

RATIONAL CREDENCE THROUGH REASONING

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Abstract

Whereas Bayesians have proposed norms such as probabilism, which requires immediate and permanent certainty in all logical truths, I propose a framework on which credences, including credences in logical truths, are rational because they are based on reasoning that follows plausible rules for the adoption of credences. I argue that my proposed framework has many virtues. In particular, it resolves the problem of logical omniscience.

1. The Question

Alongside the more traditionally recognized attitude of full belief, there is the attitude of degree of belief, or credence. Each of us has credences in countless propositions. And each such attitude, like each of our full beliefs, is either rational or irrational, either epistemically permitted or not.

When is a credence rational? And when it is, what explains why it is rational? I'm looking for conditions that aren't just necessary or sufficient (or both) for rationality, but conditions that are explanatory. An adequate sufficient condition will explain why certain credences are rational, and an adequate necessary condition will explain why other credences are irrational.

I'll focus on the role of logic in constraining and explaining rational credences. I use 'logic' here in a loose but intuitive sense. There's an intuitive difference between cases where the acquisition of empirical evidence explains why a credence is rational and cases where apriori and formal constraints of logic explain. For example, if you inspect a cloth by candlelight, it's your acquisition of certain empirical evidence through perception that explains your rationally becoming more confident the cloth is green.¹ By contrast, something about logic appears to explain why it would be irrational to stubbornly maintain higher cre-

1. From Jeffrey (1965/83, p.165), but he doesn't say perception makes credences rational.

dence in the conjunctive claim that Linda is a feminist banker than in the component conjunct that Linda is a banker.² To the extent possible, I'll set aside cases like the green cloth.

I begin with two motivations for the position I want to develop and advocate in this paper.

2. Two Motivations

2.1 First Motivation: The Standard Bayesian Approach Is Unsatisfactory

The first motivation is negative. I am dissatisfied with the very popular Bayesian approach, in particular with what Bayesians say in response to my main question. I have two reasons, a well-known problem for Bayesianism, and a new problem.

A fundamental thesis of Bayesianism, and the thesis that most clearly aims to apply norms of logic to constrain rational credences, is *probabilism*. This is the normative thesis that, necessarily, rational credences always obey the standard axioms and consequent laws that define mathematical probability. One of those axioms says the probability of any logical truth is the maximal value (standardly measured as 1), so probabilism has the immediate consequence that the rational credence in any logical truth is always certainty. This problem, usually called *the problem of logical omniscience*, is the first reason I find a Bayesian approach to my question unsatisfying. Credences other than certainty in logical truths could be perfectly rational. A common sort of example concerns the rational credence that the trillionth digit of π is a 2.³ Probabilism makes the absurd claim that the only rational credence in this is either total disbelief or certainty (measured as 0 or 1). It's plausible, though, that, given your current evidence, some other credence may be rational. In particular, a credence of 1/10 seems to be perfectly rational.

The second shortcoming of Bayesianism is more fundamental.

2. From Tversky and Kahneman (1982, p.92). See fn.23 for the original case.
3. See, for example, Savage (1967), Hacking (1967) and Titelbaum (2013, sec. 5.4).

There is a distinction drawn in traditional epistemology that has been ignored in Bayesian epistemology. This is the fundamental distinction between (a) *having* a rational belief, and (b) having adequate *support* for a belief. In jargon that is now common, this is the distinction between *doxastic justification* and *propositional justification*.⁴ (I understand 'justified' and 'rational' to mean the same thing.) It is possible to have adequate support for a belief you do not hold, or do not rationally hold because you hold it for the wrong reasons. For example, reading a scientific study could give me support for believing, or for having high confidence, that coffee is good for you, but this belief or credence could at the same time fail to be rationally held if I base it only on wishful thinking, disregarding the study. The second shortcoming that leaves me dissatisfied with Bayesianism, then, is that Bayesians never draw this distinction between having support and having a rational credence, between propositional and doxastic justification. Call this *the problem of conflated rationality*. (Some authors have made the related observation that Bayesians offer no theory of reasoning.⁵)

I see conflated rationality as the more fundamental problem, even though it does not, like the problem of logical omniscience does, identify some particular false claim made by Bayesians. It's more fundamental because it's arguably the source of other problems, including logical omniscience. Probabilism faces the problem of logical omniscience because probabilism is formulated as a necessary condition on having rational credences, i.e. on doxastic justification.⁶ But I don't see that this thesis about doxastic justification has any plausibility. I also don't think any arguments support it. In particular, it gets no support from the well-known arguments Bayesians give for probabilism. What's plausible, and what those arguments may support, is a different

4. From Firth (1978), Goldman (1979) and Conee and Feldman (1985).

5. Staffel (2013) and Broome (2013, pp.175, 208–9, 275).

6. See, for example, Joyce (1998, p.580), Hájek (2008/09, p.229), or Pettigrew (2016, p.17). See also the general view of Joyce (manuscript). There Joyce says we have a "duty" to hold "well-justified" beliefs and that well-justified beliefs must obey the laws of probability.

but analogous thesis, one that says, roughly put, that we have propositional justification to conform our credences to the laws of probability. (I'll state the proposed thesis here a little more precisely in section 3.2. I'll wait till section 5 to review the arguments for probabilism and consider exactly what they support.) If we replace probabilism with this analogous thesis about propositional justification, it seems to me much, if not all, of the implausibility of logical omniscience recedes — quite a bit recedes immediately, to my mind. But since the question of this paper is “When is a credence rational?”, it is a question about doxastic justification, and so probabilism doesn't offer a plausible answer.

Replacing probabilism with a thesis about propositional justification seems to me a much more plausible strategy for Bayesians than one that is commonly employed. This other strategy is to insist that probabilism is a thesis about what the *ideally rational* agent believes.⁷ But I don't see why it should be plausible that an ideally *rational* agent is certain what the trillionth digit of π is. Being well-informed about π 's decimal expansion seems to concern virtues other than rationality. Perhaps it concerns virtues of speed and memory, if not simply the virtue of being well-informed or even omniscient.⁸

2.2 Second Motivation: Logic and the Epistemology of Full Belief

A second, more positive motivation will help me introduce and support the proposal I want to advocate about rational credence. This second motivation is a plausible picture of the role of logic in the epistemology of full beliefs. I'll go on to propose a view on credence that

7. For example, this is a main thesis of Christensen (2004). Talbott (2015, sec. 6.1) says, “Because relaxing that assumption [logical omniscience] would block the derivation of almost all the important results in Bayesian epistemology, most Bayesians maintain the assumption of logical omniscience and treat it as an ideal to which human beings can only more or less approximate.” At the end of the paper, in section 5, I address the issue about risking losing the important results of Bayesian epistemology.

8. In another paper, a sort of companion to this one, I dedicate more space to criticizing existing approaches to solving probabilism's problem of logical omniscience, including the “ideal rationality” suggestion. See Dogramaci (2018).

aims to parallel it.

When is full belief in a logical truth, or in a logical consequence of known premises, rational? And what explains why it is rational? Here it seems much clearer what sorts of approaches are wrong and what approach is right.

It would be implausible to say there is a norm requiring rational thinkers to have full beliefs in all logical truths and consequences of what they know. That would just amount to endorsement of a problem of logical omniscience for full belief. It is not even true that rational thinkers have permission or even reason to believe each logical consequence. If a proposition is a logical consequence of things you know, that fact just by itself gives you no reason to believe this proposition.⁹ You cannot justifiably believe, say, Lagrange's four-square theorem (which says every natural number is the sum of four square numbers) only on the basis of some axioms that entail it, such as the Peano axioms.

When can you rationally believe Lagrange's theorem? That is, when have you got doxastic justification? You cannot take a great leap to the theorem, but you can rationally believe it if, and because, you (or a trustworthy testifier) worked up to it in a series of smaller steps. What the great leap/small step metaphor amounts to here is this.¹⁰ There are certain basic rules for rationally inferring certain logical consequences of what we already know. These are rules we all implicitly know as reasoners, and I call them ‘basic’ when we follow them not by following any other rules. (We may adopt a non-basic rule for, say, inferring that a given number is the sum of four squares.) Meta-logical investigation has made explicit many of the basic rules we follow and given them standard names, such as rules like Modus Ponens, Modus Tollens, Disjunctive Syllogism, Conditional Proof, and so on. (I mean to

9. Harman (1986, p.17): “One might have *no* reason to accept something that is logically implied by one's beliefs if there is no short and simple argument showing this.”

10. I'm borrowing this use of the metaphor from Schechter (forthcoming).

be naming rules for rational belief revision, rather than rules for constructing proofs. I believe the epistemic role of proofs is, in any case, only to supply reasoners with justification to believe various conditionals that can feature in rational belief revision using belief rules like Modus Ponens, such as a conditional linking the Peano axioms to Lagrange's theorem.) The familiar rule of Modus Ponens, for example, might be put like this: given rational beliefs in premises of the forms p and *if- p -then- q* , you have defeasible permission to adopt, on that basis, a belief in the conclusion of the form q . This rule, like all rules of belief revision, only gives defeasible permission. The rule says you may believe q — but the permission is defeated if you have sufficiently strong reasons against q . It's very hard to say what qualifies as "sufficiently strong" to generate such defeat, and I won't try to here. But it's our following a rule roughly like this, together with other rules, that explains when and why we have rational beliefs in logical truths and consequences of what we know.

In forming rational beliefs in logical truths and consequences, we don't merely conform to these basic rules — we *follow* them. Like with Chomsky's well-known view of the rules of syntax, explicit knowledge of the rules we follow is elusive, while the implicit knowledge that constitutes our ability to competently follow the rules is universal among us as reasoners.¹¹

My second motivation, then, for the view I will propose is just this. We form rational, i.e. doxastically justified, full beliefs in logical truths and consequences when, and because, we follow certain rules, such as Modus Ponens, Modus Tollens, and so on. It seems plausible that a more general theory is true, one that covers credences as well as full beliefs.

11. Chomsky (1965, ch. 1). I don't assume that our minds contain representations of the rules we follow, or even that there exists a reductive theory of rule-following. I allow primitivism about rule-following, as in Boghossian (2008, 2014).

3. Proposal

3.1 *Proposal: Rational Credence Is Governed by Rules for Reasoning*

Many influential epistemologists have agreed that when a belief is rationally held, that is generally because the reasoner followed the correct rules of belief formation.¹² Observing that the picture is extremely well-developed for beliefs in logical truths and consequences, Goodman famously suggested our next constructive task should be to try to identify the correct rules governing rational (full) beliefs in conclusions that aren't logical consequences of what we know.¹³ I want to apply this idea of rule-governed rationality specifically to rational credences. My general proposal is that you have a rational credence if and only if, and *because*, any reasoning that led you to that credence followed the correct rules for reasoning to credences.

3.2 *The Proposal Requires Going Beyond Traditional Bayesianism*

Bayesianism doesn't offer a plausible implementation of this project, since probabilism isn't a plausible candidate for a rule that people follow when they form rational credences. It is as plausible as the corresponding rule for full beliefs, "Be logically omniscient!". Easy to say.

Though I reject probabilism, I am, as I indicated, open to a distinct view that may extract the spirit and the good idea behind probabilism.

12. For example, Goldman (1986, 2009), Pollock and Cruz (1986/99, ch. 5), Field (2000), Wedgwood (2002), Peacocke (2004), Boghossian (2008), Horwich (2008) and Broome (2013).

Some of these authors also discuss earlier precedents, e.g. Peacocke discusses Frege (see p.1). Some authors vary terminology slightly; for example, Pollock and Cruz use 'epistemic norm' as well as 'procedural norm' instead of 'rule'.

Regarding the attribution to Broome, see sec. 12.1 and sec. 13.7 (and the rest of ch. 13); in making the attribution, note that, as elaborated in footnote 22, I'm counting "automatic processes" (Type 1 thinking) as reasoning, whereas Broome stipulatively uses 'reasoning' in a semi-technical way that excludes automatic processes and is restricted to what he calls active reasoning (Type 2). (I'll discuss the Type 1/Type 2 distinction below.)

A notable example of a vocal dissenter from the rule-following approach is van Fraassen (1989, part II).

13. Goodman (1946/53/83, secs. 3.2–3.3).

I'm open to this: the rules that generate rational credences, i.e. doxastic justification, are such that *if* a creature relentlessly and flawlessly followed them, then the creature would end up with credences that obey the laws of probability. We can, if we want, articulate that view using the language of propositional justification. Suppose we are happy — as I would be happy — to say we have propositional justification, i.e. we have support, for any conclusions the right rules would lead us to rationally believe, if we followed the rules relentlessly and flawlessly. If we say that, then I'm open to replacing probabilism with this similar, though distinct, view: every rational thinker has *propositional justification* for credences that obey the laws of probability.¹⁴ This view is parallel to the plausible view concerning full belief in logical truths and consequences. A believer in Peano's axioms may have support, i.e. propositional justification, to believe Lagrange's theorem, but there is no requirement to believe it.

While I've argued against probabilism, I will not in this paper argue against the other central principle of traditional Bayesianism. That principle says that, in response to empirical evidence, our reasoning with credences should follow *conditionalization*, which is the one rule of reasoning that Bayesianism does embrace. Conditionalization is, however, not relevant to the focus of this paper. The focus, as I said, is on the way rational credence is constrained by logic. Even if conditionalization, or something like it,¹⁵ is a fine rule for reasoning in response to the acquisition of empirical evidence, there must be other rules that allow us to meet our logical duties. This point was rightly made by Staffel in her paper arguing why we should not be content to model reasoning with credences as a matter of conditionalizing: "[M]uch reasoning with degrees of belief is done without taking into account new

evidence, so conditionalization is irrelevant in these cases. These are cases in which the reasoner forms a new credence on the basis of her existing credences, in combination with the rules of probability. Such cases surely count as reasoning, and they don't require employing conditionalization."¹⁶ Staffel says she is referring to reasoning that proceeds "in combination with the rules of probability". A goal of this paper is to argue that there indeed are such rules, but they are not any familiar feature of the Bayesian framework. The rules are yet to be made explicit. (Section 4 will try to make a start on the large project of making explicit some of the rules that govern rational credences.)

3.3 Immediate Payoffs of the Proposal

The rule-oriented framework I'm proposing has some immediate payoffs. Most significantly, it allows us to solve the problem of logical omniscience in a sensible way. We do so by incorporating an important insight of Harman's.

Harman made the obvious but very important point that it is "worse than pointless" to clutter our minds inferring trivial or uninteresting consequences.¹⁷ We must accommodate this point, and I do so by understanding the epistemic rules of correct reasoning as rules of *permission*, not obligation. I suggested a formulation of Modus Ponens as a rule of permission, and I will later sketch similarly permissive rules for credence. As Harman himself observes, a principle for avoiding clutter implies a principle for avoiding logical omniscience. Our framework of rules of permission, by addressing the clutter avoidance issue, addresses the problem of logical omniscience. Avoid clutter, and you avoid logical omniscience. One of the deepest and most stubborn problems for Bayesianism can thus be resolved by adopting a rule-

14. See again the citations in note 6 for why this view I'm open to is *not* what Bayesian authors have meant by 'probabilism'.

15. When Bayesians discuss conditionalization, they normally presuppose the subject already conforms to probabilism, so an improved rule for reasoners who violate probabilism might be in order. See Staffel (2017) for some related discussion.

16. Staffel (2013, p.3542). She credits Alan Hájek for bringing up the point.

17. Harman (1986, p.12). Friedman (forthcoming) gives a very long list of epistemologists who endorse Harman's observation. She conducts a detailed exploration of the proper exact formulation of the clutter avoidance principle and demonstrates its significant consequences.

oriented framework.

One more observation from Harman is that epistemic permissions must sometimes become obligations.¹⁸ If someone shows you that your beliefs have some absurd consequence q , you are not permitted to ignore q . I endorse Harman's suggestion that we have an obligation to believe a permitted proposition that we either do or ought to take an interest in. Later, when I sketch permissive rules for credence, we will need to keep in mind that such interests can convert these permissions into obligations.

A second payoff that we can immediately appreciate concerns another famously stubborn problem for Bayesianism. Bayesianism presupposes that rational reasoners never forget their evidence. Many Bayesians have offered revisions to Bayesianism in order to allow forgetfulness.¹⁹ But the revisions come at costs, often making the prescriptions of Bayesianism less clear-cut.

A rule-oriented framework can offer a better approach to forgetfulness. Rational reasoners only need to remember their conclusions. They can forget the premises that led to a conclusion, and continue to rationally believe that conclusion. Admittedly, failure to store all your beliefs in memory can have bad effects: you might later gain evidence that some forgotten premise was false, and now you're stuck with the conclusion. But, again as Harman observed, this is how the concept of rationality actually works.²⁰ If I form a belief that p based on testimony from Smith, and much later on learn Smith is a liar but forget that I learned p from Smith, my justification to believe p is not defeated. This is how rationality operates, and our framework allows us to say it about rational credence.

18. Harman (1986, p.15). He credits Robert Stalnaker for bringing up the point.

19. For example, Williamson (2000, ch. 10) and Titelbaum (2013, pt. 3).

20. Harman (1986, ch. 4).

4. Development: What Are the Rules for Credences?

I've motivated and advanced a general proposal, namely that credences are rational because the reasoner followed certain rules. The proposed theory won't be complete, and can't be fully tested, until we learn exactly what these rules are that generate rational credences. The rules are ones we all implicitly know, but making them explicit is an enormous project. You could write a whole book that only sketched a small sub-class of our implicit rules.²¹ Here I will only try to take some small steps toward a fuller development of the view.

A manageable task for the rest of this paper will be to focus just on rules of one certain type. Sub-section 4.1 explains that, just like the reasoning that leads us to full belief in Lagrange's theorem, much of our reasoning with credences will have properties that are associated with what some cognitive scientists call *Type 2* thinking. Given my motivations from section 2 (i.e. given that I'm interested in logical rule-governed reasoning with credences that parallels the reasoning to Lagrange's theorem), this *Type 2* thinking is what I will focus on. (I'll also qualify my commitments to the significance of the *Type 1/Type 2* distinction.)

Sub-section 4.2 then introduces a view about what we are reasoning *about* when we are reasoning with credences. The view will help us make sense of how rules for credences can have some of the paradigmatic *Type 2* features.

Then, in sub-sections 4.3 and 4.4, I'll have a try at proposing some specific rules for such reasoning with credences. Here I'll be guided by some ideas from John Broome and Ian Hacking on how to formulate plausible *requirements* on our attitudes and *rules* for reasoning and how the two interrelate.

Finally, in sub-section 4.5 I address the objection that the rules I propose are too sophisticated for many people to plausibly follow them.

21. See, e.g., Strevens (2013) or Oaksford and Chater (1998, 2007). See also North (2010) for a discussion that's not explicitly on this but is closely related to Strevens's project.

4.1 *Narrowing the Target: Reasoning that's Conscious, Active, Effortful, and Infallible*

Many cognitive scientists endorse a *dual-process* model of reasoning. The model is popular, but still controversial.²² I won't rely on its most controversial parts.

Here is one version of the model. Our thinking falls into two types. Type 1 thinking is (i) unconscious, (ii) automatic, (iii) fast, and (iv) applies heuristics that are fallible. Type 2 thinking, on the other hand, is (i') conscious, (ii') active, (iii') slow, and (iv') conditionally infallible (i.e. it's impossible for the reasoning's basis to be true and its conclusion false).

Type 1 thinking is demonstrated in the classic cases where people reliably say something logically false or probabilistically incoherent, e.g. we name the wrong cards to turn over in the Wason Task, or we say Linda is more likely a feminist bank teller than a bank teller.²³ Type 2 thinking is demonstrated when we revise those answers to say the logically or probabilistically correct thing.

I've highlighted four features that have been said to distinguish Type 1 and Type 2 reasoning on (one version of) the dual-process

model. Now, what's controversial about the model is the claim that any such features mark a deep and natural two-part division in human psychology. One theme of the critiques of the model is the observation that these various features are not even always present, or absent, together.²⁴ Friends of the model have recommended concessions and revisions in response to these criticisms.²⁵ I want to remain neutral on the psychological depth and naturalness of the Type 1/Type 2 distinction or any such two-part division of human reasoning. All I claim is that *some* of our reasoning exhibits these four Type 2 features, such as the corrective uses of Modus Ponens and Tollens just mentioned, and I claim that, if we look, we'll find rule-governed reasoning with credences that likewise exhibits these features. This is the target I want to explore. It's an appropriate target for philosophy, since our unconscious heuristics are often best uncovered with the help of empirical studies.²⁶ But philosophers can make explicit the rules that govern reasoning that exhibits these Type 2 features. From Aristotle to Gentzen, philosophers have had great success making rules that govern reasoning with full beliefs explicit, rules that have deservedly been given their own names, like 'Modus Ponens'. Now we need to turn our attention to reasoning with credences.

In the rest of this sub-section, I'll briefly clarify how I understand each of these four features associated with, as I'll still call it, "Type 2 reasoning".

What is **conscious** reasoning? Dual-process theorists are often frustratingly quiet or unclear on this. I already noted that many philosophers believe that any piece of reasoning must follow some general

22. See, e.g., Evans (2008), Evans and Stanovich (2013), Kahneman (2003) or the popular book Kahneman (2011). Kahneman reserves the word 'reasoning' for Type/System 2 belief formation, but I use 'reasoning' more inclusively for either type of belief formation. In using 'reasoning' in this inclusive way, I am following Staffel (2013, sec. 3.3), who argues that this fits the intuitive notion of reasoning. Arpaly and Schroeder (2014, chs. 1–2) offer a kind of dual-process theory with a focus on moral reasoning.

23. The Wason Task: you know four cards each have a letter on one side and a number on the other. You see these faces up, A, 8, D, 3, and are asked which cards must be turned over to settle whether this rule holds: if a card has a vowel on its letter side, it has an even number on the other side. Most people initially fail to say that the 3 must be turned over (among other errors). Originally from Wason (1966).

The Linda Problem: "Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations." Based on this profile, most people give a lower credence to Linda's being a bank teller than to Linda's being a feminist bank teller. Originally from Tversky and Kahneman (1982)

24. See Osman (2004), Kruglanski and Gigerenzer (2011), and Mugg (2016) for criticisms of the dual-process theory.

25. See, e.g., Evans and Stanovich (2013) and Carruthers (2015, ch. 7). One of the sensible recommendations Evans and Stanovich make is to retire the terminology of 'System 1' and 'System 2', even right after Kahneman (2011) made it very popular. Reasoning, they concede, doesn't take place within either of two systems or modules. (The 'Systems' terminology was first introduced by Stanovich (1999), and is still used by many.)

26. See, e.g., the empirical work of Oaksford and Chater (2007, ch. 7).

rule. But, of course, no piece of reasoning *requires* the reasoner to be *aware* of what that general rule is. And, of course, any reasoning will make the reasoner aware of its *conclusion*. So how does whether reasoning is conscious or unconscious help draw any useful distinction? What I suggest that we are sometimes but not always aware of — “conscious” of — is this: in conscious reasoning we are aware that certain considerations are serving as the basis of the conclusion we draw. We *unconsciously* reason that Linda is more likely a feminist bank teller because we are not aware that we are basing that conclusion on the representativeness of her profile as feminist. We need empirical psychologists to reveal that. But we consciously reason to Lagrange’s theorem because we must be aware that we are basing that conclusion on the proof’s assumptions.²⁷

When is reasoning *active*? Of course, we cannot voluntarily switch from believing something is true to believing it’s false, or vice versa, or switch to/from agnosticism. But we can voluntarily put an effort into reasoning from a given basis, not knowing what specific conclusions we will draw. It’s this actively guided deliberative exercise that I’ll call active reasoning. It seems to be what other authors have in mind when they declare that they intend to theorize about active reasoning. Again, the example of proving a remote theorem like Lagrange’s theorem illustrates when our reasoning has this sort of feature. Our reasoning to the theorem requires some active guidance, monitoring, and effort.²⁸

27. In support of this way of drawing the distinction, consider also the role of confabulations in processes that get attributed to Type 1: while we know *what* we’ve concluded, we often unwittingly fabricate explanations of what led us to that conclusion. (See Kahneman (2011) for an overview. See Strevens (2013) for an interesting example attributed to James Clerk Maxwell.) I claim this kind of blindness to our basis is not a feature of the type of reasoning we must use to discover a theorem like Lagrange’s.

28. Broome and Boghossian develop views of reasoning that explicitly just target *conscious* and *active* reasoning. Their characterizations of their target each overlap quite a bit with the characterizations I’m giving here. See Broome (2013, esp. chs. 12 & 13) and Boghossian (2014, esp. secs. 2, 9, 10 & 12). Arpaly and Schroeder (2014, esp. chs. 1 & 2) similarly develop a theory that targets “deliberation” which they say is essentially “conscious” and “voluntary”. And

When does reasoning count as **slow**? The source of slowness is said to consist in the fact that certain kinds of reasoning tax our working memory. (Perhaps this is an effect of being conscious and active.) Humans have a limited working memory that acts as a kind of bottleneck that slows certain reasoning. The geometric reasoning an athlete must perform to predict where a flying ball will end up may be extremely complex, but it doesn’t tax working memory. By contrast, the reasoning we use to do an arithmetic calculation, though elementary in a sense, taxes our working memory and so must proceed slowly.²⁹

Finally, while our fallible Type 1 heuristics predictably generate certain errors, Type 2 reasoning (which we usually need to catch those errors) is **conditionally infallible**. That is, it cannot take you from a true basis to a false conclusion, the traditional criterion of what we call deductively valid reasoning. Again, don’t worry if this fourth feature means that Type 2, as I’m understanding it here, doesn’t carve a deep and natural joint between two systems of reasoning in human psychology in the way some dual-process theorists once sought. My goal is to identify a kind of logical reasoning with credences that parallels logical reasoning with full beliefs as exemplified by Modus Ponens and company, and the latter are all, I’ll take it, infallible.³⁰

But there’s another worry. How is reasoning with credences either

Carruthers (2015) develops a theory of “reflective” thinking that is “conscious” (see e.g. sec. 1.2) and “purposeful (or active)” (see, e.g., sec. 1.3). None of these authors defines ‘conscious’ exactly as I have, but I only claim that my characterization is one useful one, not the only useful one.

29. See Nagel and Smith (2017, sec. 2) and especially see Carruthers (2015), who extensively develops a view of reasoning that assigns this special role to “working memory”. On Carruthers’s view, though, contrary to my suggestion above, no species of reasoning involves our being conscious of our reasons or basis for our conclusion. (He instead says that conscious reasoning involves our being conscious only of sensory states.)

30. Let this be a motivating but *defeasible* criterion of our target. In light of the apparent counter-examples to the validity of Modus Ponens and Tollens given in McGee (1985), Kolodny and MacFarlane (2010), and Yalcin (2012), we might either use some more complex, refined definition of validity/infallibility, or just decide that validity isn’t a feature of these rules after all. See Stojnić (2017) for a defense of the classical validity of these rules.

fallible or infallible, given that credences are not straightforwardly true or false, and so the reasoning cannot straightforwardly fail or succeed at preserving truth? (Credences can be measured for accuracy, but I won't propose to use accuracy measures to apply the notion of (in)fallibility to reasoning with credences.³¹) In the next sub-section, I'll show how credences can indeed be true or false, at least in a minimalist sense. Then we'll be able to see a way that reasoning with credences can be fallible or infallible. (The view I'm about to propose in the next sub-section might be disputed. So, in a long footnote, I mention an alternative feature that I'd also be happy to use to target Type 2 reasoning.³²)

This fourth feature excludes from my target the rules of so-called *direct inference*, including the Principal Principle.³³ Others have made

31. See Pettigrew (2016), Schoenfield (2015), and Horowitz (forthcoming) for a recent sample of the large literature on how to analyze the accuracy of credences.

32. There's a distinction between rules that *transmit defeat for a conclusion's justification to the premises' justification* and the rules that do not similarly transmit. Defeasibility is a property of justification: it is justification that can be lost given new evidence. All justification is defeasible, including the justification for conclusions reached by infallible rules, even a rule for believing a tautology. (All you need is a sufficiently trustworthy guru.) However, some rules do, and others don't, transmit total defeat for their conclusions' justification to their bases' justification. E.g., when you apply the representativeness heuristic to conclude that Linda is probably a feminist, you can later gain new evidence that establishes that she is certainly not a feminist without this generating any defeat for any other justification you have, including justification for the basis on which you initially judged that Linda is probably a feminist. By contrast, you cannot defeat all of your justification for believing *q* without necessarily also transmitting that defeat to your justification for believing both of *p* and *if-p-then-q*. Thus, Modus Ponens transmits defeat from conclusion to premises, as do the other rules we use on the way to inferring Lagrange's theorem. I'd be happy to take as the fourth feature of the reasoning I will focus on, then, that it uses rules that must transmit total defeat from conclusion to basis in this way. This feature is closely connected with infallibility. With Modus Ponens, it's *because* Modus Ponens is known to everyone to be infallible that it transmits defeat in this way. I suspect any such rules with this defeat-transmitting property have it because of some similar such underlying infallibility.

33. Direct inference is extensively explored by Kyburg (1974, 1982) and his critics such as Levi (1977) and Seidenfeld (1978); see Plantinga (1993, ch. 8) for

the similar suggestion that direct inference should be modeled in terms of heuristics, the Type 1 family of thinking.³⁴

4.2 What Do We Reason About When We Reason with Credences?

The reasoner who follows the rule of Modus Ponens forms a new full belief on the basis of two existing full beliefs, but what they reason *about* is the world. The reasoner may believe that it rained, and that if it rained then the snow melted, and, taking those two beliefs as her basis, she may conclude that the snow melted, for example. The reasoner's beliefs are each directed outward on the world of rain and snow, not inward on their own mental states. The conditional infallibility of the reasoning concerns the truth-conditions of these worldly contents. At the same time, the reasoner can also look inward: the reasoning can be conscious because the reasoner, looking inward, can be aware that one belief was formed on the basis of certain other beliefs.

How can we develop a parallel view about reasoning with credences? How can reasoning with credences be similarly infallible and conscious? The question has some intrinsic interest, and we also need to address it in order to apply the model of Type 2 thinking to credal reasoning. How credal reasoning can be active and slow is less puzzling, but I'll say a bit about it below too.

Just as reasoning with full beliefs is directed outward, likewise, so I say, credal reasoning is directed outward. It is about the world, not our inner mental states. In particular, when we reason with credences, what we reason *about* are the *chances* of things, such as the chances of rain or of snow. The general view I want to endorse here is this:

a useful critical review. The Principal Principle was famously introduced in Lewis (1980).

34. Strevens (2013, p.216) suggests his proposed rules of "equidynamics" could be grouped in the direct inference family. Also, as he makes clear, followers of his rules are *unconscious* of what contents are serving as their bases. See, in particular, his interesting account of James Clerk Maxwell's brilliant but unconscious reasoning. (However, in some very simple cases of direct inference, I take it we are conscious of what is serving as our basis, so those would be examples where the four features I've highlighted can come apart.)

(CC) Necessarily, an attribution of a credence is correct if and only if a corresponding attribution of a belief about chance is correct.

This biconditional, (CC), is plausibly explained by a reductive view that many have embraced: credences *just are* ordinary full beliefs with probabilistic contents.³⁵ I'm inclined to accept this reductive view. It is a simple and natural explanation of what we reason about when we reason with credences. But I will *not* presuppose the reductive view in this paper. I *will*, however, help myself to the weaker view expressed by the biconditional (CC). The convenient and natural way for us to attribute a credence is by attributing a belief about chance. To attribute a high credence that it rained, for example, we naturally say that you believe the chance that it rained is high.

Many object that it is unrealistic to suppose that, for any person or creature with a credence, we can talk about their having a belief about chance. They object that the view places implausibly high standards of conceptual sophistication on ordinary people, children and maybe animals (if they can have credences).³⁶ My reply is that the objection as-

35. For endorsements of the explicitly reductive view, see Lance (1995, sec. 4), Schiffer (2003, p.200), and Holton (2014, sec. 1).

See Weisberg (2013, sec. 3.1) for further support for the view. Weisberg doesn't outright endorse the view himself, but he is sympathetic and he proposes that Hawthorne and Stanley (2008) ought to endorse the view.

Yalcin (2007, p.1021) doesn't discuss the reductive view, but he sympathizes with the view that credences are attributed using attributions of beliefs that embed an epistemic modal (for example, 'likely') in the belief report. Weisberg and Holton both say this fact about how we use language to attribute attitudes is evidence (if inconclusive evidence) for the reductive view, and both then give further arguments in support of the reductive view.

Moss (forthcoming) shares this view about how we attribute credences. She doesn't endorse the reductive view that credences are beliefs about chances, but she has a related view. Moss agrees that when we say, 'Jones believes it's likely that Smith smokes', we are attributing to Jones an attitude of belief, just the same kind of attitude we attribute when we say, 'Jones believes Smith smokes', but the former attitude is distinguished by having a complex probabilistic content. Moss, however, says she does not want to analyze probabilistic content as I want to, as an attribution of chance (see her sec. 1.1). She instead analyzes these probabilistic contents as sets of probability spaces.

36. For example, Christensen (2004, p.19) and Frankish (2009, p.77).

sumes implausibly high standards for concept attribution. I am happy to liberally attribute the concept of chance to any creature that we want to attribute a credence to.³⁷

When I talk of chance here, I mean a specific sense of this ambiguous term, one that has been called *evidential* or *epistemic probability*.³⁸ I'll use 'evidential chance' as my disambiguating term (only because 'chance' strikes my ear — and the ears of Hawthorne and Stanley (2008, p.581–2) too — as closer to contemporary ordinary language here than 'probability'). An evidential chance is, as the name suggests, a chance that is relative to a given body of evidence. It is thus not an objective chance of the sort that appears in indeterministic scientific theories like quantum mechanics. Nor is it the psychological state of subjective chance — that is just credence. A stock example will help illustrate what evidential chance is. Suppose a fair coin was tossed, and the outcome determined whether a second toss was performed using a double-headed coin (if the fair coin came up heads) or a double-tailed coin (if the fair coin came up tails). Suppose we know all that, but we didn't watch the tosses. Then the objective chance of heads on the second toss is either 0 or 1, but the evidential chance is a half (or "half a chance", as ordinary speakers might put it). And that is also conceptually distinct from credence. The evidential chance would be a half, even if everybody adopted some other credence, or none at all.

Evidential chance is one disambiguation of how we use ordinary probabilistic language. It is not credence (subjective chance). It is the

37. Hawthorne and Stanley (2008, p.585) and Weisberg (2013, p.10) make a similar reply.

38. Plantinga (1993, chs. 8 & 9) uses the term 'epistemic probability'. Williamson (2000, ch. 9), and following him Hawthorne and Stanley (2008), use the term 'evidential probability'. These ideas derive from the earlier theory of so-called logical probability, associated with Keynes and Carnap; for an overview, see Hájek (2012).

Lance (1995, p.171), Holton (2014, p.13), and Weisberg (2013, p.10) correctly emphasize that we must focus on epistemic or evidential, rather than objective, probability or chance when we are concerned with ordinary attributions of credence.

notion we use to *attribute* a credence, like when we say that a juror believes that there is a low chance that the defendant is guilty, or when we self-attribute a credence, like when I say I believe the chance of rain is high.

With the explication of evidential chance and its role in the attribution of credence in place, via (CC), I can re-state the general thesis of this paper in the following way: you have a rational credence, n , in a proposition p — if and only if, and *because* — you reasoned that the chance of p is n , and your reasoning followed the correct rules for reasoning about chances.

And now I can make it clearer how some reasoning with credences can have the Type 2 features, including those of being infallible and conscious. Credal reasoning can be infallible when the chance claims the reasoning is about must preserve truth. If I reason that the chance of some rain is high, and therefore the chance of sunny weather all day is low, that can be infallible, because it can be impossible that the premise is true yet the conclusion is false. How can credal reasoning be conscious in the way I suggested Type 2 reasoning is conscious, i.e. how can we tell that we've based one credence on certain others? The answer may be that we can introspect our credences, and their bases, by way of self-attributions of beliefs about chance.³⁹ I can, for example, be aware that I have low credence in a sunny day, and also that this is based on my high credence in rain. I do so by self-attributing a belief that the chance of a sunny day is low, and by recognizing that this belief is based on another belief that the chance of rain is high. (CC) can also make the active and slow character of credal reasoning clearer. I might actively deliberate by thinking, "The chance of rain is high. Now what follows?" And, while monitoring and effortfully guiding my thinking along, I might slowly arrive at other conclusions — for example, that the chance of a fun picnic is low.

(One more advantage of (CC) I'll quickly add: it can help us make

39. See Dogramaci (2016) for defense of this view of how we can introspect our credences.

sense of *imprecise* or *indeterminate* credence. Imprecise credences, sometimes called 'mushy credences', are often modeled as taking a range rather than a sharp point as a value. For them, we might suppose the person believes the chance is anywhere in that range but the person has no more precise belief about the chance. Indeterminate credences have no precise value or precise range at all. For them, we might say the person believes the chance is simply high, low, or even just "low-ish", and there is no more precise fact about the person's credence.)

4.3 Rules of Type 2 Reasoning — initial examples

In his book *Rationality Through Reasoning* (from which my title is borrowed), Broome wrote: "Bayesians owe us an account of the active reasoning process by which you can bring yourself to satisfy Bayesian requirements." (p.208). He was referring to what I've called Type 2 reasoning and to our satisfying the requirements of probabilism.⁴⁰ In this sub-section and the next, I try to make some progress on this project that Broome says Bayesians owe.

The rules we're looking for now, rules that fully belong to Type 2 reasoning, are ones we expect to be characterized by our four features: reasoning with these rules is conscious, active, slow, and infallible. As a result of having these properties, the reasoning can also be expected to serve the distinctive function often attributed to Type 2 reasoning: it improves the overall accuracy of our views by overturning the mistaken conclusions sometimes reached by our other fallible rules.⁴¹

40. See p.207 for why I take Broome to be referring to Type 2 reasoning, and see p.175 for why he is referring to probabilism. Broome (2013) doesn't extensively discuss cognitive science, but he leans on the Type 1/Type 2 distinction, and his theory is targeted at Type 2 reasoning, just not by that name. See especially his comments in chapters 12.1 and 13.

41. In the clever and revisionary theory of Mercier and Sperber (2011), this function of Type 2 reasoning must be pursued socially. They argue that we reason in ways that make conscious what contents are serving as our bases so that we can publicly share our reasoning with others, and then the social tribunal evaluates and filters out the most accurate views, which then go on to become accepted by all parties. I sympathize with their theory, and intend my

How shall we uncover these rules, making our implicit knowledge explicit? At a very general level of description, the method for uncovering these rules is the method of pursuing reflective equilibrium, made famous by Goodman as the right method for investigating the correct rules of reasoning.⁴² Since these rules are ones that make it accessible to consciousness what their bases and conclusions are, we can propose various candidate rules and consult our intuitions. We can consult intuitions about whether this or that individual candidate looks like a plausible general rule, or whether this or that specific instance in which a candidate rule gets applied looks like a plausible case of good reasoning, or whether this or that package of rules appears to be coherent. And we can consider whether a proposed rule is really valid, i.e. infallible; we can look to see whether there are counter-examples that expose its fallibility, thus disqualifying it from our target of Type 2 reasoning.

Still, this advice to pursue reflective equilibrium is a little open-ended and hard to start out on. Is there a more useful strategy we can be guided by?

A more usefully specific and concrete strategy for the armchair philosopher trying to intuit the correct rules of Type 2 reasoning is demonstrated by Broome. Broome uncovers a number of highly intuitive rules by extracting them from intuitive *rational requirements*.⁴³

proposal to be ultimately compatible with it, but I can't fully explore this here.
42. The method is developed not only by Goodman (1946/53/83, ch. 3), but of course also by others, including Quine (1951) and Rawls (1971, ch. 1), who gave it its name. Cohen (1981) is a valuable defense of the correct observation that we cannot avoid using Goodman's method, even in the face of the empirical discoveries that human cognition uses heuristics and exhibits some built-in biases. I share the view of Setiya (2012, pp.39, 83): what goes by the name 'reflective equilibrium' and is most problematic is the associated coherentist epistemology, but as a characterization of a methodology for philosophy, the method of reflective equilibrium is close to a truism. Kelly and McGrath (2010), and at greater length McGrath (forthcoming), provide an extensive and thoroughly argued exploration of the virtues and limitations of much of what has been called 'the method of reflective equilibrium'.

43. See pp.247–8, 258–9, and sec. 14.2.

Broome says the rules for correct reasoning have a "grounding in requirements of rationality" (p.259).⁴⁴ By 'requirement of rationality', he means a wide-scope necessary condition on rational credence (doxastic justification), typically a synchronic condition, i.e. one that concerns a set of attitudes held at a single time. Broome says the rules of Type 2 reasoning serve to enable us to fulfill these requirements of rationality, in particular to do so when the automatic processes of Type 1 fail to get us to fulfill these requirements.⁴⁵

Here's an example. An intuitive rule for reasoning can be extracted from the intuitive synchronic requirement that corresponds to Modus Ponens.⁴⁶ The requirement is:

(M-req) You are rational only if: if you believe p , you believe *if- p -then- q* , and you are or ought to be interested in whether q , then you believe q .

I already suggested, in section 2.2, the rule that helps you fulfill this requirement, a permission on reasoning:

(M-rule) Given rational beliefs in premises of the forms p and *if- p -then- q* , you have defeasible permission to adopt, on that basis, a belief in the conclusion of the form q .

As mentioned earlier, Harman and Stalnaker observe that you have not merely a permission but a requirement to believe q , once you are, or

44. In the quoted sentence, Broome is actually saying "basing permissions" are grounded in requirements. But Broome also says each correct rule of reasoning is determined — he also says "generated" — by its corresponding basing permission. See pp.247–8, 255, 260, and sec. 14.2.

45. See pp.207 and 247–8. This characterization of the function of Type 2 reasoning doesn't conflict with, and largely overlaps with, the more traditional characterization of its function, which I mentioned above. The traditional function is to correct the errors, the false beliefs, that Type 1 reasoning can produce in us. Since the requirements Broome has in mind are requirements that we usually violate by holding some false belief, the function of helping us come to fulfill our requirements will usually coincide with the function of correcting errors.

46. Broome offers similar but slightly differently formulated proposals to what I give here (see pp.157 and 252).

ought to be, interested in whether q . Then you are obliged to engage in the reasoning licensed by (M-rule) to fulfill your requirement, (M-req).

As Broome also observes, though, there is no apparent algorithm for determining the rules from the requirements. We must also rely on our independent intuitions about correct reasoning.⁴⁷ Broome himself only proposes rules for reasoning with non-probabilistic full beliefs and intentions. He only points to, but doesn't tackle, the challenge Bayesians face of finding the correct rules for Type 2 reasoning with credences. So, let's begin tackling that challenge now.

A bunch of familiar names will appear on the list of rules for reasoning with credences. If, as (CC) implies, credences at least correlate with full beliefs about chances, then all our old rules for reasoning with full beliefs are now candidates to also qualify as a rule for reasoning with credences, including our Type 2 logical rules like Modus Ponens (M-rule), Modus Tollens, and all the rest. There are some interesting issues that arise when we consider how those rules apply to credences, or beliefs about chance, but I won't take these up in this paper.⁴⁸ My interest here isn't in how those familiar rules might also apply to credences. My interest is in rules that specifically govern reasoning with credences. What might these rules be?

Between Broome's suggestion and the existing Bayesian constraints, a good strategy has been handed to us. Let's try to mine the well-known synchronic super-requirement of Bayesianism, namely probabilism, and from that requirement and all the laws of probability, we can try to craft plausible corresponding rules for reasoning. But we'll need to replace the requirements of probabilism with more plausible requirements, ones that do not generate a problem of logical omniscience. From those improved requirements, we can then try to extract

plausible rules. These will be the rules of Type 2 reasoning, the reasoning that helps us, slowly and effortfully, fulfill our requirements of rationality, and correct the mistakes of our fallible Type 1 rules. This sub-section identifies some initial examples, and the next sub-section identifies another and suggests there exist many more.

To anticipate, the rules I'm going to propose in fact are rules that would, if carried out relentlessly and without error, fulfill the Bayesian requirements of probabilism. Thus, on my earlier suggested understanding of propositional justification, we'll have propositional justification for credences that obey the laws of probability. Though, again, remember: we have no reason, as well as no ability, to carry out the rules relentlessly. There is no reason to pursue either logical omniscience or perfect probabilistic coherence.

So then, probabilism says, again, that rational credences conform to the laws of probability. The laws of probability are, or can be represented as, the consequences of certain axioms. The canonical axioms, stated in simplified forms sufficient for our purposes here, are these three:

- (A1) Every logical truth has the maximal probability.
- (A2) The probability of a disjunction of mutually exclusive disjuncts is the sum of the probabilities of the disjuncts.
- (A3) Every proposition has a non-negative value for its probability.

I'm not going to discuss the third axiom, since to my mind it is largely true as a matter of convention. I'm also going to go along with the standard view that the maximal probability is 1, which I also take to be true as a matter of convention. These are conventions for measuring credence. Other ways of measuring would be equally effective.⁴⁹ And for convenience, let's now suppose credences take precise and cardinal values. (If this is more than a mere convenience, if it's a false idealiza-

47. See pp.256–9.

48. For example, Yalcin (2012) argues, as I mentioned, that Modus Tollens is not valid after all. In fact, it's specifically reasoning about chance that invalidates it. One of the examples: (P1) My dog is not likely to bark in the night, but (P2) if a burglar breaks in, my dog will likely bark in the night; yet we can't thereby conclude (C) a burglar will not break in.

49. See Easwaran (manuscript, ch. 4) for discussion. Pettigrew (2016, pp.78–9), for one, agrees that the use of 0 and 1 to represent minimal and maximal credences is a convention.

tion, then I must defer accounting for the complexities.)

As stated, axioms (A1) and (A2) aren't plausible requirements of rationality. We already discussed the reasons why, namely the problem of logical omniscience and what I called the problem of conflated rationality. (Axiom (A2) contributes to the problems in its own ways. For example, because of (A2), probabilism requires us to give every logical falsehood the minimal credence, even if the shortest proof of its logical falsity is unknown and very sophisticated.) Nevertheless, these axioms can point the way to more plausible candidates for requirements of rationality.

Hacking identified the plausible proposal fifty years ago.⁵⁰ Paraphrased here slightly, Hacking proposed these two requirements on rationality:

- (H1) You are rational only if: if you know that p , then you have maximal credence in p , i.e. $Cr(p) = 1$.
- (H2) You are rational only if: if you know p and q are exclusive, then your credence in the disjunction, $Cr(p \vee q)$, is the sum of your credences in the disjuncts, $Cr(p) + Cr(q)$.

(He left alone the non-negativity axiom, which, again, I think we should take to be conventional anyway.)

I think Hacking's suggestions are very plausible ways to improve (A1) and (A2) as requirements of rationality for credences. I have a slight preference for an alternative version of (H1) that I'll offer, and which some readers might also prefer: I prefer to replace 'you know' with the stricter 'you grasp a proof'. On this alternative, you may be certain only of what you prove, though we could understand proof in a broad sense that goes beyond logical and mathematical truths. (Maybe you can prove that you exist, or that you have hands.) My reason for preferring the you-grasp-a-proof version of (H1) is that some authors have recently defended the identity of ordinary (non-probabilistically

characterized) full belief and maximal credence.⁵¹ I find their defense plausible, and if they are right, that would make Hacking's you-know version of (H1) a triviality, and thus not clearly a genuine norm. As I say, though, my preference for the alternative is slight, and I will remain neutral below.

Our interest now, though, is in finding plausible rules for reasoning, rules with the characteristics of Type 2. What might these be? What rules of reasoning might correspond to the requirements of (H1) and (H2)?

Using your preferred version of (H1), the corresponding rule would plausibly be this:

- (R1) If you know [or, if you grasp a proof] that p , then you are defeasibly permitted to adopt, on that basis, maximal credence in p .

I think it's plausible that there is a rule like this, in one form or another. And it's indeed a significant psychological and epistemological fact that a component of our Type 2 reasoning should include a rule like this, one that serves to monitor and, if necessary, overturn the verdicts of our heuristics and any other fallible rules that govern how we form credences. To see this, consider again the example that highlighted probabilism's problem of logical omniscience. What is the rational credence that the trillionth digit of π is a 2? Hacking tries to resolve the problem by inserting the 'you know' condition into the requirement of rationality. Our rule-oriented approach incorporates Hacking's insight, but moves it into a rule of reasoning. This shift is significant. This shift to rules can give us an overall theory, sophisticated and also realistic, that involves two interacting types of thinking: a set of quick-and-easy-to-use Type 1 rules whose fallible verdicts may be corrected by the slow-and-harder-to-come-by calculations issued by infallible Type 2 rules. Whereas a fallible rule of direct inference tells you to have a credence of $1/10$ that the trillionth digit is a 2, the rule

⁵⁰. Hacking (1967).

⁵¹. See Clarke (2013) and Greco (2015).

represented by (R1), in one form or another, can overturn that verdict and tell you to be certain the digit is a 2. We can bring out the (conditional) infallibility of rule R1 by re-formulating it in terms of beliefs about chances, following (CC). Then the rule is: if you know [or, if you grasp a proof] that p , then you are defeasibly permitted to believe the chance of p is 1, i.e. that p is a certainty. The rule is plausibly (conditionally) infallible in this sense: it's impossible that you know [or have proved] p but the chance of p is less than 1.⁵²

Next, what rule of reasoning might correspond to the requirement of (H2)? This raises a big question. It would seem the basis of your reasoning should include at least knowledge (or proof) that p and q are exclusive. But then, should you revise your credence in the disjunction on the further basis of your credence in the disjuncts? Or instead should you revise your credence in the disjuncts on the further basis of your credence in the disjunction?

The question, and its answer, is parallel to the issue of when it's rational to reason by Modus Ponens and when it's rational to reason by Modus Tollens. I don't know the general answer here; I don't know how to fill in the exact general conditions for when to "Ponens", when to "Tollens", and when to suspend judgement. Remember that we formulated the rule of reasoning by Modus Ponens as a rule issuing defeasible permissions. You may adopt a belief in the conclusion, q , unless you have sufficiently strong reasons against q . All rules of reasoning must be formulated this way because all justification is defeasible. But I can't say (and I suspect no one can say) exactly when these defeasible justifications get defeated. The requirements of rationality only insist

52. Christensen worries that rational self-doubt will mean ordinary people will constantly have their justification for certainty defeated. That would mean, on the above proposal, that our knowledge will be defeated. See Christensen (2007) and, for another useful analysis, Smithies (2015). I see Christensen's worry as posing a deep skeptical challenge for our claims to certainty and perhaps also to knowledge, comparable to that of Unger (1975). I don't offer a response here. It might be that certainty and knowledge both fall to the skeptical challenge while rational high credences survive.

that I have one among many permissible combinations of attitudes. I must follow the right rules to successfully end up with rational attitudes (doxastically justified beliefs), but where exactly those rules lead me depends on where the strength of my evidence lies, and I don't know how to make that metaphor of "strength" non-vague.

Of course, the fact that evidence can be mixed, with fragments pointing in different directions, is part of what makes having credences useful. But even with credence, we'll still face a similar difficult question about how to fulfill a requirement like (H2). We'll have two corresponding rules of reasoning, and they will compete in the way Modus Ponens and Modus Tollens do. The rules seem to be these:

- (R2-a) If you know p and q are exclusive, and you have a rational credence of x in p and a rational credence of y in q , then you are defeasibly permitted, on that basis, to adopt a credence in their disjunction equal to the sum $x + y$.
- (R2-b) If you know p and q are exclusive, and you have a rational credence of z in the disjunction, $p \vee q$, then you are defeasibly permitted, on that basis, to revise your credences in the disjuncts so that their sum equals z .

In fact, the multiplicity of rules here is even greater. There are presumably further sub-rules, telling us things like how to revise our credence in *one* disjunct on the basis of our credences in the *other* disjunct and the whole disjunction.

There is a difficult question here that Bayesianism doesn't yet have the means to address. Bayesians tell us to fulfill a requirement, (A2), or more plausibly (H2), but there is more than one rule that allows us to fulfill that requirement. Bayesians recommend we revise credence using only a single super-rule, conditionalization. If there were only one rule, we wouldn't face this question. But we're trying to find more plausible rules governing reasoning with credences. How to apply multiple rules to our mix of evidence is as difficult a question as how to apply the classical rules like Modus Ponens and Modus Tollens to our mix of evidence in science, philosophy, and ordinary life. I don't know

the answer, but we shouldn't ignore the question.

Let me next say something about the accessibility to consciousness that rules for Type 2 reasoning are supposed to exhibit. I've suggested understanding this to mean that reasoners can not only easily bring to consciousness the content of their conclusion but also easily consciously determine what content(s) served as the reasoning's basis. Do our proposed rules have this feature? To bring out how they do, we just need to use (CC) to reformulate our rules, as I already did above for R1, by replacing any mentions of credence with mentions of beliefs about chance. It's then plausible, I think, that reasoners can easily make the corresponding contents of their reasoning's basis and conclusion — claims about chance — accessible to consciousness, and they can tell when one content served as the basis for reasoning to another as a conclusion. For example, consider an application of rule R2-a, in terms of beliefs about chances. Someone can easily be conscious of the fact that their conclusion about the $\frac{2}{3}$ chance of either Smith or Jones getting the job is based on their prior knowledge that exactly one person will get the job and their prior beliefs that Smith and Jones each stand a $\frac{1}{3}$ chance of getting the job.

That might sound like sophisticated reasoning, the kind that requires some training. You might thus wonder, are rules like R2-a and R2-b plausibly *basic* rules of reasoning? We are seeking rules that, like Modus Ponens, are basic, as opposed to rules that we've justified by using other rules. Are the R2 rules plausibly basic? I'll turn to this concern shortly, in sub-section 4.5. First, let me propose a few more rules I think are candidates for Type 2 basic rules for reasoning with credence.

4.4 *Rules of Type 2 Reasoning — conjunctions and more examples*

I've been trying to extract rules for reasoning using axioms of a mathematical theory as inspiration. But we need to be cautious about giving axioms more epistemological significance than they really deserve. What qualifies something as a well-chosen axiom is its usefulness to working mathematicians, or perhaps some kind of mathematical ele-

gance. That might not have much to do with what makes something a genuine requirement of rationality or a genuine basic rule of reasoning. Consider the dramatic differences between most axiomatic proof theories of logic and the natural deduction proof theories that provide a much better guide to the rules of reasoning for full belief in logical truths and consequences. Probabilism and the standard axioms of probability were not designed with the intention to reflect the rules for reasoning with credences.

Where can we look to find more clues to rules that we can make explicit? We can look at what we teach in classrooms. Just as we teach beginners how to write mathematical proofs by teaching them techniques that the various natural deduction rules reflect, likewise we teach beginners how to calculate probabilities by giving them a much handier toolkit of rules than just axioms like A1 or A2. Excellent textbooks often won't start off with the axioms, or even ever present them as axioms.⁵³

We have to think carefully to find, among all the things taught in the classroom, plausible candidates for genuinely basic rules of reasoning. We teach students to obey Bayes' theorem and to apply the rule of conditionalization, but these are not plausibly basic rules. We should obey Bayes' theorem and conditionalization only if and because we've used *other* rules to justify them. For example, Bayes' theorem, in one common form, is usually derived using a rule for calculating the probability of conjunctions.⁵⁴ A conjunction rule far more plausibly reflects a genuinely basic rule for us. The conjunction rule, formulated as a rule for reasoning with credences, might be this:

(R3) If you have a rational credence of x in p and a rational conditional

53. See, for example, the presentation in the popular textbook Skyrms (1966/2000, ch. 6), which elaborates 8 rules for calculating probabilities without ever calling any of them axioms.

54. And the required justification for conditionalization might be very sophisticated, like, for example, the justifications offered by Greaves and Wallace (2006) and Pettigrew (2016, pt. 4).

credence of y in q -given- p , then you're defeasibly permitted, on that basis, to adopt a credence in the conjunction $p \& q$ equal to the product xy .

Again, we could use (CC) to formulate this rule in terms of beliefs about chance, if we wish. I'll leave it as is, in terms of credences. And, again, the next sub-section will address whether it's psychologically realistic that we follow anything resembling a rule like this.

The rule mentions the attitude of conditional credence. I take conditional credence to be a genuine and unique attitude type, not reducible to or constructed out of ordinary unconditional credence. But I take it to be partly constitutive of conditional credence that it is subject to this rule as well as the corresponding requirement of rationality.⁵⁵

As with the rule for disjunctions, there is actually a multiplicity of rules lurking behind (R3), each permitting us to base certain credences and conditional credences on others so as to fulfill a corresponding wide-scope requirement of rationality. What is the corresponding requirement? It may appear to be the following, but this would be wrong:

(*) You are rational only if your credence in a conjunction equals the product of your credence in one conjunct and your conditional credence in the other conjunct (conditional on the first conjunct).

What's wrong with this proposed requirement is the same as what's wrong with treating the standard axioms as reflecting requirements of rationality: it leads to problems of logical omniscience. For example, if (*) were a requirement of rationality, then it would follow that always obeying Bayes' theorem would be a requirement too, but that is implausibly demanding.

We are avoiding the problem of logical omniscience by formulating the rules of reasoning as rules of mere permission. The rules give

55. For arguments that our psychology includes these irreducibly conditional attitudes, see Wedgwood (2013), Edgington (1995), and Lewis (1976), among other work.

you merely propositional justification for endlessly many logical conclusions, but cluttering your mind with too many of these would be unwise. If, as on Broome's approach, these permissive rules are generated by requirements, the requirements must be hedged in the right way to be made plausible. We fixed up (A1) and (A2) by using Hacking's fix, yielding (H2) and (H3). We need to likewise fix (*) somehow, but Hacking's method doesn't seem to apply. The right fix is the one we already applied to the requirement for Modus Ponens: the requirement only kicks in if you are or ought to be interested in the propositions in question.⁵⁶ A more plausible candidate for the requirement, then, might be something like this, for example (continuing with the 'H' labeling for improved requirements):

(H3) If you are or ought to be interested in the chances of p and of $p \& q$, and also in the conditional chance of q -given- p , then: you are rational only if $\text{Cr}(p \& q) = \text{Cr}(p) \times \text{Cr}(q|p)$.

We should also insert similar prefatory clauses explicitly in the full formulations of (H1) and (H2), such as "If you are or ought to be interested in the chances of p , q , and $p \vee q$, then you are rational only if, ...". In the interest of clutter avoidance, I'll not write it all out.

Once we introduce this prefatory clause, we can have a more subtle and sensible view than we could before. To illustrate, consider what we can say about the Linda experiment. My own view of people's probabilistically incoherent Type 1 reasoning about Linda is a lenient one: I'm inclined to count the verdicts of our Type 1 heuristics as (defeasibly) rational, so I'd let the subjects who assign higher credence to Linda being a feminist banker (than to her being a banker) off the hook, at least *until* their justification is defeated by reasoning using the rules of Type 2, rules like (R3). How does Type 2 reasoning get subjects *on* the hook? We are always obliged to obey (H3), but it's only once we are, or ought to be, *interested* in the relevant propositions that we become

56. Broome (2013, p.219) likewise uses a "caring clause" to avoid the problem. The basic idea, we already noted, is from Harman (1986, ch. 2).

obliged to reason using the rule in (R₃). Once we do engage in the appropriate (R₃) reasoning, the initial Linda verdict will be defeated. At that point, it will be irrational to stubbornly maintain higher credence that Linda is a feminist banker than that she is a banker. But to acquire this defeater, the reasoner must apply (R₃) to the two Linda propositions *together*. So long as the reasoner has not considered the two propositions together, then I'd allow that the reasoner still counts as obeying (H₃); that is, the reasoner can rationally maintain the initial credences generated by the representativeness heuristic, a rational credence in a conjunction and a rational lower credence in its conjunct. But Kahneman and Tversky asked subjects to produce a *ranking* of the chances of the propositions. So, if subjects followed these instructions, then even if the representativeness heuristic could generate a short-lived rational higher credence that Linda is a feminist bank teller, the considerations required to produce a ranking would, via (H₃), require revising those initial credences. Subjects become required to reason using something like the rule in (R₃), which defeats the justification for their initial credences about Linda's occupation. This sketch I've just offered of how requirements and rules can interact, generating and defeating justifications, is much more attractive, to my mind, than the traditional Bayesian view or, what's similar, the standard view held within the psychological literature.⁵⁷

I've now sketched, in (R₁)–(R₃), a number of examples of plausible rules of reasoning with credences. While there are many details to debate and work out, my goal is for these examples to point the way toward a “slightly more realistic” picture of rational credence, to

57. See Rysiew (2008) for a valuable discussion of what psychologists say about rationality here. As he observes, what's often called the “classical” or “standard” view in psychology is that any deviation from principles of classical logic and probability theory is irrational. But a sensible epistemologist, of course, doesn't conflate their subject matter with logic or probability theory. (See also Vranas (2000) for a philosopher's useful interpretation and analysis of one psychologist's non-standard position, Gigerenzer's, on the rationality of Type 1 heuristics.)

borrow from the title of Hacking's paper.

I think there are many more basic rules for reasoning with credence. I think there is a rule corresponding to a requirement to give q at least as high a credence as p when you know that *if- p -then- q* . I think there is a rule corresponding to a requirement to give propositions known to form a partition credences that sum to 1. I think there even is rule corresponding to some suitably hedged form of the law of total probability. These are just illustrative suggestions. Just as with the rules for full belief in logical truths and consequences, the dividing line between basic and non-basic rules may be vague and may differ slightly across people. (For example, is a rule to obey DeMorgan's law a basic rule? Maybe it is for some people while others need to justify it using other rules.) Sketches of plausible examples here and throughout this section are meant to clarify and support my main thesis: credences are rational if and only if, and because, they are formed using the right rules of reasoning.

4.5 *How Can Ordinary Untrained People Follow These Rules?*

In proposing that rational credence is governed by rules, including sophisticated rules belonging to Type 2, I'm making a claim not only about normative epistemology but also a claim about human psychology. These are not only the rules that make credences rational — they are rules that we actually follow (even if we are prone to performance errors, even if some rules issue conclusions whose justification is regularly defeated by other rules, and even if we find it extremely hard to make our rules explicit). This link between epistemology and psychology is unavoidable when it comes to our most basic rules of reasoning, and it is at the heart of the method of reflective equilibrium. Goodman said, “Principles of deductive inference are justified by their conformity with accepted deductive practice.”⁵⁸ The same is true of credal reasoning.

58. Goodman (1946/53/83, p.63).

Some readers might balk at various rules I've proposed we actually follow. They might think the proposals are too sophisticated to be psychologically realistic. How many people in human history, they might skeptically ask, have ever followed a rule to set their credence in a conjunction equal to the product of their credence in a conjunct and their conditional credence in the other conjunct?

This sort of skeptic might be motivated by Hacking's bold thesis that nothing resembling our modern concept of probability existed before the mid-1600s. The concept itself emerged suddenly around 1660, on Hacking's view.⁵⁹ The skeptic might further think that, even today, the concept of probability is rarely grasped, and the use of any rules resembling those I sketched even rarer.

I disagree with Hacking's view of the conceptual history. I side with those historians and psychologists who find our concept of probability featuring in the reasoning of all humans, today and through recorded history.⁶⁰ But I think that even an adherent to Hacking's view should not be skeptical of the rules I've proposed we follow — they need only restrict the attribution of those rules to those people lucky enough to

59. Hacking (1975/2006, see esp. pp.8–9).

60. On the history of the use of probability before 1660, see Garber and Zabell (1979), Hald (2003, ch. 3), and Franklin (2001). On the psychology, see Xu and Garcia (2008) and also Strevens (2013, esp. ch. 4), who discusses Xu and Garcia's work as well as other psychological studies observing probabilistic reasoning in infants. (Xu and Garcia explicitly remain neutral about whether the rules in question are Type 1 or 2: "Our findings provide evidence that infants possess a powerful mechanism for inductive learning, either using heuristics or basic principles of probability." p.5012.) See van Fraassen (1989, sec. 7.2) for a compelling portrait of how easy it really is to discern the outlines of the full mathematical concept of probability in the everyday talk and thought of ordinary people.

In an introduction written for the new edition of his book released in 2006, Hacking says more. Whereas historians like Franklin must take the psychology of probabilistic reasoning to be part of our innate, genetic endowment in the way that basic arithmetic is innate (as Carey (2009) and others have shown), Hacking takes probabilistic reasoning to be unlike arithmetic, and instead more of a local cultural artifact. Hacking says he doesn't sympathize with the view that evolutionary psychology makes probabilistic thought, as we engage in it, at all inevitable.

possess the concept of probability and have rational credences. Once you sign on to the thesis that rational credence is explained by reasoning using rules, you have to attribute some basic rules. While no real person can follow the super-rule 'Obey probabilism!', at least some people, however few or however many, can follow rules like (R1)–(R3). Admittedly, these rules are more sophisticated than Modus Ponens, but we must appeal to moderately sophisticated rules like these if we want to vindicate anything even resembling the requirements of probabilism, and I do want to preserve versions of such requirements — I specifically want to say we have propositional justification for fulfilling them. Even if, as I think, requirements resembling probabilism need substantial hedging (as (H1)–(H3) try to incorporate), we still require fairly sophisticated rules to lead us to conclusions like our considered verdict that Linda is no more likely a feminist bank teller than a bank teller. And that is a verdict that I strongly suspect we are all able, upon reflection, to see for ourselves: no one is just taking the examiner's word for it when they realize they've flubbed the Linda problem!

Though I say we *could* restrict the attribution of the more sophisticated rules for reasoning to very few people, there is a strong case for attributing these rules to all people. The case for this has been most extensively made by Gigerenzer and his collaborators.⁶¹ They argue that, when tasks are posed in the right way, ordinary people excel at reasoning that conforms to requirements like (A1)–(A3), or at least (H1)–(H3), as well as other requirements drawn from probabilism, even Bayes' theorem. We're all like Socrates's pupil in Plato's dialogue *Meno*. We can reason well and without explicit instruction, as long as we're posed the right line of questioning.

Gigerenzer's view is that, when probabilistic problems are presented together with an invitation to consider corresponding frequencies, we will avoid or correct the errors that our heuristics commit. It's important to see that this approach requires no commitment to a

61. See Gigerenzer et al. (1999) and Gigerenzer (2008), among other books and articles.

frequentist interpretation of probability or chance.⁶² The frequentist interpretation is vulnerable to fatal objections, in particular the objection that there often exists an intuitive probability when there exists no actual frequency or well-defined hypothetical frequency.⁶³ But Gigerenzer's suggestion is only that, when we *can* find corresponding frequencies for a probabilistic problem, then we will be guided to reason with our credences in ways that do obey sophisticated requirements, like (H1)–(H3). These ways of reasoning thus plausibly follow rules like (R1)–(R3).⁶⁴ I'll give an example (a hypothetical example, but one based on Gigerenzer's research).

Imagine having a dialogue a bit like the one in *Meno*. You ask your novice student, What's the chance that the next car to pass on this road will be a red BMW? If the student can't answer right away, you might ask follow-up questions. How frequently does a BMW come down this road? Or, put in other words, what fraction of all the cars that drive on this road are BMWs? And what fraction of cars on this road are red? Suppose the student can look up frequency facts like these. What Gigerenzer's program shows is that ordinary people know how to proceed from here. They know that they need another fact, either a fact about the fraction of BMWs *among* the red cars, or else the fraction of red cars *among* the BMWs. Ordinary people know that the fraction of cars that are *F&G* is a certain "fraction of a fraction". It's the fraction of *Fs* among the fraction of cars that are *G*. People can also see that it doesn't matter in what *order* that fraction-of-a-fraction is taken: you arrive at the same correct answer if you know the fraction of *Gs* among the fraction of cars that are *F*. So, if you told your student that

1/2 the cars on the road are red, and 1/6 of the red cars are BMWs, you'd expect your student will be able to reason that 1/12 of the cars are red BMWs. Equally, if you told your student that 1/4 the cars on the road are BMWs (it's a rich neighborhood) and 1/3 of the BMWs are red (the BMW owners particularly love red), you'd again expect your student will again be able to reason that 1/12 of the cars are red BMWs. And people will let this conclusion about fractions of populations — frequencies, in Gigerenzer's sense — guide them to a credence, to a judgement about the chance that the next car will be a red BMW. You don't need to be a Pascal or a Laplace to engage in this kind of reasoning.

From the observation that a fraction-of-a-fraction is commutative, in the way I just described, Bayes' theorem is a short justification away. Bayes' theorem describes the relationship between converse conditional probabilities. A simple form of it can be expressed as follows (demonstrated here by expressing the probability of a conjunction in two equivalent ways):

$$\Pr(p \ \& \ q) = \Pr(p \mid q) \times \Pr(q) = \Pr(q \mid p) \times \Pr(p).$$

That form of Bayes' theorem is something novice undergrads can, with a bit of thought, establish for themselves, in particular when they consider it in light of the following corresponding commutativity fact about fractions of fractions in populations:

$$\begin{aligned} &\text{The fraction of things, in a given population, that are } F\&G\text{s} \\ &= \text{the fraction of } F\text{s found in the fraction of the things that are } G\text{s} \\ &= \text{the fraction of } G\text{s found in the fraction of the things that are } F\text{s.} \end{aligned}$$

Zhu and Gigerenzer found that even young children do well at reasoning that obeys Bayes' theorem, as long as they conceptualize the problem with the aid of frequencies.⁶⁵ Other studies showed that adults, and again even children, can reason well on the notorious

62. See Hájek (2012) for discussion of the interpretations of probability.

63. See, e.g., Jeffrey (1977/92) and Hájek (2009).

64. I say rules "like" these since it would take further argument, which I can't give here, to establish that we do follow the fully general rule which has some instances where no accompanying guiding frequency exists. The issue here is a version of Kripkenstein's skeptical challenge about how, with only finitary behavior and dispositions, we manage to follow one general infinitary rule rather than another; see Kripke (1982).

65. See Zhu and Gigerenzer (2006), revised as Gigerenzer (2008, ch. 12).

Monty Hall problem, as long as the problem is conceptualized using the aid of frequencies. Instead of focusing on the question, “Should I switch or not switch doors?”, subjects rather need to consider a frequency question, “In how many cases will switching win?”.⁶⁶

The evidence suggests, then, that however regularly we do fail to reason by rules like (R₃) and do violate requirements like (H₃), we also have a natural ability to succeed at reasoning using rules like (R₃) and thereby obey probabilistic requirements like (H₃).

5. Conclusion: Putting Probabilism in Its Place

What is the status of probabilism? It is false, but something in the neighborhood is true. We are not required to obey laws of probability unless we are or ought to take an interest in certain propositions. But our interests are easily and often drawn to questions, as Kahneman and Tversky drew them when they asked us to rank the likelihoods of Linda’s occupations. Then we become subject to requirements. Furthermore, even when our interests are idle and no requirements apply to our attitudes, the permissive rules of reasoning still give us propositional justification for conforming to the untriggered requirements. There is nothing wrong with reasoning in ways that lead to doxastically justified conformity with probabilism, except the risk of cluttering your mind. Rational credence through reasoning is like full belief in logical consequences based on rules like Modus Ponens. It is perfectly reliable, or conditionally reliable, but reliability should never be confused with rationality.

What about the highly admired arguments in favor of probabilism? Some of these arguments have to be rejected. Two old arguments, the Dutch Book argument and the Representation Theorem argument, aim to establish probabilism, i.e. they aim to show that a rational person’s credences must obey the laws of probability.⁶⁷ I accept the objections

66. See Krauss and Wang (2003). Gigerenzer (2008, p.189) has more references.

67. See Christensen (2004) for defenses of these arguments. This section’s title is a play on that book’s title, *Putting Logic in Its Place*.

to these arguments given by others.⁶⁸

A third, newer argument for probabilism is the accuracy-based argument.⁶⁹ This argument is presented as though it establishes probabilism,⁷⁰ and I must deny that it does establish that.⁷¹ But I accept the argument’s second-to-last-step, namely that, roughly put, credences obeying the laws of probability are more accurate than other credences (“more accurate” in specific senses clarified in the cited works). I only reject the inference from this demonstration of reliability to a conclusion about rationality, namely the conclusion of probabilism. I take the argument to only show, for credence, the sort of thing we already knew for full belief: logical omniscience is more accurate than ignorance of or disagreement with logical truths and consequences. My position is the same for credence and full belief: unless we avoid the problem of conflated rationality, that is, unless we distinguish doxastic justification from propositional justification (as well as from reliability), we will end up with a problem of logical omniscience. The advocates of the accuracy argument conflate all these.

Finally, there is the strongest argument of all for probabilism and the entire Bayesian framework. This is the holistic argument that probabilism helps solve many puzzles of confirmation theory — for example, (i) the ravens paradox, (ii) the paradoxical confirmation of irrelevant conjuncts, and (iii) the difficulty of explaining the value of surprising evidence and diverse evidence.⁷²

My response is to not deny the strength of the holistic argument.

68. See Meacham and Weisberg (2011) for a demolition of the Representation Theorem argument, and Hájek (2008/09) for objections to both arguments.

69. See Joyce (1998) and Pettigrew (2016), among other defenses.

70. See Schoenfield (2015) for a variation of the accuracy-based argument that doesn’t lead to probabilism, at least not straightforwardly (see secs. 6–7).

71. See Hájek (2008/09) and Easwaran and Fitelson (2012) for other objections to the accuracy argument.

72. See the presentations in, for example, Horwich (1982) and Strevens (2017). Hájek (2008/09, 249) says “the best argument for probabilism” is that it resolves such puzzles and paradoxes and gives us an elegant and unified theory of decision and confirmation.

I want to buy the Bayesian solutions to the puzzles of confirmation theory, but there is only a small price for them. The rule-following framework permits us to imitate probabilists, permits us to conform to the laws of probability. Since we can imitate probabilists, we can co-opt their solutions to the puzzles. But we just need to do so only after we've done the requisite reasoning.⁷³

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