Choice points for a Modal Theory of Disjunction *

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Abstract

This paper investigates the prospects for a semantic theory that treats disjunction as a modal operator. Potential motivation for such a theory comes from the way in which modals (and especially, but not exclusively, epistemic modals) embed within disjunctions. After reviewing some of the relevant data, I go on to distinguish a variety of modal theories of disjunction. I analyze these theories by considering pairs of conflicting desiderata, highlighting some of the tradeoffs they point to.

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This paper investigates the prospects for a semantic theory that treats disjunction as a modal operator—or, as I will say, the prospects for a 'modal theory of disjunction'. A common application for modal theories—though not the one I will focus on here—is to the explanation of free choice inferences.[1] The idea is that it is in virtue of non-standard semantic features of disjunction that "She may be either here or there" licenses an inference to "She may be here *and* she may be there". It would be an extraordinary success if modal theories were an essential component of an account of free choice inferences.

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[1] See, among others, Zimmermann (2000); Geurts (2005); Lin (ms.); Fusco (forthcoming).

However, the prospects of such accounts are disputed, as some argue that the mechanisms that trigger these inferences are essentially pragmatic (Kratzer and Shimoyama, 2002),[2] or, more generally, optional (Fox, 2007) in a way that cannot be vindicated by explanations that revolve around modal theories of disjunction.

Given this, it is worth inquiring about a different kind of interaction between disjunction and modals.[3] Imagine (following Geurts, 2005) that 'It' names a runaway chicken and consider:

(1) It must be here or It must be there.

Roughly speaking, sentences like (1) are assertible exactly when one's epistemic state has three features: it is compatible with the chicken's being in one salient place ('here'), it is compatible with its being in another salient place ('there'), and it is not compatible with any other location.

This interpretation is not predicted if we assume that disjunction is Boolean and that the semantic value of (1) is calculated by applying the semantic value of disjunction to the semantic values of "It must be here" and "It must be there". After all, each of those sentences is false (the first is false because It might be there; the second because It might be here). So, the disjunction would be predicted false.

Variants on (1) have been much discussed in recent literature on embedded epistemic modals. Schroeder (2015) uses them to make trouble for the semantic framework of Yalcin (2007). Klinedinst and Rothschild (2012); Rothschild (2012); Moss (2015) treat them as evidence against Boolean disjunction. Dorr and Hawthorne (forthcoming) aim to reconcile these data with classical disjunction.

This paper takes this motivation for modal theories at face value. I assume that the relevant readings of sentences like (1) exist and that a modal theory of disjunction is a legitimate option for capturing them. The paper's project is to identify and evaluate some of the main modal theories.[4]

[2] On the other side of the debate, see Fusco (2014) for a critique of pragmatic accounts.

[3] Zimmermann (2000) and Geurts (2005) seek to provide uniform explanations of this phenomenon and of free-choice inferences. They seem to view all these interactions as inextricably linked.

[4] There is another kind of analysis of disjunction which could turn out to have much in common with the theories I discuss here. This is the alternative-introducing analysis of Von Ste-

This sort of project ought to matter even to those who are skeptical about modal theories. It is tempting to dismiss modal theories as revisionary accounts of core logical notions. Yet, the fact that an account is revisionary, on its own, ought to carry almost no weight in its assessment. After all, either modal theories help satisfy the goals of semantic theory (in which case, no extra points are docked for being revisionary) or they don't (in which case, their being revisionary ought to be reflected in specific, identifiable flaws).

This is the plan: §1 offers a deeper look at the data that motivate modal theories. §2 presents an off-the-shelf semantic theory for epistemic modals to serve as background. §3 presents some recent modal theories of disjunction on which my analysis focuses. §4 contains my main analysis: I consider three pairs of contrasting constraints on a theory of disjunction. For each pair, I show that none of the theories from §3 satisfies both constraints.

The existence of these conflicts is not intended as a *reductio* of modal theories. The only moral that the present discussion warrants is that there are important tradeoffs in designing a modal theory. Judgments about how to weigh the different constraints are subtler, less clear cut and less stable than judgments about what desiderata are jointly satisfiable. For this reason, different interpretations of these tradeoffs are possible: perhaps, some constraints are more important than others, or, perhaps, the background framework needs structural changes. Or maybe, these conflicts show something problematic about the prospects for modal theories. I leave the choice of interpretation up to the reader.

1 Motivation

What are the data that motivate modal theories of disjunction? Let's start with a fictional scenario:

> **Search and Rescue.** Suppose your friend Meg parachuted herself in some random point on the Iberian peninsula. You know that she has not moved since then: she is injured and her radio equipment

chow (1991); Alonso-Ovalle (2006); Aloni (2007); Roelofsen (unpublished), among others. A detailed analysis of the relationship between these theories and the modal theories I am to discuss will need to take place elsewhere.

> is damaged. You want to find her, but you have no further information concerning her whereabouts. An approximate probability model is as follows: it is .85 likely that Meg is in Spain; .15 that she is in Portugal; .001 likely that she is in Andorra and about .00001 that she is in Gibraltar.

In such a situation, consider:

(2) Either Meg is in Spain or it is likely that she is in Portugal.

Most (though not all) speakers find (2) to be acceptable in the context of Search and Rescue.[5] By 'acceptable', I do not just mean that it is passable, grammatical English. I mean, more strongly, that it seems true (to the relevant speakers in the relevant context).

What makes (2) remarkable is that, given just the provided probability model in Search and Rescue, one can also accept:

(3) It is not likely that she is in Portugal.

This is surprising: if disjunctive syllogism were valid, we would expect (2) and (3) to entail the categorical conclusion that Meg is in Spain. But that conclusion does not seem to follow; after all, there is a very serious probability that Meg is in Portugal.[6]

The phenomenon is not limited to probability operators but extends to *might* and *must*. Suppose we learn with certainty that Meg did not land in one of the small countries (perhaps, we conducted a thorough search of Andorra and Gibraltar). In this context, (4) is acceptable:

(4) Either Meg is in Spain or she must be in Portugal.

So far, it may appear that this phenomenon is restricted to *mixed* disjunctions (i.e., disjunctions with one non-modal disjunct and a modal disjunct). Examples from Geurts (2005); Schroeder (2015); Klinedinst and Rothschild (2012); Rothschild (2012) and Moss (2015) suggest otherwise. To see this, let us stick

[5]Though some speakers find it to be less than perfect, they agree that it is much better than "It is likely that Meg is in Portugal or she is in Spain".

[6]The apparent failure of this form of disjunctive syllogism is noted in Klinedinst and Rothschild (2012), who credit the observation to Yalcin, and in Schroeder (2015).

to the more informed context in which we know that Meg is not in Gibraltar and not in Andorra. In that context, all of the following are acceptable.

(5) Meg must be in Spain or she must be in Portugal.

(6) Meg might be in Spain or she must be in Portugal.

(7) Meg might be in Spain or she might be in Portugal.

As with (2), disjunctive syllogism seems to fail for (5). I might accept (5) and "Meg might be in Portugal". I might deduce from the second claim (plus background knowledge) that it is not the case that Meg must be in Spain. Given disjunctive syllogism, I could further conclude that Meg must be in Portugal. But, intuitively, this inference seems invalid.

The phenomenon I have been describing is not to be confused with the familiar fact that one can assert a disjunction without being able to assert either of its disjuncts. As an example of the familiar fact consider:

(8) The number of residents of New York state is either even or odd.

The disanalogy with the modal case is that disjunctive syllogism does not seem to fail for non-modal sentences. In the specific case, the set of premises consisting of (8) and "It is not the case that the number of resident of NY state is odd" does entail that the number of residents of NY state is even.

Some informants report that the intended reading of these disjunctions is clearer if the particle *else* is used. Furthermore, some theorists have suggested to me that only with *else* do these sentences sound ok. I agree with the informants, but not with the theorists. There are lots of naturally occurring examples of this sort use of disjunction that do not involve *else*. Here is a small selection:

(9) They suggest that he must be worried about the fate of his ships, or he must be in love, but he rejects both of those ideas. [Shakespeare Navigators Website, scene summary for *The Merchant of Venice* Act 1, Scene 1]

(10) It must be the way that you move inside my head (Yeah, yeah) Or it must be the songs that you sing to me in bed (Yeah, yeah) [John Legend, lyrics to *Must be the way*, 2004]

(11) It must be fear or it must be a desire to carry through other measures. [Proceedings and debates of the constitutional convention held in 1867 and 1868 in the city of Albany, p.329]

(12) It must bear a causal relation, or it must be an effect, or it must contain an attribute common to two items, or it must be a uniform concomitant, or what not. [William James, *The Principles of Psychology*, 1890]

The examples could be multiplied (one only needs to google the string "or it must be"). They are frequent, natural and completely ordinary speech.

Although I have emphasized the relationship between disjunction and epistemic modals, it is easy to find similar examples with other kinds of modals. Here is an example with deontic modals (see fn. 19 or Kolodny and MacFarlane (2010), and Dorr and Hawthorne (forthcoming)):

(13) Either Mary is in Spain or we should send out search parties to Portugal.

It is coherent to accept (13), deny that we should send out search parties to Portugal (if there is a cost to sending out a search party in vain, it might be imprudent to send them to Portugal, given that Meg is likely to be in Spain), and yet not be in a position to conclude that Mary is in Spain. Appreciating these deontic uses helps establish that a fully general explanation of the interaction between modals and disjunction extends beyond the epistemic domain. In further support of this point, Klinedinst and Rothschild (2012) produce several examples involving the modal *would* in clearly non-epistemic interpretations. In light of the variety of examples, the relationship between epistemic modals and disjunction, which is what I focus on here, should be viewed as an essential step in an integrated theory of modality and disjunction, but not as the only significant case.

Finally, there are many examples that involve different modals in the two disjuncts. One such example, from Klinedinst and Rothschild (2012), is:

(14) John should practice the piano or his recital will be a disaster.

Another, naturally occurring example of a similar kind is:

(15) Anybody wishing to get involved in politics at a national level quite

obviously has to have access to large sums of money or it is likely that they will not succeed. [`historylearningsite.co.uk`; page on Political Action Committees in the US]

These mixed examples are particularly interesting because they tell against wide-scoping approaches. Wide-scopers maintain that the logical form of (5), "Meg must be in Spain or she must be in Portugal", is not correctly represented by: [$\Box$(*Spain*) *or* $\Box$(*Portugal*)] but rather by $\Box$[*Spain or Portugal*]. The resulting account of the meaning of (5) is reasonable: it correctly predicts the conditions at which we find it acceptable and explains why disjunctive syllogism fails (despite superficial appearances, once we scope out the modal, the relevant inference is no longer disjunctive syllogism).

However, even setting aside familiar objections (Yalcin, 2012; Silk, 2014), wide-scoping does not seem adequate to the task at hand. We have already noted two kinds of cases that in which it is implausible to wide-scope a modal: disjunctions of epistemic modals with different quantificational force (like (6)) and disjunctions of modals with different flavors (like (14) and (15)).

Even if wide-scoping is inadequate, there is a different kind of conservative approach that might work. Perhaps, these interactions between modals and disjunction are instances of modal subordination (Roberts (1989) is the *locus classicus* on modal subordination; Dorr and Hawthorne (forthcoming) apply the idea to examples like those in §1). According to the subordination approach, the domain for the right disjunct of (5) ("she must be in Portugal") can be restricted by propositions made salient by context. It is reasonable to suppose that the first disjunct is part of the context. Modal subordination might be understood as a kind of anaphoric relation between modal domains and previous elements of the discourse, or it might be a *sui generis* phenomenon. Be that as it may, the conservative account maintains that it is through modal subordination that the second disjunct of, say, (2) and (5) is interpreted as "if she is not in Spain, she must be in Portugal". Crucially, everything in the subordination account is compatible with classical, Boolean disjunction.

I do not have decisive objections against subordination approaches. I can, however, gesture towards two challenges. One: in some of the examples, earlier modals must also inherit restriction from later ones. In "Meg must be in Spain or she must be in Portugal" disjunctive syllogism seems to fail for both disjuncts, suggesting that both disjuncts need to be restricted. That is, we might need a proposition made salient by the second disjunct to restrict the first disjunct. This backwards restriction is not immediately predicted by standard subordination approaches. Two: the subordination approach makes it a bit of a mystery why the relevant data seem to be systematically associated with conjunction and disjunction, but not with other ways of connecting clauses. Consider this discourse:

(16) Either she is in Spain or we should send search parties to Portugal. But we should not send search parties to Portugal because she is likely to be in Spain.

A subordination account ought to explain why the first occurrence of *should*, but not the second, can get restricted by “She is not in Spain”. None of these challenges refute the idea that what we see in the data reviewed above is a species of modal subordination. But hopefully they make it clear that both positions in this debate need development.

2 Domain semantics

Before developing some modal theories of disjunction, let’s lay out a baseline compositional formal semantics for epistemic modality—essentially the domain semantics of Yalcin (2007) and MacFarlane (2011, 2014).

Assume that we have possible worlds drawn from some set W. Let L be a propositional modal language, augmented with a disjunction operator ‘*or*’ not to be confused with Boolean disjunction. Upper case italic variables A,B,C range over sentences of the language. Lower-case variables p,q,r range over sets of worlds. Sentences are evaluated relative to a point of evaluation, consisting of (i) a context (ii) an information state s (a set of worlds) and (iii) a world w. Non-modal sentences are assigned a compositional truth-value (1 or 0) directly by each possible world w. Modals get these clauses:

(17) $[\![\Diamond A]\!]^{c,s,w} = 1$ iff $\exists v \in s, [\![A]\!]^{c,s,v} = 1$

(18) $[\![\Box A]\!]^{c,s,w} = 1$ iff $\forall v \in s, [\![A]\!]^{c,s,v} = 1$

Note that modals do not depend on the context parameter at all, but they do depend on the information parameter.

There are a few different notions of validity that might be considered in this system. Validity might be preservation of truth at a point of evaluation:

> **Point consequence:** $A_1,...,A_n \models_p B$ iff there is no point c,s,w such that $[\![A_x]\!]^{c,s,w} = 1$ (for $1 \leq x \leq n$) but $[\![B]\!]^{c,s,w} = 0$.

According to Yalcin (2007), point consequence is not the correct notion of consequence for the present formal framework. Yalcin's preferred alternative requires an auxiliary definition:

> **Acceptance:** A is accepted by s (in c) iff for all $w \in s$, $[\![A]\!]^{c,s,w} = 1$

One can then define consequence as preservation of acceptance (see also Veltman, 1996, where this is one of three notions of consequence defined in the context of update semantics).

> **Informational consequence:** $A_1,...,A_n \models_i B$ iff there is no context c and state s such that A is accepted by s in c (for $1 \leq x \leq n$) but B is not.

One quick way of seeing the difference between point and informational consequence is to appreciate one of the key points of Yalcin (2007): $\ulcorner A \;\&\; \Diamond\neg A\urcorner$ is point consistent but informationally inconsistent.

As the reader can verify by herself, any argument that is point valid is also informationally valid. We will use this result to establish that an argument is point invalid if it is informationally invalid.

I mentioned earlier that some of the theories of disjunction I will consider make non-trivial use of the context parameter. These theories can also appeal to context-relative notions of consequence:

> **Point consequence in c:** $A_1,...,A_n \models_{p,c} B$ iff there is no pair s,w such that $[\![A_x]\!]^{c,s,w} = 1$ (for $1 \leq x \leq n$) but $[\![B]\!]^{c,s,w} = 0$.

Read $A_1,...,A_n \models_{p,c} B$ as "the argument with premises $\{A_1,...,A_n\}$ and conclusion B is point-valid in context c". Similarly for informational consequence:

> **Informational consequence in c:** $A_1,...,A_n \models_{i,c} B$ iff there is no state s that accepts A_x in c (for $1 \leq x \leq n$) but does not accept B in c.

These notions can help explain why certain inferences, though invalid, can appear forceful in specific contexts (an invalid inference might preserve acceptance, or truth at a point of evaluation, if the context is held fixed).

3 The space of modal theories

What options are there for a semantics for disjunction that accommodates the data of §1 within the domain semantics framework? §§3.1-3.3 introduce three proposals from the recent literature (where necessary I adapt them to the framework of §2); §3.4 introduces some promising hybrids.

3.1 Dynamic disjunction

Let us start with the implementation of dynamic disjunction by Klinedinst and Rothschild (2012).

(19) $[\![A \text{ or } B]\!]^{c,s,w} = 1$ iff either $[\![A]\!]^{c,s,w} = 1$ or $[\![B]\!]^{c,s_{\neg A},w} = 1$.

Here and in the following 's_A' denotes the result of updating s by ruling out from s all the worlds that do not satisfy A. (19) diverges from standard Boolean disjunction because it shifts the information parameter against which the second disjunct is evaluated.

To understand how it operates, think back about examples like our (4), i.e. "Either Meg is in Spain or she must be in Portugal". According to (19), (4) is roughly the Boolean disjunction of "Meg is in Spain" and "*if she is not in Spain*, she must be in Portugal". Consequently, one can accept (4) and also accept that it's not the case that Meg must be in Portugal, without thereby being committed to accepting that Meg is in Spain.

Although the semantics in (19) has some distinctive advantages (see below §4.3), it needs to be generalized to apply to cases in which both disjuncts are modals. Klinedinst and Rotschild make a proposal to address those cases, but I will argue (§4.3) that it also has some shortcomings that might justify a symmetric treatment of the two disjuncts.

Two new symmetric accounts have been presented in Moss (2015) and Lin (ms.). Neither Moss nor Lin work within the domain semantics framework. I have chosen to translate their theories within that framework for two reasons. First, the results I will derive from the translations are reasonably robust (almost all of them can also be derived in the original settings). Second, in both cases, it is interesting to ask how the semantics for disjunction works in isolation from the other elements of the theory. It goes without saying that it is also essential to evaluate these options within the full theoretical context in which they arise.

3.2 Lin's case reasoning semantics

Let me start by presenting an adaptation of Lin's (ms.) semantics into the domain semantics framework. Say that two sets p and q *cover* the information state s iff $s \subseteq p \cup q$. In this case, call the set $\{p,q\}$ a *covering* of s.

(20) $[\![A \text{ or } B]\!]^{c,s,w} = 1$ iff there is a covering $\{p,q\}$ of s such that:

(i) s_p accepts A and

(ii) s_q accepts B

To understand this semantics, consider its application to (5), "Meg must be in Spain or she must be in Portugal", in a context in which we know that Meg is somewhere on the Iberian peninsula, that she is not in Andorra and that she is not in Gibraltar. (inside the formalism, assume that s contains some worlds in which Meg is in Spain and some in which she is in Portugal.) The semantics predicts that (5) is true (for $\langle c,s,w\rangle$ as given in the scenario) because s can be covered by two sets p and q one of which settles that she is in Spain, and the other settles that she is in Portugal. Thus, i_p accepts "Meg must be in Spain"; i_q accepts "Meg must be in Portugal".

It is central to Lin's goals that (20) contributes to a semantic explanation of free choice inferences.[7] Recall, however, that we did not demand that a modal theory of disjunction account for free choice inferences. Lacking that ambition, we can entertain a simple modification of (20) that has a slightly more classical logic:

(21) $[\![A \text{ or } B]\!]^{c,s,w} = 1$ iff there is a covering $\{p,q\}$ of s such that:

(i) s_p accepts A or s_p accepts B

(ii) s_q accepts A or s_q accepts B

[7] Here is a partial illustration of the explanation (Lin's explanation differs somewhat because his semantics for '$\Diamond$' is not the domain semantics of §2): $\ulcorner \Diamond A \text{ or } \Diamond B \urcorner$ requires (a) that s_p accept $\ulcorner \Diamond A \urcorner$ and (b) that s_q accept $\ulcorner \Diamond B \urcorner$. Now suppose we also accept that neither s_p nor s_q is empty. It then follows that s accepts $\ulcorner \Diamond A \ \& \ \Diamond B \urcorner$

$\ulcorner$*A or B*$\urcorner$ is true just in case you can divide s in two (possibly overlapping) sets and each accepts at least one of the disjuncts (the metalinguistic disjunction in (i)-(ii) is to be understood as Boolean disjunction). The key difference is that the two covering sets might accept the same disjunct. The extra feature of (21) is that disjunction introduction is informationally valid on (21), but not on (20) (I say a bit more about the connection between accounting for free choice inferences and invalidating disjunction introduction in §4.2).

3.3 Moss's semantics

By modifying clauses (i) and (ii) in this way, we have taken the first step towards a modal theory in the style of Moss (2015). To get all the way there, we must change (21) some more. First, Moss does not use covering sets but partitions (and not necessarily binary partitions). For some sets to cover s, they have to be exhaustive of s. For some sets to partition s, they have to be mutually exclusive and exhaustive of s. Second, in Moss's system, partitions are not quantified over, but instead are values of covert variables.

More precisely, modals like *might*, *must* and logical connectives like *or* come with a covert index whose value is a partition of the set of possible worlds (in turn, this partition induces a partition of s). To make this index explicit, I write 'or_n'. Let '$\pi_{c,n}$' denote the partition associated with index n in context c. Given all this, we can formulate a Moss-inspired semantics for 'or_n' as follows:

(22) $[\![A\ or_n\ B]\!]^{c,s,w} = 1$ iff $\forall p \in \pi_{c,n}:\ s_p$ accepts A or s_p accepts B.

A major upshot of having partitions be the values of covert variables is that disjunctions turn out to be context-sensitive. Fixing s and w, the truth-value of a disjunction might still vary, depending on what partition context supplies. §4.2 below discusses how to justify Moss's view that disjunction is context-sensitive.

3.4 Variants and Hybrids

Our cast of characters is almost complete. The theories in the last group are modifications of the accounts in §§3.2-3.3. They are obtained by replacing the notion of acceptance with the notion of truth at a point. For example, (21) could be modified to:

(23) $[\![A \textit{ or } B]\!]^{c,s,w} = 1$ iff there is a covering $\{p,q\}$ of s:

(i) either $[\![A]\!]^{c,s_p,w} = 1$ or $[\![B]\!]^{c,s_p,w} = 1$

(ii) either $[\![A]\!]^{c,s_q,w} = 1$ or $[\![B]\!]^{c,s_q,w} = 1$

Similarly, (22) could change to:

(24) $[\![A \textit{ or}_n\ B]\!]^{c,s,w} = 1$ iff $\forall p \in \pi_{c,n} : [\![A]\!]^{c,s_p,w} = 1$ or $[\![B]\!]^{c,s_p,w} = 1$.

Another way of forming hybrids is to combine Moss's use of partitions with Lin's idea of existential quantification, so as to get a semantics that quantifies over partitions.[8]

(25) $[\![A \textit{ or } B]\!]^{c,s,w} = 1$ iff $\exists \pi, \forall p \in \pi : s_p$ accepts A or s_p accepts B.

The semantics in (25) uses partitions but it is context insensitive (for this reason, I did not include the partition-denoting index in '*or*'). Finally, we can implement both of the changes I just described to obtain:

(26) $[\![A \textit{ or } B]\!]^{c,s,w} = 1$ iff $\exists \pi, \forall p \in \pi : [\![A]\!]^{c,s_p,w} = 1$ or $[\![B]\!]^{c,s_p,w} = 1$

In the next section, I analyze the semantic theories I presented here on the basis of a few design principles. It will help then to have terminology that enables us to express general claims about several theories at once. With this in mind, I say that a theory is:

- **asymmetric** if it treats disjuncts asymmetrically, like (19); all other theories presented in this section are **symmetric**.
- **partition-based** if it appeals to partitions; **covering-based** otherwise (note that asymmetric theories do not invoke either concept, so this is a distinction within the family of symmetric theories).
- **indexical** if the partition (/covering) is picked out by context; **existential** if it is existentially quantified over.
- **acceptance-based** if it invokes the notion of acceptance [like (21),(22), (25)]; **truth-based** if it invokes the notion of truth at a point of evaluation [like (19), (23), (24) (26)].

[8] This idea was suggested to me as an option in independent conversations with Daniel Rothschild and Seth Yalcin (p.c.).

I note briefly that the distinction between partition- and covering- based theories will not play a major role in my discussion. Although partition- and covering-based theories are not equivalent, they seem to diverge only in rather abstract and recherché cases. An exception is the theory in (20), which does not make much sense as a partition theory. In general, I will concentrate on theoretical differences that are likely to show up in more concrete predictive distinctions.

4 Analyzing modal theories

Having distinguished a variety of modal theories disjunction, the natural questions are: How to sort through them? Which ones are most promising? A good way of answering these questions is to consider pairs of contrasting constraints—pairs of constraints such that none of the theories from §3 satisfies both. The general moral is that if we want to hold on to a modal theory, some theoretical tradeoffs and some explaining away of prima facie plausible constraints is inevitable. In each case, then, I note some possible escape routes—what a proponent of a theory T might say to explain away the constraints that T does not satisfy.

4.1 Contingency vs. Schroeder's Constraint

It seems plausible that many disjunctions with non-modal disjuncts are contingent. It is hardly a deep insight that sentences like (27) vary in truth-value from world to world.

(27) Either Jill is an actress or she is a singer.

However, not all of the theories identified in §3 yield this. Say that a sentence A is *world-invariant* iff for every c, s, w, w' $[\![A]\!]^{c,s,w} = [\![A]\!]^{c,s,w'}$ (else it is *world-variant*). The received view is that a sentence is contingent exactly if it is world-variant.

Fact 1: world-invariance. *According to all the acceptance-based theories in §3, all disjunctions are world-invariant.*

Proof: It is easy to verify by inspection that the properties that are required for the truth of a disjunction on any acceptance-based theory are world-invariant. In particular, claims about whether information states accept or fail to accept sentences do not vary with the world of evaluation.

This fact sets up an argument against acceptance-based theories. If A is world-invariant, then the proposition it expresses must not be contingent. But we just established that on any acceptance-based theory, all disjunctions are world invariant. So, no disjunctions are contingent. This conclusion is implausible and an initial strike against acceptance-based theories.

The conclusion could be resisted by positing two semantic entries for disjunction.[9] According to the first, which is most naturally applied when the disjuncts are not modal, disjunction is the classical Boolean operator. According to the second, most naturally applied to the case of modal disjuncts, disjunction gets a modal interpretation. This is more or less the route of Moss (2015, §3). It is important to anticipate, however, that Moss can offer a justification for positing two entries that is not available in the current framework. The result of this bifurcated view is that disjunctions of non-modal sentences are world-invariant and hence (under our current assumptions) contingent.

By contrast, truth-based theories allow contingent disjunctions, thus offering some hope to those who demand a single lexical entry for disjunction. In fact, such theories have a stronger property:

Fact 2: Boolean reduction. *Truth-based theories reduce to Boolean disjunction (denoted by '$\vee$') in the special case of disjunctions with non-modal disjuncts.*

Proof: Every truth-based theory entails that disjunctions with non-modal disjuncts are true just in case at least one of the disjuncts is true at the world of evaluation. Consider, for example, the theory in (24) (the others are similar). Let $\ulcorner A\ or_n\ B \urcorner$ be a disjunction whose disjuncts are non-modal. Then $[\![A\ or_n\ B]\!]^{c,s,w} = 1$ iff for every $p \in \pi_{c,n}$: $([\![A]\!]^{c,s_p,w} = 1$ or $[\![B]\!]^{c,s_p,w} = 1)$. But right side of the biconditional depends only on whether $\ulcorner A \vee B \urcorner$ holds at w.

[9] Another option would be to deny that contingency is to be understood as world-variance. This might fail, for example, on some interpretations of two-dimensional semantics. But it is not clear that two-dimensionalist techniques apply here, so it is not clear how relevant this response is.

Boolean reduction is a stronger property than contingency, so it is violated by acceptance-based theories. This seems to be an advantage for truth-based theories.

However, the advantage that these considerations provide to truth-based theories comes at a price. There is a plausible constraint that is only satisfied by acceptance-based theories. Schroeder (2015) discusses examples like (4) (repeated here) as part of an argument against the semantic framework of §3.

(4) Either Meg is in Spain or she must be in Portugal

Schroeder notes that it should be possible to accept sentences of the form $\ulcorner$*A or* $\Box B\urcorner$ without thereby accepting either disjunct. Indeed, Schroeder points out, this is the main reasons why disjunctions are useful. Call this *Schroeder's constraint.* Given the framework in §2, this constraint is violated if disjunction is Boolean.[10] Can any modal theories from §3 do better? Only, it turns out, if they are acceptance-based.

Fact 3: Schroeder's Constraint. *All of the acceptance-based theories (and none of the others) satisfy Schroeder's Constraint.*

> *Proof:* to establish the result about acceptance-based theories, it is enough to produce a mixed disjunction of the form $\ulcorner$*A or* $\Box B\urcorner$ and a model that accepts that sentence without accepting either disjunct. So let $s = \{w_1, w_2\}$ and A, B be non-modal sentences such that A is true at w_1 but not at w_2 while B is true at w_2 but not at w_1. (for indexical theories, we need the additional stipulation that context provides a non-trivial partition, like $\pi = \{\{w_1\}, \{w_2\}\}$.) Given this, $\ulcorner$*A or* $\Box B\urcorner$ is accepted by s (updating s on the first element of the partition accepts A while updating on the second element of the partition accepts $\ulcorner\Box B\urcorner$), but neither of the disjuncts are.
>
> For the negative result about truth-based theories, the argument is mostly Schroeder's. Suppose that $\ulcorner$*A or* $\Box B\urcorner$ is accepted by an arbitrary s (in an arbitrary context). Suppose further that $\ulcorner\Box B\urcorner$ is not accepted by s (in c). Then, for some w, $[\![\Box B]\!]^{c,s,w} = 0$. Since $\ulcorner\Box B\urcorner$

[10] Schroeder also notes that this problem persists if, instead of the simple framework from §2, which was based on Yalcin (2007), the semantic framework reflects the more sophisticated approach in Yalcin (2011).

is world-invariant, it must be false for any w. Since ⌜A *or* $\Box B$⌝ is accepted by s, $[\![A \text{ or } \Box B]\!]^{c,s,w} = 1$ for all $w \in s$. Let w be an arbitrary world. Any partition, must include a non-empty subset p such that $[\![\Box B]\!]^{c,s_p,w} = 0$. Fix a p with this property, since $[\![\Box B]\!]^{c,s_p,w} = 0$, we must have $[\![A]\!]^{c,s_p,w} = 1$ (or else the disjunction would be false). Since A is non-modal and hence information invariant, this implies $[\![A]\!]^{c,s,w} = 1$. Since w was arbitrary, A is accepted by s (in c), which establishes that it's impossible for ⌜A *or* $\Box B$⌝ to be accepted without it also being the case that at least one disjunct is accepted.

A proponent of a truth-based theory of disjunction might respond by adopting Moss's (2015) treatment of mixed disjunctions. To interpret mixed disjunctions, posit a covert modal operator, to be added to the non-modal disjunct at the level of syntactic representation that interfaces with the semantics. This operator defaults to epistemic necessity (I represent it as '$\blacksquare$'): then "Either Meg is in Spain or she must be in Portugal" is interpreted as the fully modal disjunction '$\blacksquare(Spain)$ *or* $\Box(Portugal)$'. The result of this modification is that Schroeder's constraint can be satisfied.

One type of challenge for this response is internal to the framework I set up. In Moss's (2015) system, there is a specific motivation for positing the covert modal. Non-modal sentences have semantic values of a different logical type than modal sentences. The former denote sets of worlds, the latter denote sets of credences. For this reason, there are two disjunction operators (similarly for the other logical constants): one inputs a pair of sets of worlds; the other inputs a pair of sets of credences. Mixed disjunctions, however, do not fit either logical type, so their non-modal disjunct gets shifted to the type appropriate to modal claims. The problem is that in the present framework we have not assigned different types of semantic values to these sentences.

Another challenge is raised by Yalcin in an unpublished commentary on Moss (2015) (delivered at the 2014 Rutgers Semantics Workshop). Yalcin objects that positing of '$\blacksquare$' in mixed disjunctions might make the resulting sentences too strong, and hence their negations too weak. Someone who denies "Either Meg is in Spain or she must be in Portugal" appears to be committed to denying "Meg is in Spain". But if the mixed disjunction gets enriched with a covert necessity operator, we can, at best, infer the negation of "Meg must be in Spain".

Yalcin's objection suggests to me that we might instead want to posit covert modals that default to possibility operators '$\blacklozenge$' instead of covert necessity operators. Incidentally, this move would align the account of mixed disjunctions with the predictions of the earlier generation of modal theories (such as Zimmerman's 2000 and Geurts's 2005). What holds me back from full endorsement of this alternative is that it does not meet Schroeder's constraint. According to the relevant truth-based theories, one can't accept $\ulcorner \blacklozenge A$ *or* $\Box B \urcorner$ without either accepting $\ulcorner \blacklozenge A \urcorner$ or accepting $\ulcorner \Box B \urcorner$. There is no space to elaborate much further on this dialectic, but I will make a quick observation and a promissory note. The observation: it is much harder to satisfy Schroeder's constraint for mixed disjunctions of the form $\ulcorner \Diamond A$ *or* $\Box B \urcorner$ than it is for disjunctions of the form $\ulcorner A$ *or* $\Box B \urcorner$. Evidence for this is that not even acceptance-based theories can satisfy the analogue of Schroeder's constraint for $\ulcorner \Diamond A$ *or* $\Box B \urcorner$. The promissory note: at this level, the culprit might well be the account of acceptance. If we really want to insist on Schroeder's constraint, we need a more complicated account of acceptance on which the mere compatibility of A with s is not enough to entail that A is accepted by s. The technical work needs to be done, but it's a matter of extracting the more complicated account of acceptance in Willer (2013, §2.2) and integrating it with a modal theory of disjunction.

4.2 Idempotence vs. Context Sensitivity

Idempotence is the principle that $A \models (A \text{ or } A)$. Idempotence is widely accepted as a key structural principle in the logic of disjunction, even though it is not easily tested against speakers' intuitive judgments (instances of $\ulcorner A$ *or* $A \urcorner$ sound odd for obvious pragmatic reasons). My main result is that Idempotence fails on indexical theories.

Fact 4: Idempotence Failure. *Every indexical theory violates idempotence (on both point and informational consequence).*

> *Proof:* We argue that every indexical theory violates idempotence for informational validity. Consider a model on which A holds at some worlds in s but not all of them. Evaluate $\ulcorner \Diamond A \urcorner$. Suppose that context determines a partition $\{p,q\}$ such that p is the set containing all the A-worlds and q is its complement. Note that q must not

be empty, and contains only ⌜$\neg A$⌝-worlds. In this model ⌜$\Diamond A$⌝ is accepted, because it is true at every world in s. However, ⌜$\Diamond A$ *or* $\Diamond A$⌝ is not accepted, because s_q contains no A-worlds. That suffices to show that idempotence is not informationally valid, which further entails that it is not point valid.

It follows immediately that every indexical theory violates disjunction introduction ($A \nvDash A$ *or* B), since every counterexample to idempotence is a counterexample to disjunction introduction. Some proponents of modal theories of disjunction view the failure of disjunction introduction as defensible and, in fact, integral to the motivation for the modal analysis. Specifically, those who are motivated by the prospect of explaining free choice inferences need to say that ⌜$\Diamond A$⌝ could be true even if ⌜$\Diamond A$ *or* $\Diamond B$⌝ is not, since the latter, but not the former, entails ⌜$\Diamond B$⌝. This line does not work in the case of idempotence, which is why idempotence makes for a better constraint than disjunction introduction.

By contrast with indexical theories, existential theories mostly satisfy idempotence.

Fact 5: Idempotence for existential theories.
a) theories that are existential and truth-based satisfy idempotence (and, in fact, or-*introduction) on all notions of consequence.*
b) theories that are existential and acceptance-based satisfy idempotence on informational (but not on point) consequence.
c) the asymmetric theory also satisfies idempotence on all notions of consequence

Proof of a): Among the theories we considered, two are existential and truth-based: the covering-based (23) and the partition-based (26). Start with the argument for the latter. Suppose that $[\![A]\!]^{c,s,w} = 1$. Consider the trivial partition $\{s\}$. Every (trivially) updated point of evaluation still verifies A, so $[\![A \text{ or } A]\!]^{c,s,w} = 1$. For the covering-based case, quantify over the trivial covering $\{s, s\}$. Note that these arguments do not depend on the fact that the two disjuncts are identical, which means that our assumption and choice of covering/partition also shows $[\![A \text{ or } B]\!]^{c,s,w} = 1$, i.e. *or*-introduction is valid.

Proof of b): I only presented the covering-based (21), but it is easy to create a partition-based variant of it. Here I prove the claim for the partition-based variant: the covering-based case is similar. To show that idempotence is an informational consequence, suppose that A is accepted by s. On the trivial partition $\{s\}$, updating s on any partition element accepts A (trivially). So ⌜*A or A*⌝ is accepted by s. To show that this entailment fails on point consequence, suppose that for a non-modal A, $[\![A]\!]^{c,s,w} = 1$ where s contains some ⌜$\neg A$⌝-world v. v must belong to some partition element, say, p. But then i_p does not accept A, which means that $[\![A \text{ or } A]\!]^{c,s,w} = 0$.

Proof of c): I pointed out that, on the asymmetric theory in (19), ⌜*A or B*⌝ is roughly equivalent to the Boolean disjunction of A and ⌜If $\neg A, B$⌝. So, the truth (at $\langle c,s,w\rangle$) of the first disjunct suffices to establish the truth (at $\langle c,s,w\rangle$) of the disjunction. So idempotence is point-valid. This argument does not, however, support disjunction introduction if the disjunct being introduced is the second disjunct.[11]

Insisting on idempotence as a theoretical constraint makes for a strong argument in favor of existential or asymmetric theories.

A possible reply available to the proponent of indexical theories is to point out that idempotence is valid in many contexts, even if it is not valid in general. Recall from §2, that the indexical theorist has context-relativized notions of validity in her arsenal. Roughly, an argument is valid in context c iff it is valid while we hold fixed the features of the context. Idempotence is valid in all those contexts in which disjunction is associated with a trivial partition.

Although I agree that contextualized notions of validity might help explain the force of some inferences, I am unconvinced by this reply. There just seems to be *no* context in which idempotence fails. So, absent some metasemantic principle that entails that trivial disjunctions like ⌜*A or A*⌝ are always associated with trivial partitions, idempotence is problematic for the indexical theories I presented so far.

[11]Disjunction introduction is valid on informational consequence. It can fail on point consequence if we introduce the right disjunct. Here is a proof of the invalidity: there are c,s,w with $[\![\Diamond A]\!]^{c,s,w} = 1$ but $[\![A \text{ or } \Diamond A]\!]^{c,s,w} = 0$. For let $s = \{w, v\}$ with A true at v but not at w.

Nonetheless, the reply raises the important question whether disjunction ought to be treated as a context sensitive operator. Moss (2015, §1.3) argues convincingly that some of the patterns we observed are not stable across contexts. Contrast these two disjunctions:

(28) This coin must land heads or it must not land heads.

(29) Meg is in Spain or Meg is not in Spain

Both existential and asymmetric theories predict that (28) is a logical truth, just like (29). This prediction is probably too strong. Suppose that there is a bucket of coins; some the coins are fair, others are two-headed; others yet are two-tailed. In some contexts, one can hear (28) as a claim that the relevant coin is one of the two-headed coins or one of the two-tailed ones. On the relevant reading, one might disagree with (28) by saying something like "neither thing must happen, I'm pretty sure it's a fair coin". If so, we should be wary of the implication that the tautological reading of (28) is available in all contexts. Existential theories make this prediction and that seems to be a problem. Instead, indexical theories predict more modestly and more plausibly that (28) should be valid in some contexts and not in others. And if that is true, the truth-conditions predicted by existential theories are both too permissive (because they entail that (28) is always a logical truth) and too inflexible (because they do not make room for context sensitivity).

Once again we have a direct contrast: some of our theories satisfy idempotence, while some others are context-sensitive. None have both properties. Moreover, it's unconvincing to defend the failure of idempotence by appealing to the context-sensitivity of modals and disjunction.

This contrast is more easily solved than the previous one. I propose that we reject the dichotomy I have set up between existential and indexical theories. Some possible theories are both indexical and existential. Suppose that context instead of supplying a partition $\pi_{c,n}$ supplies a *domain* $\Pi_{c,n}$ of possible partitions. The doctrine behind this supposition is that it is too strong to expect context to always provide a unique partition for each connective. Then we can define theories that are both existential and indexical, like:

(30) $[\![A \textit{ or}_n B]\!]^{c,s,w} = 1$ iff $\exists \pi \in \Pi_{c,n}, \forall p \in \pi$: $[\![A]\!]^{c,s_p,w} = 1$ or $[\![B]\!]^{c,s_p,w} = 1$

(31) $[\![A \textit{ or}_n B]\!]^{c,s,w} = 1$ iff $\exists \pi \in \Pi_{c,n}, \forall p \in \pi$: s_p accepts A or s_p accepts B

If we impose no constraints on $\Pi_{c,n}$, Idempotence will still fail. But if we do require that $\Pi_{c,n}$ always include the trivial partition, Idempotence is valid *simpliciter* (as opposed to valid in some but not all contexts). At the same time, including the trivial partition in $\Pi_{c,n}$ does not validate (28).

4.3 Order Effects vs. Doubly Modal Disjunctions

Drawing on a common theme in dynamic semantics, Klinedinst and Rothschild (2012) note that (32) does not seem to mean the same thing as (4).

(4) Meg is in Spain or she must be in Portugal.

(32) Meg must be in Portugal or she is in Spain

The asymmetric analysis, (19), captures this by design. Recall the informal idea is that (4) is roughly equivalent to the Boolean disjunction of "Meg is in Spain" and "If she is not in Spain, she must be in Portugal". The order effect arises because the disjuncts play different roles: the first disjunct plays a role in the interpretation of the second, but not vice-versa.

This is an important advantage for asymmetric theories, because it is hard for symmetric theories to predict this. Perhaps, indexical theories have ingredients that can help. The structure of the relevant partitions might vary with context and it is possible to claim that the very sentence one utters (whether it be (4) or (32)) affects the context. But other than pointing to this abstract possibility, it is hard to say something very substantive about the relevant metasemantic principles.

In my view, the best argumentative course for proponents of symmetric theories is to offer an indirect argument. That is, to argue that the explanatory advantage of asymmetric theories is counterbalanced by empirical or theoretical considerations that seem to favor symmetric theories.

I mentioned earlier that the asymmetric theory faces an initial difficulty with disjunctions with two modals. We encountered many examples with this structure, for example:

(5) Either Meg must be in Spain or she must be in Portugal.

(14) John should practice the piano or his recital will be a disaster.

In these cases, the second disjunct is not intuitively restricted by the negation of the first. Instead, it is restricted by the *prejacent* (i.e. the embedded clause) of the first disjunct. Klinedinst and Rothschild (2012, p.158) note this point and amend the theory with the idea that, instead of updating the information parameter with $\ulcorner\neg A\urcorner$ (where A is the first disjunct), we update instead with $\ulcorner\neg A'\urcorner$ where A' is a sub-clause of A. In both (5) and (14), this sub-clause is the prejacent of the modal.

Even with this change, the theory is still not general enough. Consider a disjunction of the form $\ulcorner(\Diamond A\ \textit{or}\ \Diamond B)\ \textit{or}\ \Box C\urcorner$, say:

(33) The note she is singing might be C, or it might be D. Or else it must be C sharp.

The second disjunction needs to be restricted with $\ulcorner\neg A\ \&\ \neg B\urcorner$, but this is not a sub-clause in the first disjunct.

Moreover, even the amended theory predicts an asymmetry between:

(5) Meg must be in Spain or she must be in Portugal.

(34) Meg must be in Portugal or she must be in Spain.

But there seems to be no such difference.

One last point: if the argument for indexical theories (§4.2) works, it also applies to the asymmetric account as I have sketched it here. For example, the asymmetric semantics does not seem to capture the relevant interpretation of (28), "This coin will land heads or this coin will not land heads", on which that argument turned. Proponents of asymmetric theories would need to identify a relevant dimension of context sensitivity.

Summing up this discussion, we should avoid drawing the conclusion that the asymmetric theory is superior just because of the asymmetric data. But, in view of the lack of systematic pragmatic explanations for those data, we should also avoid drawing the conclusion that symmetric account are superior. One thing that seems clear, however, is that the asymmetric theory has a clear advantage over modal theories that do not treat disjunction as context sensitive at all.

5 Conclusion

It's time to draw some defeasible conclusions about that the way forward for modal theories of disjunction. One: proponents of modal theories ought to prefer theories that are *both* existential and indexical. Two: they need new technology to craft a more complex notion of acceptance, so as to allow their theory to satisfy Schroeder's constraint in full generality (regardless of whether the background theory is acceptance-based or truth-based). Three: If the technology permits (and absent external reasons to approach the matter otherwise), they should favor truth-based theories since these allow a unified treatment of modal and non-modal disjunctions. Four: they must either develop systematic pragmatic principles that can retrieve the asymmetric data within a symmetric theory or tackle the challenges for asymmetric accounts.

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