Impossibility and Impossible Worlds

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Introduction

Possible worlds have found many applications in contemporary philosophy: from theories of possibility and necessity, to accounts of conditionals, to theories of mental and linguistic content, to understanding supervenience relationships, to theories of properties and propositions, among many other applications. Almost as soon as possible worlds started to be used in formal theories in logic, philosophy of language, philosophy of mind, metaphysics, and elsewhere, theorists started to wonder whether impossible worlds should be postulated as well. To take just one example, possible-worlds theories of mental content associate sets of worlds with beliefs (or perhaps entire belief systems): the content of a belief is (or is represented by) the set of possible worlds where that belief is true. But what should we say about beliefs that cannot possibly be true: false logical or mathematical beliefs, for example, or beliefs in metaphysical impossibilities? It would be natural to represent these beliefs with sets of impossible worlds. If James thinks that 87 is a prime number, the set of worlds associated with his beliefs includes worlds where 87 is prime, for example. If Jane is undecided about the principle of excluded middle, her belief worlds should include some at which the principle is correct and some where it is incorrect. And so on.

There are some direct arguments that we should take impossibilities and impossible worlds seriously: we seem to talk about, count, and compare impossible scenarios just as we talk about, count, and compare possible ones, and if that talk is taken at face value then it commits us to impossible scenarios. But most of the arguments for accepting that there are impossible worlds involve pointing to the value of theories that postulate them. If theories postulating impossible worlds offer us satisfactory explanations and understanding, especially of otherwise puzzling phenomena, that gives us reason to endorse the existence of impossible worlds. (Not indefeasible reasons: better theories that reject the existence of impossible worlds may come along, after all.)
This methodology is not uncontroversial, of course, and some might think that success of theories employing impossible worlds would only be evidence that they are useful heuristics or have some other lesser value. Debates about impossible worlds have typically centred on the quality of the theories that employ them, so this chapter will begin with a survey of some of the more important applications impossible worlds have figured in.

In this chapter, I will first introduce some of the ways theories of impossible worlds have been used to deal with problems that arise for more standard possible-worlds analyses in philosophy. Then I will discuss some questions about the nature and extent of impossible worlds. Lastly, I will turn to some important objections to the project of employing impossible worlds in our philosophical theorising.

Some Uses for Impossible Worlds

The main reason philosophers have been interested in impossible worlds is the potential that they will play a useful role in good theories of other phenomena. One fruitful strategy for finding applications for impossible worlds is to look at some of the places possible worlds have been employed in philosophical theories, and to see whether limitations of possible-worlds approaches can be overcome by expanding the theories to encompass impossible worlds as well. Let us turn to some of the main uses. (For more detailed presentations of a range uses of impossible worlds, see Berto 2013 and Nolan 2013).

One famous application of the theory of possible worlds has been in offering a theory of counterfactuals: conditional sentences like "If the conveyor had not short-circuited, the factory would not have burned down". While there is controversy about what counts as a counterfactual conditional, paradigm cases include conditionals about what would have happened, had things gone otherwise than they did. Famously, just determining the truth-values of antecedents and consequents of these conditionals is not always enough to see whether they are true or false. A popular approach to counterfactuals, pioneered by Stalnaker 1968 and Lewis 1973, is to treat their truth conditions as involving other possible worlds: roughly, a counterfactual is true provided that in the nearest possible worlds where the antecedent is true, the consequent is true
as well. (In the possibilities most relevantly like ours where the conveyor did not short circuit, the factory did not burn down.)

Restricting attention to possible worlds gives intuitively the wrong result when we are asked to consider counterfactuals about what would have happened if something impossible had happened. Stalnaker's and Lewis's theories both predict that whenever the antecedent of a counterfactual is impossible, the counterfactual as a whole is true. Thomas Hobbes, famously, devoting time to discovering a method of squaring the circle (something we now know is impossible). "If Hobbes had squared the circle, he would have made a famous mathematical discovery" seems true (and we might say so if we are trying to explain Hobbes's motivation for his attempt). But "If Hobbes had squared the circle, he would have become a werewolf" seems false: even skilled geometers do not turn into werewolves. Counterfactuals with impossible antecedents ('counterpossibles') can still be handled in something like a Lewis-Stalnaker framework if impossible worlds are used as well, and a counterfactual is true provided that in the nearest worlds (possible or impossible) where the antecedent is true, the consequent is true as well. Hobbes-squaring-the-circle worlds are impossible, but the ones where he does so but does not turn into a werewolf are more relevantly similar to our own than ones in which he squares the circle, and also becomes a werewolf. (Routley 1989 is the first I know of to extend a Lewis-Stalnaker-style semantics to impossible worlds, and this approach to counterpossibles has been defended by Nolan 1997, Vander Laan 2004, and Brogaard and Salerno 2013, among many others.)

Perhaps the earliest systematic use for impossible worlds was in providing the semantics for various logical systems: modal logics, and then logics representing the language of psychological attitude attributions (talk about belief, desire, etc.) The earliest use of worlds that are in some sense "impossible" is in possible worlds semantics for modal logics that employ an "accessibility" relation: in models where some worlds are not accessible from the actual world, those worlds are not possible "from" the actual world (though those worlds are still standardly labelled "possible worlds"). Accessibility relations on possible worlds seem to have made their first appearance in journal publications in Prior 1962 and Kripke 1963, though see Copeland 2002 for a fascinating pre-history of the notion.
The earliest published use of impossible worlds in a stronger sense I have been able to find is Kripke 1965, who employed "non-normal worlds" to provide the semantics for a range of modal logical systems including the Lewis systems S2 and S3. Logics of belief and desire using impossible worlds came soon after: Creswell 1970 used "non-classical worlds" to represent differences in belief attributions, and Hintikka 1975 employed "impossible possible worlds" for a similar purpose. (See also Creswell 1973 and Rantala 1982 for some other early contributions to this program.) One feature of systems such as Cresswell 1970 that has become standard is that the model-theoretic definition of logical validity requires truth preservation only at all possible worlds in all models: that is, there is no possible world in any model where the premises are all true but the conclusion not true. This feature allows us to keep a robust notion of logical consequence, while not requiring that all impossible worlds are logically well-behaved.

As far as a pure model-theory goes, so called "worlds" need not have much of a connection to ways things cannot be, but philosophical motivations for these logics can be given in terms of genuine impossibilities, via explaining why the formal models are appealing ones for capturing the ideas behind the logics. An example of using the semantics of inaccessible "possible" worlds not just as a piece of formalism but as being about possible and impossible worlds is Salmon 1984, employing principles about worlds to argue against the modal logic S4.

Probably the main use of possible worlds in philosophy has been in theories of mental and linguistic representation. At the beginning of this chapter, an example of using impossible worlds to improve on a possible-worlds account of belief contents was given. Impossible worlds also play a fruitful role in improving on the "possible worlds semantics" tradition in semantics for natural languages (see Partee 1989 for a number of advantages of using possible worlds in semantics). Using only possible worlds has some limitations: it is hard to avoid running together distinct claims that are necessarily true, or distinct claims that are impossible. But we want to explain differences in meanings between these claims when explaining the meaning of mathematical language or philosophical language, for example. See Nolan 2013 pp 364-366 for a more detailed discussion of the use of impossible worlds in a theory of linguistic and mental representation.
Finally, impossible worlds seem to have a role to play in the metaphysics of the non-representational world. Possible worlds have been used in a wide variety of metaphysical theories from the 1960s: Montague 1969 is an important founding paper in this trend, and it appears throughout the metaphysical writings of David Lewis. Many of the same areas can benefit from theories employing impossible worlds. Apart from theories of counterfactuals discussed above, metaphysical topics recently treated with theories of impossible worlds include explanation in general and metaphysical explanation in particular (Kment 2014), theories of essence (Brogaard and Salerno 2013 p 646-648) and the nature of omissions (Bernstein 2016). Nolan 2014 lists a number of other areas that seem to call for metaphysical treatments using resources more fine-grained than possible worlds: though of course there are other resources available besides analyses in terms of impossible worlds.

The Nature of Impossible Worlds

Before deciding whether to adopt a commitment to the existence of impossible worlds, it is reasonable to ask what impossible worlds are supposed to be, and what features they are supposed to have. There is little agreement about either of these questions, though the disagreements about what kinds of things impossible worlds are do have parallels with better-known debates about what possible worlds would be.

Perhaps the most common type of account of possible worlds takes them to be abstract objects of some sort or other. One might take possible worlds to be sets of sentences, or sets of propositions, or each world to be a single, maximal proposition, or each world to be a maximal property, or maximal state of affairs, or possible worlds might even be sui generis abstract entities. Such views require a specification of what it is for a sentence (or claim, proposition, etc.) to be true according to one of these abstract objects: notice that a claim can be true according to a world without in any sense being in fact true. The family of views that treat possible worlds as abstract objects are often labelled "abstractionists". It is natural for abstractionists about possible worlds to be the same sort of abstractionists about impossible worlds. As well as sets of sentences that can all be true together, there are sets of sentences that
cannot be all true together. As well as maximal possible propositions, there are propositions that
cannot be true, including maximal propositions that cannot be true. As well as maximal states of
affairs that can be instantiated, there seems no obvious bar to supposing there are maximal states
of affairs that cannot be instantiated. And so on. There are sometimes metaphysical puzzles
about the details, but the general shape of abstractionist theories of impossible worlds is clear.
(Abstractionist theories of impossible worlds include those presented by Mares 1997, Vander
Laan 1997 and arguably Zalta 1997, while a more recent abstractionist approach is defended in
Jago 2014.)

David Lewis famously claimed that possible worlds were objects of the same kind as our
physical universe (Lewis 1986), and while this "concretism" remains a minority view of possible
worlds, it retains some influential defenders. Concretism about impossible worlds is an even less
popular view, since on the face of it they would be things that cannot exist, but do exist, and the
inconsistent ones at least would be things "whereof you speak truly by contradicting yourself"
(Lewis 1986 p 1, p 7 fn1 3). Despite these hurdles, several forms of concretism, which treat
impossible worlds as in some sense the same kind of thing as our own cosmos, have been
defended. Yagisawa 2009 is the most famous defence of this view, though Kiourti 2010
and Vacek 2013 each defend a different development of a concretist view about impossible worlds.

Some theorists have suggested that we conceive of possible worlds and impossible worlds as
being different kinds of entities. The most popular "mixed option" is to treat possible worlds as
concrete, but impossible worlds as abstract objects that are in some sense constructions from
possible worlds. Restall 1997 was an early suggestion of this combination, but it has recently
been advocated by Berto 2010, who argues that it combines benefits of concrete realism about
possible worlds, such as a reductionist account of modality, with the benefits of postulating
impossible worlds.

There are other options for theories of impossible worlds than treating them as existing abstract
objects or existing concrete universes. One is to treat them as non-existent objects of one sort or
another: see, for example, Priest 2005. There are also approaches that refuse to accept that
impossible worlds are anything at all, existing or non-existing. One could treat talk about
impossible worlds as a *convenient fiction* for regimenting our talk about what cannot happen, embracing either a fictionalism or instrumentalism about impossible worlds. Or one could offer a *paraphrase* of talk of impossible worlds into a theory that was not committed to such things.

Beyond questions about what kind of entity impossible worlds are, there are other questions about impossible worlds to be sorted out. One of the main choice points is whether to treat impossible worlds as closed under some logic or not: that is, whether there is some interesting logic $L$, and associated consequence relation $\models_L$, such that whenever a set of propositions $\Gamma$ is true according to a world, and there is a proposition $A$ such that $\Gamma \models_L A$, $A$ will also be true according to that world. When impossible worlds are the inaccessible worlds of Kripke-style semantics for modal logics, or the non-normal worlds of semantic treatments of systems like S2 or S3, the non-modal formulas true according to the worlds are closed under classical (propositional or predicate) logic.

For some other applications, impossible worlds are all treated as if they are closed under classical logic, or whatever other logic that the actual world is supposed to be closed under: some theorists only accept the existence of metaphysically impossible worlds that are still logically possible (see e.g. Kment 2014) These theorists see the use for impossible worlds where e.g. metaphysical necessities do not obtain, or some facts that flow from the essence of objects obtain but others do not. Allowing for metaphysically impossible worlds permits e.g. non-trivial counterfactuals about what would be the case were the metaphysical facts different, and for representing ignorance of logically ideal agents who are ignorant or mistaken about some metaphysical necessities.

More radical still than these options are theories according to which all worlds are closed under an interesting and substantial logic, but that logic is considerably weaker than the (strongest) logic that the actual world is closed under. For example, the worlds might only be closed under First Degree Entailment, which ensures e.g. that conjunctions are true according to a world when their conjuncts are, but does not ensure that there are no contradictions true according to a world, or that there are no failures of excluded middle at such worlds. This approach is called the "Australasian Plan" by Priest 1997a and following him Berto 2009. The Australasian plan has
some advantages, for example that the truth-value, at any world, of many truth-functional compounds can be defined in terms of the truth-values at those worlds of the propositional constituents of those compounds. It also would enable us to rely on some standard logical principles, in full generality, for working out truth according to worlds that we might be considering when evaluating counterfactuals or evaluating belief contexts. Priest 1992 is one influential philosophical motivation for an Australasian-plan approach to impossible worlds.

Most radical of all is the option of treating impossible worlds as not, in general, being closed under any logical consequence relation at all (apart from identity: when B is true according to a world, B will be true according to that world). Approaches like this typically hold that logical consequence is connected to truth-preservation at possible worlds rather than truth-preservation at all worlds: recognising a wide range of impossibilities does not automatically require a revisionary conception of which worlds are logically possible. Naturally, even if impossible worlds in general are not all closed under any particular logical consequence relation, individual worlds might still be: indeed, there can be individual impossible worlds closed under classical logic if the only impossibilities there are e.g. metaphysical ones. This approach was labelled the "American plan" for impossible worlds by Priest 1997, on the grounds that several of its defenders are American (Zalta 1997, Vander Laan 1997) or "honorary Americans" (Nolan 1997). One argument put forward for this generosity with impossible worlds in Nolan 1997 p 547 is that otherwise a theory of impossible worlds looks like an awkward halfway house, with some impossibilities corresponding to impossible worlds and some (apparent) impossibilities not even being found among the impossible worlds (e.g. cases where A is true according to an impossible world, B also being true according to that impossible world, but A&B failing to obtain at that world).

The dispute between Australasian-plan approaches and American-plan approaches is likely to be settled by looking more carefully at what is required for good theories using impossible worlds. I think that once the full range of applications are tackled, sufficiently anarchic impossible worlds will need to be appealed to that no interesting logical consequence relation will be found that they are all closed under. Champions of the Australasian plan will no doubt expect that this degree of freedom in our theorising about impossible worlds is unnecessary. (Interestingly,
Graham Priest himself seems to have shifted camps: his remarks in Priest 1997 suggest his sympathy is with the Australasians, but Priest 2005 includes "open worlds" that need not be closed under any substantial consequence relation, in order to handle the full range of phenomena involving intentional attitudes like belief.)

**Objections to Postulating Impossible Worlds**

Sustained philosophical attention to impossible worlds is a relatively recent phenomenon, and as might have been predicted early discussions have tended to be by people trying to make the case for postulating them or using them in one philosophical inquiry or another. However, some criticisms of the project of employing impossible worlds, either for specific purposes or in general, have appeared in the literature. In this section I will focus on two fairly general criticisms of the use of possible worlds, rather than criticisms of specific applications: though see Williamson forthcoming for a recent criticism of the use of impossible worlds for a specific application: the theory of counterpossible conditionals.

Robert Stalnaker's influential "Impossibilities" (Stalnaker 2003) contains many concerns about impossible worlds, and since it is in the form of a dialogue it is not clear which of them Stalnaker is advancing, or even thinks can be made to work. I will focus here on just one of the important concerns that Stalnaker's dialogue suggests (though perhaps he would not it in the way I am about to).

The concern is whether any argument for postulating "impossible worlds" would be self-defeating. Suppose we showed that for a range of tasks we postulated possible worlds, we need more worlds than e.g. Stalnaker would countenance: suppose we needed to postulate worlds that were not closed under classical consequence to provide a theory of belief, for example, or worlds where contradictions were true to handle some counterfactuals. Why suppose such worlds represent *impossibilities*, rather than conclude that we were too restrictive about what *possibilities* we recognise? For example, if beliefs serve to distinguish between possibilities, and we have to genuinely distinguish in belief between worlds where 123 is prime and 123 is composite, wouldn't this just show that both of those are genuine possibilities after all? (Compare
Mortensen 1989, who accepts that every proposition is possible, roughly on the grounds that we can treat any proposition in the way ordinary theories hold we can only treat possible propositions.) Stalnaker suggests that arguments for impossible-world theories are self-defeating: at best, they are arguments for only using possible worlds, but revising how generous our theory of possible worlds should be. (At least if I have interpreted Stalnaker correctly here: I take the exchange on pp 62-67 to suggest this, particularly the remarks about "logical" and "illogical" space on pp 62-63).

A defender of impossible worlds should provide a response to this challenge (whether or not it was what Stalnaker had in mind). Why are the new worlds postulated impossible in any interesting sense, if they play the same theoretical roles as possible worlds? Why not just count them as more possible worlds, and say that we previously underestimated the extent of what is possible? One immediate response to this challenge could be to point out relevant differences in how impossible worlds are deployed. If impossible worlds play some but not all the roles possible worlds are supposed to, it is worth keeping a distinction in place (however it is labelled). Possible worlds, for example, play a distinctive role in connection with the (non-epistemic, non-deontic) modal operators: it is not enough that there be an impossible world according to which a proposition $p$ obtained to ensure that $p$ was genuinely possible. (Perhaps we can introduce new operators, $M'$ and $L'$, perhaps, so that $M'(p)$ is true whenever $p$ is true according to some world, possible or impossible, and $L'(p)$ is true when and only when $p$ is true according to every possible and impossible world. On the face of it, though, that would no more correspond to a genuine sense of possibility and necessity than introducing new operators $M''(p)$ and $L''(p)$ so that $M''(p)$ is true no matter what $p$ is, and $L''(p)$ is false no matter what $p$ is, would show that all propositions were really possible!) Likewise, in many impossible-world theories, logical validity is a matter of truth-preservation at possible worlds, not all worlds.

A second response to the challenge that goes a little deeper would be to defend an understanding of possibility and necessity so that we have good reason to treat something as impossible even if it is thinkable or representable in language, if it makes sense to consider it as a supposition for counterfactual reasoning, or even if it plays some of the other roles traditionally assigned to possibilities. There are many substantial theories of possibility and necessity which could play
this role. Perhaps a necessary condition to be possible is that a proposition not logically imply a contradiction, for example, and that this could be argued for: then if we could also show that some contradictions are thinkable (and different contradictions differently thinkable), supposable for non-trivial counterfactual reasoning, make a non-trivialising contribution to a fiction, and so on, then that combination would be a principled position which provides a response to the Stalnakerian concern about collapse. Or perhaps it is a necessary condition on a claim’s being possibly true that it does not violate any analytic rules: a philosophical position which defended that claim and also showed how some statements which do violate those rules can be believed, or non-trivially supposed, or non-trivially contribute to a fiction, etc., could be used to argue that not every apparent impossibility is just a possibility in wolf’s clothing. In general, there are many ways that a theory of the difference between possibility and impossibility can draw the line so that the distinction does not collapse, even if impossible worlds do some of the work traditionally thought to be the exclusive province of possible worlds. But a general theory of the divide between possibility and impossibility owes us some such answer to this Stalnakerian challenge.

Another interesting challenge to the use of impossible worlds for philosophical theorising has been offered by Bjerring and Schwarz (2017). Their concern is based around the question of how fine-grained the distinctions need to be for impossible worlds to perform the theoretical tasks those worlds are often assigned, especially in the philosophy of mind and language. Impossible worlds, if they are to add anything, must allow the drawing of distinctions that are not marked as differences among possible worlds: e.g. two sentences true at all the same possible worlds will have to be true at different impossible worlds, if they are to be associated with different contents. (E.g. if I am to be able to believe that 2+2=4 without believing that there is a square root of -1.) But then when we look at how much contents seem to come apart, very few of the sentences true at the same possible worlds will be true at the same impossible worlds. (There seem to be very few other mathematical propositions I must believe if I believe that 2+2=4.) At the limit, you might think that distinct sentences must always be associated with different contents. But if contents are distinguished this finely, then impossible worlds seem to lose a lot of their explanatory power. For example, if we model worlds in full generality as associating arbitrary sets of sentences with the truth-value "true", with the complement of those sets treated as not true at the relevant world, then our model of the meaning of sentences just consists of representing
the meaning of a sentence as being true whenever a set of sentences it belongs to is the set of true sentences: not a very illuminating analysis of truth conditions!

Bjerring and Schwarz also argue that it is more difficult than it appears to motivate an "intermediate" position on mental content: once you start relaxing constraints on belief worlds beyond possible ones, it is hard to keep belief worlds closed under any very interesting logic. So one line of response defenders of impossible worlds have engaged in is to look for principled points to restrict the flexibility of the impossible worlds needed e.g. for mental content. (See Jago 2014, especially chapter 8). While it makes sense to limit how fine-grained impossible worlds are for some purposes, my own suspicion is that to handle all the relevant phenomena we will occasionally need impossible worlds that draw very fine distinctions indeed.

One place we might need very fine distinctions is in handling the beliefs of very unusual individuals, such as those in the grip of odd religious or metaphysical theories. Perhaps normally when someone believes a conjunction they believe the conjuncts. But someone convinced that a god is beyond logic might believe e.g. that their god both exists and does not exist, in a way that it does not follow that their god does not exist: and so does not believe that their god does not exist.

So I think the spirit of Bjerring and Schwarz's challenge will be difficult to answer. There are plausible purposes for which a theory of impossible worlds will need to allow that very similar sentences, true at all the same possible worlds and many of the same impossible worlds, nevertheless differ in whether they are true according to some particularly impossible and unusual worlds. A defender of impossible worlds who accepts this owes us an account of how such a system of impossible worlds can still play a non-trivial explanatory role. I am optimistic that this can be done, but an adequate response to this general challenge will have to be left to another occasion.

**Conclusion**
Employing possible worlds in our theorising runs into significant limitations when we need to distinguish between necessary equivalents. Impossible worlds offer us a straightforward way to keep the benefits of theories that employ possible worlds without running into the problems that stem from the fact that our theories seem to need distinctions that do not correspond to differences between possible worlds. Employing impossible worlds is not the only way to tackle the problem of doing justice to distinctions that go beyond modal differences: but at the current state of philosophical development there are few if any rivals that have been developed that offer a unified response to all the puzzles that impossible worlds can help with. (Though see Dučí et al. 2010 for an introduction to one systematic rival program. Work in the relevance/relevant logic tradition also offers many treatments of phenomena that trouble theories built with possible-worlds resources, but the model theory behind most relevance-logical treatments is uses points of evaluation that behave just like Australasian-plan impossible worlds, so it seems better to classify relevance/relevant approaches as implementations of impossible-worlds approaches rather than as rivals to using impossible worlds.)

Theorists who employ impossible worlds disagree with each other on many questions about impossible worlds: their extent, their nature, and how best to employ them in theories of other philosophical phenomena. If past philosophical developments are any guide, these internal debates will no doubt continue. However they are resolved, the use of impossible worlds in philosophical theorising is rapidly becoming as well entrenched as invoking possible worlds. The expansion of philosophical focus from the actual to include the merely possible will continue into appreciation for the impossible as well.

References


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