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A Preliminary Experimental Verification of Violation of Bell Inequality in a Quantum Model of Jung Theory of Personality Formulated with Clifford Algebra

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ABSTRACT

We comment some recent results obtained by using a Clifford bare bone skeleton of quantum mechanics in order to formulate the conclusion that quantum mechanics has its origin in the logic, and relates conceptual entities. Such results touch directly the basic problem about the structure of our cognitive and conceptual dynamics and thus of our mind. The problem of exploring consciousness results consequently to be strongly linked. This is the reason because studies on quantum mechanics applied to this matter are so important for neurologists and psychologists. Under this profile we present some experimental results showing violation of Bell inequality during the MBTI test in investigation of C.V. Jung's theory of personality.

Key Words: experimental verification, violation of Bell's Inequality, quantum model, Jung theory, personality, Clifford algebra.

1. Introduction

Some recent results deserve here some further comment and consideration**.**

1) By using the Clifford algebra one of us [(1,2)] has recently obtained two results that seem to be of importance.

According to a procedure previously introduced from Y. Ilamed and N. Salingaros [1], he started giving proof of two existing Clifford algebras, the Si that has isomorphism with that one of Pauli matrices and the $N_{i,\pm 1}$ where N_i stands for the dihedral Clifford algebra. The salient feature is

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that he showed that the $N_{i,\pm1}$ may be obtained from the Si algebra when we attribute a numerical value (+1 or −1) to one of the basic elements (e_1, e_2, e_3) of the Si. He utilized such shown result to advance a criterium under which the Si algebra has as counterpart the description of quantum systems that in standard quantum mechanics are considered in absence of observation and quantum measurement while the $N_{i,\pm1}$ attend when a quantum measurement is performed on such system with advent of wave function collapse. The physical content of the criterium is that the quantum measurement with wave function collapse induces the passage in the considered quantum system from the Si to $N_{i,+1}$ or to the $N_{i,-1}$ algebras, where each algebra has of course its proper rules of commutation. He re-examined the von Neumann postulate on quantum measurement, and gave a proper justification of such postulate by using the Si algebra. Soon after he studied some applications of the above mentioned criterium to some cases of interest in standard quantum mechanics, analyzing in particular a two state quantum system, the case of time dependent interaction of such system with a measuring apparatus and finally the case of a quantum system plus measuring apparatus developed at the order n=4 of the considered Clifford algebras and of the corresponding density matrix in standard quantum mechanics. In each of such cases examined, he found that the passage from the algebra Si to $N_{i,\pm1}$, considered during the quantum measurement of the system, actually describes the collapse of the wave function. Therefore he concluded that the actual quantum measurement has as counterpart in the Clifford algebraic description, the passage from the Si to the $N_{i,\pm1}$ Clifford algebras, reaching in this manner the objective to reformulate von Neumann postulate on quantum measurement and proposing a self-consistent formulation of quantum theory.

In substance, on the basis of such results , we may say that it was reached mathematical proof of two existing stages of our reality, an ontic state of irreducible indeterminism, often called in standard quantum mechanics as the state of potentiality of a given quantum system, and a stage marked instead from actualization that is to say ….the reduction of the basic potentiality to a level of made aware actualization of some explored property or quantum variable of such investigated quantum system. The first stage is described by the proper Clifford algebra S_i as well as the second is described instead by the different Clifford algebra $N_{i,\pm1}$.

Let us express in detail that such obtained results change very little the conceptual framework of standard quantum mechanics. It is known from more than eighty years ago that quantum mechanics, according in particular to Von Neumann in 1955, predicates that we have two fundamentally different types of evolution for a quantum system. First there is the casual (reversible) Schrödinger equation, and the second, there is the non casual (irreversible) change due to a measurement. We also know very well that such standard interpretation has given origin in such years to a tight debate that of course was not able to lead to a final conclusion on such matter. The basic reason is that von Neumann did not show that reality actually follows such steps. He only postulated the two previously mentioned indications of Schrödinger causal (reversible) time evolution and of the non causal (irreversible) change due to a measurement, respectively. The novel feature is that we give now mathematical proof of two such existing Clifford algebras linked respectively, the S_i to the first kind of quantum time evolution and the $N_{i,\pm1}$ to the non casual (irreversible) change due to a measurement. In substance, we give finally proof of what in von Neumann was only a postulate and such result, according to the demonstrative privilege that mathematics has always had in science and in particular in physics, should represent an actual advance in the knowledge that we have on this matter. The results my be found in detail in [1]

As any mathematical or physical new approach, also the present results are open to interpretations.

We enunciate the following statement:

The first algebra, the S_i, refers to the representation of a particular situation in quantum mechanics where the observer has not been called to measure and to decide, as example on the state of a given two-state system. So, it relates the standard quantum mechanics. Through an operation that mathematically is represented by the N_i algebra, the observer finally decides to perform a quantum measurement and to specify which state is the one that will be or is being observed. In conclusion, when it happens that the so called wave function reduction or collapse of wave function we have a transition from the S_i algebra to the N_i algebra.

Note that we have been forced to use the following phrases "the observer has not been called to measure and to decide" and "Through an operation that mathematically is represented by the N_i algebra, the observer finally decides". The term "decision" is recurrent in both such phrases. The first point is that we know now the algebra relating the system when the subject does not decide to perform a measurement and we know that there exists another algebra that relates the subject when he decides to perform the measurement.
Let us explain in more detail.

The used term Decision is the key word here. Quantum measurement is an operation that mathematically is represented by the N_i algebra. The profound discrimination between such two algebras indicates that a quantum measurement is not only a physical interaction between two systems(the measuring apparatus and the measured system) but, in accord in some manner with Schneider [2], we cannot avoid to add a basic other feature . A quantum measurement is fundamentally an interaction between languages, perception, and cognition. In other terms, we cannot escape to fix one time for all that a quantum measurement is a semantic at, just using here Schneider words.

We state precisely: Discrimination between S_i and N_i algebra indicates that a measurement is a cognitive act. It does not exist a measurement without a cognitive task. It is not important if we read directly the result of the measurement on the instrument or if instead it is read automatically , it is not important if the measuring apparatus is macroscopic or not, it is fundamentally important to accept that any measurement is conceived at its source on the basis of a cognitive –semantic act. Any measuring instrument is realized at its source so to perform a semantic-cognitive act and without such basic condition we have not a measurement. A measuring device is a structure whose counterpart t is the matter of our perception and of our mental operations. We cannot ignore that such operation of measurement cannot run if we have not previously established the mathematical symbols , the semantic and semiotic functions, in brief … the cognitive performance, that enables us

subsequently to express the results of the measurement.

The arising conclusion is that the shown mathematical results given in [1] evidence that quantum mechanics is a two-faced Janus … from one face looking to basic phenomenology of matter and by the second face looking at our conceptual entities , to our mind , and thus to our consciousness.

2) In the previous point, we have arrived to conclude that quantum theory includes in itself not only the description of the reality at the microphysical level. It also envelops the cognitive performance that is required to conceive reality. This is the reason because studies on quantum mechanics are so important for neuroscience and psychology.

To support this thesis, we have the previous mentioned theorems but we have also a further proof that one of us (1,2) has recently obtained.[3] .

Let us start considering the following argument. In 1932 von Neumann showed a result that is of crucial importance for us. In brief, this author constructed a quantum matrix logic on the basis of quantum mechanics.

Also even if highly promising, this result, however, cannot be considered so central and determinant for our purposes. Actually, in order to obtain a novel feature, we have to show that the result that was obtained from von Neumann may be inverted.

In fact, in the previous mentioned papers [3], one of us (1,2) was able to show that not only a quantum matrix logic may be constructed on the basis of quantum mechanics but exactly the inverted situation. He showed that quantum mechanics may be derived on the basis of logic. Arriving to give proof that quantum mechanics derives from logic, one completes the circle of our reasoning. He reaches the highest possible support to the thesis that quantum theory is the first "physical theory" of cognition of our mind and that we think in a quantum probabilistic manner.

This is the objective that was reached in [3]. Stated that quantum mechanics runs about two basic foundations, the first being the irreducible indeterminism and the second being the quantum interference, starting with his usual basic Clifford elaboration, this author constructed a Clifford logic approach. Than, following the scheme introduced in the previous point (1), and thus using the two theorems relating respectively the S_i and the N_i algebras, the author demonstrated that, according to such Clifford algebraic scheme, the origins of the most fundamental quantum phenomena as the indeterminism and the quantum interference, derive not from the traditional physics itself but from the logic.

As statement, the only admissible consequent conclusion is that quantum mechanics relates cognitive-conceptual entities and that we think in a quantum probabilistic manner.
Of course we have to outline here with greatest emphasis that the excellent logic Yuri Orlov, starting with 1977 and when he was in prison Camp 37-2 in Urals in USSR as dissident, started to study this problem [4]. He introduced a so called Wave Calculus based upon Wave logic. He

did not use the Clifford algebra but arrived to similar conclusions on the logical origins of quantum mechanics.

There is still some other comment to add.

If we have logical origins of quantum mechanics as consequence we have a logical relativism in this theory. How is that we have not such logical relativism in classical physics? What is the reason because we have instead such strong constraint in quantum mechanics? We give here an answer that of course is in accord with Orlov. The explanation is as it follows:

There are stages of our reality in which it results impossible to unconditionally defining the truth. Logic, language and thus cognition enter with a so fundamental role in quantum mechanics because there are levels of our reality in which the fundamental features of cognition and thus of logic and language, and thus the conceptual entities, acquire the same importance as the features of what is being described. At this level of reality we no more may separate the features of matter per se from the features of the cognition, of the logic and of the language that we use to describe it. Conceptual entities non more are separated from the object of cognitive performance.

As correctly Yuri Orlov outlined in his several papers, the truths of logical statements about dynamic variables relating matter structure become dynamic variables themselves in quantum mechanics.

Therefore our statement is that the cognition becomes in itself an immanent feature that operates symbiotically with the matter phenomenology that traditional physics aims to represent.

This is the profound reason because we have to apply quantum mechanics at cognitive level. Quantum mechanics is the first "physical theory" of cognition. It enables us to approach the first and fundamental principle that interfaces mind and matter.

There are levels of reality in which, as described by quantum mechanics, we no more may separate the features of matter per se from the features of the cognition, of the logic and of the language that we use to describe it. This is the basic reason because we think in a quantum probabilistic manner and this is the reason because quantum mechanics is so important in neuroscience and psychology.

In conclusion, by the previous discussion, we have reached the results that we have exposed in the points (1) and (2). It is rather evident that they touch the basic problem about the structure of our cognitive and conceptual entities and thus on our mind. The problem of exploring our consciousness seems to us to be consequently strongly linked. It is our personal view that such studies need to be strongly encouraged and this is the reason because in the present paper we relate about some further and recently results that we have obtained about the possibility of quantum mechanics to adequately represent mental states.

2. A Rough Scheme of Quantum Mechanics with Clifford Algebra

Let us give a brief statement of our Clifford algebraic approach to quantum mechanics. We use Clifford algebra to represent a bare bone skeleton of quantum mechanics. Let us give an example of our approach.

Let us introduce three basic algebraic abstract elements e_i, $i=1,2,3$, having the following basic features:

1) $e_i^2=1$ and 2) $e_i e_j = -e_j e_i = i e_k$ with $i,j,k=1,2,3$, $ijk=$ permutation of 1, 2, 3 and $i^2=-1$ (2.1)

We see that the axioms 1) and 2) introduce the two basic requirements that we invoke for quantum mechanics: ontic potentiality/irreducible indeterminism and non commutativity. The first axiom in fact considers an abstract entity, e_i, but at the same time fixes that its square is 1. This is to say that to each e_i with $i=1,2,3$, under particular conditions in such an algebra, may correspond or the value +1 or the value -1. For each e_i we have the ontological potentiality to link one of such possible numerical values. The second axiom introduces non commutativity for e_i ($i=1,2,3$).

The abstract elements e_i are marked by irreducible, intrinsic indetermination. Consequently, we may calculate their mean values, $<e_i>$, considering the probabilities for +1 or for -1 values, and writing

$$<e_1>=(+1)p(+1)+(-1)p(-1), <e_2>=(+1)p(+1)+(-1)p(-1), <e_3>=(+1)p(+1)+(-1)p(-1) \quad (2.2)$$

where $p(+1)$ and $p(-1)$ represent the probabilities for +1 and -1 values, respectively, with $p(+1)+p(-1)=1$. The quantum like features of this algebra may be synthesized in the following equation that we discussed in our previous work where of course a detailed explanation of our Clifford elaboration may be found [1]:

$$<e_1>^2+<e_2>^2+<e_3>^2 \leq 1 \quad (2.3)$$

In this manner a quantum mechanical scheme may be represented by such algebra. We may introduce the well known Pauli matrices at order n=2 as representative for the basic elements e_i. This is an important operation since, from one hand, it helps us to identify some hidden features of our algebra, and, on the other hand, it introduces for the first time the possibility of a self-referential operation. Let us proceed with the aid of an example. Let us suppose that in the operation of a progressive description of some entity or structure, we have arrived at the condition that two dichotomous variables A and B are actually required in order to characterize it. We may use the matrix representation of the basic elements e_i and we may realize some new algebraic elements given by the direct product of matrices. In this case, we will have new basic elements in the following manner:

$E_{oi} = I \otimes e_i$ and $E_{io} = e_i \otimes I$ being I the unit matrix, $i = 1,2,3.$ (2.4)

Note that E_{0i} and E_{i0} will satisfy the same rules that were given in 1) and 2) for e_i. In detail we will have that

$E_{0i}^2 = 1$, $E_{0i}E_{0j} = iE_{0k}$, and $E_{i0}^2 = 1$, and $E_{io}E_{jo} = iE_{k0}$. (2.5)

It is important to observe that we will have also that $E_{0i}E_{j0} = E_{j0}E_{i0}$ for any (i, j) and $i = 1,2,3; j = 1,2,3$.

As required, we have now two dichotomous variables, E_{0i} and E_{i0} , $i = 1,2,3,$ to describe the given process. Let us consider still that e_i are the basic elements of our algebra given at order n=2 while E_{0i} and E_{i0} are the same basic elements but at order n=4.

2a. The arrangement of an experimental situation

Let us start by considering the following experimental situation. We have an abstract or material entity that we call S that is constituted by a pair of separated sub entities S_1 and S_2 on which we may perform four experiments that we call respectively a_1, a_2, a_3, and a_4.Let us still consider that each of the experiments $a_i\ (i = 1,2,3,4)$ has two possible outcomes, or $+1(r_+)$ or $-1(r_-)$. Still, continue to admit that some of these experiments may be performed together, respectively on S_1 and S_2, and we will call them coincidence experiments $a_{ij}\ \ (i, j = 1,2,3,4)$. The experiment a_{ij} has four possible results that are:

$a_i(r_+)a_j(r_+), a_i(r_+)a_j(r_-), a_i(r_-)a_j(r_+), a_i(r_-)a_j(r_-)$ (2.6)

We may also introduce the expectation values for such coincidence experiments. We call them E_{ij}, and according to the definition, we have that

$E_{ij} = (+1)p(\ a_i(r_+)a_j(r_+)) + (-1)\ p(a_i(r_+)a_j(r_-)) + (-1)p(\ a_i(r_-)a_j(r_+) + (+1)p(a_i(r_-)a_j(r_-))$ (2.7)

Obviously, p_{ij} means the probability that the coincidence experiment a_{ij} gives the outcomes $r_i r_j$ while, generally speaking, p_i will represent the probability that the single experiment a_i will give outcome $r_i\ (i, j = +,-)$

This is a basic scheme that in several our previous papers we have discussed in the framework of the so called Clifford algebra by which we have realized a rough or “bare bone skeleton “of quantum mechanics [1]]. We will not discuss further such elaboration here addressing the reader to the above quoted papers for a close examination.

In the forthcoming steps of this paper we will describe the physical conditions in which by using the (2.6) and the (2.7), we may derive the celebrated Bell inequality which states explicitly

$$|E_{13} - E_{14}| + |E_{23} + E_{24}| \leq 2 \tag{2.8}$$

Summarizing, we have an entity S constituted by two separated components entities S_1 and S_2 .We may perform an experiment a_1 on S_1 obtaining as result r_+ or r_-. We may still perform an experiment a_2 on S_1 still obtaining as result or r_+ or r_-. We may perform an experiment a_3 on S_2 and it may be also similar to a_1 on S_1 with possible results r_+ or r_-, and finally an experiment a_4 on S_2 that may be similar to a_2 on S_1 with possible results r_+ or r_-. Now, the experiment a_1 may be performed in coincidence with the experiments a_3 and a_4, and thus we denote such coincidence experiments by a_{13} and a_{14} respectively, and thus obtaining E_{13} and E_{14}. We may also perform the coincidence experiments a_{23} and a_{24} obtaining E_{23} and E_{24}. All such expectation values are considered in the previous (2.8).

In quantum mechanics, we choose the set of observable properties of a quantum entity to which we are interested. These constitute the state of the entity. We also define a state space, which delineates the possible states of the entity. A quantum entity is described using not just a state space but also a set of measurement contexts. The algebraic structure of the state space is given by the vector space structure of the complex Hilbert space: states are represented by unit vectors, and measurement contexts by self-adjoint operators.

The crucial notion on which we may fix our consideration is the notion of quantum entanglement. With reference to entity S and to the two composing subentities S_1 and S_2 one says that a quantum entity is *entangled* if it is a composite of subentities that no more can be factorized in their components that of course can be identified only by a separating measurement. When a measurement is performed on the entangled entity, its state changes probabilistically and this change of state is called *quantum collapse*.

In pure quantum mechanics, if H_1 is the Hilbert space representing the state space of the first subentity, and H_2 the Hilbert space representing the state space of the second subentity, the entangled state will be represented by $H_1 \otimes H_2$ The tensor product determines new states with new properties. In brief we have a profound difference: in quantum mechanics we may consider the space of the composite system not the Cartesian product, as in classical physics, but the tensor product, and it introduces the existence of new states with new properties.

Entanglement was recognized early as one of the key features of quantum mechanics. Entanglement can be described as the correlation between distinct subsystems and such correlation cannot be created by local actions on each subsystem separately. The advantage given by quantum entanglement relies on the crucial premise that it cannot be reproduced by any classical theory [5]. Despite the fact that the possibility of quantum entanglement was acknowledged almost as soon as quantum theory was discovered, it is only in recent years that consideration has been given to finding methods to quantify it. Historically the Bell inequalities are seen as a means of determining whether a two quantum state system is entangled.

It is now known that the larger the violation of the Bell inequality is , the more the entanglement is present in the system This leads to the perception that the Bell inequalities represent a measure of entanglement in such systems.

In this manner we arrive to the conclusion that we can use the violation of Bell inequality as an experimental indication for the presence of a quantum structure. If Bell inequalities are satisfied for a set of probabilities connected to outcomes of the previously considered experiments, there exists a classical Kolmogorovian probability model. In such model the probability can be explained as due to a lack of knowledge about the precise state of the system under consideration. If, on the other hand, Bell inequalities are violated, as shown in [6], no such classical Kolmogorovian probability model exists. Quantum states arise as having ontological potentiality and thus intrinsic irreducible indeterminism. Probabilities in this case are involved as non classical and thus become the non classical probabilities, that is to say, the quantum probabilities that characterize the sphere of quantum ontological processes. This the reason because it is so important to examine the (2.8) .

3. The problem of the Self

We have to consider now the problem of the Self. May we introduce a mathematical-physical model of the Self?

Also if it is well known that the first psychological studies and physics went both in psychology at the first starting of this discipline, to day they are seen together so infrequently. May be that when physics is considered so linked to mathematics as it is the case of the present elaboration, both fields seem so abstract that describing one in terms of the other is seen soon from psychologists or neurologists with some prejudice and considered not able of giving some direct advantage Freud developed his results using symbols, analogies, figures in the world of the arts and of the literature but never he used mathematics or physics. Instead, there are eminent figures of mathematicians that have given fundamental contributions having had so much to say about the workings of mind [7], and Descartes gave the first psychological legacy to physical knowledge by his Cogito ergo Sum...

In this paper we would be able to indicate some result in the direction of mapping the structure of the self by using quantum mechanics: to present some modeling example aiming to match the human experience of selfhood.

In modeling the Self we outline here his first nature that is reflectivity. Self is by its nature self-referential. .It is at once subject and object, observer and observed of itself as well as of the others. This attitude has often lead psychologists to consider dualistic theories. Self-observation is the key concept here. Lefebvre's mathematical approach to social psychology is often referred to as reflexive theory – It is related to the possibility of taking into account subjects' self-image(s). We aim to outline here that the centuries-old philosophical and psychological ideas that man has an image of the self containing an image of the self obtains a new advance in the mathematical-physical model of the subject possessing reflection that we outline here. One assumption underlying the model is that the subject tends to generate patterns of behavior such that some kind of similarity is established between the subject himself and his second order image of the self.

Still, quantum mechanics is based on its basic formulation of intrinsic and irreducible indeterminism.

Would psychologists speak about indetermination or inter-determination? Many disorders of the Self are considered to be based on the divarication between the subjective and objective features of the self. Often psychologists indicate that in hallucinations, as example, dreams, imaginations, the subjective and objective features separate. In the intrinsic undependability of self-observation, a dose of intrinsic and irreducible indetermination arises for us all and we have unconscious as relevant counterpart. At the extreme limits we have the whole spectrum of psychopathology. So, the importance of a model arises.

In the case of the Self, we are accustomed to conceive the simplest features of observer and observed that in our interpretation become the inside and outside, respectively. The fact that they are separate and at the same time have unity appears impossible to us but actually it is due to an artifact of our traditional point of view on this matter. This is precisely the question with all dualism in psychology. However this is a matter that may be overcome accepting a less ingenuous and less modest vision of our reality. Think as example about the concept of quantum entanglement in quantum mechanics or consider $E_{0i}E_{j0} = E_{j0}E_{i0}$ of our algebraic basic scheme. They give rise to the new algebraic basic set E_{ji} or E_{ij} .

Using our Clifford algebraic formalism, for the first time we have also introduced a self referential mathematical formalism. To explain such a referential mathematical operation, let us return to our basic algebraic scheme but evidencing what V.A. Lefebvre [8] recently outlined. Following Lefebvre, as we know, the central topic of Western philosophy, starting with John Locke, was the problem of representing mentally one's own thoughts and feelings. Actually, it is a very difficult concept to represent. This is the reason to use here a pictorial representation, the same figure that V.A. Lefebvre introduced to describe his formulation [8]. Tentatively we may express self attitude through the reflection. A subject having reflection may be conceived as a miniature human figure with the image of the self inside his head. We recover it here in the following figure (Fig. 1) with Clifford algebraic formulation. It represents with care the subject with reflection. We prefer to call it the Clifford algebraic picture of a subject having perception of itself. In this figure, following V.A. Lefebvre, we may say that inside the subject's inner domain, there is an image of the self with its own inner domain. An image of the self is traditionally regarded as the result of the subject's conscious constructive activity.

Let us analyze how the Clifford mathematical operation given in (2.4) realizes this formulation. It is the faithful correspondent of the self-picture given in figure in which, in fact, E_{0i} , for example, or also E_{i0} , contain in their inside that image of itself that is e_i . We may conclude that, at least for our present possibilities of understanding what the self is and its self-perception represents, we have for the first time identified a basic algebraic scheme and the corresponding mathematical operations to represent it.

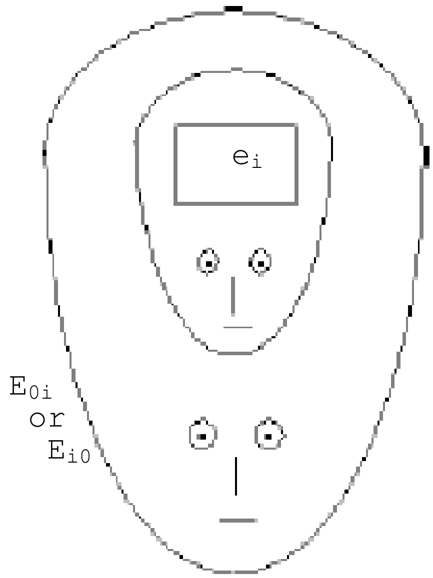

Fig.1 Self

Let us shift for an instant from our view point of bare bone skeleton of quantum mechanics, all based on the use of abstract algebraic entities of Clifford algebra to the standard quantum mechanics. Here we have the spin this time conceived instead as quantum physical observable. We have to evidence here that previously other authors outlined the role of spin as self-referential variable [9] and its possible role on consciousness. They introduced the spin-mediated consciousness theory. We consider the basic e_i elements in our Clifford algebraic formulation as abstract entities, and this concept of abstraction is of fundamental importance for the whole body of our elaboration also if in standard quantum mechanics they are usually connected to spin. In points (1) and (2) we outlined that quantum mechanics relates conceptual entities, and we have several times evidenced that the final approach of our elaboration is that there are stages of our reality in which we no more may separate the "object" from the cognitive feature that we have about it. Consequently, matter is interfaced with cognitive feature. This could be one of the profound reasons because in their papers in [9] it was evidenced the so important role for the spin also arriving to give explanation of its role at the neurophysiological level.

4. A Quantum Model of Jung Theory Realized with Clifford Algebra

4a A Brief Review of Jung and His Theory.

Now we will state first a brief, hurried and for this also approximate exposition of Jung theory just to take into accounts some of the basic concepts that we will use in detail in the next section. Carl Gustav Jung was born in the small Swiss village of Kessewil in date July 26, 1875,. His father was Paul Jung, a country parson, and his mother was Emilie Preiswerk. Very soon he discovered philosophy, and this led him to forsake the strong family tradition and to study medicine becoming a psychiatrist.

In December 1900 he began working at the Psychiatric Institute in Zurich, the Burghölzli, directed by Eugen Bleuler. In the winter of 1902-1903 Jung was in Paris to attend the lectures of Pierre Janet. In 1903 he married Emma Rauschenbach (1882-1955), who remained with him until his death. In 1905 he became a lecturer at the University of Zurich, where he remained until 1913. Between 1904 and 1907 he published several studies on verbal tests of association and in 1907 the book Psychology of dementia praecox.

The scientific activity of Jung is manifested by the concept of "complex". For Jung, complexes make up the basic structure of the psyche. They are central themes or content areas that are powerful, emotionally charged, and connected to archetypes. Complexes organize and influence our feelings, thoughts, perceptions, and behavior. The self in Jungian theory is one of the archetypes. It signifies the coherent whole, unified consciousness and unconscious of a person - 'the totality of the psyche. The Self, according to Jung, is realised as the product of individuation, which in Jungian view is the process of integrating one's personality. For Jung, the self is symbolised by the circle (especially when divided in four quadrants), the square, or the mandala.

What distinguishes Jungian psychology is the idea that there are two centers of the personality. The ego is the center of consciousness, whereas the Self is the center of the total personality, which includes consciousness, the unconscious, and the ego. The Self is both the whole and the center. While the ego is a self-contained little circle off the center contained within the whole, the Self can be understood as the greater circle.

Generally speaking in Jung theory we have a dynamics with very interrelated relationships. When relationships weakens or break, the other complexes become autonomous, and arrogate to themselves the possibility of direct action, by a process of dissociation which is the source of psychological problems.

Jung's study of the Ego also led to his laying the foundation for the study of psychological types Jung was fascinated with the concept of classifying people according to their particular personality traits and preferences. As we will consider in detail in the following section, based on his observations, exposed in *Psychological Types*, he identified two psychological attitudes – Introversion, in which psychological energy is directed inward and Extraversion, in which it is directed outward - and four psychological Ego-functions - Intuition, Sensing, Thinking and Feeling. He explained that each of us exhibits both attitudes and all four functions at times, but that we each prefer one of the attitudes and one function from each of the Intuition/Sensing and Thinking/Feeling alternatives. As we become more whole and integrate more unconscious material into our personality, however, we may, at key developmental points in the lifespan, become more adept at using our inferior functions.

This work later formed the basis for the development of the Myers-Briggs Type Indicator, currently the most popular personality typing system in the world, and that we will use in the present paper.

Initially close to the ideas of Sigmund Freud, he finally withdrew in 1913 after a process of conceptual differentiation culminated with the publication in 1912 of Libido: symbols and transformations. In it he expounded his guidance, analytical research by broadening the individual's personal history to the history of human society. The unconscious is not just the individual, produced by the removal, but in the individual there is also a collective unconscious that is expressed in archetypes.

4b A Quantum Model in Jung Theory

Now we may pass to consider a possible theory of personality. In Jungian theory, the Self is one of the archetypes. The coherent whole unifies consciousness and unconscious of a person. As previously said, the Self, according to Jung, is realised as the product of individuation, which in Jungian view is the process of integrating one's personality.

Let us consider now in detail some basic features of Jung theory. As previously mentioned, we have four basic psychological functions, Thinking, Feeling, Sensing, Intuiting and two Attitudes (Introversion and Extraversion). Certainly, if we claim here that such psychological function are linked and inter-related with attitudes in humans, we do a so general and unspecific statement that all the psychologists will agree. However, an interesting indication could be to advance such so phenomenological approach, attempting to give to the basic four psychological functions, to the attitudes and to their possible interrelationship, a theoretical formulation so that we may experiment about, and obtain precise and quantitative results.

The question that we pose in detail here is the following: could psychological functions be quantum entangled with attitudes? If such kind of possible correlations should be evidenced, we certainly will obtain first of all a further evidence of the effective role explained from quantum mechanics in brain and mind processes, and, in addition, a new quantum model of Jung theory of personality should arise, this time based on the principles of a well defined physical theory. It should represent an actual advance.

We have to introduce here a necessary and precise statement.. The first idea to use two qubits for Jung's theory of personality is due to Reinhard Blutner, and Elena Hochnadel. They started their work based on this excellent idea in 2009 [10] In this paper we proceed now giving a Clifford algebraic elaboration of the same matter, thus confirming it and advancing with the experimental results that we have obtained.

Let us indicate the Feeling by F, the thinking by T, the sensing by S and the Intuition by I. Still we call E the extroversion and I_1 the introversion. Our approach should be well known to the reader by this time. We introduce now some Clifford basic elements.

We call the Thinking function (T) by E_{03}. It is a dichotomous variable that as previously explained may admit values or +1 or –1. $E_{03} \rightarrow +1$ means that the subject is Thinking. $E_{03} \rightarrow -1$ means that he is Feeling.(F).

So we have that

$$T = -F = E_{03} \tag{4.1}$$

This is the quantum Clifford algebraic scheme for rational functions.

Now we introduce the irrational functions. We call the Sensing (S) by the Clifford basic element E_{01} to which again are linked the values ± 1. $E_{01} \to +1$ means that the subject is Sensing while instead $E_{01} \to -1$ means that he is Intuitive (I). So we have

$$S = -I = E_{01} . \tag{4.2}$$

These are the four psychological functions characterized by our quantum algebraic scheme.

Let us now introduce the attitudes of the Self, calling E extroversion and I_1 introversion. Let us consider another algebraic Clifford Element

$$E = -I_1 = E_{30} \tag{4.3}$$

$E_{30} \to +1$ means extroversion, otherwise $E_{30} \to -1$ means introversion.

Finally, let us consider another Clifford basic element. It takes in consideration states of explicit intermediation between extraversion E and Introversion I_1. We call it M , and We pose

$$M = E_{10} \tag{4.4}$$

with the realization that it assumes $E_{10} \to +1$ when the subject is in a state of equal superposition of pure extroverted and pure introverted condition while instead we have $E_{10} \to -1$ otherwise.

In this manner we have realized two basic features. The first is that by introducing the (4.1), we have fixed that the rational functions are opposites from each other and, considering the (4.2), we have admitted that also the irrational functions are opposites from each other.

Obviously, consider that, using the (4.1) and the (4.2), we enter by Clifford algebra in a quantum bare bone skeleton of quantum mechanics. This is to say that rational as well as irrational functions now become to be considered having an irreducible intrinsic indetermination in their state. This is to say that the person has an ontological potentiality, a quantum superposition of alternatives, to be T or F becoming actually T or F when his Self is submitted to direct self or outside direct observation. The algebraic theorems given in point (1) fix the algebraic structure of such passages with S_i representing the situation when the person has an ontological potentiality, a quantum superposition of alternatives, and $N_{i,\pm 1}$ when the Self is submitted to perform self or outside direct observation. The same thing happens for psychological functions S and I being the person in a superposition of such states (Operating the Clifford algebra S_i) and becoming actually

S or I (operating in this case the algebra $N_{i,\pm 1}$) Obviously, the selection of the state T or F, and, respectively, S or I is only a matter of probability that is enhanced in favouring one psychological function respect to the other in dependence of the inner structure of his Self and of the context in which the self is under direct observation.

This is the quantum scheme of the approach. It profoundly reformulates Jung theory under an ontological as well as epistemological profile. We have previously explained in detail such contents in points (1) and (2). As example, an important implication of our elaboration is that both superior and inferior functions coexist, and it is only a matter of our inner developed structure and of the instantaneous context that, probabilistically speaking, one function results prevailing on the other in our subjective dynamics.

Fixed such important conceptual points, let us attempt to give soon some result confirming possibly that we are formulating a theory in a correct direction. Let us calculate the expectation value (mean value, of T, F, S, and I). Looking at our basic relation of Clifford algebraic scheme of quantum mechanics given in the (2.3), we obtain immediately that

$$<T> = \cos\vartheta,\ <F> = -\cos\vartheta,\ <S> = sen\vartheta,\ <I> = -sen\vartheta \tag{4.5}$$

where ϑ is an arbitrary angle ranging from $-\pi$ to π

Let us schematize the results of the (4.5) in Fig.2. We obtain the behaviors of the expectation values for such psychological functions.

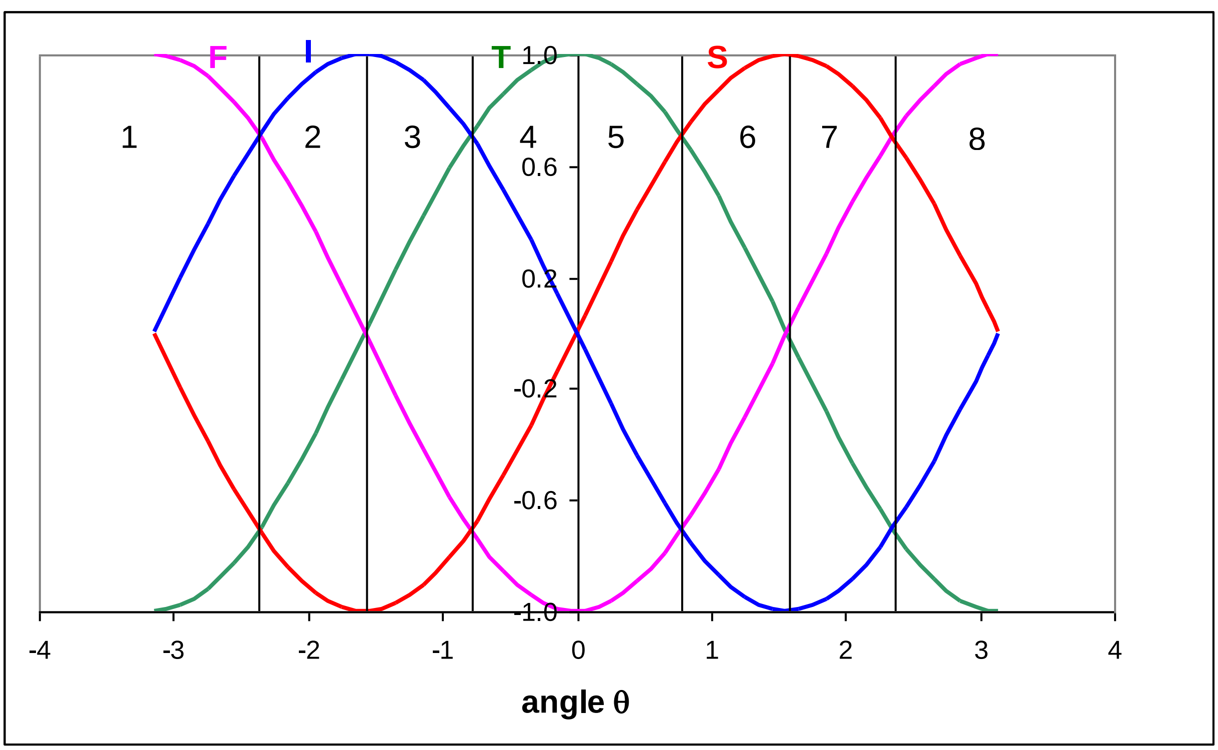

Fig. 2 Expectation values of the four psychological functions.

It is easily observed that we obtain eight corresponding sections:

1) F>I>S>T
2) I>F>T>S
3) I>T>F>S
4) T>I>S>F
5) T>S>I>F
6) S>T>F>I
7) S>F>T>I
8) F>S>I>T

They are in perfect accord with Jung theory. There is no doubt that this quantum approach reproduces perfectly the eight different proportions that were identified also by Jung theory when he characterized the superior and secondary psychological functions of a subject. Remember that he outlined that we just have them in different proportions. We have a superior function which we prefer and it is best developed in us, and a secondary function of which we are aware and we use in support of our superior function. The personality of a person conflicts if the Self has to realize two opponent functions in the same attitude. Here it is one of the interesting features of such obtained results obtained.

The interest is on one hand under the profile of the scientific investigation but, on the other hand, it is of great interest also under the applicative diagnostic perspective. Let us explain such concept in detail. As we know, we may experimentally estimate the values of T, F, S, I as always performed in the standard cases by the test. The important difference is that we know the corresponding analytical expressions as predicted by the (4.5). Consequently, for each subject investigated we may now reconstruct his pattern in the Fig.2 and thus establishing this profile in the case of normal subjects as well as in the cases of pathological conditions, differentiating also the possible different stages of the considered pathology. This is a perspective that in our opinion delineates a possible advance of valuable interest.

This last comment completes our exposition on the Jung four psychological functions as elaborated by a bare bone skeleton of quantum mechanics using the Clifford algebraic approach.

Now the attitudes of the Self: The different attitudes of the Self may be extraversed or introversed and they have been quantum mechanically algebraically expressed by us in the (4.3) and in the (4.4). According to our quantum language, as previously for the four psychological functions, also here the situation is now conceptually reformulated respect to traditional Jung theory. We may have pure extroversed or pure introversed states but we may also have the ontological true potentiality, signed from irreducible indeterminism, of potential superpositions of extroverted and introverted states. Here, this feature is particularly enhanced since we have a precise algebraic element that characterizes it. Again we have two different Clifford algebraic structures, given respectively by S_i and by $N_{i,\pm1.}$ Remember, in particular, that we have here also the Clifford algebraic element

$$M = E_{10}$$

to which we attribute the numerical value of –1 if the subject always collapses to a possible state of extraversion or intraversion while it still remains to be +1 if the subject remains in an uncollapsed state of equal superposition of pure introvetrted and pure extroverted states. Also in this case we may calculated the mean values obtaining

$$< E >= cos\varphi \quad , \quad < I >= -cos\varphi \ , \quad < M >= sen\varphi \tag{4.6}$$

Under the view point of the experimental investigation we may repeat here all that we have previously outlined for the psychological functions. We may explore the attitudes of the Self and his balancing. It is relevant to outline here further the importance of such acquired possibilities under the basic theoretical profile of the elaboration as well as in the case of analyzing possible implications under the clinical profile.

Now a step one: It may be useful to repeat here the notion of quantum entanglement that we have also prospected previously. Using very simple terms we may say that quantum entanglement is a pure quantum phenomenon in which the states of two or more objects or entities anyway separated, remain linked together so that one object can no longer be described without considering its counterpart. A quantum interconnection maintains between the two components also for any space distance separation between the two separated objects, leading to a net correlation between measurable observable properties of such two or more components. We need to re outline here that such very extraordinary property of correlation at distance relates only quantum entanglement that is exhibited only from systems subjected to the principles and to the rules of quantum mechanics. We need the previously mentioned Bell inequality. If it is violated, we have quantum entanglement.

Our attempt is to verify if or not Jung theory has a possible quantum formulation. By this way we may admit that human subjects in some conditions realize quantum entanglement in the sense that psychological functions are entangled with Self-attitudes. We may write Bell inequality linking psychological functions and attitudes. With clear evidence of the used symbolism, we write in this case the Bell inequality in this manner

$$\left|E(M,T) - E(M,S)\right| + \left|E(E,T) + E(E,S)\right| \leq 2 \tag{4.7}$$

E states for expectation value. M,T,S,E state respectively for intermediation and attitudes M , and E and T and S for psychological functions. This last result completes our exposition. Again we retain to be important to re-outline here that the first elaboration of this matter was given by Reinhard Blutner, and Elena Hochnadel [10].

4. Materials and Methods

All we know about the MBTI that is to say the Myers-Briggs Type Indicator. We may use MBTI to classify the personality of the subject adopting some predefined sentences.

We decided to use the MBTI to submit the (4.7) to experimental verification in order to evaluate if or not we may speak about quantum entanglement between psychological functions and attitudes in human subjects. We decided to perform an experiment that we thought in the following manner. Using the sentences given in the MBTI we prepared possible pairs of sentences $(M,T),(M,S),(E,T),(E,S)$ coupling them in a computer archive. Male and female normal subjects were selected with age ranging from twenty to thirty years old. Each subject was subject to simultaneous sentences (M,T), soon after (M,S), then (E,T) and finally (E,S), each pair of sentences given to subject after a short time from the other. Each pair of sentences was selected at random by the computer from the previously arranged archive and given to the subject. In this manner we calculated $E(M,T),E(M,S),E(E,T),E(E,S)$ for each subject. For each person we repeated the experiment three times selecting at random every time the pairs of sentences. Each administration was given to the subject after a period of at rest for the subject of about 15 minutes.

5. Results

We are in the condition to confirm some results. A group of three psychologists, specialized in the administration of psychological tests, were active in the experiment. One of them found that the Bell inequality was violated in the 59% of the investigated cases, the other psychologist found instead Bell violation in the 63% of cases, and the third psychologist found a violation in the 72% cases. Such results agree in a satisfactory manner with those that we obtained in a previous preliminary experimentation that we performed. The experimental indication seems quite clear. Subjects showed in percent a violation of Bell inequality and this is to say that in such case psychological functions and attitudes in these subjects gave quantum entanglement.

It emerges a quantum model of personality theory. Under a strict psychological profile, a plastic behaviour is observed where psychological functions, attitudes and their quantum entanglement explain a decisive role in the subject mental dynamics. Therefore it becomes very interesting to deepen what is the role of quantum entanglement in such dynamical profile, and this the object of our subsequent current research.

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