Will done Better: Selection Semantics, Future Credence, and Indeterminacy

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Statements about the future are central in everyday conversation and reasoning. How should we understand their meaning? The received view among philosophers treats will as a tense: in ‘Cynthia will pass her exam’, will shifts the reference time forward. Linguists, however, have produced substantial evidence for the view that will is a modal, on a par with must and would. The different accounts are designed to satisfy different theoretical constraints, apparently pulling in opposite directions. We show that these constraints are jointly satisfied by a novel modal account of will. On this account, will is a modal but doesn’t work as a quantifier over worlds. Rather, the meaning of will involves a selection function similar to the one used by Stalnaker in his semantics for conditionals. The resulting theory yields a plausible semantics and logic for will and vindicates our intuitive views about the attitudes that rational agents should have towards future-directed contents.

1. Introduction

Our topic is the semantics for statements about the future in English. In particular, we focus on sentences involving the English auxiliary will, such as:

(1) Cynthia will pass her exam.

Sentences like (1) are uniquely interesting. An account of their meaning faces challenges from a number of philosophical domains: semantics, epistemology, and metaphysics.

The semantic challenge is generated by a tension in the linguistic behaviour of will. On the one hand, will has the characteristic marks of a modal operator. On the other, will fails to display the standard scope interactions of modals. For example, unlike must or might, will commutes freely with negation. That is, ‘It will be the case that it doesn’t
rain’ and ‘It is not the case that it will rain’ have the same truth conditions, despite the difference in relative scope between will and negation.

The epistemological challenge comes from the role of will-statements in everyday thinking and deliberation. We are often uncertain about the propositions expressed by will-claims, and at least sometimes this uncertainty seems rational. An adequate account of will should assign will-sentences contents towards which we do, and rationally may, have attitudes of this sort.

The metaphysical challenge comes from considerations about the open future. Past facts are settled, while at least some of the facts about the future seem not to be. Both the claim that the future is open and the nature of the relationship between the metaphysics and the semantics of the future are disputed. But the following seems uncontroversial: whatever the truth about the metaphysics, our semantic theory should avoid ascribing widespread error to ordinary speakers.

The existing literature is split between two general approaches, roughly mapping onto the divide between philosophers and linguists. Philosophers invariably treat will as a tense, i.e. an operator whose semantic function is to shift the time of evaluation of a clause. This view, often combined with a supervaluational account of the truth value of will-claims, is well-positioned to accommodate the epistemological challenge and can be developed so as to meet the metaphysical challenge. But it is problematic from a linguistic point of view. Most linguists (though not all of them) treat will as a modal—i.e. as an expression that manipulates a world parameter. Though typical modal accounts of will are well-positioned to accommodate the linguistic challenge, they flounder in the face of the epistemological challenge.

We propose a new theory of will that draws together elements from these two views and improves on all existing accounts. Here is a sketch. Following the dominant view in semantics, we hold that will is a modal. But will differs from standard modals like must or may, which work as quantifiers. The best analogy for will is the selection function meaning that Robert Stalnaker uses in his semantics for conditionals. That is, will selects a unique world from a distinguished set of worlds. Roughly, the selected world is ‘the world instantiating the one actual complete course of history’, among the ones that are compatible with history up to now. The approximate truth conditions of (1) are:

(2) In the actual complete course of history, Cynthia passes her exam.
Hence our semantics presupposes that there is a ‘unique’ actual course of history. At the same time, it might be indeterminate which possible world instantiates the actual course of history. As a result, it might be indeterminate which world will selects, and will-statements may have indeterminate truth values. We make room for this combination of views by distinguishing two levels of theorizing: on the one hand, the compositional semantic analysis of will; on the other, the proper treatment of the indeterminacy that (on some metaphysical views) affects will-statements. These levels are often conflated. We think it’s crucial that they be kept distinct. This yields a view that combines several desirable features: (i) it yields a plausible semantics and logic for will; (ii) it generates contents for will-statements towards which we can be rationally uncertain; and (iii) it makes room for (though doesn’t require) the metaphysical claim that the future is open.

Here is an overview of the paper. In §2 we outline some plausible constraints for a semantics for will. In §3, we give an informal overview of the account, which is stated in full in §5. (§4 spells out our metaphysical assumptions.) In §6 we explain how to define a notion of truth that makes room for indeterminacy. Finally, we check that our account yields the desired logical and epistemological predictions (§§7-8).

One last preliminary point: some uses of will have a so-called ‘volitional’ reading, i.e. they work as injunctions to the hearer to bring about the prejacent. For example, on its volitional reading (1) is an injunction to the hearer to see to it that Cynthia passes her exam. In this paper, we refrain from making claims about these uses, and about their connection to the more ordinary future-directed uses.

2. Semantics for the future: three constraints

Any plausible account of will, we believe, ought to respect three constraints.

2.1 The modal character of will

Our first constraint is that will should have a modal meaning. By this we mean that will manipulates a possible world parameter,\(^1\) similarly

\(^1\) It doesn’t matter to our account whether this world parameter is assumed to be represented in the object-language via world variables, or in the metalanguage via an index coordinate. When stating our semantics, we choose the latter option.
to modal auxiliaries like *must* or *might*. For example, here is a toy modal meaning for *will*:

\[ \Box \text{will} A \] is true at \( w \) and \( t \) iff for all worlds \( w' \) that are open possibilities at \( w \) and \( t \), \( A \) is true at \( w' \) and at some \( t' \geq t \)

(For now, just take ‘open possibilities’ to be possibilities that, for all that is settled at the time of utterance, might instantiate the future course of events.) The modal view contrasts with a temporal view, on which *will* manipulates exclusively a time parameter and no world parameter. Again, for illustration, here is a toy temporal view:

\[ \Box \text{will} A \] is true at \( w \) and \( t \) iff for some \( t' \geq t \), \( A \) is true at \( t' \) and \( w \)

To be clear: we understand the modal view as compatible with the claim that, in addition to the world parameter, *will* manipulates a time parameter. What distinguishes modal from nonmodal analyses is whether *will* manipulates a world parameter at all.

The linguistics literature has provided three pieces of evidence for the modal view. Taken together, they seem to us compelling.

The first piece of evidence is morphological. According to a widely accepted view (Abusch 1997, 1998; Condoravdi 2002; Kaufmann 2005), *will* shares morphology with the modal *would*. In particular, *will* and *would* have in common a modal morpheme, often represented as ‘WOLL’: *will* is present + WOLL; *would* is past + WOLL. The assumption of common morphology allows us to explain otherwise puzzling semantic facts. For example, it explains why we can replace *will* with *would* in indirect reports of past utterances of *will*-sentences. If, on Tuesday, Harriet says ‘I will come to work tomorrow’, then on Wednesday we would report Harriet’s utterance by saying ‘Harriet said she would come to work today’.

The second piece of evidence for the modal view is that *will* (on a par with other expressions that normally induce future reference, like *going to*) may have epistemic readings (Palmer 1987; Encç 1996). These readings generally require a stative predicate (like *be*) in the prejacent.\(^2\) Here is an example:

\((3)\) John will be in London by now.

Notice two facts about \((3)\). First, as is made clear by the modifier *by now*, the prejacent of \((3)\) has its reference time in the present. Hence, at the time of utterance, it is settled whether John is in London.

\(^2\) Following common (and medieval) usage, we use the term *prejacent* to denote the clause that *will* takes scope over.
Second, in (3), *will* works as a marker of evidentiality: roughly, it signals that the speaker is inferring John’s location on the basis of a body of evidence. To see this, notice that (3) is infelicitous if uttered by someone who is looking directly at John, even if both are indeed in London. Both these facts are hard to explain on a purely temporal view. By contrast, views on which the languages of uncertainty and prediction are both modal in nature seem ideally placed to account for the data.  

Third, as Peter Klecha (2013) has recently argued, *will* allows for modal subordination. Roughly, modal subordination is the phenomenon whereby, in discourses containing several modals, earlier modals may restrict the domain of later modals (Roberts 1989). As an example, consider the following discourse:

(4) Jane might come to the party. Sally would come too.

The occurrence of *would* in the second sentence is naturally understood as restricted to worlds where Jane does come to the party. A natural explanation for this is that its domain of quantification is somehow anaphoric to the worlds that witness the *might*-claim in (4). Klecha points out that, similarly to *would* and other modals, *will*-sentences can inherit restrictions from previous modal elements of the discourse.

(5) If the supplies arrive tomorrow, it will be late in the day. They will contain three boxes of cereal.

(6) The supplies might arrive tomorrow. It will be late in the day.

In this respect, *will* patterns with modals like *might* and *would*, and not with tenses, like the past tense:

(7) a. If the supplies arrive tomorrow, it might be late in the day. They might contain three boxes of cereal.

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3 An anonymous reviewer challenges our claim that *going to* has the relevant present-directed readings. We agree that it is sometimes more natural to use *will* in these constructions. However, our informants uniformly agree that both of the following sentences sound good in the appropriate contexts:

(i) The swordfish is going to be ready now.

(ii) The swordfish is not going to be ready yet.

Given that present-directed *going to* can have a similar meaning as *will*, and given that the next argument for a modal view does apply to *going to*, we think that there is about as much reason for a modal analysis of *going to*.
b. # If the supplies arrived yesterday, it was late in the day.
They contained three boxes of cereals.

(8) a. The supplies might have arrived yesterday. It would have been late in the day.
b. # The supplies might have arrived yesterday. It was late in the day.

These facts hold in languages other than English. This speaks against a view that tries to accommodate the modal character of *will* by claiming that the English *will* is ambiguous between a modal and a nonmodal meaning. (More on this in §2.4.)

The evidence in favour of a modal view seems quite strong to us. At the same time, not everyone finds it convincing. To motivate our project, we don’t have to take the evidence to be definitive. All we need is that it be strong enough to make the modal view a serious contender.

2.2 Scopelessness
Our second constraint is that *will* is scopeless with respect to an important class of other linguistic items. By this we mean that changes in the relative syntactic scope between *will* and these other items don’t make a difference to the truth conditions of *will*-sentences. This is a remarkable feature of *will*, and one that is not generally shared by modal expressions.

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4 An anonymous reviewer suggests that Klecha’s modal subordination argument might fail for Romance languages. According to the reviewer, in those languages, modal subordination only obtains if *will* is translated in the conditional mood. This does not appear to be quite right. We have surveyed nine Italian and two French informants, asking them to rate discourses like (5)-(6), both in the indicative and conditional mood, and (7)-b and (8)-b. A large majority of our informants accepts the translations of (5)-(6) with the future indicative (though there is an overall preference for their variants in the conditional mood). Moreover, and crucially, nearly all of them strongly prefer the future indicative variants over the past tense variants, i.e. the translations of (7)-b and (8)-b.

5 Modal analyses are a majority among linguists, but not universally accepted. Arguments against them are found in Comrie (1989) and Kissine (2008). In addition, von Stechow (1995) extensively develops a nonmodal view. We believe that many anti-modal arguments can be resisted: a good starting point is Portner’s (2009, pp. 239-40) critique of Kissine’s arguments.

6 The point is widely acknowledged in the literature, since at least Thomason (1970). See also the discussion of excluded middle for *will*-sentences in Copley (2009) and of the interactions between future operators and negation in MacFarlane (2014, p. 216).
For present purposes, it is enough to observe scopelessness with respect to negative items, as illustrated by:\(^7\)

\((9)\)  
\(a. \) It will not rain.  
\(b. \) It is not the case that it will rain.

\((9)-a\) and \((9)-b\) are truth conditionally equivalent. The situation is similar with different preajcents, and when clauses like \((9)-a\) and \((9)-b\) are embedded in other environments. In short, \textit{will} appears to commute freely with ordinary English negation. This observation is strengthened by considering items that lexicalize negation, such as \textit{doubt} (which, following common assumptions, we understand as \textit{believe that not}) and \textit{fail}: \((10)-a\) and \((10)-b\) are truth conditionally equivalent:

\((10)\)  
\(a. \) I doubt that Sam will pass his logic exam.  
\(b. \) I believe that Sam will fail his logic exam.

For a comparison with an auxiliary that is not scopeless, consider minimal variants of \((10)-a\) and \((10)-b\) that involve a deontic modal. Suppose we’re talking about the obligations that Sam must fulfil in order to stay enrolled in his degree. It is clear that \((11)-a\) is \textit{not} truth conditionally equivalent to \((11)-b:\)

\((11)\)  
\(a. \) I doubt that [in order to graduate] Sam must pass his logic exam.  
\(b. \) I believe that [in order to graduate] Sam must fail his logic exam.

The lack of scope interactions with negation immediately yields an interesting logical constraint:

\textbf{Will Excluded Middle} (preliminary take):  
\(\neg will A \lor will not A\) is a logical truth.

For now, we informally gloss ‘logically true’ as ‘true whenever uttered’. In §7, we derive the validity of this schema, given our semantics and two standard formal concepts of consequence.

\(^7\) Perhaps the scopelessness of \textit{will} extends further. We think it is likely that \textit{will} is scopeless with respect to comparative expressions. (See the discussion of comparatives and conditionals in Korzukhin 2014.) For reasons of space, we limit ourselves to considering negation.
2.3 *The cognitive role of future statements*

Future-directed statements play an important role in our cognitive economy. It is a platitude that ordinary agents are uncertain about the future. Assuming that credences attach to propositions, it seems natural to understand ‘being uncertain about the future’ as ‘having non-extreme degrees of belief towards the propositions that are expressed by *will*-claims’. Moreover, at least in some cases, these non-extreme degrees of belief seem also rational. A semantics for *will* should yield contents for *will*-claims that are appropriate inputs to our theories of attitudes. For illustration, consider the following case:

**Sports Fan:** Suppose that Cynthia comes to work each day wearing a Warriors cap, a Giants cap, or no cap, depending on a random draw (with each option having equal probability). You are certain that for each of the three caps, it is an open possibility that Cynthia wears that cap tomorrow. What degree of belief should you assign to the proposition that tomorrow she’ll wear a Warriors cap?

Presumably ‘1/3’ is a rationally permissible answer. In some theoretical settings, it may even be required: if some version of Lewis’s (1986c) principal principle is a requirement of rationality and all your evidence is of the admissible variety, ‘1/3’ would appear to be the only rational answer. Similarly, it seems that the fair odds for a bet on the truth of that proposition are 1 to 2.

These claims seem to be truisms, yet they are surprisingly hard to vindicate on a family of existing semantics for *will*, i.e. modal accounts. For illustration, consider the toy modal semantics mentioned in §2.1. This theory treats *will* as a universal quantifier over the open possibilities at the point of utterance. (This account captures what Prior 1967 calls the ‘Peircean future tense’.)

The problem for this semantics is that if Warriors-cap futures and Giants-cap futures are both possible, you should have zero credence in the propositions expressed by each of (12) and (13):

(12) Cynthia will wear a Warriors cap tomorrow.

(13) Cynthia will wear a Giants cap tomorrow.

To see why the theory makes this prediction, recall that you are certain that all the headgear options are open possibilities at the time of utterance. It follows that you are certain that the truth condition for
each of (12) and (13) does not hold. In the next section, we show that virtually every theory that treats will as a universal quantifier faces this problem.

2.4 Surveying the options

It is difficult to satisfy all three constraints. For one thing, the first two seem to be in direct tension with each other. If will is a modal, as the first constraint requires, we expect it to have nontrivial scope interactions, in violation of the second constraint. And indeed, basic modal analyses of will predict that switching the relative scope of will and negation does have truth conditional effects. For illustration: on Kaufmann’s (2005) account, will is a universal quantifier over (roughly) the worlds realizing the most likely courses of future history. On this theory, by switching around will and not, we get the two non-equivalent readings:

will > not: all the most likely futures do not satisfy the prejacent.
not > will: not all the most likely futures satisfy the prejacent.

The linguistics literature offers attempts to reconcile the first two desiderata. A prominent example is the modal analysis of Copley (2009). Building on work by von Fintel on generics (1997), Copley claims that will-sentences presuppose that their domain is homogeneous with respect to the prejacent. For an occurrence of will in a sentence of the form \( \neg \text{will } A \) to have a denotation, its domain must contain only \( A \)-worlds or only \( \neg A \)-worlds. We have concerns about the stipulative character of this proposal. But we can set them aside, because Copley’s theory, like all existing modal theories, runs into a more basic problem: it fails to address our third constraint. The propositions that modal theories deliver are not propositions that we can plausibly have non-extreme credences in.

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In a similar spirit, Belnap et al. (2001, p. 160) object to the Peircean that there is a difference between a bet that it will rain tomorrow and a bet that it is inevitable that it will rain. One can win the former without winning the latter. Also, the problem is structurally analogous to one that has recently received attention in the counterfactuals literature. (See Hawthorne 2005; Edgington 2008; Moss 2013; Schulz 2014.)

Notice that the constraint we are discussing is not about cognition, but about the contents that are the final output of the semantics. One may be sceptical about the connections between possible worlds semantics and a theory of cognition. But it is standard to assume that compositional semantics should deliver contents that are suitable objects for propositional attitudes. For example, classical models of assertion in semantics (e.g. Stalnaker’s 1978) assume that contents of assertions and contents of attitudes may be represented via formal objects of the same kind, and that the former may be used to update the latter.
To see the problem, consider (1) again:

(1) Cynthia will pass her exam.

Despite their important differences, existing modal theories share a common core. They treat will as a universal modal whose domain is a subset of the set of worlds that are ‘open’ at the time of utterance—i.e. worlds that, for all that is settled at the time of utterance, may capture the future course of events. (More about this in §4.) This subset consists of the worlds that are maximal relative to a contextually supplied ordering. Individual accounts contribute different interpretations of the ordering. The domain might consist of the maximally likely open worlds (Kaufmann 2005), or of the maximally normal open worlds (Copley 2009), or of the worlds that maximally match the speaker’s knowledge (Giannakidou & Mari 2015) (to mention only a few of the available options). The resulting truth conditions for (1) are:

(1) is true iff, for all w s.t. w is one of the best open worlds, Cynthia passes her exam in w.

Suppose, however, that you are certain that Cynthia’s passing or failing the exam are both represented within the set of best open worlds. Suppose, that is, that Cynthia passes her exam at some best open worlds and fails it at some others. In this case, existing modal views require that your credence in (1) be zero. On those views, (1) says that all the best worlds are worlds where Cynthia passes, while you’re certain that in some best worlds she passes, and in some others she doesn’t. But this prediction is obviously wrong. You ought to (and generally do) assign positive credence to the content expressed by (1)—witness the fact that you should (and would) be disposed to accept at least some bets on it.

Copley’s (2009) assumption that the domain is presupposed to be homogeneous with respect to the prejacent does not help here. On this view, (1) suffers from presupposition failure in the scenario we have described. It is unclear what credence, if any, one should assign to the content of a sentence in a context that violates the sentence’s

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9 Arguably, plenty of natural language cases fall in this category. Here is one that seems uncontroversial to us. Suppose that coin tosses are genuinely indeterministic, that there is a .5 chance that the coin that you’re going to toss will land tails, and that you believe this. Then consider:

(i) The coin I’m about to toss will land tails.

The set of closest worlds used to evaluate (i), by any of the metrics used in the literature, will presumably include both heads- and tails-worlds.
presuppositions. It seems both irrational and unusual to assign them ordinary positive credences. For instance, consider the proposition expressed by ‘The King of France is bald’. It seems irrational to assign positive credence to that proposition while also being certain that France is not a monarchy. And indeed, ordinary agents have no temptation to do so. By contrast, it is routine for agents to assign positive credences to will-claims in situations of uncertainty about the future. Hence the contents delivered by existing modal theories of will are inadequate.

Let us now peek quickly at the philosophical literature. By far the most popular view among philosophers is what Prior calls ‘Ockhamist semantics’. Ockhamists don’t ascribe any modal character to will. For them, ‘It will rain’ is true (in a world w and at time t) if and only if there is a moment t+ in the future course of w (i.e. after t) such that it rains (in w and at t+). (Context might further narrow the interval during which t+ is situated.) The obvious problem with Ockhamism is its inability to satisfy our first desideratum. The Ockhamist has no story about the relationship between will and would, about predictive uses of will, or about modal subordination.

It might appear that classical supervaluationism is an exception to this pattern. Classical supervaluationists (e.g. Thomason 1970, 1984; Belnap & Green 1994) complement the Ockhamist semantics with the idea that a sentence like (1) is true simpliciter just in case it is true at the time of utterance in every open future. But supervaluationism is not a modal theory in the sense that matters to us here. Though it has a modal element, this element is not distinctive of the lexical entry for will, but appears in the ‘global’ definition of truth. For this reason, classical supervaluationism is unable to account for the evidence for a modal treatment of will we summarized in §2.1. This is not to say that supervaluationist theories are entirely on the wrong track. Our own account brings together a modal analysis of the compositional contribution of will with a supervaluationist-inspired picture of indeterminacy.\footnote{There are also important points of contact between the present theory and the selection-based account of conditionals in branching time in Thomason & Gupta (1980). In addition, Todd (2016) has recently defended a variant of the Ockhamist view on which all unsettled will-claims are false (see also Schoubye & Rabern 2016 for criticism of Todd’s view). We defer a direct comparison between our theories to future work.}

One final option is to claim that will is ambiguous between a modal and a nonmodal meaning. The modal meaning explains why will seems to satisfy the first desideratum. The nonmodal meaning explains why it seems to satisfy the second and third desiderata. We won’t...
attempt a full refutation of the ambiguity option; but we notice that it
has two major disadvantages. First, it systematically over-generates.
For example, it predicts a true and a false reading for:

(14) The probability that Cynthia will wear a Warriors cap is 0.

(14) is true on the modal meaning and false on the nonmodal mean-
ing, so we should be able to hear it as true. Perhaps there are man-
oeuvres to be made to block this reading, but we leave it to the
ambiguity theorist to explain what they are. Second, an account of
will that does not exploit ambiguity seems obviously preferable on the
usual grounds of simplicity and theoretical unity. So, by giving an
account of will that satisfies all desiderata we provide an indirect ar-
gument against the ambiguity view.

3. Overview: selecting the future

We present our full account of will in the next few sections, but it is
helpful to illustrate the central ideas without the formalism. We start
by adopting some (but not all) of the insights associated with branch-
ing time frameworks. At every moment in time, we suppose, there are
multiple possible histories that fully coincide with respect to the past
and diverge with respect to the future. As time passes, histories that
had previously coincided up to a point part ways, ‘making true’ dif-
f erent courses of events. In diagram form:

\[
\begin{array}{c}
\text{\scriptsize{t_0}} \\
\text{\scriptsize{t_1}} \\
\text{\scriptsize{t_2}} \\
\text{\scriptsize{t_3}} \\
\text{\scriptsize{t_4}} \\
\end{array}
\]

\text{Figure 1}

This picture is often combined with substantial metaphysical claims
about the nature of possible worlds and the indeterminacy of the
future. Importantly, our account is neutral between the main
background options. (We clarify this in §4.) All we need is that, given any world \( w \) and time \( t \), we can determine the historical alternatives to \( w \) at \( t \). Here is how we do it:

Two worlds \( w \) and \( v \) are historical alternatives at \( t \) iff \( w \) and \( v \) match perfectly in their history (i.e. iff they match perfectly in matters of particular fact) up to \( t \).

The notion of a perfect match in matters of particular fact is borrowed from David Lewis (1979a, 1983). Two worlds that perfectly match in matters of particular fact up to a certain point in time are duplicates—indiscernible copies of each other—up to that point.\(^{11}\)

Note that our definition of historical alternatives involves no reference to a notion of openness. This is key to our later vindication (§4) of the claim that our semantics is neutral about the open future hypothesis.

Now, consider again (12), repeated here:

(12) Cynthia will wear a Warriors cap tomorrow.

As a first step, we assume that \textit{will} is a modal. Like all modals in natural language, it is interpreted against a background set of possibilities. Following Kratzer’s terminology (1977, 1981a, 1991b), we call this set the modal base. For the particular case of \textit{will}, the modal base in a given context is the set of historical alternatives to the world of the context, at the time of the context. For example, in the scenario we described, the modal base of (12) includes worlds where Cynthia wears a Warriors cap, worlds where she wears a Giants cap, and worlds where she wears no cap. In diagram form:

\[\begin{array}{c}
t' \\
\text{Warriors cap} \\
\downarrow \\
\text{Giants cap} \\
\downarrow \\
\text{no cap} \\
\end{array} \]

\[\begin{array}{c}
t \\
w \\
\uparrow \\
v \\
\uparrow \\
z \\
\end{array} \]

\textbf{Figure 2}

\(^{11}\) We think that a metaphysical notion of duplication, like the one we just invoked, is clear enough to put to work in defining the modal base of \textit{will}. This follows Lewis (1979a), who deploys it in his official statement of the criteria for similarity used in counterfactual semantics. Alternatively, one may define a notion of indistinguishability based on a canonical language, as in Thomason (1984). We do not need the extra flexibility afforded by Thomason’s notion.
Standardly, the modal base is the domain of quantification of the modal. But, as we anticipated, our account is not quantificational. Instead, we propose that will singles out one world within the modal base, and evaluates the prejacent at that world. Intuitively, the selected world represents the ‘way things will actually be’—in other words, the historical alternative that will actually be realized. So, (12) is true just in case, in the selected world, Cynthia wears a Warriors cap tomorrow.

The explanation for the scopelessness of will (and consequently the validation of will-excluded middle) flows immediately from the fact that the prejacent is always evaluated relative to a single world. (See §7.)

Our semantics for will presupposes that, at the time of utterance, there is a unique, fully specified way things will actually be. (In the jargon introduced by Belnap & Green 1994, this is the assumption that there is a ‘thin red line’ that marks the complete course of actual history.) This assumption is controversial. Theorists in the branching time tradition object that, in the context of future-directed discourse, we have no right to speak of ‘the way things will actually be’, or of ‘the actual world’. On the one hand, it might be that the future is open—that there is no fact of the matter, at the current time, about what way things will turn out. On the other, even if the future is not open, it is not clear that a semantics for natural language can legitimately presuppose a metaphysical claim of this sort.

Even if one agrees with these concerns, we don’t think that the compositional semantics for will needs to be changed. We distinguish what information is needed by the compositional semantics from what information the world is able to supply. We assume that the compositional semantics requires as input a unique world of utterance. Like all parameters used in semantic computations, the value of this parameter is supplied by the context. At the same time, we leave it open that it may be indeterminate what context the utterance takes place in, and hence which world is supplied to the semantics.
For illustration, consider again (12). Perhaps, at the time of utterance, it is indeterminate whether the actual world is a Warriors-cap-world, a Giants-cap-world, or a no-cap-world. If so, it is indeterminate which context the utterance of (12) takes place in. The context might be the context of Figure 3, or it might be a context in which some other world (for example, v, as in Figure 4) is selected.

Let us highlight an important point. We grant that it may be indeterminate which world an utterance takes place in; moreover, we said that the modal base of will is determined as a function of the world and the time of the context. But it doesn’t follow that the modal base of will is indeterminate. The reason is that, given the way that we have defined historical alternatives, all worlds that are candidates for being the world of the context have the same historical alternatives. So we will be able to speak of the modal base of will in a context, even if it is indeterminate what context the utterance takes place in.

The next few sections execute the plan we just sketched. §4 specifies some metaphysical background. §5 presents our compositional semantics, including an analysis of will-conditionals. §6 elaborates our treatment of indeterminacy; §7 shows how our account yields the logical and linguistic predictions we identified; §8 shows how our account yields appropriate predictions about the cognitive role of will-statements.

4. Metaphysical background

Our account is neutral on a number of metaphysical issues connected to branching. In this brief section we explain how.

First, supporters of branching time often claim that possible worlds literally share initial temporal segments. (See e.g. Thomason 1970; Belnap & Green 1994; Belnap et al. 2001.) The point at which two worlds branch is the point at which the initial segment ends. By contrast, opponents of branching argue that worlds with identical histories up to a point are qualitatively identical, but still have no parts in
common. (See e.g. Lewis 1986a.) We understand the claim that two worlds \( w_1 \) and \( w_2 \) are historical alternatives at a time \( t \) as the weak claim that there is a perfect match between matters of particular fact between \( w_1 \) and \( w_2 \) up to \( t \). This is compatible both with genuine overlap and with mere indistinguishability.

Second, the branching framework is often associated with the claim that the future is ‘open’. The relevant contrast here is with past events, which are taken to be fixed in a way in which the future is not. There are a number of ways to explain the relevant concept of openness. Following Barnes & Cameron (2009), we choose one that is noncommittal between different metaphysical theses.

**Openness**: (at least some) contingent facts about how things will be are presently unsettled.

Some writers (for example, Belnap & Green 1994) adopt Openness as the starting point of the enterprise of giving a semantics for *will*. Others (like MacFarlane 2003, 2008, 2014) take it as a methodological *desideratum* that a semantics for the future should not decide between different metaphysical options about Openness.

We are not committed to any of these claims. Unlike Belnap and Green, we don’t assume Openness. Our apparatus is compatible with both Openness and its denial. Unlike MacFarlane, we don’t endorse the neutrality of the semantics as a methodological constraint. As it happens, however, our semantics for *will* does turn out to be metaphysically neutral about Openness—in the sense that both the defender and the opponent of Openness are able to use it. The reason is that we separate the design of the compositional semantics from the account of indeterminacy. As a result, both the supporter of Openness and its opponent can adopt the compositional meaning we assign to *will*. They will diverge on whether they accept the suggestion that it is indeterminate which context an utterance takes place in. (See §6.) But we do not claim that the metaphysical neutrality is, in itself, a reason to accept our account.

5. **Semantics**

5.1 **Setup**

Let us start by introducing some notation. We use italicized capital letters (‘\( A \)’, ‘\( B \)’, etc.) as metalinguistic variables over sentences; and boldface letters (‘\( A \)’, ‘\( B \)’, etc.) as metalinguistic variables over sets of
worlds. We use ‘propositions’ and ‘sets of worlds’ interchangeably, but everything we say is meant to be compatible with theories according to which propositions merely determine sets of possible worlds, without being identical to them.

As is standard in semantics, we define an interpretation function of the form,

\[
[\cdot]_{\text{parameters}.g}
\]

Such a function assigns truth values compositionally to sentences relative to a series of parameters and an assignment function, conventionally denoted by ‘g’. (The latter is just a function that assigns objects to syntactic indices, and is needed to handle variables.) Different theories employ different parameters. The interpretation function is also relativized to a context, but to remove clutter we avoid explicit mention of the context unless strictly needed.

We also make some specific assumptions about \textit{will}. First, \textit{will} is a sentential operator, i.e. an operator that takes a full clause as argument. This is a simplification, but one that is harmless given our purposes. Second, as we flagged in §3, \textit{will} takes as argument a modal base, i.e. a set of worlds that are used for the interpretation of the modal.\footnote{This understanding of modal bases is slightly simplistic. Modal bases are officially functions from worlds to propositions. (See von Fintel & Heim 2011 for discussion.)} In particular, the modal base of \textit{will} is the set of historical alternatives to the world of the context. Syntactically, we assume that modal bases are the semantic values of covert pronouns that work as arguments of modals. We represent these pronouns as ‘\textit{fi}’ , and their values, sets of worlds, as ‘\textbf{F}_i’ . For shorthand, we generally represent modal bases in LFs just as a subscript of modals; hence we write ‘\textit{will}_{f}’ rather than the more extended ‘\textit{will} [f]’.

### 5.2 Semantics for will

Our semantics for \textit{will} is based on an extended analogy with Stalnaker’s (1968) semantics for conditionals.\footnote{After finishing this paper, we discovered that the idea that \textit{will} has a selection semantics is also briefly entertained in a recent paper by Kratzer (2016). Schulz (2014) has recently defended an interesting variant of Stalnaker’s semantics. Roughly, conditionals quantify over a set of worlds, but they also select (via a choice function) an arbitrary world within that set. We lack space for a full comparison here. Let us just state without argument or development that, on the most natural implementation, building the analogy with Schulz’s system rather than Stalnaker’s would require us to consider indeterminacy at the level of the compositional semantics, which we are reluctant to do.}

\footnote{\textit{This understanding of modal bases is slightly simplistic. Modal bases are officially functions from worlds to propositions. (See von Fintel & Heim 2011 for discussion.)}}
As in Stalnaker’s semantics, we assume that the interpretation of will involves appeal to a selection function, denoted by ‘s’. A selection function maps a pair of a world w and a proposition A to a ‘selected’ world w’. Intuitively, s selects the world w’ that is ‘closest’ to the starting world w while at the same time verifying proposition A. For the case of conditionals, and counterfactuals in particular, there is much literature on how exactly the metric of closeness should be construed.\(^{14}\) We don’t need to settle these issues here. We can adopt any of the metrics that have been proposed for counterfactual conditionals.\(^{15}\)

Selection functions are characterized by two important constraints:

- **Inclusion**: if A is non-empty, \(s(w, A) \in A\).
- **Centering**: if \(w \in A\), \(s(w, A) = w\).

Inclusion says that the world selected must verify the input proposition (provided that some world does verify the input proposition). Centering says that, if the input world verifies the input proposition, then the world selected is the input world itself. Inclusion and Centering are the only constraints we impose on selection functions, which can then be defined as follows:\(^{16}\)

A function \(s : W \times \mathcal{P}(W) \rightarrow W\) is a selection function iff

\[
\text{i. if } A \text{ is non-empty, } \ s(w, A) \in A, \text{ and } \\
\text{ii. if } w \in A, \text{ then } \ s(w, A) = w. 
\]

At this point, we’re ready to state the meaning of will. We assume that interpretation is relativized to three parameters:\(^{17}\) a world of evaluation \(w\), a selection function \(s\), and an assignment \(g\).

\[
[\text{will}_f A]^{w,s,g} = 1 \text{ iff } [A]^{s(w,g(f)),s,g} = 1 
\]

To simplify the notation, we will just write \(F\) instead of \(g(f)\) throughout the paper.

\(^{14}\) For some sample proposals concerning the metric of closeness for counterfactuals, see Lewis (1979b); Kratzer (1981b); Hiddleston (2005).

\(^{15}\) As will soon be evident, the choice between different metrics only matters for will-conditionals. All we need to settle the selected world in all other cases is the centering condition. (See below.)

\(^{16}\) Stalnaker imposes some extra constraints on selection functions. We leave it open whether these extra constraints should apply to will; nothing hinges on these for our purposes.

\(^{17}\) Recall that, to avoid clutter, we omit the context parameter.
Let us make some comments about this compositional semantics.

First, the basic effect of will is to shift the world at which its prejacent is evaluated. This feature is shared with standard semantic accounts of modals in natural language, like must. The difference is that modals usually introduce quantification over the world of evaluation parameter, while will replaces the world of evaluation with another one picked via the selection function.

Second, the entry in (15) does not reflect any temporal shift. It is easy to introduce temporal shift, letting will quantify existentially over times. (Accordingly, interpretation is relativized to an extra parameter for times.)

\[(\text{will}A)^{w,t.s,g} = 1 \iff \exists t' \geq t, \ [A]^{s(w,F),t',s,g} = 1\]

This said, throughout the discussion we stick to the entry in (15). This is mostly for simplicity. The central innovation we introduce is the appeal to selection functions. Other elements of the meaning of will can stay in the background. Moreover, there are reasons to think that a full-blown semantics for will exploits time in a way that is more complex than simple existential quantification over temporal instants.\(^{18}\) So the account in (16) would need update and clarification anyway.

Third, this semantics has an interesting consequence for unembedded occurrences of will: as it turns out, will is semantically vacuous with respect to the modal parameter. Recall that the modal base of will defaults to the set of historical alternatives to the world of the context.\(^{19}\) Furthermore, the initial world of evaluation defaults to the world in which the utterance takes place.\(^{20}\) Hence, when will is unembedded, the world that works as the input to the selection function is a member of the modal base. In this situation, the centering assumption entails that the world returned by the selection function is always the

---

\(^{18}\) In particular, as argued by Abusch (1998) (see also Condoravdi 2002), it seems that our best semantics for tense should quantify over intervals rather than instants. In the context of this theory, the semantic effect of will (as well as of other modals) would not be to shift the time of evaluation, but rather to extend forward the time interval at which the prejacent is evaluated. (For a proposal in this vein, see also Kaufmann 2005.)

\(^{19}\) It might be valuable to consider a variant of our semantics that assigns to will an epistemic modal base. We do not do so here, but for another account of the future on which will has a partly epistemic meaning, see Giannakidou & Mari (2015), who discuss the case of Greek and Italian.

\(^{20}\) This is via the definition of truth at a context, which fixes the value of w to the world of the context. See §6 for details.
world of evaluation itself. Thus, in its unembedded occurrences, *will* merely ‘overwrites’ the world of evaluation parameter with itself.

\[(\text{[will}_\text{f} A])^{w,s,g} = 1 \iff [A]^{w,s,g} = 1\]

Thus, when *will* occurs unembedded, our semantics effectively collapses on the simple Ockhamist semantics, which treated *will* as a mere tense.

Why bother, then, with the complexities of our selection function semantics? There are many good reasons. They mainly relate to the fact that, on our account, *will* has a modal base. This opens up the possibility of accounting for *will*-conditionals (adopting the popular assumption that *will*-clauses function as restrictors), modal subordination (via anaphoric links between the modal bases of the different modals), and epistemic readings (by assuming that the modal base can have different flavours). The selection function account also allows for a vindication of the *will*-would connection.

Giving a full-fledged account of all these phenomena would take too long. But below we give a brief sketch. Even from these remarks, it should be clear that our account provides the tools for vindicating the modal character of *will*.

### 5.3 Applications

#### 5.3.1 Conditionals and modal subordination

A selection function semantics for *will* allows for a natural account of *will*-conditionals. This also provides the tools required for an account of modal subordination.

Following a longstanding tradition (see e.g. Lewis 1975; Kratzer 1991a, 2012) we assume that the function of *if*-clauses is to restrict modal bases—to rule out of the modal base the worlds that are incompatible with them. (This effect is modelled by intersecting the modal base with the set of worlds individuated by the *if*-clause.)

There are many ways to implement this semantic effect. Building on work on quantifier domain restriction by Kai von Fintel (1994), we choose a simple one that dovetails well with our assumption that the object-language syntax contains a variable referring to the modal base. Nothing hinges on this particular choice of implementation.

We assume that *if*-clauses work as assignment shifters (similarly to lambda-binders in a system in the style of Heim & Kratzer 1998). At a
syntactic level, *if*-clauses are co-indexed with the relevant modal base variable. For example, the LF of (18) is in (19):

(18) If John goes to London, he will meet with Matthew.

(19) [If John\(_1\) goes to London] \(_4\) will\(_{f_4}\) [he\(_1\) meet with Matthew].

At a semantic level, conditionals are interpreted via a rule that instructs us to perform assignment shift, mapping the modal base variable to a set of worlds determined by intersecting the old modal base with the proposition expressed by the antecedent. Formally:

\[
\frac{}{\text{recall that } F=g(f) \text{ and } A=\{w : [A]^{w,s,g} = 1\}}
\]

To illustrate this, consider again (18). Given modal base \(F\), let \(g^*\) be the assignment that coincides with \(g\) except at index 4, which is mapped to the set of worlds in \(F\) at which John goes to London (i.e. \(g^* = g[4 \rightarrow F \cap \text{[John goes to London]}]\)). Then we predict:

\[
\frac{\text{[(If } A) (MODAL\_f B)]^{w,s,g} = [MODAL\_f B]^{w,s,g[f \rightarrow F \cap A]}}{\text{[(18)]}^{w,s,g} = [\text{will}_{f_4} [\text{he}_1 \text{ meet Matthew}]]^{w,s,g*}}
\]

Informally, and simplifying, the resulting truth conditions of (18) are:

\[
\frac{}{\text{[(18)]}^{w,s,g} = \text{true iff John meets Matthew at } v, \text{ where } v \text{ is the world that is selected when } s \text{ is given as input the set of the historical alternatives (to } w) \text{ where John goes to London.}}
\]

Notice that the selected world need not coincide with the actual world or with the world of evaluation. In particular, for any \(w\) such that John does not go to London at \(w\), the world selected by \(s\) taking \(w\) as input must be different from \(w\) itself. In this case, our Stalnakerian semantics and semantics in the Ockhamist tradition diverge.

This treatment of conditionals also yields a straightforward account of modal subordination. Consider again Klecha’s example:

(5) If the supplies arrive tomorrow, it will be late in the day. They will contain three boxes of cereal.

We can predict the relevant interpretation of (5) by assuming that the modal base variables associated with the two occurrences of *will* are co-indexed:

(23) If the supplies arrive tomorrow, it will\(_{f_3}\) be late in the day. They will\(_{f_4}\) contain three boxes of cereal.
The antecedent *if the supplies arrive tomorrow* shifts the value of the relevant index. But, given co-indexing, both occurrences of *will* are interpreted in the scope of the relevant supposition.\(^{21}\)

5.3.2 *The will-would connection*

Our treatment of *will* also allows us to vindicate the morphological connection between *will* and *would*. The precise nature of this connection depends on one’s views about the meaning of *would*.

On the one hand, if one assumes a Stalnakerian semantics for *would*, then the connection is immediately vindicated: *will* and *would* turn out to have exactly the same meaning—*modulo* differences in what possibilities are in the modal base in the two cases. Of course, Stalnaker’s semantics for *would*, and in particular the principle of Conditional Excluded Middle that it entails, are controversial. But, first, notice that all the arguments that we gave above for the scopelessness of *will* carry over to the case of *would*. Moreover, the literature has provided plenty of further arguments in support of Conditional Excluded Middle. (See e.g. von Fintel & Iatridou 2002; Williams 2010; Klinedinst 2011.\(^{22}\))

On the other, the connection is not straightforward if one adopts a Lewisian semantics for *would*. In this case, one will have to explain why *will* deploys a selection function while *would* has universal quantificational force. While this is a nontrivial task, a selection function account of *will* is better placed to fulfil it than an Ockhamist semantics.

5.3.3 *Epistemic readings of will*

Recall the example we used to introduce epistemic readings of *will*:

\[(3)\] John will be in London by now.

*Will* in (3) is not used to talk about the future, but rather has an epistemic reading. Predicting how and when *will* receives an epistemic reading goes beyond the scope of this paper. These questions connect to general phenomena in the semantics of modality, and its relationship with tense and aspect, that we can’t cover here.\(^{22}\) But we do want

\(^{21}\) We assume that the effects of the shift operated by the conditional antecedent extend beyond the boundaries of individual sentences. An assumption of this sort seems required by any account of modal subordination.

\(^{22}\) For some relevant discussion, see Condoravdi (2002); Condoravdi & Deo (2008); Khoo (2015).
to note an interesting analogy between *will* and other modals concerning the availability of epistemic and non-epistemic readings.

It is well-known (see, for example, Condoravdi 2002) that the reference time of the prejacent correlates with the flavour that is assigned to the modal. In particular, prejacents with a reference time in the past or in the present correlate with epistemic readings, while prejacents with a reference time in the future correlate with non-epistemic readings. Here are some examples:

(24) a. John must be in London by now. (✓epistemic/#deontic)
    b. John must go to London tomorrow. (#epistemic/✓deontic)

(25) a. Cynthia must be wearing a cap today. (✓epistemic/#deontic)
    b. Cynthia must wear a cap tomorrow. (#epistemic/✓deontic)

(26) a. Sam must have gone to Chicago last April. (✓epistemic/#deontic)
    b. Sam must go to Chicago next April. (#epistemic/✓deontic)

We note that *will* is similarly asymmetric:

(27) a. John will be in London by now. (✓epistemic/#historical)
    b. John will go to London tomorrow. (#epistemic/✓historical)

(28) a. Cynthia will be wearing a cap right now. (✓epistemic/#historical)
    b. Cynthia will wear a cap tomorrow. (#epistemic/✓historical)

(29) a. Sam will have gone to Chicago last April. (✓epistemic/#historical)
    b. Sam will go to Chicago next April. (#epistemic/✓historical)

Of course, if we are right, there are significant differences between *will* and modals like *must*. Only the latter have a quantificational semantics
and only the former are analysed via selection functions. But these examples point to an analogy at a different level.

One natural way to account for the contrasts in (24)–(29) is that different prejacents somehow force a different choice of modal bases for the modals. For example, Condoravdi (2002) suggests that modal claims whose prejacents have a reference time in the present rule out non-epistemic modal bases via a constraint requiring that the modal base be sufficiently diverse. In any case, the pattern in (24)–(29) suggests that, whatever one says for the case of must will be exportable to will.

6. Truth, validity, and indeterminacy

§5 offers a compositional semantic account of will, but does not fix the truth conditions of will-claims. To get the latter, we must define a notion of truth at a context. (This is the stage of the theory that MacFarlane 2003 calls ‘postsemantics’.)

Adopting a definition of truth at a context requires us to take sides in the debate about the indeterminacy of statements about the future. To make the presentation more concrete, we adopt a specific account of how indeterminacy affects the semantics, i.e. the one defended by Barnes & Cameron (2009). According to this account, each context determines a single actual world, but it is indeterminate which context the utterance takes place in.

In a standard contextualist framework, built on Kaplan (1989), truth at a context is defined by fixing the values of index parameters to the

---

23 This is the constraint endorsed by Condoravdi, roughly stated:

**Diversity condition:** If \( \gamma^{MODAL} \) has a non-epistemic modal base \( M \), then there are worlds \( w \) and \( v \) in \( M \) such that \( A \) is true at \( w \) and false at \( v \)

24 See also Iacona (2014), who sketches an Ockhamist picture whose treatment of indeterminacy is parallel to ours. Barnes & Cameron’s view is part of a broader family of views that draw inspiration from supervaluational accounts of indeterminacy but retain a bivalent semantics. For more work in this direction, see, among many, McGee & McLaughlin (1995); Dorr (2003); Barnes & Williams (2011). We contribute to this tradition by showing that it makes openness compatible with a standard Kaplanian picture of context and illuminates the division of labour between the semantics and the metaphysics of the open future. A close relative of our point of view is Wilson’s (2011) Kaplanian interpretation of the proposal in Saunders & Wallace (2008). As Wilson puts it (p. 364), Saunders and Wallace’s background assumptions ‘entail that where there are multiple complete branches, there are multiple contexts and hence multiple distinct utterances’. Though we are neutral on the relevant assumptions, we do think it is fruitful to think along these lines.

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coordinates of the context. (Following Lewis 1980, we take contexts to be concrete situations of utterance.) Here is a formal definition:

**Truth at a Context:** $A$ is true as uttered at $c$ iff $[A]^{w,s,c} = 1$

In a slogan: $S$ is true at a context $c$ just in case $S$ is true at the circumstances fixed by $c$. Traditional supervaluationist accounts (for example, Belnap & Green 1994) reject Kaplan’s definition of truth at a context. They maintain that a context of utterance does not fix which world is to count as actual. These accounts replace truth at a context with a new definition that allows for sentences that are neither true nor false at the context of utterance. Unlike traditional supervaluationists, we endorse the simple definition of truth at a context given above. Hence, on our account, every sentence is either true or false at a context. (This part of our account is independent of our compositional analysis of *will*, which is also compatible with a standard supervaluationist postsemantics.)

How, then, can we satisfy the theorists who maintain that the future is genuinely open? We assume that if the future is open, it is indeterminate which context the utterance takes place in, and hence it is indeterminate which truth value the sentence has. An important consequence of this hypothesis is that both the defender and the opponent of Openness are able to help themselves to our framework, including the definition of truth (and the two notions of validity that we give below). Their disagreement is moved out of the semantic apparatus entirely: the defender of Openness denies, and the opponent of Openness claims, that a concrete situation of utterance determines a unique context.

The notion of truth at a context is important for a number of reasons. One of them is that (following Kaplan himself) we can use it to define a plausible notion of validity. On this notion, an argument is valid just in case it preserves truth at a context: no context makes the premises true and the conclusion false.

**Validity:** $A_1, ..., A_n \models_1 B$ iff, for any context $c$ such that $A_1, ..., A_n$ are true at $c$, $B$ is also true at $c$.

We should flag an important consequence of our moving the indeterminacy outside the semantics: unlike traditional supervaluationism, our logic for the relevant fragment of the language is allowed to be straightforwardly classical. It will also be helpful to appeal to a second

---

25 For completeness, here is a sample definition of truth at a context that fits the traditional supervaluationist’s *desiderata*: $A$ is true as uttered at $c$ iff for all worlds $v$ that are historically possible in $c$, $[A]^{v,s,v} = 1$. 
notion of validity, one that captures preservation of truth at a point of evaluation.

Validity\(_2\): \( A_1, \ldots, A_n \vdash_2 B \) iff, for any triple \((w, s, g)\) such that \( [A_1]^{w,s,g} = 1, \ldots, [A_n]^{w,s,g} = 1, [B]^{w,s,g} = 1 \).

A single sentence \( A \) is valid\(_1\) just in case \( \models_1 A \); similarly for validity\(_2\). Note that every argument that is valid\(_2\) is valid\(_1\). The converse however does not hold: any context that makes \( \models_2 A \) true, also verifies \( A \), but there are points at which \( [\neg A]^{w,s,g} = 1, [A]^{w,s,g} = 0 \). (This can happen only in points such that \( w \notin g(f) \).) When \( A \) and \( B \) are such that \( A \vdash_1 B \) and \( B \vdash_1 A \), we say that they are equivalent\(_1\) (similarly for equivalent\(_2\)). Here too we have that every equivalent\(_2\) pair is equivalent\(_1\) (though not vice versa). These connections between these logical notions are important. Although validity\(_1\) (and equivalence\(_1\)) are more significant notions in the overall architecture of the theory, we can establish them by way of establishing validity\(_2\) (and equivalence\(_2\)).

7. Consequences for the logic of \textit{will}

Having acquired a notion of validity, we are ready to explore the main consequences of our apparatus. We start by vindicating our second desideratum, i.e. the scopelessness of \textit{will}. First, we notice that the semantics satisfies the excluded middle property:

\textbf{Will Excluded Middle:} \( \Gamma \text{ will}_f A \lor \text{ will}_f \neg A \vdash \) is valid\(_2\) (hence valid\(_1\)).

We use ‘\( \lor \)’ for disjunction to highlight that the sentence is valid on the assumption that \textit{or} is boolean disjunction.\(^{26}\) The argument for excluded middle also establishes a related fact:

\textbf{Negation Swap:} \( \Gamma \neg \text{ will}_f A \vdash \neg \text{ will}_f A \) and \( \neg \text{ will}_f A \vdash \neg \text{ will}_f \neg A \) are equivalent\(_2\) (hence equivalent\(_1\)).

This explains why we do not perceive different scopes for negation despite the fact that \textit{will} is a modal.

\(^{26}\) \textbf{PROOF:} let \((w, s, g)\) be an arbitrary point of evaluation. We have that \( \Gamma \text{ will}_f A \lor \text{ will}_f \neg A \) is true at \((w, s, g)\) iff either \( A \) is true at \( s(w, F) \) or false at \( s(w, F) \). But the right-hand side of the biconditional is always true (since, for any set of worlds \( S \) and any world \( w \), it is always the case that \( w \) either belongs or doesn’t belong to it). Hence, \( \Gamma \text{ will}_f A \lor \text{ will}_f \neg A \) is true at \((w, s, g)\). Since \((w, s, g)\) was arbitrary, \( \Gamma \text{ will}_f A \lor \text{ will}_f \neg A \) is true at any point of evaluation.
The analysis also entails that will-conditionals satisfy a principle of conditional excluded middle. Interestingly, this holds whether or not conditionals in general satisfy this principle.

**Compositional CEM for will-Conditionals:** For any point \( \langle w, s, g \rangle \),
\[
[(If \ B)(willf \ A)]^{w,s,g}_{w,s,g} = 1 \text{ or } [(If \ B)(willf \ not \ A)]^{w,s,g}_{w,s,g} = 1.
\]
Note that this and all the following principles about conditionals are restricted to the case in which the conditional antecedent is compatible with the modal base.

Since Compositional CEM holds at any point of evaluation, we get:

**Postsemantic CEM for will-Conditionals:**
\[
\Gamma(If \ B)(willf \ A) \lor (If \ B)(willf \ not \ A)^\perp \text{ is valid}_2 (\text{hence valid}_3).
\]

Relatedly, there is only one way of negating the consequents of conditionals.

**Narrow Negation Swap in Conditionals:** \( \Gamma(If \ B)(willf \ not \ A)^\perp \) and \( \Gamma(If \ B)(not \ willf \ A)^\perp \) are equivalent_2 (hence equivalent_3).

This is a trivial consequence of the non-conditional negation swap, because \((if \ B)\) merely operates on \(g\). More importantly, we can derive:

**Wide Negation Swap in Conditionals:** \( \Gamma(If \ B)(willf \ not \ A)^\perp \) and \( \Gamma(Not \ (If \ B)(willf \ A)^\perp \) are equivalent_2 (hence equivalent_3).

This completes our illustration of the basic logical implications of our semantics.

### 8. Belief and doubt in will-claims

#### 8.1 Probabilities of simple will-claims

Recall the cognitive problem from §2.3. Ordinary agents are uncertain about the future. On one natural way of understanding this uncertainty, this means that ordinary agents have non-extreme degrees of belief in the propositions expressed by will-claims. Moreover, at least in some cases, it seems that this uncertainty is rationally permissible, if

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27 Whether \( \Gamma(If \ B)(willf \ A)^\perp \) is true at \( \langle w, s, g \rangle \) boils down to the truth of \( A \) at \( \langle w, s, g[f \rightarrow F \land B] \rangle \). If it is not true, it must be false, but in that case, \( \Gamma(If \ B)(willf \ not \ A)^\perp \) must be true.

28 PROOF: \( [(not(If \ B)(willf \ A))]^{w,s,g}_{w,s,g} = 1 \iff [(If \ B)(willf \ A)]^{w,s,g}_{w,s,g} = 0 \iff [(not(If \ B)(willf \ A))]^{w,s,g}_{w,s,g} = 1 \iff [(If \ B)(not \ willf \ A)]^{w,s,g}_{w,s,g} = 1 \). Together with Narrow Negation Swap, this equivalence entails Wide Negation Swap.

---
not rationally required. An adequate theory of will should vindicate this intuition.

This problem should be distinguished from a different, important problem surrounding belief in future claims. Suppose that the future is objectively open, in the sense defined above. In particular, suppose that branching theorists in the style of Thomason (1970), Belnap & Green (1994), and Belnap et al. (2001) are right about the metaphysics of branching: there are several possible futures, each of which shares the segment that we occupy at the present time. In this case, it is unclear what we mean when we say that the probability of an open proposition—say, the proposition that Cynthia will wear a Warriors cap tomorrow—is \( r \). If it is genuinely open whether Cynthia will wear a Warriors cap, then there are (at least) two ‘equally real’ futures. In one of them she wears the cap, while in the other she does not. This problem has received attention in philosophy of physics (in particular, by defenders of the Everett interpretation of quantum mechanics—see, among others, Wallace 2014). We are not contributing to this discussion here. Rather, we are taking for granted that it somehow makes sense to assign non-extreme credences to propositions about the future (and hence, derivatively, to the claims that express them).\(^{29}\)

Let us then return to our Sports Fan example. Recall: every day, Cynthia tosses a fair die and, on the basis of the outcome, decides whether to wear a Giants hat, a Warriors hat, or no hat. Consider a rational agent who assigns credence \( \frac{1}{3} \) to each of the three possibilities. Against this background, what we want to show is that the proposition expressed by (12) (repeated below) on our account also gets credence \( \frac{1}{3} \).

\[(12) \text{ Cynthia will wear a Warriors cap tomorrow.}\]

Before we start, let us remind you that this is a nontrivial task. In fact, as we pointed out in §2.4, existing modal accounts fail this task. The reason is that these accounts declare \( \Box \text{will} \ A \) true just in case \( A \) is true at every best future. This semantics makes (12) false, and hence predicts that a rational agent who is aware of the openness of the future should assign credence zero to the proposition it expresses.

\(^{29}\) If you’re sceptical about these claims, you may take our arguments in this section to provide a good litmus test for our semantics of will. What we’re going to show is that if our theories of credences warrants assigning the attitudes that seem intuitive offhand, then the semantics delivers contents that vindicate the intuitive assignment of credences.
Let us spell out some basic assumptions. In keeping with standard possible worlds semantics for attitudes, assume that credences are defined over sets of worlds. In particular, assume that an agent’s credences at a given time may be modelled by a probability function $\mu$ satisfying the usual constraints. For example, let $\mu$ model an agent’s credences at the current point in time. Let $\mu(A) = 1/3$, where $A$ is the set of worlds where Cynthia wears a Warriors cap. Our task is to check that $\mu(\text{PROP}_W) = 1/3$, where $\text{PROP}_W$ is the content our semantics associates to an utterance of (12).

For current purposes, we can take the content expressed by the utterance of a $\text{will}$-sentence at a given context to be the set of worlds such that the utterance is true as evaluated at those worlds. Formally:

Content of $A$ at $c$: $\|A\|_c = \{w : [A]^{w,c,s,g} = 1\}$

In what follows, we suppress reference to the context to avoid clutter.

It is easy to see that this yields exactly the verdict we need. Take our example: $\|\text{Cynthia will wear a Warriors cap}\|$ is just the set of worlds in which Cynthia wears a Warriors cap. On the assumption that the credence that our agent assigns to Warriors-cap-worlds is $1/3$, she will also assign credence $1/3$ to the proposition expressed by (12). More generally, letting $\|B\|_F$ denote $\|B\| \cap F$, we obtain:

Transparency: For any prejacent $A$, $\|\text{will}_{f}A\|_F = \|A\|_F$.\(^{30}\)

When we restrict consideration to the worlds in the modal base, unembedded $\text{will}$-sentences and their prejacents are true at exactly the same worlds.

8.2 Probabilities of complex $\text{will}$-sentences

The probabilities of Boolean compounds of $\text{will}$-sentences work out as one would intuitively expect. Transparency and Negation Swap immediately entail:

For all $F$, $\|\text{not will}_{f}A\|_F = \{w : w \text{ is } \neg A\text{-world}\} \cap F = \|\text{not } A\|_F$

In our example, $\|\text{It is not the case that Cynthia will wear a Giants cap}\| = \|\text{Cynthia will not wear a Giants cap}\|$. Both propositions have probability $2/3$ (according to $\mu$). Transparency entails similar results.

\(^{30}\) Proof: Suppose $v \in F$. Then: $v \in \|\text{will}_{f}A\|$ iff $[[\text{will}_{f}A]]^{v,s,g} = 1$ iff $[A]^{(v,F),s,g}[\neg F] = v$ iff $v \in \|A\|$. The first equivalence follows from our definition of content; the second from the truth conditions of $\text{will}_{f}$; the third from $s(v,F) = v$; which in turn follows from Centering and $v \in F$. 

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for conjunction and disjunction, so that:  
\[ \| \text{will}_F (A \& B) \|_F = \| \text{will}_F A \|_F \cap \| \text{will}_F B \|_F \]  
and  
\[ \| \text{will}_F (A \lor B) \|_F = \| \text{will}_F A \|_F \cup \| \text{will}_F B \|_F . \]

The case of conditionals is more complex. It is well-known that standard possible worlds semantics for conditionals fail to vindicate, in general, the intuitive assignments of probabilities to conditional sentences.\(^{31}\) Since our account of will incorporates a standard semantic account of conditionals, it shares this feature. To get a sense of the problem, consider again the Sports Fan scenario, and take the conditional:

\((30)\) If Cynthia wears a cap, she will wear a Warriors cap.

Recall that Cynthia decides to wear a Warriors cap, a Giants cap, or no cap, depending on a random process that makes each of the three options 1/3 likely. Accordingly, suppose that \((30)\) is evaluated against the toy modal base we described in §3, and consisting of a Warriors-cap-world, a Giants-cap-world, and a no-cap-world. It seems natural to say that your degree of belief in \((30)\) should be (or, at the very least, may be) 1/2. But our semantics can’t vindicate this result.

To see this, consider the content of \((30)\):

\[ \| \text{If cap, will}_F \text{ Warriors cap} \| = \]
\[ \{ w : \| \text{If cap, will}_F \text{ Warriors cap} \|^{w,s,g} = 1 \} = \]
\[ \{ w : \| \text{will}_F \text{ Warriors cap} \|^{w,s,g[f \rightarrow F \cap \text{cap}]} = 1 \} = \]
\[ \{ w : \| \text{Warriors cap} \|^{g(w,F \cap \text{cap}),s,g[f \rightarrow F \cap \text{cap}]} = 1 \} \]

A world belongs to this proposition if the selection function maps to it when that world and the restricted modal base are given as input. It is easy to show that the proposition in \((30)\) cannot have probability 1/2. The basic point is that, given that in our model we have only three worlds, each of which has probability 1/3, no proposition (i.e. no set of worlds) can have probability 1/2.\(^{32}\)

\(^{31}\) See Lewis (1979c), (1986b); Hájek & Hall (1994). It is often assumed that the intuitive probabilities of conditional sentences should match the conditional probabilities of the consequent, given the antecedent. But one doesn’t need to endorse this general thesis (which has been called into question; see e.g. Kaufmann (2004)) to see the problem. All we need is that there are examples in which such an assignment is plausible, such as \((30)\) in the main text.

\(^{32}\) PROOF: The modal base contains three worlds: the Warriors-cap-world \(w\), the Giants-cap-world \(v\), and the no-cap-world \(z\); each has probability 1/3. \(w\) is a member of \(\| (30) \|\) and \(v\) is
One could respond that the problem depends on the fact that we have used a modal base that is too simple. If we add enough worlds to the modal base, we will be able to identify a content that gets probability \( \frac{1}{2} \). This is correct, but the point illustrated by our toy example will still hold for some conditional or other, provided that we stick to finite modal bases.\(^{33}\) Alan Hájek (1989, 2012) has pointed out that for any (nontrivial and finite-ranged) probability function \( Pr \), the conditional probability values assigned on the basis of \( Pr \) outnumber the unconditional probability values assigned by \( Pr \). Hence there will always be some conditional probability value that doesn’t find a match in the probability of any proposition—exactly as happens in our example, where no proposition has probability \( \frac{1}{2} \). (See also Hall 1994 for an extension of Hájek’s argument to the countable case.)

Before closing, let us gesture towards a way of refining our ideas that will yield better results for conditionals. We build on a line of thinking about conditionals that has been developed over the past three decades (van Fraassen 1976; McGee 1989; Jeffrey & Stalnaker 1994; Kaufmann 2009, 2015; Bradley 2012; for some criticisms of Kaufmann and an alternative approach see Khoo 2016). So far, we have assumed that the credences of a rational agent are distributed over an algebra of possible world propositions. But there is a second dimension of uncertainty, which this model doesn’t capture: an agent may be uncertain about which world is selected by the selection function. Go back to our example and consider the no-cap-world \( z \). What is the value of the selection function, when the relevant arguments are \( z \) and the proposition that Cynthia wears a cap? Both the answers ‘The Warriors-cap-world \( w \)’ and ‘The Giants-cap-world \( v \)’ seem to be open epistemic possibilities. Being uncertain between these two answers, of course, would mean being uncertain about which of two candidates for the selection function (call them ‘\( s_w \)’ and ‘\( s_v \)’) is the ‘correct’ one.

\(^{33}\) It is controversial that modal bases of modals in natural language should be finite. However, it is not implausible that the credences of ordinary subjects are only defined over a finite number of subsets of the modal base, given standard cognitive limitations. For a more realistic variant of Hájek’s argument, consider a subject whose credences are only defined over a threefold partition of logical space, i.e. the one that includes the three propositions that Cynthia wears a Giants cap, a Warriors cap, or no cap, plus the relevant Boolean combinations. Depending on the selection function, this subject must have credence \( \frac{1}{3}, \frac{2}{3}, \) or undefined credence in (30).
To register this kind of uncertainty alongside uncertainty about which world is actual, we need to refine the elements of the underlying algebra. Rather than worlds, we may use pairs consisting of a world and a selection function. This also involves modifying our notion of content: we need to take contents to be sets of pairs of a world and a selection function. (See Bradley 2012 for a more extensive development of the idea; Bradley’s approach is in the tradition stemming from van Fraassen 1976. See also Jeffrey & Stalnaker 1994, and Kaufmann 2009, 2015).

2D content of at $c$: $\langle \{A\} \rangle = \{ \langle w', s' \rangle : [A]^{w', s', \& c} = 1 \}$

It is straightforward to see how, on the new picture, (30) may be assigned probability 1/2. We now have an algebra of six possibilities, consisting of the pairs:

$$\{ \langle w, s_w \rangle, \langle v, s_v \rangle, \langle z, s_w \rangle, \langle w, s_v \rangle, \langle v, s_v \rangle, \langle z, s_v \rangle \}$$

It is easy to show that a probability distribution that assigns to each of these pairs probability $1/6$ assigns probability $1/2$ to (30).

9. Conclusion

Traditionally, will has been treated as a tense in philosophy, and as a modal in large sectors of the linguistics literature. Linguists are right: there is strong evidence that will is a modal. At the same time, all existing modal theories fail to deliver some important desiderata. In particular, they cannot be integrated with our intuitive attitudes towards the future. We have suggested that will is indeed a modal, but doesn’t have a quantificational semantics. Rather, will selects the ‘one actual future’ out of the set of historical alternatives at the time of utterance. Besides validating the evidence for the modal character of will, this account predicts a range of important logical interactions for will and dovetails well with intuitions about the cognitive role of future statements—thus doing better than any other theory on the market.34

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