

Virtuous Insightfulness

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Insight often strikes us blind; when we aren't expecting it, we suddenly see a connection that previously eluded us—a kind of 'Aha!' experience. People with a propensity to such experiences are regarded as insightful, and insightfulness is a paradigmatic intellectual virtue. What's not clear, however, is just what it is in virtue of which being such that these experiences tend to happen to one renders one intellectually virtuous. This paper draws from both virtue epistemology as well as empirical work on the psychology of problem solving and creativity to make some inroads in accounting for insightfulness as an intellectual virtue. Important to the view advanced is that virtuously insightful individuals manifest certain skills which both cultivate insight experiences (even if not by directly bringing them about) and enable such individuals to move in an epistemically responsible way from insight experience to epistemic endorsement.

1. Insight and insightfulness

On 3 July 2012, Yitang Zhang, a 50-year maths lecturer, was taking a stroll in the backyard of a friend's house in Colorado before leaving for a concert. He had been working tirelessly for years on one of the most elusive problems in pure mathematics, the twin prime conjecture, one of the 'holy grails' of pure mathematics. The twin prime conjecture says that there are infinitely many primes numbers, p , such that $p + 2$ is also a prime number—i.e., such as (3, 5), (5, 7), etc. Leading experts in analytic number theory have been attempting to prove this since the middle of the 19th century, but with no success. In fact, by 2012, no one had even proven that there is *any number* such that there are an infinite number of primes which differ by only that number (this later conjecture is called the

bounded gaps conjecture). And what's more is that no one had any obvious leads as to how to make headway, though it was clear to all that if anyone could provide a proof for the bounded gaps conjecture, this would be an enormous breakthrough towards proving the twin prime conjecture.

When Zhang was taking that walk in his friend's backyard in the summer of 2012, he had made, to that point, no substantial contribution to mathematics and was virtually unknown by experts working in the field. He'd not even published a paper since 2001, and had for a long time struggled to find any academic employment. But, as Zhang later told a reporter, that evening in July 2012 a solution to the bounded gaps conjecture suddenly came to him, and 'I immediately realized that it would work.' He wrote up the proof and sent it to the most prestigious journal in the field, *Annals of Mathematics*¹ and the publication of his proof left the experts shocked; it was praised as one of the greatest and most surprising breakthroughs in number theory, described as 'astounding' by number theorist Daniel Goldson in an interview with *Quanta* magazine, and further as 'one of those problems you weren't sure people would *ever* be able to solve².'

So what exactly 'happened' to Yitang Zhang in his friend's back yard? And why did it happen to *him*? Let's begin with the 'what' and then move to the 'why'. One mundane way to describe the situation is that what Zhang did was to form a belief—viz., a belief about the bounded gaps conjecture—which was both true and important. While this description is literally accurate, it fails (rather badly) to capture an important epistemic dimension of Zhang's moment of clarity. After all, like most mathematicians working in number theory, Zhang *already* believed the conjecture was true—he believed this years before his discovery. Most number theorists believed it as well.

In an interview with the *New Yorker* following the publication of his proof, Zhang spoke more of the remarkable experience he had that evening. He admitted that he wasn't consciously *trying* to solve the bounded gap conjecture at the moment the solution struck him. Rather, what he was actually doing in his friend's back yard was smoking a cigarette and watching for deer, hoping he might spot one in the woods past a fence, and then he suddenly realised there was a way to prove the bounded gaps conjecture. He said in that interview, of the experience he had: 'it's hard to say what it is ... Something very special ... I knew [immediately] that, even though

¹See Zhang (2014) for the published proof.

²<https://www.quantamagazine.org/20130519-unheralded-mathematician-bridges-the-prime-gap/>

there were many details to fill in, we should have a proof. Then I went back to the house³.’

There is plenty to say about Zhang’s epistemic achievement, though let’s focus on what took place on 3 July. While Zhang worked remarkably hard in the years leading up to his discovery, we can imagine others working similarly hard, but who—after staring at the deer in the yard—(or, after staring at whatever the leading number theorists were staring at that evening) were struck by no thought of any importance. Zhang in that circumstance surely had a remarkable *insight*. But what is an *insight*?

First, a disambiguation. Consider a parallel to assertion. In one sense, an assertion is a speech *act*; in another sense, an assertion is what is *issued* by the speech act of asserting. As assertion admits of a speech act/object distinction⁴, likewise, insight admits of a cognitive act/object distinction. In its most general and abstract sense, an act or *episode of insight* is a token episode in which one grasps of something *X*, its *relatedness* to something else, *F*, where *F* can be another object, property, etc. Call this, for short, *grasping under a relational mode*. Insight, in the second (object) sense, is what such a cognitive episode *issues*, the deliverance of an act of grasping of something its relatedness to something else⁵.

Working with this very generic account of an episode of insight (which I’ll continue to refine), we might expect that what such an episode issues—insight in the object sense—is best understood as a true belief *that* (for instance) some *X* is *F*. In Zhang’s case, this would translate to: believing not simply that the bounded gaps conjecture is correct (something he already believed), but rather believing that a certain *proof* for the bounded gaps conjecture is correct. But this cannot be entirely right, either.

To appreciate why, consider a helpful example from Stephen Grimm (2012), involving grasping, of a necessary truth, its necessity, in a way that is required to know the necessary truth a priori. As Grimm observes, I might believe on someone’s (say, a school teacher’s) testimony that a certain truth is a necessary truth. Provided I lack any defeaters, I thus plausibly *know* on your say so that something is a necessary truth, and yet, as Grimm (2012)

³<http://www.newyorker.com/magazine/2015/02/02/pursuit-beauty>

⁴For a helpful recent discussion of this distinction, see MacFarlane (2014).

⁵One might wonder whether this simple account rules out anything like an ‘atomic’ or non-relational insight—viz., just an insight *that p*. The answer, in short, is yes. But this is a benefit of the account. A mere *belief* of something, that it is so, doesn’t entail an insight (in the object sense) on the view I’m proposing, even though one’s grasping something under a relational mode can entail that one believes of something that it is so. See the ensuing discussion of Grimm (2010) and Schaffer (2001) for some reasons why.

writes, it's possible 'even though I now have knowledge of a necessary truth, I nonetheless fail to see or grasp or in some way *appreciate* its necessity⁶.' If that's the case, I fail to know the necessary truth a priori. In order to do *that* I need to grasp, *of the proposition*, its necessity⁷. Compare now with insight: just as knowing a priori that something is a necessary truth involves a grasping of the proposition its necessity, similarly, grasping of something, its relatedness to something else, isn't going to be secured simply by having a belief *that* something is related to something else in a particular way. Such a belief can be acquired (e.g., testimonially) without the kind of grasping that's essential to insight⁸.

Here is a further very general point about insight: though Zhang's insight was profound, important, original, etc., insights *as such*, needn't be. Often times, I might have an entirely typical episode of insight when simply ruling out a relevant alternative under unremarkable circumstances. Consider, as Jonathan Schaffer (2001) has argued, that in order to properly count as ruling out a relevant alternative (i.e., whether the milk in the fridge has been consumed) in an epistemically satisfactory way, I must not *merely* believe or even know of the relevant alternative *that* it is false⁹. I must do something further: I must come to regard it as false while also appreciating the alternative *as* relevant, and without this appreciation, the alternative remains live even if my belief that the relevant alternative is false persists.

Let's move now from insight to *insightfulness*. Individuals can be more or less insightful. When they are, this will surely have *something* to do with their dispositions to insightful episodes (episodes with the features described), the episodes which issue insights. But here some care is needed. Insightfulness, at least as a term of epistemic praise—viz., as an intellectual virtue—isn't simply a matter of having *many* insights.

After all, we aren't likely to regard an individual as especially insightful if her cognitive history includes an well-above average number of insights (relative to the mean) provided the insights themselves are ones which

⁶My italics.

⁷Compare here with Jonathan Schaffer's (2001) suggestion that in order to rule out a relevant alternative, you need to not merely believe of some relevant alternative that it is false, but you must further grasp it as false *as* an relevant alternative.

⁸See Pritchard (2009) for a related point regarding understanding-why.

⁹For some seminal discussions of relevant alternatives in epistemology, see Dretske (1970) and Lewis (1996). For a more sophisticated contemporary version of this kind of view, see Pritchard (2010).

are on the whole utterly ‘shrugworthy’¹⁰: unimportant, superficial and/or which fail to point beyond mundane ‘common knowledge’¹¹.

I want to suggest now four dimensions along which an *individual* can be plausibly evaluated as *more insightful*, with references to features of that individual’s insights conceived as the deliverances of the individual’s grasping under a relational mode¹². These four dimensions involve *depth*, *non-triviality*, *originality*, and *frequency*. These points won’t be uncontroversial, so I want to briefly defend them before moving on to connect insight with intellectual virtue in more sophistication and, ultimately, with feeling.

Firstly, depth. An insight is deep to the extent that what is grasped under a relational mode is (in the context of discovery) *unexpectedly* so related, viz., as when two areas of physics are brought together in a surprising way¹³. A paradigmatic example here is James Maxwell Clerk’s late 19th century discovery of electromagnetic force, when previously electricity and magnetism were regarded to be two entirely separate kinds of force. Leon-

¹⁰Thanks to Mark Alfano for suggesting this term. Note that the converse of shrugworthy is *interesting*, though the matter of what is interesting is itself a substantive question. For some notable discussion on this point, see Hidi and Baird (1986) and Stace (1944).

¹¹Of course, some insights are important *in a domain* even if not important outside of that domain. Here we can imagine an individual, perhaps with Asperger’s Syndrome, whose cognitive interest and inspiration are piqued only in the case of discussing bridges. A lack of interest might undercut this individual’s inclination to be insightful outside of the context of discussing bridges. But, this inclination is *not* undercut in the case of discussing bridges. In cases like this, we might say that the individual is insightful in a very domain-specific way. Thanks to Emma C. Gordon for discussion on this point.

¹²Note that these four dimensions along which an individual can plausibly be evaluated as more insightful are not meant to be necessary and sufficient conditions for virtuous insightfulness. More weakly, they are just dimensions along which propensities to insight episodes can be plausibly evaluated as more or less good, from an epistemic point of view. The noting of these dimensions is compatible with granting that some virtuously insightful individuals might do better along some dimensions than others.

¹³I’m using ‘unexpectedness’ as an intuitive way, though I don’t think a detailed account of this (as it bears on the dimension of depth) will be promising. For example, a subjective account of unexpectedness seems to make insights too easy to come by; obvious connections will be unexpected for someone who is especially narrow-minded or poor at appreciating connections. By contrast, an objective account of unexpectedness will likely be too restrictive. For example, take Itô’s lemma (also referred to as the Itô–Doebelin theorem), which was proved by Noboru Itô in 1944. However, it was later discovered that Wolfgang Doebelin proved the same lemma four years earlier, in 1940, but sealed his results in an envelope which was not opened until 2000. Had it turned out that Doebelin’s results were established when he proved the lemma in 1940 (rather than sealed away in an envelope), the lemma’s proof would (four years later) perhaps be unsurprising in the mathematical community when Doebelin proved it, and Doebelin’s drawing the connections he did would not longer be ‘unexpected’ (objectively) but expected.

hard Euler's famous identity, $e^{i\pi} + 1 = 0$, which connects five fundamental mathematical constants, is also a classic example of a deep insight, and one that is often referred to as a paragon of mathematical beauty¹⁴. We are inclined to regard an individual as *more* insightful, the deeper her insights.

Another dimension relevant to insightfulness is *non-triviality*. Even deep insights can be trivial. Take, for example, spurious correlations. One who reads papers daily might suddenly grasp that there is an utterly unexpected strong correlation (over 95% from 1999-2009) between railway train death collisions and barrels of US crude oil imports from Norway¹⁵, a relationship previously unappreciated. Like Maxwell's discovery of electromagnetic force, one's discovery of this spurious correlation has the trappings of depth. Surely though an individual whose insights are nearly exclusively about spurious correlations¹⁶ (or about unexpected relationships between various patterns of blades of grass) will not be regarded as equally insightful as another individual whose insights are equally deep but also important, or meaningful¹⁷.

Thirdly, and probably most controversially, *originality* matters; specifically, the originality of the deliverances of one's episodes of insight is a dimension that—at least, if our patterns of attributing insightfulness are a guide—relevant to the insightfulness of the individual. If (as a contingent matter of fact) many other individuals have had the same insights as you, this seems to negatively affect our willingness to attribute insightfulness. A corollary of this point is that at least *one* feature (originality) relevant to our

¹⁴For a discussion of the connection between deep insight and mathematical beauty, see Rota (1997).

¹⁵The source for this data is the US Department of Energy and Centers for Disease Control and Prevention.

¹⁶One might be inclined to think that grasping spurious correlations would give rise to merely 'spurious' (i.e., not genuine) as opposed to genuine but trivial, insight. I think, however, that we must distinguish between two kinds of correlations which fall under the banner of 'spurious correlations', one kind of which gives rise to trivial albeit genuine insights, another which gives rise to spurious or non-genuine insights. In the former case, a correlation is in fact present, and the subject has successfully identified the relevant pattern which happens to be a pattern that lacks any deep explanation—viz., a *coincidental correlation*. Compare the grasping of such a coincidental correlation with a different case in which one mistakenly regards oneself as having identified a correlation or pattern when no such pattern is present. The latter is a spurious or non-genuine insight; no actual pattern has been grasped or appreciated; the former case, a trivial albeit genuine insight. Thanks to an anonymous referee at *Episteme* for raising this point.

¹⁷It's doubtful that triviality has an absolute measure, independent of interests. The example I've given above of a spurious correlation is one which that takes for granted actual interests.

judgments of insightfulness is relative; it is a dimension of insightfulness which doesn't have an absolute measure¹⁸.

Fourthly, frequency. An individual who has many episodes of insight is (defeasibly) more insightful than one with fewer. This claim is compatible with an assessment that an individual A who, has very few insights, is comparatively more insightful than another, B, who has more, provided A's insights are evaluated more highly than B's on the other three dimensions. The point about frequency is just that, all else equal, more insights make one more insightful—in the mundane sense that more generous acts makes one more generous.

2. What makes insightfulness an intellectual virtue? A dilemma

The previous section articulated several key marks of *insightfulness*, as a function of the kinds of insights issued by an individual's episodes of insight. This section explores what makes insightfulness, as a character trait, an intellectual virtue, and in doing so, raises a kind of dilemma.

One very natural approach to vindicating insightfulness as an intellectual virtue will be to follow *virtue responsibilism* and offer a motivationalist account of insightfulness as an intellectual virtue (e.g., Battaly 2008; Montmarquet 1993)¹⁹. On this account, insightfulness is an intellectual virtue not because of the epistemic value of the insights issued by insightful people, but because of the epistemic value of the characteristic *motivation* of insightful people. Compare here with openmindedness: Heather Battaly remarks: 'What makes open-mindedness, so construed, an intellectual virtue? What makes it an intellectual, rather than a moral, virtue is its *motivational component*. Even though it need not track the truth, it is

¹⁸One reason this line will be controversial is that it, in a certain way, might seem *unfair*. To appreciate this point, consider an example: suppose you've got 50 individuals on desert islands, and each generates at the same time, the same deep insight. My suggestion is that we are inclined to regard each as insightful *to a less extent* than we would were the local circumstance different, and each individual's insight were original. Again, though, this point is meant to be an artefact of our normal patterns of attributing insightfulness, which are perhaps not fair (in the sense that it is, along at least one dimension, outside of one's control whether that individual is likely to be regarded as insightful in one's social community).

¹⁹For an alternative kind of motivationalist account, framed in terms of personal worth, see Baehr (2011).

characterized by a *motivation for truth*²⁰.'

Even if motivationalist accounts are plausible in the case of some intellectual character virtues, in the case of insight, however, motivationalist accounts seem to get the wrong result across a range of cases where insightfulness appears to be intellectually virtuous in the presence of intellectually *vicious* motivation²¹. To appreciate this point, just imagine two versions of a case, both of which the agent in question has little care for the truth, for responsible inquiry, lacks curiosity, is intellectually dishonest, etc. Now, in the first version of this case, suppose that over the course of a lifetime, this individual fails to generate any original insights, whereas, in the second version of the case, the individual (holding fixed the vicious motivations) has a range of insights which are original, deep, etc. Two key points: firstly, intuitively, *despite* whatever vices the individual in the second case has, she also appears to possess one kind of intellectual *virtue* which the agent in the former case lacks²². Even more, it's hard to see how the agent in the second case's vicious motivation actually detracts in any from the epistemic goodness of her insightfulness. If this is right, though, then to the extent that insightfulness is an intellectual virtue, this isn't going to be accounted

²⁰See however Carter & Gordon (2014) for some criticism of both virtue responsibilist and reliabilist attempts to account for openmindedness as a virtue in terms of a connection with truth.

²¹A further potential problem for a responsibilist vindication of insightfulness as an intellectual virtue is that episodes of insight do not seem to be cognitive performances, even under a wide reading, which the agent is responsible for bringing about; rather, they *happen to one* (often, when one is thinking of something else entirely, i.e., see Segal (2004) on incubation).

²²A virtue responsibilist might be inclined to insist that the above example confuses intellectual virtue with a mere knack, skill or gift. The individual who despite lacking a motivation for truth reliably generates deep insights clearly has a kind of skill or gift—viz., perhaps a fine-grained skill that permits this individual to spot connections or patterns with ease, which in turn facilitates the kind of grasping that is characteristic of insight. However, this concession is compatible with ascribing to such an individual the quality of insightfulness despite the underlying lack of interest in the truth. The responsibilist who wishes to maintain that the case in question features mere skill would presumably require a story for how possessing the right motivations would raise the skill to the level of virtue in other circumstances. The responsibilist could well have such a story, though the intuition this case pair is meant to elicit is that such a story wouldn't be a straightforward one; at least, the example is meant to reveal a peculiarity about insight, in so far as it cannot be accounted for within a responsibilist model as elegantly as other kinds of character virtues. Thanks to an anonymous referee at *Episteme* for requesting clarification on this point.

for from within the motivationalist model²³.

A rival paradigm for accounting for what makes a trait an intellectual virtue is *virtue reliabilism* (Greco 1999; 2003; 2010a; 2012; Sosa 1991; 2009; 2011), according to which what accounts for why a given trait is an intellectual virtue is its propensity to deliver true beliefs and avoid error. On this model, to the extent that insightfulness is an intellectual virtue, it will be in virtue of insightfulness's reliability in issuing true beliefs.

This view also quickly runs into a jam. Take as a starting point Mark Alfano's (2012, 246) remark that 'If someone has an original insight even once a week, that might qualify her as insightful'. This seems about right. Consider Elon Musk, who has invented PayPal, Tesla Motors, SpaceX, and holds many patents. Musk might very well have an original insight per week, and even if he has several 'false positives' which he scraps each week²⁴, this intuitively doesn't seem to undermine our willingness to regard him as (very) insightful nonetheless. Put another way: on the supposition that Musk has an original insight each week, that very plausibly *suffices* for his being virtuously insightful, the point intimated by Alfano's observation; we needn't (for instance) inquire further about his track record. This latter point is however at tension with the virtue reliabilist account of what makes a trait an intellectual virtue; the virtue reliabilist fails to account for why insightfulness, qua intellectual virtue, shouldn't be approached through the lens of a batting average.

3. Insight problem solving, feeling and virtue

A tempting knee-jerk conclusion to draw is that if insightfulness can't be accounted for as an intellectual virtue on either a motivationalist or reliabilist model—that is, with reference to characteristic motivation or propensity to issue true beliefs—then it's simply not a *bona fide* intellectual

²³As Mark Alfano has pointed out to me, a similar point can be made with reference to what Nietzsche called 'abysmal insights', insights which Nietzsche regarded as painful and unwanted, into the depths of the pettiness and contemptibility of the human soul.

²⁴False positives could involve cases where one merely *thinks* that one has grasped some connection under a relational mode but in fact has not done so. For example, I might have an 'insight' to the effect that a particular suspect is guilty, when the suspect is innocent. Or, even if the suspect really is guilty, I might merely *think* I've grasped this, when actually my regarding the suspect as guilty is the result of an undetected bias. Note that on some non-factive views of insight, these 'false positives' count as genuine insights, provided they aid to render my world more *intelligible*. This is a view that has been advanced by Wayne Riggs (2004).

virtue. This would be far too quick. In this section and the next, I want to show how insightfulness can be vindicated as an intellectual virtue, in a way that draws from both virtue epistemology as well as empirical psychology.

To this end, it will be helpful to begin by reviewing some empirical work on *insight problem solving* in psychology. Consider the following classic insight problem: the ‘nine-dot problem’ (e.g., Chronicle, Ormerod, and MacGregor 2001; Kershaw and Ohlsson 2004), according to which one is instructed to connect all nine of the dots below, using just four straight lines, but without lifting one’s pencil/pen from the paper. (Try this, though don’t look yet at the solution below²⁵).

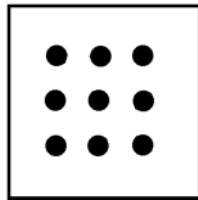


Figure 1: The nine-dot problem

There is a certain phenomenology common to the discovery of solutions to insight tasks such as the one above (e.g., Sternberg and Davidson 1995; Metcalfe and Wiebe 1987). For instance, once the participant realises that the lines can go beyond the square that is formed by the nine dots, there is with this (and the discovery of other insight problem solutions) a kind of ‘aha’ experience.

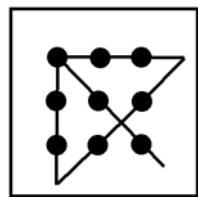


Figure 2: Solution

One aspect distinctive of episodes of insight is that the ‘aha’ experience

²⁵While apprehending a solution to the nine-dot problem is not something we’d regard as a *deep* or especially interesting insight, it is on my view at least an insight nonetheless, and so of the sort the psychological structure of which is instructive (as insight problems of this sort have been well studied).

is not gradual, but sudden. For instance, in a study by Janet Metcalfe and David Wiebe (1987), participants completed classic insight problems along with ‘non-insight’ problems (i.e., such as problems where one must simply apply an algebra rule). In each case, participants recorded ‘patterns-of-warmth’ ratings, which reflect the subjects’ feelings of approaching a solution. Patterns-of-warmth ratings differed significantly in the case of insight versus non-insight tasks. As they found ‘Algebra problems and noninsight problems showed a more incremental pattern over the course of solving than did insight problems ... the phenomenology of insight-problem solution was characterized by a sudden, unforeseen flash of illumination’²⁶.

Secondly, a key feature of the psychology of insight is termed *incubation*, where a break in attention to the task at hand can effectively facilitate the solution process (e.g., Dodds, Ward, and Smith 2003; Hélie and Sun 2010), though the mechanisms by which this occurs are debated²⁷. In a study by Eliaz Segal (2004), insight tasks were completed more effectively by individuals who took a break after reaching a self-reported ‘impasse’ than did those who did not.

Thirdly, there is some neuroscientific evidence that the kind of ‘aha’ experiences associated with participants attaining solutions to insight tasks correspond with different neural states in the case of insight versus non-insight tasks. As Jung-Beeman et al. (2004) note, the right anterior temporal area is associated with making connections across distantly related information during comprehension; and, further, fMRIs in their 2004 study revealed ‘increased neural activity in the right hemisphere anterior superior temporal gyrus in the case of individuals gaining solutions to insight relative to noninsight tasks’ (Jung-Beeman et al. 2004).

These empirical features of insight solutions, as reported in cases of classic insight problems in psychology, suggest a picture on which (i) episodes of insight are typically (ii) preceded by incubation; (iii) are characterised by a distinctive phenomenology (i.e., feelings of approaching an answer which differ from non-insight cases) and neurophysiology (relative to non-insight solutions); and that the distinctive phenomenology or ‘aha’ experience is both (iv) sudden and (v) unforeseen.

Note firstly, that this picture aligns well with the account offered by Yit-

²⁶(Metcalfe and Wiebe 1987, 238).

²⁷As Segal (2004) has proposed, the mechanism function of the incubation period is best understood as simply diverting the solver’s releasing her mind from the grip of a false organizing assumption.

ang Zhang of his own experience. Zhang's incubation period (when his attention was diverted from the bounded gaps conjecture and focused on the deer in the yard) was suddenly interrupted by a realisation that he described as 'immediate' and 'something special'.

I want to now (perhaps unsurprisingly) suggest that a satisfactory account of what makes insightfulness an intellectual virtue should make *some* essential reference to how is that what is *distinctive* of episodes of insight—i.e., namely, the kinds of 'eureka'²⁸ moments which typically are preceded by incubation—are connected in some way to our epistemic ends or goals. But, in order to make this argument, however, I shall present an objection to the strategy just outlined; the account of insightfulness as a virtue I want to defend emerges as a response to the anticipated objection.

The objection to accounting for what makes insightfulness an intellectual virtue, *in terms of* how the psychology of insight is connected to some epistemic end (i.e., truth) is that such an account will face what we can call (for lack of a better term) the 'bunny problem'. Consider that rabbits' hare-trigger instinct to flee from danger—an instinct which often saves their lives when predators are near—is nonetheless very unreliable at detecting danger, even when working normally in an environment suited to its function²⁹. Their 'danger' detectors generate mostly (around 90%) false positives, which is why when a bunny's 'sense of danger' is triggered, this is nearly always in the absence of genuine danger. Of course, having an over-primed disposition to flee is evolutionarily advantageous for bunnies, and this is so even though the sense of danger detector is woefully unreliable at distinguishing genuine danger from 'false' danger³⁰. In short, the bunny's sense of danger doesn't discriminate danger from non-danger.

Unfortunately for us, though, the key psychological features associated with insight are triggered in cases of *correct* insights (i.e., where one's grasping under a relational mode is apt, and generates some epistemically valuable 'cognitive contact with reality') but also in cases of *incorrect* insights (i.e., where one sees an unexpected connection, rendering a body of information or task intelligible, but which is *not* accurate), as when

²⁸I am, in this paper, using 'Aha' and 'eureka' experiences interchangeably, which is following precedent in this literature. I do this with some reservation, though, as 'eureka' seems to capture experiences that belong to a narrow class of insights with special practical importance to the agent, whereas 'Aha' seems germane to a broader class of insight experiences.

²⁹I borrow this example from Peter J. Graham.

³⁰Compare with D. Kelly's (2011) remarks about the untrustworthiness of feelings of disgust, given the propensity of such feelings to issue false positives.

we pre-emptively celebrate an ‘insight’ (i.e., ‘Aha, it’s the butler... wait, nevermind’³¹.) This is why, as Stephen Grimm (2010) notes, from the *inside*, genuine (factive) understanding feels indistinguishable from what he calls *subjective understanding* which is merely a grasp of how specific propositions interlink, and which does not depend on their truth but rather on their forming a coherent or intelligible picture³². Coherent pictures of course, can depart dramatically with how things actually stand and thus be of scant *epistemic* value.

4. Insightfulness as a virtue, redux

Compare Yitang Zhang with British Mathematician Andrew Wiles, who believed, in 1993, that he’d proven Fermat’s Last Theorem, which states that no positive integers a , b , and c can satisfy the equation $a^n + b^n = c^n$ for any integer n such that $n > 2$. This is harder than it looks³³; no one had proven it in the more than 350 years since it was first proposed, though many had tried.

Wiles’ announcement in 1993 followed an insight he described as ‘beautiful ... I was electrified. I knew that moment the course of my life was changing³⁴’. But Wiles had overlooked a gap in his proof. After his series of lectures in Cambridge, he began receiving e-mails from a reviewer, Princeton’s Nick Katz, with questions about the details of the proof. Most of

³¹There might be different kinds of botched insights, some more epistemically valuable than others. The kind of botched insight I’m interested in (as illustrated by the examples I use), by contrast with paradigmatic apt insights, are ones in which the agent, suitably informed, appreciates the error from within her epistemic framework. Wiles, for instance, working from within his own understanding of mathematics, made a mistake he could himself later appreciate. Compare this however with a very different sort of botched grasping, one that we might be inclined to regard as epistemically defective, but which the agent herself would not, from within the standards of evaluation of her own framework. For instance, compare the defectiveness of Ptolemy’s insights, in the *Amalgest*. They served the aim of rendering Ptolemy’s world picture intelligible (even if not accurate), and were not defective *from within* his own model. There is a sense in which Wiles’ grasping (from 1993, not 1994) is thus botched in a different way than was Ptolemy’s. For those who are inclined to release a grip on metaepistemological absolutism (e.g. Boghossian (2006)), it will look as though the latter kind of ‘botched’ grasping is not botched as all, even if the former is. Thanks to Modesto Gómez Alonso for stimulating discussion on this point.

³²For discussion on this point, see Gordon (2016).

³³The proof, which Wiles eventually correctly put forward in 1995, was so complicated, and drew from so many different areas of mathematics, that only a few mathematicians were capable of verifying it at the time. See Singh (1997) for an overview.

³⁴<https://plus.maths.org/content/fermats-last-theorem-and-andrew-wiles>

Katz's questions, Wiles could answer, but one of the questions revealed what he appreciated as a serious problem³⁵. By 1994, Wiles was on the brink of giving up, when he was finally able to patch the gap, and prove Fermat's Last Theorem successfully, in 1995.

Wiles' remarkable felt experience in 1993, which he took to be beautiful, was in an important respect defective, a kind of botched grasping (even if a highly skilled botched grasping)³⁶. In order to give a plausible account of what makes insightfulness an intellectual virtue in terms of how the psychological features of insight are connected to some epistemic end (i.e., truth, understanding, etc.) it is necessary to discriminate between the kinds of 'eureka' feelings which typically follow incubation in cases of *correct* insight (i.e., Zhang's in 2012 and Wiles' in 1994) from the subjectively indistinguishable 'eureka' feelings which follow incubation in cases of *mistaken* insights (i.e. Wiles in 1993).

I want to now suggest how such an account of insightfulness as an intellectual virtue might do exactly this. The model I want to suggest connects virtue and feeling in two different directions: from preparation and incubation to feeling, and from feeling to judgment or verification. The model I suggest draws from the basic 'four stage' model of creative problem solving as originally introduced in Graham Wallas' (1926) seminal text in the philosophy of creativity *The Art of Thought*. On Wallas' view, the feeling of illumination is (in the case of creative problem solving) best appreciated as the third step in a four-step creative process anteceded by (i) preparation and (ii) incubation and followed by what Wallas calls (iv) verification. What I want to suggest, in a nutshell (which I will flesh out), is that the virtuously insightful individual is skilled at *bringing about* (even if not directly or intentionally) 'eureka'/illumination experiences specifically by excelling in the anteceding stages of preparation and incubation, and further, is skilled in a very specific way which I'll explain, at moving *from* these felt experiences to judgment or verification, where the aim of the final stage is truth (or more broadly, understanding). For ease of reference, call the first stage the *cultivation stage of virtuous insight* and the second the

³⁵The problem was a very subtle and technical one. According to biographer Singh (1997, 405-06), Wiles described the magnitude of the problem as follows: 'It was an error in a crucial part of the argument involving the Kolyvagin-Flach method, but it was something so subtle that I'd missed it completely until that point. The error is so abstract that it can't really be described in simple terms. Even explaining it to a mathematician would require the mathematician to spend two or three months studying that part of the manuscript in great detail'.

³⁶Of course, Wiles' grasping was not without some epistemic value. See fn. 31 for discussion of varieties of botched grasping.

management stage of virtuous insight.

4.1. Cultivation stage of virtuous insight

An initial worry is that despite what Zhang did in the previous stages, it was a matter of fortune that he had the insight he did when staring at the deer, thinking of something else entirely, and not even at that moment *trying* to solve any problem. The underlying intuition here is that it was in a straightforward sense out of his *control* that he had the illuminating experience he did. And if it's out of his control, then it can't be explained by any virtue of Zhang's at the preparation and incubation stage, but rather, by luck³⁷. Or so this line of worry goes.

Here, an analogy from the literature on unreliable achievements will be helpful. Consider, for example, *artistic achievements*. To borrow an example I've advanced in previous work³⁸, an artist for instance might produce her signature pottery by using methods that very often produce cracked and broken failures. In order to produce a new and successful work, she may, in ordinary circumstances instance, have to make several attempts. 'It may be that ordinarily, her ways of making pottery are *unsafe*. Nevertheless, we would not, for this reason, hesitate to say that her successful works are, in many cases, *artistic achievements*' and in doing so credit her success in the main to her ability. A similar intuition holds in the case of athletic achievements. As John Greco (2012) remarks:

[...] different abilities require different degrees of reliability. Thus Kobe Bryant is a great free-throw shooter because he is successful around 85% of the time. Jeter is a great hitter because he is successful around 30% of the time (2012, 17).

³⁷This line has two kinds of sources of support. Firstly, prior to John Greco's (2012) more recent articulation of virtue epistemology, Greco (2008; 2010b) offered a *contrastive* account of the relationship between a success's dependence on ability as opposed to luck. In particular, Greco's view was that if a success is not saliently explained by ability, then it's lucky. To the extent that we're not inclined to regard instances of insight as saliently explained by skill (given their being not subject to direct control) we might be led to regard the success as lucky. Secondly, the above line receives direct support from what is now a minority view in the philosophy of luck in epistemology, in particular, *lack of control accounts* (Zimmerman 1987; Riggs 2009) according to which successes are lucky provided they are relevantly out of the subject's control. Cf., Pritchard (2005) for a detailed discussion and an alternative modal model.

³⁸See Carter, Jarvis & Rubin (2015).

Even if the artist's successful pot and Jeter's hit are unsafe—viz., they don't occur in most nearby worlds where we hold fixed the initial conditions—we nonetheless credit the success to the ability, even if it is not an ability to intentionally bring about the outcome at will more often than not.

To the extent that the successful potter and Derek Jeter's respective successes are due to their abilities, it is because, in light of their dispositions, they do much better than the *mean* in attaining success. For instance, it is because of Jeter's practice and form that he hits above the mean, even though he can't directly bring about it that he gets a hit. Likewise, I want to suggest, even though it was beyond Zhang's control to bring about his being struck with the 'eureka' experience he had, his being so struck is in an important respect creditable to his skills at the antecedent *preparation* and *incubation* stages, and that this point holds more generally.

Firstly, regarding the incubation stage. While Browne and Cruse (1988) have reported some effect of choosing relaxing (over other forms of) activity during incubation, several other studies have suggested that activity choice during incubation makes little difference (e.g., Dodds, Ward, and Smith 2003; Olton and Johnson 1976).

Preparation however is different. Skill at the preparation stage can, in one obvious sense, come as a result of manifesting one's *other* task-relevant intellectual virtues, i.e., by being intellectually tenacious, intellectually courageous, intellectually openminded, etc. in ways that bear on the problem task³⁹. This should not be surprising, and is clearly the case in our pet examples of Zhang, and also Wiles, each of whom worked tirelessly

³⁹One might object that it is somehow illicit to advert to other intellectual virtues in the service of explicating a different virtue. Perhaps, as this line of worry goes, doing so (in order to explicate the cultivation stage of virtuous insightfulness) undermines the case for thinking that insightfulness is a *distinct* virtue, but rather, that it simply 'reduces' in some way to other more basic intellectual virtues. I think a useful analogy to consider, in replying to this objection, is the famous Reid/Hume divide between anti-reductivists and reductivists in the epistemology of testimony. Is testimony itself distinctive as an epistemic *source*, or does it reduce to more basic sources? If the latter, then testimony is simply different from memory, perception and other basic epistemic sources. I'm entirely open to the thought that insightfulness be understood, as Humean reductivists think of testimony, along the lines of a 'derivative' intellectual virtue, in so far as the explication I'm offering for what makes an individual virtuously insightful makes needed recourse to the exercise of other intellectual virtues. However, even embracing this kind of Humean interpretation needn't involve denying that insightfulness is an intellectual virtue *at all*. In order to make that case, one must argue that it is criterial of any bona fide virtue that its explication make reference to no other virtue. This would be a very strong criterion. (After all, both Plato and Aristotle have in different ways regarded the virtues as unified).

prior to their breakthroughs⁴⁰. However, there are also more fine-grained features at the preparation stage which generate a higher likelihood of success relative to the mean. For example, results reported by Silveira (1972) have shown a positive correlation between the timing of one's interruption in insight tasks. In Silveira's experiment, she had experimental participants work on a chain-link insight task, and the participants were interrupted either 3 or 13 minutes after starting. Then, following a 30 incubation period, the subjects returned to the task to work, for either 32 or 22 minutes, whereas control subjects worked continuously for 35 minutes. Silveira found that whereas those interrupted 3 minutes in did only slightly better than control subjects, those interrupted 13 minutes in did significantly better than in the control case. As Dodd et al (2003, 7) noted, a conclusion to be drawn here is that 'incubation effects may be more apparent when the person solving the problem has had more time to become fixated prior to taking a break from the problem'. Of course, individuals can be better or worse at facilitating these conditions, and part of what makes an individual perform better relative to the mean is her resilience, focus and commitment to the task, traits an individual can be responsible for cultivating.

Putting this all together: while the illuminating experience of being struck with a 'eureka' moment is not typically brought about by deliberate direct control—these are things which we say *happen* to one, seemingly out of the blue—they are not a matter of luck, but rather, can be properly attributed to an insightful individual's virtues in light of how the insightful individual performs better than the mean in the (to use Wallas' model) preparation and incubation stages anteceding the moment of insight⁴¹.

⁴⁰For a detailed discussion on Wiles' preparation, see Singh (1997).

⁴¹While properly managing these antecedent steps plausibly increases the propensity to insight experiences, it might be that there are certain rare cases where little effort is required at these antecedent stages. For example, the mathematical genius Srinivasa Ramanujan had what Freeman Dyson called 'some sort of magic tricks that we don't understand'—viz., a capacity to inexplicably intuit deep mathematical results, for example concerning mock modular forms, without supplying proofs for these results. In the case of Ramanujan's insights about mock modular forms, several of these were later proven to be accurate by mathematical tools that were not available in Ramanujan's time. Such cases are plausible outliers in which no further effort at the preparation and/or incubation stage would increase the propensity of insight experience. Thanks to an anonymous referee at *Episteme* for noting this case.

4.2. Management stage of virtuous insight

§4.1 outlined how the virtuously insightful individual can, by excelling in the antecedent stages of preparation and incubation, perform better than the mean in generating the ‘eureka’ experience, despite such an experience being out of the individual’s direct performative control.

But of course, as was noted at the end of §3, the ‘eureka’ experience is but the *third* of four stages of the Wallas model of creative problem solving. And, as I suggested the move from the third stage to the fourth stage—i.e., judgment or endorsement—can be done more or less skilfully. Recall again here, the ‘bunny problem’, where a rabbit’s ‘hare-trigger’ instinct is needed to avoid danger, it is woefully unreliable; the hare-trigger response to perceived threat is a poor predictor (approximately 10%) of danger. And the point about the subjective indistinguishability between eureka experiences which accompany accurate versus botched grasping reveal a similar kind of problem of us: like the bunny, the feeling in cases of botched grasping is indistinguishable from the feeling of apt grasping.

In this section, I want to propose an account of how the virtuously insightful individual can be appreciated as skilled at moving *from* these felt experiences that follow preparation and incubation to judgment or verification (i.e. the fourth stage), where the aim of this stage is truth (or more broadly, understanding).

The first step here will be to consider more carefully the epistemic *structure* of the connection between the third and fourth stages, that is between the ‘eureka’ feeling, and the ensuing judgment or *endorsement* of the content issued by the grasping. Here, it will be helpful to draw from one established and increasingly popular model within which this kind of transition can be sharpened.

According to the *perceptual model of emotional experience* (e.g., Elgin 1999; Prinz 2004; Roberts 1988; Johnston 2001) emotional experience constitutes reason or evidence for evaluative judgement in a similar way to the way in which perceptual experience constitutes reason or evidence for empirical judgement⁴². One class of evaluative judgments are *epistemic* evaluations. And so one way to think about the epistemic significance of the kinds of ‘eureka’ feelings associated with insight are, within this model, specifically, as reasons or evidence for epistemic evaluations. On this view, the trans-

⁴²This presentation of the perceptual model owes to Michael Brady (2013, 67) who challenges the model and embraces instead a kind of ‘proxy’ evidence view of the epistemic role of emotional experience.

ition from stage three to four (i.e., from feeling to judgment) is a transition from feeling as evidence for an epistemic evaluation—an endorsement—of the insight.

Thinking about the transition from stage three to four within this model makes evident the place for the virtuously insightful individual to exercise skill. Just as, in the perceptual case, an individual can afford too much or too little epistemic weight to one's perceptions in forming empirical judgments⁴³, so likewise, an individual can afford too much or too little epistemic weight to one's emotional experiences (at the 'eureka stage') in epistemically evaluating the insight. For example, the virtuously insightful individual, upon having the 'eureka' experience, has fewer false negatives relative to the mean, in that the virtuously insightful individual does not ignore the kind of felt connections between information when these are epistemically valuable (i.e., of the sort that are likely to lead to true belief or understanding). Likewise, the virtuously insightful individual is appropriately *cautious* in moving from emotional experience to endorsement.

To give an example of the exercise of such caution, consider the following case, where cognitive biases are at play. Take, for example, *belief bias* (i.e., Klauer, Musch, and Naumer 2000) according to which individuals tend to give disproportionate positive evaluations of the logical strength of arguments the more believable their conclusions are judged to be. Many insights are graspings which *depend* on appreciation of argument cogency; naturally, one potential cause of botched grasping (i.e., a 'eureka' experience which does not generate *apt* insight) is belief bias. The virtuously insightful individual is more attuned than most in her *sensitivity* to the kinds of biases she might be afflicted with which could defeat the evidential weight of the 'eureka' experience. The virtuously insightful individual accordingly is best understood as occupying a kind of Aristotelian mean between affording too little or too much evidential weight to the kind of felt experience at stage 3, in her transition from the insight to endorsement. Put another way: the virtuously insightful individual is better than the mean at issuing *safe* endorsements, conditioned upon her felt experiences of insight.

⁴³At one limit is the individual who 'believes everything she sees', and is as such often deceived; at the other limit is the individual who deprives herself from perceptual evidence gained by others by the vice of over-cautiousness.

5. Concluding remarks

‘Aha’ experiences happen to some more than others—as if out of the blue—but virtuously insightful individuals are better than others at cultivating the right precursors to these episodes, and importantly, at moving in a responsible way from these felt experiences to endorsed insights. In this paper I hope to have shown how both the psychology of problem solving as well as the resources of virtue epistemology, paired together, offer some explanation for why this is so.⁴⁴

⁴⁴Thanks to Mark Alfano, Michel Croce, Modesto Gómez Alonso, Emma C. Gordon and Kate Elgin for helpful comments on a previous version of this paper. I am also grateful to an anonymous referee and the Associate Editor at *Episteme* and an especially insightful audience at the Cognitive Irrationality Project at the University of Basel, and in particular to Anne Meylan, Melanie Sarzano and Marie van Loon for organising the event.

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