The Structure of Spatial Localization

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INTRODUCTION

Material objects, such as tables and chairs, have an intimate relationship with space. They have to be somewhere. They must possess an address at which they are found. Under this aspect, they are in good company. Events, too, such as Caesar's death and John's buttering of the toast, and more elusive entities, such as the surface of the table, have an address, difficult as it may be to specify. A stronger notion presents itself, though. Some entities may not only be *located at* an address; they may also *own* (as it were) the place at which they are located, so as to exclude other entities from being located at the same address. Thus, for certain kinds of entities, no two tokens of the same kind can be located at the same place at the same time. This is typically the case with material objects. Likewise, no two particularized properties of the same level or degree of determinacy can be located at the same place at the same time (although particularized properties of different degree, such as the red of this table and the color of this table, can). Other entities seem to evade the restriction. Two events can be perfectly co-located without competing for their address. Or, to use a different terminology, events do not *occupy* the spatial region at which they are located, and can therefore share it with other events. The rotation of the Earth and the cooling down of the Earth take place at exactly the same region.

Some of these facts and hypotheses have important bearings as to matters of identity. For instance, co-localization seems to be a sufficient condition for identity in the case of material objects, but not in the case of events. That entities of different kind have different relations to space is in itself an interesting state of affairs. But other philosophical questions—at a deeper level—are worth exploring. What are the relationships between an entity and the space at which it is located? What is the metaphysical structure of localization? What its modal status? What follows is an attempt to address some of these questions and to work out at least the main coordinates of the logical structure of localization. We shall also highlight some of the main interactions between the notion of localization and other notions in nearby areas, such as the notions of part and whole. Among other things, this will enable us to show that neither mereology nor topology, taken alone or jointly, permit an adequate representation of some very basic spatial structures: the notion of spatial localization deserves an independent place in spatial representation. In the long run, this is bound to tie in with some highly controversial metaphysical issues concerning the true nature of space, and particularly the dispute between absolutistic and relational conceptions. Here, however, we shall try to remain neutral with respect to such issues. We shall follow common sense in distinguishing between an entity and “its” region of space, but we shall avoid any commitment as to the ultimate ontological status of the latter.

HAVING AN ADDRESS

Spatial things are spatial insofar as they cannot but be somewhere. That is to say, it must be possible to assign them an address of some sort. People have addresses, and by extension every material object can be assigned an address. These addresses, in turn, may induce an address assignment to whatever events may involve people and other objects. Numbers and sets (at least the empty set) are by contrast addressless.

Elementary as it might be, the notion of an address involves intricacies that bear witness to the complexity of localization. We can distinguish, to begin with, a *dispositional* and an *actual*, non-dispositional notion of address. John lives in Manhattan, and even if he drives somewhere outside the Big Apple, he still retains an address there. This is the customary notion of an address. It is intrinsically dispositional, and is partly grounded in non-spatial facts, such as conventions, or communicative intentions in general. By contrast, when John travels from Manhattan to Paris, or simply takes a walk in Manhattan, his actual address changes. He may very well retain his dispositional address, but as a matter of fact he is now somewhere else than he was before he started travelling or walking. This may be a slight departure from the ordinary notion of an address, but the extension is obvious. (Think of the taxi-driver asking for John's address when John is calling from a restaurant.)

On a different line, we may further distinguish a spatially *minimal* and a broader, *non-minimal* notion of an address. Every time John moves (including when he only moves, expands, or contracts parts of his body), his minimal address changes. This holds clearly in the non-dispositional sense, and can be extended to the dispositional sense. On the other hand, John may move around Manhattan and still be in Manhattan. His moving does not necessarily carry along a change in his address, if this is understood in a broad sense. (This may well be a matter of focus. Think of an astronaut specifying the position of John.)

A third classificatory criterion draws upon the semantic structure of the singular terms used for denoting addresses. *Spatially unstructured* terms include proper names of addresses or definite descriptions whose semantic complexity is not spatially relevant. Consider, respectively, 'John lives in Manhattan' and 'John lives in the Empire State Building', or 'John lives in the most expensive building of all'. *Spatially structured* terms are either proper names or definite descriptions whose semantic components are overtly spatial. The interesting cases are provided here by coordinate systems or street numberings: 'John is at 40° North (of the Equator) and 74° West (of Greenwich)'; 'He is at 5th Avenue and 34th Street'; 'He is at 77 5th Avenue'.

Address terms may also be used projectively, so to speak. For instance, the address of a fly at a certain time might be given in terms of the fly's projection on the squares of a chess board: 'The fly is now at b7'. These and similar cases, however, when the address name makes reference to a place that is not the place where the object is really located (either actually or dispositionally), need not be of our concern here. Also, for simplicity we shall ignore the case of indexical spatial expressions such as 'here' or 'three feet from there', or of any other expression that essentially involves a viewpoint, such as 'to your right' or 'behind the Empire State Building'.

BEING LOCATED

One understands addresses in the dispositional sense in terms of the actual notion of an address: to have a dispositional address α implies having the possibility of actually (non-dispositionally) being at α. To systematize the actual notion of an address (the address you have when you are where you are), it is convenient to rely on the spatially minimal notion of an address. This, in fact, is sure to be common to whatever actual address you might have. Your actual, spatially minimal address gives your *exact* localization.

The exact localization of an object is the region of space taken up by the object (modulo any worries as to whether the boundary belongs to the object or to the complement, or indeed whether the object has an exact boundary). Thus, John is not exactly located in Manhattan, but he is exactly located in the space "carved out" of the air, or of whatever medium he might be in (water, if he is swimming; concrete, if he was nasty to his godfather). This notion of exact localization thus requires that spatial regions be included in the *prima facie* ontology (although, as we mentioned, we want to remain neutral with respect to their ultimate ontological status). Indeed, in a preliminary characterization, (exact) localization is a relation whose second term is always a region of space. Even if we can talk of John's being located *in a column* in the basement of his godfather's country house, this would not be the sense in which we are using the term here. Instead, John would be located in a space carved *inside* that column. On the other hand, the first term of the localization relation can be whichever sort of entity you have in your spatial ontology—spatial regions included. We can speak of John's being located at region *r*; but we can also speak of John's body, of the sum of his present intradermal events, or even of region *r* itself being located at *r*.

Note that reference to exact localization has the derivative (but not negligible) advantage of inhibiting all issues of vagueness: no matter how vague the boundary of an object is—if you allow for vague boundaries at all—the spatial region at which the object is exactly located is perfectly determinate (relative to the object).

PRINCIPLES OF LOCALIZATION

In terms of exact localization, assumed as a primitive relation, we can begin to unfold a more detailed picture of locative relations by exploiting the part–

whole structure of spatial entities. As a matter of fact, regions of space form a mereologically well-behaved structure—a prime example of a domain ordered by the *part of* relation. We shall assume the same to hold for every spatial entity, i.e., for whatever sort of entity can be located at a region of space. In doing so, we shall ignore the difficulties arising from the consideration of tensed part–whole principles, and we shall presume that standard extensional mereology—as rooted in the work of Les;´ niewski [1916] or Leonard and Goodman [1940]— will fit the purpose [1]. That is, let 'P' symbolize parthood (proper or improper), and let 'O' be the overlap relation, defined as usual as sharing of a common part:

DP1 $\quad \mathrm{O}(x, y) =_{\mathrm{df}} \; z(\mathrm{P}(z, x) \quad \mathrm{P}(z, y)).$

Then, the part–whole theory that we shall assume is given by the following two axioms:

AP1 $\quad \mathrm{P}(x, y) \leftrightarrow \quad z(\mathrm{O}(z, x) \rightarrow \mathrm{O}(z, y))$

AP2 $\quad \; x(\varphi(x)) \rightarrow \; x \;\; y(\mathrm{O}(x, y) \leftrightarrow \; z(\varphi(z) \quad \mathrm{O}(z, y))).$

By AP1, parthood amounts to inclusion of overlappers: this ensures that P is reflexive, antisymmetric, and transitive (a partial ordering) whereas O is reflexive and symmetric, but not transitive. By AP2, every satisfied condition φ picks out an entity consisting of all and only those things that satisfy φ. We assume no restriction on the existence of such mereological sums, regardless of whether they are homogeneous and spatially connected. Among other things, these principles guarantee that parthood satisfies a supplementation principle to the effect that if an object has a proper part, it has a second part disjoint from the former:

TP1 $\quad \mathrm{PP}(x, y) \rightarrow \; z(\mathrm{P}(z, y) \quad \neg \mathrm{O}(z, x)),$

where 'PP' (proper parthood) is defined in the obvious way:

DP2 $\quad \mathrm{PP}(x, y) =_{\mathrm{df}} \mathrm{P}(x, y) \quad \neg \mathrm{P}(y, x).$

In addition, we assume reciprocal parthood to be tantamount to identity:

DP3 $\quad x{=}y =_{\mathrm{df}} \mathrm{P}(x, y) \quad \mathrm{P}(y, x),$

which we take to be governed by the usual axioms. By TP1, this implies extensionality: no two non-atomic entities can have exactly the same proper parts.

On these grounds, we can now articulate a minimal theory of localization based on a specific primitive, say ‘L’. Intuitively, as we said, we want this to express the relation of *exact* localization, so that ‘L(x, y)’ will read ‘x is exactly located at y’. However, with the help of mereology we can immediately expand the set of available locative relations by saying that x is *wholly* located (WL) at y just in case x is located at some part of y; x is *partly* located (PL) at y just in case some part of x is located at y; and x is *generically* located (GL) at y just in case some part of x is located at some part of y:

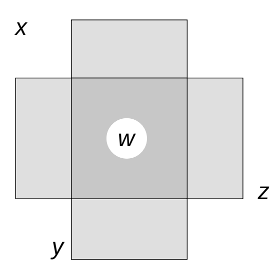
DL1 $\quad \mathrm{WL}(x, y) =_{\mathrm{df}} \; z(\mathrm{P}(z, y) \quad \mathrm{L}(x, z))$.
DL2 $\quad \mathrm{PL}(x, y) =_{\mathrm{df}} \; z(\mathrm{P}(z, x) \quad \mathrm{L}(z, y))$.
DL3 $\quad \mathrm{GL}(x, y) =_{\mathrm{df}} \; z \;\; w(\mathrm{P}(z, x) \quad \mathrm{P}(w, y) \quad \mathrm{L}(z, w))$.

Thus, exact localization is a special case of a more general notion of localization: within certain obvious limits, if ‘L’ expresses the notion of minimal address, ‘WL’ corresponds to the wider, non-minimal notion (John is wholly located in Manhattan), ‘PL’ to its dual (Manhattan is partly located at the region occupied by John), and ‘GL’ to the general case. This is somewhat captured by the following immediate consequences of DL1–DL3:

TL1 $\quad \mathrm{L}(x, y) \rightarrow \mathrm{WL}(x, y) \quad \mathrm{PL}(x, y)$
TL2 $\quad \mathrm{WL}(x, y) \quad \mathrm{PL}(x, y) \rightarrow \mathrm{GL}(x, y)$

(The converse of TL1 also holds, but not that of TL2: 96th Street is generically—but neither wholly nor partly—located at the region occupied by Central Park.)

By way of illustration, consider a cross-shaped region x, its overlapping bars y and z, and their common part w. Then we have, *inter alia*: WL(y, x), WL(w, y), PL(x, y), PL(y, w), GL(x, y), GL(y, x), GL(y, z), and GL(z, y).



Of course, in a domain comprising only regions of space (so that both arguments of the L-relation range over entities of the same kind), the conceptual cluster of localization collapses onto plain mereology *via* the transforms $\mathrm{L}(x, y) \Leftrightarrow x=y$, $\mathrm{WL}(x, y) \Leftrightarrow \mathrm{P}(x, y)$, $\mathrm{PL}(x, y) \Leftrightarrow \mathrm{P}(y, x)$, and $\mathrm{GL}(x, y) \Leftrightarrow \mathrm{O}(x, y)$. In general, however, this is not the case, as there is more to the ontology of localization than just spatial regions. (Here, and in the following, we are speaking of spatial regions in a very general fashion, so as to include three-dimensional, voluminous regions as well as points and other boundary-like elements. Correspondingly, our general domain of discourse may include, next to three-dimensional bodies, also entities of fewer dimensions. It would also be interesting, albeit beyond our present purposes, to see whether and to what extent the theory of temporal and spatio-temporal localization can be obtained from the theory of spatial localization by simply exploiting the intuition of time as a fourth dimension.)

Turning now to the specific axioms for ‘L’, the following set constitutes a minimal requirement (further postulates will be considered in the next sections):

AL1 $\quad \mathrm{L}(x, y) \quad \mathrm{L}(x, z) \rightarrow y=z$
AL2 $\quad \mathrm{L}(x, y) \quad \mathrm{P}(z, x) \rightarrow \mathrm{WL}(z, y)$
AL3 $\quad \mathrm{L}(x, y) \quad \mathrm{P}(z, y) \rightarrow \mathrm{PL}(x, z)$
AL4 $\quad \mathrm{L}(x, y) \rightarrow \mathrm{L}(y, y)$.

Axiom AL1 is a sort of extensionality principle, to the effect that a single entity cannot be exactly located at two distinct regions (hence, in particular, two distinct regions cannot be exactly co-located): L is a functional relation AL2 and AL3 fix the intuitive bridge from the theory of localization to its mereological background: if x is exactly located at y, then every part of x is wholly located at y, and x is partly located at every part of y. This ensures the following patterns of monotonicity:

TL3 $\quad \mathrm{WL}(x, y) \quad \mathrm{P}(z, x) \rightarrow \mathrm{WL}(z, y)$
TL4 $\quad \mathrm{WL}(x, y) \quad \mathrm{P}(y, z) \rightarrow \mathrm{WL}(x, z)$ provided $\mathrm{L}(z, z)$
TL5 $\quad \mathrm{PL}(x, y) \quad \mathrm{P}(z, y) \rightarrow \mathrm{PL}(x, z)$
TL6 $\quad \mathrm{PL}(x, y) \quad \mathrm{P}(x, z) \rightarrow \mathrm{PL}(z, y)$
TL7 $\quad \mathrm{GL}(x, y) \quad \mathrm{P}(y, z) \rightarrow \mathrm{GL}(x, z)$ provided $\mathrm{L}(z, z)$
TL8 $\quad \mathrm{GL}(x, y) \quad \mathrm{P}(x, z) \rightarrow \mathrm{GL}(z, y)$.

As for AL4, it guarantees that L behave as a reflexive relation whenever it can: all (and only) those things at which something is located—i.e., on the intended interpretation, all spatial regions—are located at themselves. Thus, although we are not assuming that everything is located somewhere (which would be a way of characterizing a world inhabited exclusively by spatial entities), we want to ensure that this holds at least of all regions. Conversely, note that we are not assuming here that every region is the region of something, i.e., a region at which something is located—except for the region itself. A region may be empty. We'll come back to this shortly.

From AL1–AL3, using DL1–DL3, we can see first of all that the choice of 'L' as a primitive for localization is formally unbiased (as it should be): any one of the other three relations would do, as per the following equivalences:

TL9 $\mathrm{L}(x, y) \leftrightarrow \quad z(\mathrm{P}(z, x) \rightarrow \mathrm{WL}(z, y))$
TL10 $\mathrm{L}(x, y) \leftrightarrow \quad z(\mathrm{P}(z, y) \rightarrow \mathrm{PL}(x, z))$
TL11 $\mathrm{L}(x, y) \leftrightarrow \quad z(\mathrm{O}(z, x) \rightarrow \mathrm{GL}(z, y))$

or also:

TL12 $\mathrm{L}(x, y) \leftrightarrow \quad z(\mathrm{WL}(x, z) \leftrightarrow \mathrm{WL}(y, z))$
TL13 $\mathrm{L}(x, y) \leftrightarrow \quad z(\mathrm{PL}(y, z) \leftrightarrow \mathrm{PL}(x, z))$
TL14 $\mathrm{L}(x, y) \leftrightarrow \quad z(\mathrm{GL}(y, z) \leftrightarrow \mathrm{GL}(x, z))$.

Moreover, the following consequences of AL1–AL3 show that reference to DL1–DL3 is actually only one among a number of equivalent ways of introducing the three derived relations in terms of 'L':

TL15 $\mathrm{WL}(x, y) \leftrightarrow \quad z(\mathrm{P}(x, z) \quad \mathrm{L}(z, y))$
TL16 $\mathrm{PL}(x, y) \leftrightarrow \quad z(\mathrm{P}(y, z) \quad \mathrm{L}(x, z))$
TL17 $\mathrm{GL}(x, y) \leftrightarrow \quad z(\mathrm{O}(x, z) \quad \mathrm{L}(z, y))$
TL18 $\mathrm{GL}(x, y) \leftrightarrow \quad z(\mathrm{O}(y, z) \quad \mathrm{L}(x, z))$.

The basic picture of logical relationships among localization and parthood relations is then completed by the following corollaries:

TL19 $\mathrm{WL}(x, y) \leftrightarrow \quad z(\mathrm{O}(x, z) \rightarrow \mathrm{GL}(z, y))$
TL20 $\mathrm{WL}(x, y) \leftrightarrow \quad z(\mathrm{GL}(x, z) \rightarrow \mathrm{GL}(y, z))$
TL21 $\mathrm{PL}(x, y) \leftrightarrow \quad z(\mathrm{GL}(x, z) \rightarrow \mathrm{O}(z, y))$
TL22 $\mathrm{PL}(x, y) \leftrightarrow \quad z(\mathrm{GL}(y, z) \rightarrow \mathrm{GL}(x, z))$

TL23 $\mathrm{GL}(x, y) \leftrightarrow \quad z(\mathrm{P}(x, z) \quad \mathrm{PL}(z, y))$
TL24 $\mathrm{GL}(x, y) \leftrightarrow \quad z(\mathrm{P}(z, x) \quad \mathrm{WL}(z, y))$.

All of these, as we said, follow directly from AL1–AL3. Using also AL4 we ensure further that L is both antisymmetric and transitive:

TL25 $\mathrm{L}(x, y) \quad \mathrm{L}(y, x) \rightarrow x=y$
TL26 $\mathrm{L}(x, y) \quad \mathrm{L}(y, z) \rightarrow \mathrm{L}(x, z)$.

Since L is reflexive on all regions, relative to the sub-domain of regions exact localization is therefore a well-behaved partial ordering. Little inspection then shows that the same also holds of complete as well as of partial localization:

TL27 $\mathrm{WL}(x, y) \quad \mathrm{WL}(y, x) \rightarrow x=y$
TL28 $\mathrm{WL}(x, y) \quad \mathrm{WL}(y, z) \rightarrow \mathrm{WL}(x, z)$
TL29 $\mathrm{PL}(x, y) \quad \mathrm{PL}(y, x) \rightarrow x=y$
TL30 $\mathrm{PL}(x, y) \quad \mathrm{PL}(y, z) \rightarrow \mathrm{PL}(x, z)$.

By contrast, general localization is neither transitive nor antisymmetric. The best we can say is that it is reflexive and symmetric among regions—but this is obvious: with respect to regions, general localization is neither more nor less than mereological overlap.

REGIONS

We have seen that the limited reflexivity of L can be used to pick out a defining property of spatial regions. Accordingly, we can define a 'region' predicate (R) by setting:

DL4 $\mathrm{R}(x) =_{\mathrm{df}} \mathrm{L}(x, x)$.

Focusing now on this predicate, at least two more structural facts are worth noting. First, AL4 together with AL3 make sure that being a spatial region is a dissective property: regions have only regions as parts:

TL31 $\mathrm{R}(x) \quad \mathrm{P}(y, x) \rightarrow \mathrm{R}(y)$.

Second, the property of being a spatial region is closed under (mereological) product, in the sense that

TL32 $\quad \mathrm{R}(x) \quad \mathrm{R}(y) \rightarrow \mathrm{R}(x \times y)$,

where

DP4 $\quad x \times y =_{\mathrm{df}} \iota z \quad w\ (\mathrm{P}(w, z) \leftrightarrow \mathrm{P}(w, x) \quad \mathrm{P}(w, y))$.

This reflects an important sense in which spatial regions form a mereologically well-behaved domain. However, AL1–AL4 are not sufficient to ensure that a similar fact extends to mereological sums. A domain built up from two atomic regions x and y satisfies the axioms even if $x+y$ is left out of the extension of ‘R’. Hence, to do justice to the intuition that the sum of any two regions is itself a region, we need to add a further, independent axiom:

AL5 $\quad \mathrm{R}(x) \quad \mathrm{R}(y) \rightarrow \mathrm{R}(x+y)$,

where

DP5 $\quad x+y =_{\mathrm{df}} \iota z \quad w\ (\mathrm{O}(w, z) \leftrightarrow \mathrm{O}(w, x) \quad \mathrm{O}(w, y))$.

More generally, define the operator σ of general mereological fusion:

DP6 $\quad \sigma x(\varphi(x)) =_{\mathrm{df}} \iota z \quad w\ (\mathrm{O}(w, z) \leftrightarrow \quad y(\varphi(y) \quad \mathrm{O}(w, y)))$.

(By AP2, this exists whenever φ is a satisfied condition, and by AP1 + DP3 it is always unique.) Then AL5 may be strengthened so as to cover infinite sums:

AL5' $\quad x(\varphi(x)) \quad x(\varphi(x) \rightarrow \mathrm{R}(x)) \rightarrow \mathrm{R}(\sigma x(\varphi(x)))$.

If there are φers, and if all φers are regions, then putting them all together is sure to give you a region. In particular, the sum of all regions is itself a region—the universal region. The rationale for AL5 and AL5' is, incidentally, related to the fact that the ontological neutrality of the operation of mereological sum cannot prevent regions from being part of hybrid sums, such as John + John’s region. Thus, although for regions dissectivity is automatically granted downwards (by TL31), closure under sum is independently necessary for constructing regions out of component regions. TL31 says that regions are necessary in order to build up regions mereologically; AL5' ensures that they are sufficient.

Likewise, note that nothing in the axioms assumed so far implies that spatial regions form a non-atomistic, dissective domain, in the sense that every region has some proper part which is a region. If desired, that must also be added as a further independent axiom. More precisely, we can express the assumption that space is dissective downwards and/or upwards by means of the following axioms, respectively:

AL6a $\mathrm{R}(x) \rightarrow \quad y(\mathrm{R}(y) \quad \mathrm{PP}(y, x))$
AL6b $\mathrm{R}(x) \rightarrow \quad y(\mathrm{R}(y) \quad \mathrm{PP}(x, y))$.

An even stronger axiom would be density: the mereological nesting of regions always yields "remainders" that are themselves regions:

AL7 $\mathrm{R}(x) \quad \mathrm{PP}(y, x) \rightarrow \quad z(\mathrm{R}(z) \quad \mathrm{PP}(z, x) \quad \neg\, \mathrm{O}(z, y))$.

THE REGIONS OF THINGS

Consider again adding the postulate that every entity be located at some region. As we mentioned, this would obviously be very strong, and would have drastic consequences for the ontology: every entity lacking spatial localization (such as the number 7, say) would be excluded from the domain. Even so, it may be convenient, for the purpose of further investigating the structure of spatial localization, to confine ourselves to such restricted domains. If we are only talking about spatial entities, we may as well make that explicit. Let us therefore make this additional assumption, at least provisionally, and for methodological reasons:

AL8 $\quad y(\mathrm{L}(x, y))$.

Given any entity x, we can now speak of its region $\mathrm{r}(x)$—the region of space where x is located:

DL5 $\mathrm{r}(x) =_{\mathrm{df}} \iota y(\mathrm{L}(x, y))$.

The unicity of $\mathrm{r}(x)$ follows directly from the extensionality postulate, AL1. (Thus, given the immediate consequence:

TL33 $\mathrm{L}(x, y) \leftrightarrow y = \mathrm{r}(x)$,

in the presence of AL8 we could have defined 'L' in terms of 'r', taking the latter as a primitive notion for the formal analysis of localization.) From AL1, together with AL4, it follows further that r is idempotent and distributes over sums:

TL34 $r(r(x)) = r(x)$
TL35 $r(x+y) = r(x)+r(y)$,

whence

TL36 $r(x+y) = r(x+r(y)) = r(r(x)+y) = r(r(x)+r(y))$.

That is, objects and their regions do not pile up to form other regions. By contrast, the analogues for the operation of mereological product may not hold unless the domain is restricted so as to include only regions. John and r(John) have no parts in common, although obviously r(John) overlaps r(John). In fact, note that if we confine ourselves to the domain of regions, besides idempotence and distributivity we also have expansiveness:

TL37 $R(x) \rightarrow P(x, r(x))$.

In view of TL34–TL36, this means that in the domain of regions the r operator behaves as a closure operator satisfying the (mereologized version of) the standard Kuratowski axioms for topological closure. However, this is not very interesting in itself, since TL37 can of course be strengthened to:

TL38 $R(x) \rightarrow x=r(x)$,

which marks a complete collapse of the distinction between an entity and its region. Not a surprise: the notion of an object's region is of interest only when the object is not itself a region.

On a different line, observe that the basic postulates for the localization relation imply that mereological relations between things be mirrored by the mereology of the corresponding regions:

TL39 $P(x, y) \rightarrow P(r(x), r(y))$
TL40 $O(x, y) \rightarrow O(r(x), r(y))$.

In fact, reference to regions allows us to draw a direct bridge from mereology to localization:

TL41 $P(x, y) \rightarrow WL(x, r(y))$
TL42 $P(x, y) \rightarrow PL(y, r(x))$
TL43 $O(x, y) \rightarrow GL(x, r(y))$
TL44 $O(x, y) \rightarrow GL(y, r(x))$.

These implications are trivial in case both x and y are spatial regions. In the general case, however, TL41–TL44 highlight a fundamental pattern of interaction between these two conceptual domains: mereological structure induces a locative structure, but locative structure has no (immediate) consequences on mereology. In particular, as it is to be expected, the converses of TL41–TL44 do not generally hold. At most, we have the following:

TL45 $\mathrm{WL}(x, y) \rightarrow \mathrm{P}(\mathrm{r}(x), y)$
TL46 $\mathrm{PL}(x, y) \rightarrow \mathrm{P}(y, \mathrm{r}(x))$
TL47 $\mathrm{GL}(x, y) \rightarrow \mathrm{O}(\mathrm{r}(x), y)$
TL48 $\mathrm{GL}(x, y) \rightarrow \mathrm{O}(y, \mathrm{r}(x))$.

In other words, locative structure induces a mereological structure (only) in the corresponding domain of regions. (The converses of TL45–TL47 also hold on the assumption that y be a region.)

ENTERING TOPOLOGY

One should not suppose that all regions are connected, or of a piece. Arguably, if object x is scattered, $\mathrm{r}(x)$ will be. Now, this kind of general principle cannot be formulated in terms of L and P only, and requires the basic framework to be supplemented by a topological machinery of some sort. More generally, the availability of topological notions makes it possible to pick out topologically salient parts (interior part, tangential part) and properties or relations (self-connectedness, separatedness, external connection) in terms of which peculiar patterns of spatial localization can be defined. For example, we can say that an object x is *internally* (*tangentially*) *located* at a region y if and only if x is located at some interior (tangential) part of y.

Following in the footsteps of Whitehead [1929], this can be made precise along the following lines. Let us use ‘C’ for the basic topological relation of connection. This is to be understood in the standard topological sense, modulo a mereological rather than set-theoretic framework. Intuitively, two things x and y are topologically connected just in case they either overlap or abut each other (i.e., intuitively, just in case either x overlaps the boundary of y or y overlaps the boundary of x). With this interpretation in mind, we take ‘C’ to be governed by the following basic axioms:

AC1 $\mathrm{C}(x,\ x)$
AC2 $\mathrm{C}(x,\ y) \rightarrow \mathrm{C}(y,\ x)$
AC3 $\mathrm{P}(x,\ y) \rightarrow \quad z\ (\mathrm{C}(z, x) \rightarrow \mathrm{C}(z, y))$.

AC1+AC2 say that C is reflexive and symmetric, while AC3 guarantees that connection be distributive over parthood.

Using ‘C’, other simple mereotopological notions are immediately defined—e.g., external connection, tangential parthood, interior parthood:

DC1 $\mathrm{EC}(x,\ y)\ =_{\mathrm{df}}\ \mathrm{C}(x, y) \quad \neg\, \mathrm{O}(x, y)$
DC2 $\mathrm{TP}(x,\ y)\ =_{\mathrm{df}}\ \mathrm{P}(x, y) \quad z\ (\mathrm{EC}(z, x) \quad \mathrm{EC}(z, y))$
DC3 $\mathrm{IP}(x,\ y)\ =_{\mathrm{df}}\ \mathrm{P}(x, y) \quad \neg\, \mathrm{TP}(x,\ y)$.

As is clear from DC1, all of these run afoul of plain mereology unless we assume C to coincide with O. Even if we restrict ourselves to a domain of regions, the possibility remains that two regions be only externally connected, i.e., intuitively, connected only through a common boundary.

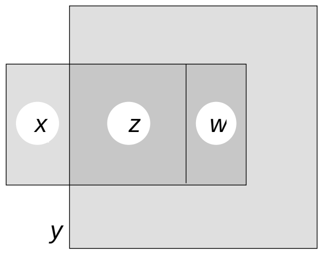
Let us stress that this account of ‘C’ is rather standard, in that it reduces to a standard mereotopology if the variables are restricted to the domain of spatial regions [2]. Below we shall consider the possibility of pursuing different interpretations; but, for the moment, let us focus on this simple framework. Following the pattern of DL1–DL3, we can then refine the picture of localization relations in the desired way, by introducing corresponding relations of external location, tangential whole/partial location, and internal whole/partial location:

DL6 $\mathrm{EL}(x, y) =_{\mathrm{df}}\ z(\mathrm{EC}(z, y) \quad \mathrm{L}(x, z))$
DL7 $\mathrm{TWL}(x, y) =_{\mathrm{df}}\ z(\mathrm{TP}(z, y) \quad \mathrm{L}(x, z))$
DL8 $\mathrm{TPL}(x, y) =_{\mathrm{df}}\ z(\mathrm{TP}(z, x) \quad \mathrm{L}(z, y))$
DL9 $\mathrm{IWL}(x, y) =_{\mathrm{df}}\ z(\mathrm{IP}(z, y) \quad \mathrm{L}(x, z))$
DL10 $\mathrm{IPL}(x, y) =_{\mathrm{df}}\ z(\mathrm{IP}(z, x) \quad \mathrm{L}(z, y))$.

It is obvious that EL is not included in any of the relations available so far, whereas TWL and IWL are special cases of WL, and TPL and IPL special cases of PL. Thus, the basic features of localization expressed by TL1–TL2 can be supplemented by the following:

TL49 $\mathrm{EL}(x, y) \rightarrow \neg\, \mathrm{GL}(x, y)$
TL50 $\mathrm{TWL}(x, y) \quad \mathrm{IWL}(x, y) \rightarrow \mathrm{WL}(x, y)$
TL51 $\mathrm{TPL}(x, y) \quad \mathrm{IPL}(x, y) \rightarrow \mathrm{PL}(x, y)$.

By way of illustration, assuming for simplicity that w, x, y, and z are all regions, in the following figure we have *inter alia* EL(x, y), TWL(z, y), TPL(y, z), IWL(w, y), and IPL(y, w).



Again, this is somewhat trivial and reduces to pure mereotopology if we confine ourselves to a domain of regions; but, as we shall see, the full-blown picture is much richer and presents several interesting features.

On this basis, the specific axioms relating ‘C’ and ‘L’ are essentially five:

AL9 $\quad L(x, y) \quad IP(z, x) \to IWL(z, y)$
AL10 $\quad L(x, y) \quad IP(z, y) \to IPL(x, z)$
AL11 $\quad L(x, y) \quad TP(z, x) \to TWL(z, y)$
AL12 $\quad L(x, y) \quad TP(z, y) \to TPL(x, z)$
AL13 $\quad z\ (C(z, x) \to C(z, y)) \to (R(x) \to P(x, y))$.

The first four of these are patterned after AL2–AL3 and simply extend to the present case the basic relationships between L and the other relations of localization. In fact, AL2–AL3 are now derivable from AL9–AL12. Moreover, we can immediately infer facts analogous to TL3–TL8 and TL15–TL24. For instance, we have:

TL52 $\quad IWL(x, y) \quad IP(z, x) \to IWL(z, y)$
TL53 $\quad IPL(x, y) \quad TP(z, y) \to IPL(x, z)$
TL54 $\quad TWL(x, y) \quad TP(z, x) \to TWL(z, y)$
TL55 $\quad TPL(x, y) \quad TP(z, y) \to TPL(x, z)$,

and so on. We also have the analogues of TL39–TL48. In particular, the regions of connected entities are themselves connected:

TL56 $\quad C(x, y) \to C(r(x), r(y))$.

(We leave it open for the moment whether the converse should also hold.) As for AL13, it is a more substantial axiom: it says that the converse of AC3 holds too whenever the first argument of the parthood relation is a region. This has the consequence of making parthood among regions characterizable in terms of their connection relations, which in turn implies that C is extensional among regions:

TL57 $\quad \mathrm{R}(x) \quad \mathrm{R}(y) \rightarrow (x{=}y \leftrightarrow \quad z\,(\mathrm{C}(z, x) \leftrightarrow \mathrm{C}(z, y)))$.

Such a characterization is of course intuitive as long as regions form a dissective domain, with no "atoms" (AL6a). Otherwise, to avoid the absurdity that every atom x be part of its complement $\sim x$, defined as

DP7 $\quad \sim x =_{\mathrm{df}} \sigma z\,(\neg\,\mathrm{O}(z, x))$,

it would be necessary to insert in the antecedent of AC5 the requirement that x be non-atomic, i.e., $\quad z(\mathrm{PP}(z, x))$.

At this point, we only need to establish the exact relationship between 'C' with the theory of localization. It is given by the following:

TL58 $\quad \mathrm{L}(x, y) \leftrightarrow \mathrm{R}(y) \quad\quad z\,(\mathrm{C}(z, \mathrm{r}(x)) \leftrightarrow \mathrm{C}(z, y))$.

Thus, x is exactly located at y just in case y is a region that is connected exactly to those things that are connected with the region of x. Likewise for the other relations of localization.

We need not go any further into the formal details. The properties of the topology-based relations defined by DL6–DL10 are rather obvious, given DC1–DC3, and in most cases they reflect corresponding properties of the mereological structure in the background. Also, although the overall picture could still be extended in several ways, the philosophical import of such extensions would be rather limited, and will not be pursued here.

CONNECTION, LOCALIZATION, AND PARTHOOD

We have three basic spatial relations now: connection (for topological aspects), parthood (for mereology), and localization. A number of facts highlighting the basic relationships among these notions have been listed. But the general question of the mutual interdependence between topology, mereology, and localization is still open: do we need all three of them, or can we dispense with some? In the domain of regions, localization reduces to mereology by TL45–TL48, and mereology reduces to topology by AL13, so only the latter is actually needed. This is in itself an important fact. But what about the general case?

Surely, if the domain is not restricted to spatial regions, localization does not reduce to and requires both mereology and topology. We have just seen that. But suppose we start with topology instead. We have relied on a standard interpretation of 'C', whereby two things are connected if they are either overlapping or abutting. However, one might be tempted here to go beyond such an accustomed interpretation and rely on a more general notion of topological connection, so as to explain away localization in terms of the resulting topological theory. If x is located at r(x)—one could argue—this would be *because* x is connected with r(x). This is intriguing, but of course it means little unless we can explain what this more general interpretation of 'C' looks like. In particular, what should the relevant notion of connection between x and r(x) amount to? Overlap is excluded: an object does not share parts with the region at which it is located—unless of course the object is itself a region, or a mereological sum including regions among other things. Moreover, two objects can be partly or wholly co-located without sharing any parts: if you put a stone inside a hole, the former does not become part of the latter (and surely enough, if the stone fills the hole perfectly, it does not become *identical with* the hole). Abutting is also excluded, if this is understood as the relation of *external* connection holding between contiguous entities. To fix ideas, we must therefore think of the relevant relation as a sort of *internal* connection. But this would take us into a circle. For what kind of relation is this, if not a relation of localization? We face here an apparent limit of topology as a general theory of space. Topology—and, more generally, mereotopology—falls short of expressing one fundamental metaphysical fact about space, namely embedding in space. The *analysis situs* overlooks the fact that objects are *situated.*

This is not the whole story, though. For consider again TL56: if two objects are connected, then so are their regions. Why not assuming the converse as well? Why not regard connection of regions as a sufficient condition for connection of corresponding entities? This would mark an extension of the standard interpretation of 'C'. But it would be definite enough, and it would strengthen the tie between localization and topology: If we read 'C(x, y)' as 'the region of x and the region of y are either overlapping or abutting'

[3], then localization *is* connection minus overlap minus abutting—or so one could argue. For consider again *x* and its region, r(*x*). On the standard interpretation of 'C' employed in the previous section, they are not connected. But on the new interpretation at issue, they would be. Now, we just saw that *x* and r(*x*) don't overlap. And we don't want to say that they are externally connected either. This would indeed follow from DC1, but if we strengthen TL56 to a biconditional, the definition of 'EC' must obviously be emended:

$$\text{DC1}' \quad \text{EC}(x, y) \quad =_{\text{df}} \quad \text{C}(x, y) \quad \neg\,\text{O}(\text{r}(x), \text{r}(y)).$$

(Otherwise, via DL6, exact localization and external location would coincide, which is absurd.) So again *x* and r(*x*) would be connected, non-overlapping, and non-abutting. But now we would have an account of each of these notions. *Ergo*, we would after all have an account of the relation between *x* and r(*x*)—and that is purely mereotopological.

Again, the argument is intriguing, but it fails to establish the purported conclusion. What follows, really, is that localization can be explained in terms of C, P, *and* r. But of course we know that regions are essentially linked to localization. The region of *x* is the region at which *x* is located (DL5), and *x* is located at its regions and nothing else (TL33). So, assuming r is tantamount to assuming L, really, and the limits of mereotopology show up again. (This is in fact the reason why, on the standard interpretation of 'C', an equivalence such as TL58 does not amount to a characterization of localization in topological terms: reference to 'R' plays a crucial role in the right-hand side of the biconditional, and 'R' is defined via 'L'.)

One final point is in order. It is clear that on this extended interpretation of 'C', mereology and topology would also retain an independent status. For on the one hand, C would be a proper extension of O by assumption. On the other, with TL56 strenghtened to a biconditional, everything connected to an entity *x* would be connected to its region r(*x*), and yet the former would not be part of the latter. (Likewise, if a stone is inside a hole, then everything connected to the stone would be connected to the hole, but the stone would not be part of the hole. Its region is part of the region of the hole, but that is all.) Going back now to the standard interpretation of 'C' employed here, the situation is significantly different. If connection is really only a matter of overlap or abutting, then the stone and the hole are arguably disconnected. But what about the stone and its region? And what about other cases of (partial) co-localization—for instance, what about the rotating and the cooling down of the Earth, are they connected? Is there a sharing of boundaries there? If the answer is no in all cases, then an important consequence follows. For then we have no more counterexamples to the reducibility of parthood to connection. In other words, there are no pairs of entities x and y such that x is not part of y even though everything connected with x is connected with y. Accordingly, the restriction R(x) could be dropped from the consequent of AL13, and the reduction of mereology to topology would be complete (as work in the tradition of Whitehead has suggested). Thus, although we started from mereology, in the end we would be left with a topology-based theory of localization. If, on the other hand, we leave open the possibility that at least some such cases of co-localization involve boundary-sharing (if not overlap), then the reduction falls short, and the final picture is truly three-fold: we need topology, we need localization, and we need mereology. Which of these accounts is correct is perhaps a matter of stipulation. We prefer the open choice, and consequently the three-fold picture.

ON SHARING AN ADDRESS

We have been dealing so far with localization understood as a relation between an object and a region of space. But we can also refer to the notions introduced above to express the fundamental ways in which two objects can be related *to each other* in terms of localization. Thus, for every L-relation ΣL, with Σ = W, P, G, E, IC, IP, TC, or TP, define the corresponding relation of region-location:

DL11 $\mathrm{R\Sigma L}(x, y) =_{\mathrm{df}} \mathrm{\Sigma L}(x, \mathrm{r}(y))$.

As the following equivalences show, any relation thus defined is equivalent to a relation expressible via P, C, and r (compare TL45–TL48 and the obvious topological analogues):

TL59 $\mathrm{RWL}(x, y) \leftrightarrow \mathrm{P}(\mathrm{r}(x), \mathrm{r}(y))$
TL60 $\mathrm{RPL}(x, y) \leftrightarrow \mathrm{P}(\mathrm{r}(y), \mathrm{r}(x))$
TL61 $\mathrm{RGL}(x, y) \leftrightarrow \mathrm{O}(\mathrm{r}(x), \mathrm{r}(y))$
TL62 $\mathrm{REL}(x, y) \leftrightarrow \mathrm{EC}(\mathrm{r}(x), \mathrm{r}(y))$
TL63 $\mathrm{RTWL}(x, y) \leftrightarrow \mathrm{TP}(\mathrm{r}(x), \mathrm{r}(y))$

TL64 $\mathrm{RTPL}(x, y) \leftrightarrow \mathrm{TP}(\mathrm{r}(y), \mathrm{r}(x))$
TL65 $\mathrm{RIWL}(x, y) \leftrightarrow \mathrm{IP}(\mathrm{r}(x), \mathrm{r}(y))$
TL66 $\mathrm{RIPL}(x, y) \leftrightarrow \mathrm{IP}(\mathrm{r}(y), \mathrm{r}(x))$.

But on the face of it, DL11 represents a fundamental key for extending the theory of localization developed so far. For it makes it possible to relax the constraint that localization is essentially a relation between objects and regions. By DL11, we can speak of an object being wholly, partly, or generically located at/with another object, and likewise for the other relations. (We can say, for instance, that John is internally wholly region-located (RIWL) in his godfather's country house.)

Now, one more relation that can be defined along these lines is *exact co-localization*, which so far we have been using intuitively. This is obtained by just dropping the modifier 'Σ' from DL11:

DL12 $\mathrm{RL}(x, y) =_{\mathrm{df}} \mathrm{L}(x, \mathrm{r}(y))$.

The link with TL59–TL66 is given by the fact that sameness of localization amounts to identity of corresponding regions:

TL67 $\mathrm{RL}(x, y) \leftrightarrow \mathrm{r}(x)=\mathrm{r}(y)$.

This is in itself an interesting notion, and rather well-behaved:

TL68 $\mathrm{RL}(x, x)$
TL69 $\mathrm{RL}(x, y) \rightarrow \mathrm{RL}(y, x)$
TL70 $\mathrm{RL}(x, y) \quad \mathrm{RL}(y, z) \rightarrow \mathrm{RL}(x, z)$.

That is, RL is reflexive, symmetric, and transitive (an equivalence relation). Moreover, we immediately have the analogues of DL1–DL3:

TL71 $\mathrm{RWL}(x, y) \leftrightarrow \quad z(\mathrm{P}(z, y) \quad \mathrm{RL}(x, z))$
TL72 $\mathrm{RPL}(x, y) \leftrightarrow \quad z(\mathrm{P}(z, x) \quad \mathrm{RL}(z, y))$
TL73 $\mathrm{RGL}(x, y) \leftrightarrow \quad z \quad w(\mathrm{P}(z, x) \quad \mathrm{P}(w, y) \quad \mathrm{RL}(z, w))$

as well as of AL2–AL3:

TL74 $\mathrm{RL}(x, y) \quad \mathrm{P}(z, x) \rightarrow \mathrm{WL}(z, y))$
TL75 $\mathrm{RL}(x, y) \quad \mathrm{P}(z, y) \rightarrow \mathrm{PL}(x, z))$.

(Likewise for the analogues of the topological extensions in DL6–DL10 and AL9–AL12.) This is as it should be. If the statue is co-located with the bronze, then the head of the statue is wholly located within the (region of the) whole bronze, and the whole statue is partly located at the (region of the) bronze of the head. This is not enough, though. For we also want to say that the sum consisting of the statue *and* the bronze is co-located with the statue, and also with the bronze. This cannot follow from the available axioms for L, which are consistent with the possibility that the sum of two co-located objects have a different localization than the objects themselves. To rule this out, we therefore need an additional axiom:

AL14 $x(\varphi x) \quad x(\varphi x \rightarrow \mathrm{L}(x, y)) \rightarrow \mathrm{L}(\sigma x(\varphi x), y)$.

This is a sort of summing principle to the effect that fusion of any number of entities that are located at a region y is also located at y. (Exercise: how is this related to AL5'?) This is obvious if we want L to be strong enough to support the full strength of the underlying mereological structure: if the sum of two things is nothing over and above the two things (the sum *is* those things, as David Lewis [1991:81] put it), then its behavior with respect to localization cannot run afoul of the behavior of its constituent objects. If a statue and the bronze it is made of are located at the same region, then so is their mereological sum. Given this, we immediately obtain the desired property of RL:

TL76 $x(\varphi x) \quad x(\varphi x \rightarrow \mathrm{RL}(x, y)) \rightarrow \mathrm{RL}(\sigma x(\varphi x), y)$
TL77 $y(\varphi y) \quad y(\varphi y \rightarrow \mathrm{RL}(x, y)) \rightarrow \mathrm{RL}(x, \sigma y(\varphi y))$.

An important consequence is that something may be co-located with some of its proper parts. Thus, co-localization falls short of identity, as we well know. In this respect, it is rather like the relation of coincidence used for instance by Brentano in his account of boundaries: boundaries are *located in* space but do not *occupy* space, and can therefore be perfectly co-located with one another [4]. However, there is an important difference here. Boundaries do not occupy space insofar as they do not *take up* space—they are lower-dimensional spatial entities. Here, in contrast, we are interested in a general feature of spatial localization, namely, that more entities can share the same address, and this feature applies to all sorts of entities regardless of their dimensionality. Every spatial entity has an address; but to have an address does not mean to be the exclusive *owner* of it.

OWNING AN ADDRESS

Take again definition DL4, and the axiom that justifies it, AL4: spatial regions are those entities that are located at themselves. Thus, there is always something located at a region. Thus, regions are never empty.

"Full" as space may be in this sense, the possibility remains that some (or even all) spatial regions be empty in the ordinary sense. Consider then the alternative: space is full in the stronger sense that, for every region y, *something else* than y is located there. We may state this in the form of an axiom as follows:

AL15 $\mathrm{R}(y) \rightarrow \quad x(x \neq y \quad \mathrm{L}(x, y))$.

This would support the claim that localization supplies a means for picking out regions of space. Or, somewhat more emphatically, it would be in agreement with the view that the individuation of spatial regions is parasitic upon the individuation of the objects located at them—a view advocated by Strawson [1959] among others. But would AL15 really give us a full space in the ordinary sense? And under what interpretation?

Note that, by AL1 and AL4, the entity x located at region y in AL15 cannot be a region itself: it must be something of a different kind—something that cannot occur as a value for the second argument of the L relation. So, one intuition of the ordinary notion of fullness is captured by the proposed axiom, namely that filling up space involves something *else* than just space. However, to adequately capture the ordinary notion, we must also clarify the relationship between filling up a region of space (occupying it, as we may also say) and simply being located at it. Every entity x is exactly located at exactly one region, its region $\mathrm{r}(x)$. But we know that the converse need not hold: different entities may share the same exact location. Material objects seem to have the property of excluding other material objects from the region they are located at. For them, localization is exclusive—they occupy the regions where they are located (the relationship with their regions is one-one). But we saw that this need not hold for all entities. Events, for instance, do not prevent other entities from sharing the same region (a fact attesting the inadequacy of any attempt to use co-localization as a criterion for event identity just as for object identity, as Davidson pointed out in several occasions[5]). Thus, the distinction between localization and occupation is not without content. And if we take "fullness" to imply spatial occupation rather than mere localization, then AL15 needs some adjustment by replacing 'L' with a suitable predicate 'OC' expressing spatial *occupation*.

As a tentative step, we may exploit the foregoing suggestion and define this notion of occupation as exclusive localization:

DL13 $\mathrm{OC}(x, y) =_{\mathrm{df}} \mathrm{L}(x, y) \quad z(\mathrm{L}(z, y) \rightarrow z{=}x)$,

or, by exploiting the notion of co-localization (RL):

DL13' $\mathrm{OC}(x, y) =_{\mathrm{df}} \mathrm{L}(x, y) \quad z(\mathrm{RL}(z, x) \rightarrow z{=}x)$.

This is defective in several ways, though. For one thing, we know that L is reflexive on spatial regions (and RL refexive *simpliciter*), so in general we would have to insert in the first conjunct of these definitions the proviso that $x{\neq}y$, and in the antecedent of the second conjunct the proviso that $z{\neq}y$. Indeed, in view of AL14, we would actually need the provisos that $\neg\mathrm{O}(x, y)$ and $\neg\mathrm{O}(z, y)$. Secondly, also the consequent of the second conjunct is to be strengthened to an overlap proviso, i.e., $\mathrm{O}(z, x)$. Otherwise nothing would occupy anything. (Suppose x is John and z the sum of John with the event of his being there: the two are co-located and distinct, and yet we want to say that John truly occupies r(John).) Thirdly, and more generally, DL13–DL13' fail to fix a suitable range for the quantified variable z. To draw from a suggestion by David Wiggins [1968: 93], no two things *of the same kind* can be located in the same place at the same time (Wiggins uses 'occupy' in the sense of our 'located'); but a statue and the bronze it is made of may very well be exactly coincident in space, at least for some portions of their respective lives. Hence, exclusive localization (dismissing only regions of space) is too strict a condition for any interesting notion of occupation—the notion whereby material objects do, and events do not, occupy the space at which they are located. What we need is a *relative* notion, which captures the sortal nature of Wiggins's criterion. And the natural solution is to use a definition schema. Where φ is any condition, we may say that an object φ-occupies a certain region if and only if it is the only φ-er that is located at that region. Taking all of this into account, we arrive at the following:

DL13" $\mathrm{OC}_{\varphi}(x, y) =_{\mathrm{df}} \mathrm{L}(x, y) \quad \varphi x \quad z(\mathrm{L}(z, y) \quad \varphi z \rightarrow \mathrm{O}(z, x))$,

which immediately implies:

TL78 $\mathrm{OC}_{\varphi}(x, y) \quad \mathrm{OC}_{\varphi}(z, y) \rightarrow z{=}x$.

Together, DL13" and TL78 provide the desired characterization of the distinguishing feature of occupation *versus* localization, at least within the limits set above.

Of course, being defined in terms of exact localization, spatial occupancy is in fact *exact* spatial occupancy. A chair occupies exactly the region of space at which it is exactly located. Weaker notions of occupancy can then be defined matching the weaker notions of localization defined above (DL1–DL3 and DL6–DL10), but we need not go into these routine details. Suffice it to remark that there is a slight asymmetry in this regard, as localization and occupation have opposite part–whole structures. Something x is partly located at a region y if some part of x is located at y; but if this part actually occupies y (relative to some relevant condition φ), we would rather say that x *wholly* occupies y; by contrast, x *partly* occupies y if x occupies only part of y, in which case we rather say that x is wholly located at y. Thus, the correct analogues of DL1–DL2 for occupation are:

DL14 $\mathrm{WOC}_\varphi(x, y) =_{\mathrm{df}} \ z(\mathrm{P}(z, x) \quad \mathrm{OC}_\varphi(z, y))$
DL15 $\mathrm{POC}_\varphi(x, y) =_{\mathrm{df}} \ z(\mathrm{P}(z, y) \quad \mathrm{OC}_\varphi(x, z))$,

which in turn imply:

TL79 $\mathrm{WOC}_\varphi(x, y) \rightarrow \mathrm{PL}(x, y)$
TL80 $\mathrm{POC}_\varphi(x, y)) \rightarrow \mathrm{WL}(x, y)$.

There is a deeper asymmetry between occupation and localization in this connection. Occupation is first and foremost exact occupation: to understand what it is for something to occupy a region requires in an essential way the capacity to compare the respective shapes and dimensions of the occupying thing and of the occupied region. By contrast, to understand what it is for a certain thing to be located at a certain region requires the competence for somebody to find a place (an actual address) for that thing, but we have seen at the beginning that this leaves room for some flexibility as to the actual extension of the relevant place. We can distinguish a minimal and a non-minimal address, with all the intermediate degrees allowed by this contraposition. The notions of whole, partial, generic, but also tangent, internal, external localization are meant to capture the variety of ensuing plausible relations; yet some of the corresponding notions of occupation would hardly enjoy any plausibility, except from a purely algebraic perspective.

What would it be for something to internally occupy a region of space? And to externally occupy one?

With these provisos, two further facts about the notion of occupation characterized by DL13" and TL78 are worth noting. First, if φ is the property of being a region (R), then every region is actually such as to φ-occupy itself (by AL1):

TL81 $\quad OC_R(x, x)$.

This may be regarded as a case of *sui generis* occupation, for spatial regions can hardly be said to *occupy* anything. But the connection with material objects is not uninteresting: objects cannot share an address with other objects; regions cannot be located at *other* regions. This is also expressed by the following:

TL82 $\quad OC_R(x, y) \rightarrow x=y$,

whence we immediately infer that OC_R is both symmetric and anti-symmetric, and collapses to identity.

The second fact concerns the relationship between occupation and the notion of full space. As we saw, the requirement that every region be the region of something (AL15) does not capture this notion, for the relevant concept of "fullness" involves something more than mere localization—it requires occupation. However, now we see that occupation is not an absolute notion, as it crucially depends on the discriminating condition φ. This means that the strengthening of AL15 can take the following form

AL15' $\quad R(y) \rightarrow \quad x(x{\neq}y \quad OC_{\varphi}(x, y))$,

where the choice of a suitable φ is still in need of further qualification. For instance, we can take φ to be the property of being a material object, or a part thereof. But this means that the fullness of space goes beyond the confines of a theory of space—it requires a theory of objects in the first place. (Observe incidentally that Wiggins' principle is not self-evidently a necessary truth. We may take TL78 as a sort of extensionality principle apt for the characterization of material objects. Yet nothing in theory rules out the possibility that two—or, for that matter, infinitely many—distinct indistinguishable material objects be perfectly co-located. If occupation is exclusive localization, relative to things of a kind, then material objects are illustrious candidates to the role of spatial occupants. But it is not the task of a theory of localization to determine *whether* such entities are actually such as to occupy space. Nor should the theory be concerned with the task of explaining *why* material objects occupy space, if they do, and why they are therefore impenetrable by other material objects—as discussed for instance by Locke, *Essay*, II, iv, and commented upon by Leibniz.)

ROOMMATES

We have seen that occupation implies localization, but not *vice versa.* And we suggested that this distinction is not without content, that over and above regions of space several other kinds of entities may be alleged as plausible "roommates" sharing one and the same address. Now, we still need to take a closer look at what patterns of co-localization can be distinguished. For surely enough, if occupation is not an absolute notion, neither is co-localization.

We have, for instance, the case of categorially non-homogeneous roommates, as illustrated by Wiggins' opposition between a statue and the bronze it is made of, or by Locke's original example of a living organism and its constituent mass of matter. And we have homogeneous roomates of lower dimensionality, as illustrated by Brentano's view on surfaces and other boundary-like entities: such entitities are located in space but do not take up any space, and can therefore be exactly co-located with one another. (We skip over the ontological intricacies involved in the notion of a boundary, which of course tie in with the general question of whether spatial regions should include lower-dimensional elements as well—points, lines, planes.[6]) Moreover, we have at least two further cases of full-fledged homogeneus roommates, entities that seem to truly escape Wiggins' restriction that entities of the same kind cannot be co-located. On the one hand, we have events, as with the Davidsonian example of the rotation of the Earth and its cooling down [7]. Some authors, for instance Hacker [1982], have actually suggested this as a primary distinguishing feature of events in comparison to objects: the latter, but not the former, occupy the space at which they are located. On the other hand, we have various sorts of immaterial or otherwise ethereal—yet genuinely spatial—entities. Thus, for instance, in his commentary of Locke, Leibniz suggested that two shadows or two rays of light may interpenetrate (*New Essays*, II-xxvii-1). J. M. Shorter [1977] envisaged the intriguing example of two intersecting clouds produced by two distinct "cloud projectors". David Lewis [1991: 75] has two angels dancing forever on the head of a pin (two totally distinct and yet perfectly co-located proper parts of the "total angelic content" of their shared region). And we suggested considering holes: we saw that holes can be interpenetrated by other things, as when you put a stone in the middle of a doughnut; but holes can also be interpenetrated by other holes, i.e., things of the very same kind, as when you put a chunk of Gruyère with a small hole inside a bigger hole in a bigger piece of Emmenthaler. The former hole does not become part of the latter. Rather, it is partly co-located with it, i.e., exactly co-located with a part of it [8]. (It is RWL-located in the second hole, in the terminology of DL11.)

We thus have at least four distinct reasons for speaking of roommates, depending on the sort of entities that we consider. The first two cases are relatively uninteresting. Categorially heterogeneous entities need not compete for localization. And lower-dimensional entities *cannot* properly compete: if they don't take up any space, how could they claim any real spatial estate? (An exception might be geographical entities, such as regional or national territories; but the sort of competition involved in occupying this sort of two-dimensional entities is not of our concern.) The third and fourth case provide clearer cases of full-fledged, categorially homogeneous entities that can share an address with their peers. Yet again we must distinguish. In the case of events, the reason seems to lie in their somewhat difficult relationship with space. We said that occupation is first and foremost exact occupation. But events can hardly be said to be spatially located, let alone exactly located. Davidson, and after him Lombard [1986], have suggested that an event's exact location is the location of the minimal part of the event, i.e., of the part occupied by the minimal participant of the event. (At each instant of time, the exact location of John's rising of his arm is the spatial region occupied by John's arm.) Since the same participant, or the same part of a participant, can be involved in more than one event at the same time, events cannot on this account occupy the space where they occur. On the other hand, Hacker—and before him Quinton [1979]—have objected that the question 'Where did .. . happen?' is rarely a question about a minimal area in this sense, and may actually be meaningless for various classes of events (e.g., for most social events or purely relational changes). If so, then events would simply run afoul of our account of localization in terms of minimal addresses: in the case of objects everything can be reduced to a matter of *exact* localization, but when it comes to events, the only applicable notion is that of generic, non-minimal localization—and this provides no grounds for occupation. No matter how we look at it, the possibility for events to share their place of occurrence with other events seems to be a natural consequence of their loose spatial nature, and only vanishes with one's willingness to reify them and assimilate events to material objects.

Holes are quite a different story (and likewise for shadows, light rays, clouds, and perhaps even angels). Their relation to space is much more definite, as is their relation to material objects. Like material objects, holes have clear spatiotemporal properties, such as shape, size, and duration (so much so that an author like David Armstrong [1968: 282] used holes against the thesis that primary qualities are definitory of material objects). Yet holes are obviously not material objects, for they lack any material consistency. And they are not regions of space either; for holes can move, as happens anytime you move a piece of Swiss cheese, whereas regions cannot. Holes are, so to speak, halfway between objects and regions. They are *reified* spatial regions: they can change position without losing their identity; but they are not sufficiently reified to prevent other things (including other holes) to share their address. Holes do not occupy space because they are spacious, they are made of space, and share with space its most distinguishing feature—interpenetrability.

PATTERNS OF LOCALIZATION

By way of conclusion, recall then our initial stipulation: localization is a relation between spatial entities of whatever sort (as first term) and spatial regions (as second term). This is a convenient simplification, and it allows us to pinpoint some major structures of localization. But we might want to allow the second term to also range over other entities than just regions. After all, the interpenetrability of holes is paradigmatic of an important pattern of spatial localization that runs afoul precisely of this restriction—containment. In many cases, to be contained in an object is to be located (wholly or partly) in a hole of that object. And to be located in an object's hole is to be located *outside* the object, at a region disjoint from the object's region.

There are two distinct issues here. On the one hand, we may want to speak of an object being located in another object. We have already seen that such an account can be covered by the theory of localization outlined above *via* the system of relations defined in DL11 and DL12. The stone is located in the hole because it is located at the hole's region. On the other hand, the interesting extension of the theory involves allowing things to be located at other things without there being any corresponding relationship between their respective regions. The stone is in the bowl, but there is no connection between the stone's region and the bowl's. And a worm can be inside a piece of cheese without there being any connection between the region of the worm and that of the cheese: the worm lies outside the cheese, strictly speaking, and yet it is located inside it. Indeed this is generally to be expected given the constraints on spatial occupancy: if *x* and *y* are both material objects—or, more generally, objects that occupy their spatial regions—then the regions they occupy must be discrete unless the object themselves overlap. In this second case, there seems to be no straightforward bridge from L to the extended relation of localization, call it 'LOC'.

Now, there is no doubt that the general features of this extended relation are to be investigated for the purpose of a general picture of localization. We may still want to draw a bridge between LOC and L by exploiting the idea that the peculiar cases of LOC are akin to containment, and containment is typically a case of localization-in-a-hole. This would suggest setting:

DL16 $\mathrm{LOC}(x, y) =_{df} \mathrm{RL}(x, y) \quad z\,(\mathrm{H}(z, y) \quad \mathrm{RL}(x, z))$,

or, more generally:

DL17 $\Sigma\mathrm{LOC}(x, y) =_{df} \mathrm{R}\Sigma\mathrm{L}(x, y) \quad z\,(\mathrm{H}(z, y) \quad \mathrm{R}\Sigma\mathrm{L}(x, z))$,

where 'H' is the *hole in* relation and 'Σ' picks out the relevant patterns of localization as in DL11 (i.e., $\Sigma =$ W, P, G, etc.). But of course this is only good within certain limits and for certain approximations. For one thing, we need a whole theory of holes (and of the hole–host interaction) to support the interpretation of 'H'. And, secondly, not every peculiar case of LOC is reducible to localization-in-a-hole, no matter how flexible we remain with respect to the relevant notion of hole (we might want to say that the containing part of the bowl is a hole in the bowl, for instance, at least for the present purposes). In some cases, hole localization is simply not a sufficient condition for the intended interpretation of LOC: a worm in a hole of the cheese is a worm in the cheese, but a fly in a hole in the bowl (a small perforation in the bottom, for instance, not the main containing hollow) is not a fly *in the bowl*. Or, to use a related example from Vandeloise [1994: 173], the bulb is in the socket, but the bottle is not in the cap—or so one could argue. In other cases, localization in a hole falls short of being a necessary condition for LOC: the flowers are in John's hands, but there is no hole involved here unless we stipulate otherwise.

We may thus want to say that extending L to a broader notion involves abandoning the geometrical structure of localization, or at least leaving it open. Spatial localization as explained by the family of L-relations is but one major pattern of localization, and extending it along the lines of DL16–DL17 is only a minor step ahead. The variety of interesting patterns is presumably much wider, and appears to be a rewarding subject for further independent exploration.[9]

NOTES

[1] For a general overview of extensional mereology, as well as of its tensed and modal extensions, we refer to Simons [1987].

[2] There is no such thing as *the* standard mereotopology, but a variety of theories mostly inspired to Whitehead's account in [1929]. See *inter alia* Clarke [1981, 1985]; Randell & Cohn [1989]; Randell *et* al., [1992a, 1992b]; Smith [1993]. For our purposes, however, we need not go into any detailed examination of the available variants. For a first appraisal, see Eschenbach & Heydrich [1993] and Varzi [1994].

[3] This is the interpretation that we employed in Casati & Varzi [1994] (Appendix); see also Varzi [1993].

[4] Brentano [1976]; compare Chisholm [1992/93]. In fact, it is interesting to observe that recent axiomatic treatments of Brentano's theory of coincidence are based on postulates comparable to our TL68–TL70, TL74 and TL77. See e.g. Smith [1995a, 1995b].

[5] See e.g. Davidson [1969]; the idea that two distinct events cannot exactly share the same spatiotemporal region has been put forward among others by Quine [1950] and Lemmon [1967].

[6] For some thoughts on this we refer to Varzi [1996].

[7] Of course, a different analysis of event identity might deny that the rotation and the cooling down of the Earth are two distinct events. This is precisely the view of Quine and

Lemmon referred to in note 5.

[8]We discuss this in Casati and Varzi [1994], ch. 7. Chisholm [1973:590] also considers shadows and holes as counterexamples to the Locke–Wiggins principle.

[9]Many thanks to Peter Simons and Barry Smith for their comments on an earlier draft of this paper.

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