolated system, determinism, reversibility, equilibrium, and symmetry represent properties of deductive systems, not of nature. Deductionism assimilates nature to artifacts (models, equations), and tacitly holds that physical systems are such artifacts. Properties of mathematical or deductive systems are thereby sometimes falsely ascribed to natural systems. This may also lead to excluding consideration of other properties than those defined in the deductive program (for example, properties of self-organization when matter is viewed implicitly as passive) [Bruiger 2014, 2016].

A deductive system has the connotation of being timeless, self-contained and fixed. This misleadingly suggests that changing laws are not to be expected any more than unforeseen variables are to be expected in a theory deemed complete. Through geometricized time (the block universe), with its associated determinism, deductionism
may actually hinder the development of physics [Cahill 2003]. We must re-examine change from a point of view in which it is fundamental, rather than seeking either unchanging laws or a transcendent meta-time [Smolin 2013] in which laws may vary, but which itself is fixed. We must free ourselves from the sheer logical closure of deductionism that may lead to false expectations, particularly of a final theory.

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