

This is the accepted version of the following article: Danne, Nicholas. "Book Review: *Autonomy Platonism and the Indispensability Argument*. By Russell Marcus. Lanham, Md.: Lexington Books, 2015. Pp. xii + 247." *Metaphilosophy* 48, no. 4 (July 2017): 591-594, which has been published in final form at [<http://onlinelibrary.wiley.com/doi/10.1111/meta.12250/full>]. This article may be used for non-commercial purposes in accordance with the Wiley Self-Archiving Policy [<http://olabout.wiley.com/WileyCDA/Section/id-828039.html>]

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The debate over mathematical indispensability typically pits fictionalists, nominalists, and Platonists in a three-way conflagration about the ontological status of numbers, and the practice of justifying that ontology from mathematics' applicability to empirical science. In *Autonomy Platonism and the Indispensability Argument*, Russell Marcus rejects the justification of mathematical ontology from scientific methodology. He defends instead an *a priori*, fallibilist justification of mathematical knowledge (13) with a corresponding ontology of modally necessary mathematical objects (171). Mathematical knowledge is fallible, Marcus argues, because we acquire it by a fallible intuition that some mathematical propositions are true and possess their truth necessarily (171, 191). Consequently, the ubiquitous application of numbers to science proves the less interesting, Marcus explains, the more we realize how many mathematical objects find no empirical correlate (37), and once we accept that mathematics applies to our world because it applies "to all possible worlds" (36). He defines "autonomy platonism" as any mathematical platonism that justifies a mathematical ontology without appealing to mathematics' applicability to empirical science (159).

Despite its intuitionistic and epistemological focus, neither indispensabilists nor nominalists should overlook *Autonomy Platonism and the Indispensability Argument*. The first nine (of twelve) chapters provide some of the clearest and most readable exegeses of the indispensability problem since Mark Balaguer's *Platonism and Anti-Platonism in Mathematics* (1998). Marcus performs his first of many favors in Chapter 1: a painless and quote-free rendering of Benacerraf's dilemma, with foundational responses by Quine and Hartry Field. Chapters 2 and 6 likewise perform the uncommon convenience of distinguishing Quinean from Putnamian indispensability, respectively. Putnamian indispensability, Marcus claims, is "inference to the best explanation" for mathematics' applicability to the sciences (115), whereas Quinean mathematical ontology follows from the theorist's commitment to "speaking most seriously" (90) after "modeling [her] best theories . . ." (115). In Chapter 4, Marcus argues that Quine yields "the strongest" of indispensability arguments (78), because of the resistance that first-order quantification presents to the "weasel" nominalist who would treat the mathematical

terms of scientific theories as useful fictions (91). Autonomy platonism preserves the spirit, if not the method, of Quine's linguistic honesty, Marcus affirms, when he claims that "we are speaking *most* seriously in pure mathematics" (89, emphasis mine).

But Marcus's affinity for Quinean metaphysics remains scant. Chapter 3 critiques Quinean confirmational holism with a non-disconfirmation argument by Elliot Sober (63). Marcus corroborates Sober, not by claiming that numbers are unlike "empirical posits" and therefore unsusceptible to empirical disconfirmation, but rather by arguing that Quinean "naturalism" employs the scientific method which does not test numbers (63), and so inevitably results in theorists declining to "question" any of their mathematical beliefs (62). Chapter 5 catalogues seven "unfortunate consequences" of Quinean indispensability (102), even granting the assumption (rejected by Marcus in Chapter 3) that Quinean indispensability is valid and sound (95). In my opinion, this list of seven consequences proves useful for distinguishing sundry indispensabilists from each other, by clarifying which of them bites which of the seven bullets.

Shifting focus away from Quine, Chapter 7 criticizes an "equivocation" suppressed by the Putnamian explanationists (138). Not all explanations entail ontological commitment, Marcus claims, for example explanations in physics involving "frictionless planes" (137). He refers to ontological explanations as "metaphysical," and to idealized or non-ontological explanations as "epistemic . . ." (141). Thus when Putnamian explanationists assert that scientific theorems *do* entail mathematical ontology, Marcus accuses the explanationists of equivocating between the metaphysical and epistemic meanings of 'explanation'. The Putnamians either endorse a metaphysical explanation and revert to Quinean indispensability, or they posit a mathematical ontology for "no good reason" (141) — i.e., the reason withheld from other epistemic explanations. With the foregoing critiques of Quinean and Putnamian indispensability in place, Marcus presents a formal argument for autonomy platonism in Chapter 8 (149), and a *reductio* against ontologically *contingent* platonism in Chapter 9 (165).

Chapter 10 contains Marcus's survey and defense of intuition. He calls mathematical intuition "an experience which can yield a belief" (171). When an agent intuits that " $7+5=12$ " (190), that intuition "lead[s] to or confirm[s] [her] belief that seven and five are twelve" (171). Because mathematical intuitions are "fallible," moreover, Marcus denies that they amount to any metaphysically or controversially "spooky" faculty of human agents (217; cf. 173, 177). To

preclude spookiness and “mysticism” (179) about mathematical knowledge, Marcus endorses a “naturalize[d] epistemology” that references contemporary neuroscience, and he claims that “[m]athematical intuition must be compatible with a mature psychology” (178). He declines, however, to reduce mathematical intuition to brain processes (180). As I interpret Marcus, an empirically complete account of intuition would not explain intuition *of the a priori* (180), and as he argues in a similar vein, “[p]roviding the neural correlates of mathematical thought” does not say anything at all about “semantic” theories that posit abstract objects as referents of “mathematical sentences” (181).

Hence Marcus envisions an “utterly uncontentious” account of intuition:

We have sense experiences of ordinary objects. . . . [C]ombined with our ability to reason, they lead us to particular mathematical beliefs. We reflect on our experiences and our particular mathematical beliefs, developing, eventually, full-blown mathematical theories (169).

Theories and intuitions, Marcus elaborates, stand in “reflective equilibrium” (170): “[t]he intuitions are constraints on the system-building, and the systems are constraints on the intuitions” (183-184). Marcus denies, however, that intuition *justifies* mathematical beliefs. As he argues in Chapter 11, mathematical beliefs are justified by the equilibrium in which they obtain, and he compares such benignly “circular” justification to that undergirding the ethical theory of John Rawls, and the logical deduction defended by Nelson Goodman (205). Marcus ends *Autonomy Platonism and the Indispensability Argument* by summarizing his conclusions in Chapter 12.

I offer two, minute criticisms of Marcus’s excellent book. Firstly, Marcus seems to take for granted the falsity of the Eleatic Principle, which claims that no entities whatsoever exist that are non-causal (187; see also 2, 20, 27, 31, 34, 163, and 191). Mark Colyvan argues carefully in *The Indispensability of Mathematics* (2001) that the Eleatic Principle remains a live option for indispensabilists, and while Marcus acknowledges this conclusion as one of the “unfortunate consequences” of indispensability (102), he does not provide any argument that numbers cannot themselves be causal.

Secondly, Marcus should do more to distinguish his proposed faculty of mathematical intuition from the Keynesian intuition (not mentioned by Marcus) of probability relations (PR’s). John Maynard Keynes posits a human faculty of acquaintance with a logical relation between

propositions a and b , namely the relation that proposition a ‘makes probable’ proposition b (*A Treatise on Probability*, I, I, §3). My question is, how does Marcus know that intuiting “ $7+5=12$ ” is not just an elliptical way of becoming acquainted with the Keynesian PR that proposition a (“ $7+5=12$ ”) *makes probable* proposition b (“seven and five make twelve, and necessarily so”)? That such a modal inference for these particular a and b is invalid (cf. 172) and nowhere suggested by Keynes is beside the point. The point is that the Keynesian can unjustifiably *believe* in autonomy platonism just as Marcus (perhaps justifiably) does, but that Keynesian and Marcusian intuitions remain indistinguishable to the agents who exercise them. The degree of rationality that a Keynesian infers for believing b from evidence a depends on her background knowledge of mathematical expressions resembling a , and theorems of mathematical knowledge resembling b (*Treatise*, I, II, §11; III, XIX, §2). Thus like Marcus, Keynesians can endorse autonomy platonism by reflecting on a few experiences and positing mathematical objects as the referents to theorems rationalized from those reflections. As Marcus admits that intuitions may yield false beliefs (13), so I interpret Keynesians to regard their mathematical theories (b propositions) as less than certain. So who is the wiser about which intuitions — Keynesian or Marcusian — are in play? Granted, Keynesians perceive PR’s *via* infallible intuition, a faculty that Marcus might call ‘spooky’. But because acquaintance with a PR is not necessarily an intuition *of* numbers, but only an intuition of a probable relation between propositions, Marcus should give acquaintance theories more attention than he does (cf. 176-177).

In conclusion, *Autonomy Platonism and the Indispensability Argument* contributes clarity and resourcefulness to a trenchant debate. I will return to this book to distinguish Putnam from Quine, and to reanalyze Colyvan’s arguments, for the foreseeable future.