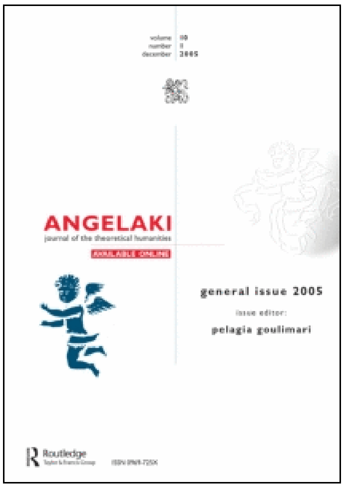

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Deleuze, Leibniz and Projective Geometry in *the Fold*

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simon duffy

DELEUZE, LEIBNIZ AND PROJECTIVE GEOMETRY IN *THE FOLD*

Explications of the reconstruction of Leibniz's metaphysics that Deleuze undertakes in *The Fold: Leibniz and the Baroque* focus predominantly on the role of the infinitesimal calculus developed by Leibniz.[1] While not underestimating the importance of the infinitesimal calculus and the law of continuity as reflected in the calculus of infinite series to any understanding of Leibniz's metaphysics and to Deleuze's reconstruction of it in *The Fold*, what I propose to examine in this paper is the role played by other developments in mathematics that Deleuze draws upon, including those made by a number of Leibniz's near contemporaries – the projective geometry that has its roots in the work of Desargues (1591–1661) and the "proto-topology"[2] that appears in the work of Dürer (1471–1528) – and a number of the subsequent developments in these fields of mathematics. Deleuze brings this elaborate conjunction of material together in order to set up a mathematical idealization of the system that he considers to be implicit in Leibniz's work. The result is a thoroughly mathematical explication of the structure of Leibniz's metaphysics. What is provided in this paper is an exposition of the very mathematical underpinnings of this Deleuzian account of the structure of Leibniz's metaphysics, which, I maintain, subtends the entire text of *The Fold*.

Deleuze's project in *The Fold* is predominantly oriented by Leibniz's insistence on the metaphysical importance of mathematical speculation. What this suggests is that mathematics functions as an important heuristic in the development of Leibniz's metaphysical theories. Deleuze puts this insistence to good use by bringing together the different aspects of Leibniz's metaphysics with the variety of mathematical themes that run throughout his work. Those aspects of Leibniz's metaphysics that Deleuze undertakes to clarify in this way, and upon which this paper will focus, include: (1) the definition of a monad; (2) the theory of compossibility; (3) the difference between perception and apperception; and (4) the range and meaning of the pre-established harmony. However, before providing the details of Deleuze's reconstruction of the structure of Leibniz's metaphysics, it will be necessary to give an introduction to Leibniz's infinitesimal calculus and to some of the other developments in mathematics associated with it.

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leibniz's law of continuity and the infinitesimal calculus

Leibniz was both a philosopher and mathematician. As a mathematician, he made a number of innovative contributions to developments in mathematics. Chief amongst these was his infinitesimal analysis, which encompassed the investigation of infinite sequences and series, the study of algebraic and transcendental curves[3] and the operations of differentiation and integration upon them, and the solution of differential equations: integration and differentiation being the two fundamental operations of the infinitesimal calculus that he developed.

Leibniz applied the calculus primarily to problems about curves and the calculus of finite sequences, which had been used since antiquity to approximate the curve by a polygon in the Archimedean approach to geometrical problems by means of the method of exhaustion. In his early exploration of mathematics, Leibniz applied the theory of number sequences to the study of curves and showed that the differences and sums in number sequences correspond to tangents and quadratures, respectively, and he developed the conception of the infinitesimal calculus by supposing the differences between the terms of these sequences infinitely small (see Bos 13).

One of the keys to the calculus that Leibniz emphasized was to conceive the curve as an infinitangular polygon.[4] Leibniz based his proofs for the infinitangular polygon on a law of continuity, and he used the adjective *continuus* for a variable ranging over an infinite sequence of values. In the infinite continuation of the polygon, its sides become infinitely small and its angles infinitely many. The infinitangular polygon is considered to coincide with the curve, the infinitely small sides of which, if prolonged, would form tangents to the curve, where a tangent is a straight line that touches a circle or curve at only one point. Leibniz applied the law of continuity to the tangents of curves as follows: he took the tangent to be continuous with, or as the limiting case ("*terminus*") of the secant. To find a tangent is to draw a straight line joining two points of the curve – the secant – which are

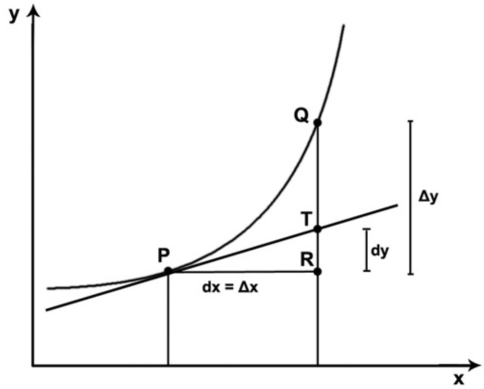

Fig. 1.

separated by an infinitely small distance or vanishing difference, which he called a differential.[5] The Leibnizian infinitesimal calculus was built upon the concept of the differential. The differential, dx, is the difference in x values between two consecutive values of the variable (at P. See Fig. 1.), and the tangent is the line joining such points.

The differential relation, that is, the quotient between two differentials of the type dy/dx, serves in the determination of the gradient of the tangent to the circle or curve. The gradient of a tangent indicates the slope or rate of change of the curve at that point, that is, the rate at which the curve changes on the y-axis relative to the x-axis. Leibniz thought of the "dy" and "dx" in dy/dx as "infinitesimal" quantities. Thus dx was an infinitely small non-zero increment in x and dy was an infinitely small non-zero increment in y.

Leibniz brings together the definition of the differential as it operates in the calculus of infinite series, in regard to the infinitangular triangle, and the infinitesimal calculus, in regard to the determination of tangents to curves, as follows:

> Here dx means the element, that is, the (instantaneous) increment or decrement, of the (continually) increasing quantity x. It is also called difference, namely the difference between two proximate x's which differ by an element (or by an inassignable), the one originating from the other, as the other increases or decreases (momentaneously).[6]

The differential can therefore be understood, on the one hand, in relation to the calculus of infinite series as the infinitesimal difference between consecutive values of a continuously diminishing quantity, and, on the other, in relation to the infinitesimal calculus as an infinitesimal quantity. The operation of the differential in the latter actually demonstrates the operation of the differential in the former, because the operation of the differential in the infinitesimal calculus in the determination of tangents to curves demonstrates that the infinitely small sides of the infinitangular polygon are continuous with the curve.

In one of his early mathematical manuscripts entitled "Justification of the Infinitesimal Calculus by That of Ordinary Algebra," Leibniz offers an account of the infinitesimal calculus in relation to a particular geometrical problem that is solved using ordinary algebra.[7] An outline of the demonstration that Leibniz gives is as shown in Fig. 2.[8] Since the two right triangles, ZFE and ZHJ, which meet at their apex, point Z, are similar, it follows that the ratio y/x is equal to $(Y-y)/X$. As the straight line EJ approaches point F, maintaining the same angle at the variable point Z, the lengths of the straight lines FZ and FE, or y and x, steadily diminish, yet the ratio of y to x remains constant. When the straight line EJ passes through F, the points E and Z coincide with F, and the straight lines, y and x, vanish. Yet y and x will not be absolutely nothing since they preserve the ratio of ZH to HJ, represented by the proportion $(Y-y)/X$, which in this case reduces to Y/X, and obviously does not equal zero. The relation y/x continues to exist even though the terms have vanished since the relation is determinable as equal to Y/X. In this algebraic calculus, the vanished lines x and y are not taken for zeros since they still have an algebraic relation to each other. "And so," Leibniz argues, "they are treated as infinitesimals, exactly as one of the elements which . . . differential calculus recognises in the ordinates of curves for momentary increments and decrements" (545). That is, the vanished lines x and y are determinable in relation to each other only in so far as they can be replaced by the infinitesimals dy and dx, by making the supposition that the ratio y/x is equal to the ratio of the infinitesimals, dy/dx. When the relation continues even though the terms of the relation have disappeared, a continuity has been constructed by algebraic means that is instructive of the operations of the infinitesimal calculus.

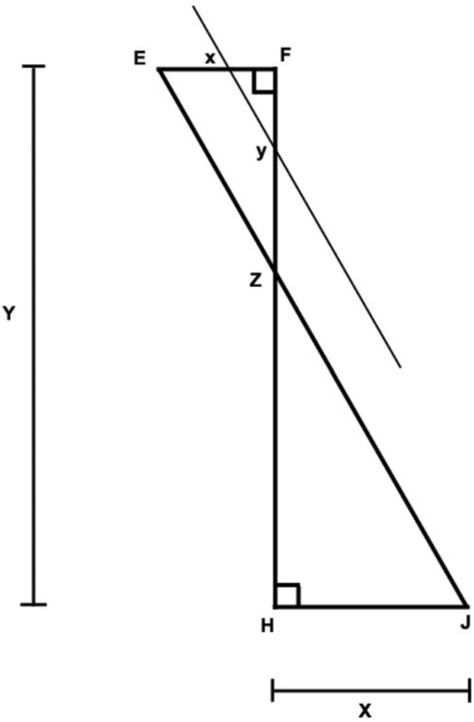

Fig. 2.

What Leibniz demonstrates in this example are the conditions according to which any unique triangle can be considered as the extreme case of two similar triangles opposed at the vertex.[9] Deleuze argues that, in the case of a figure in which there is only one triangle, the other triangle is there, but it is there only virtually.[10] The *virtual* triangle has not simply disappeared, but rather it has become unassignable, all the while remaining completely determined. The hypotenuse of the virtual triangle can be mapped as a side of the infinitangular polygon, which, if prolonged, forms a tangent line to the curve. There is therefore continuity from the polygon to the curve, just as there is continuity from two similar triangles opposed at the vertex to a single triangle. Hence this relation is fundamental for the application of differentials to problems about tangents.

In the first published account of the calculus,[11] Leibniz defines the ratio of infinitesimals as the quotient of first-order differentials, or the associated differential relation. He says that "the differential dx of the abscissa x is an arbitrary quantity, and that the differential dy of the ordinate y is defined as the quantity which is to dx as the ratio of the ordinate to the subtangent" (Boyer 210) (see Fig. 1). Leibniz considers differentials to be the fundamental concepts of the infinitesimal calculus, the differential relation being defined in terms of these differentials.

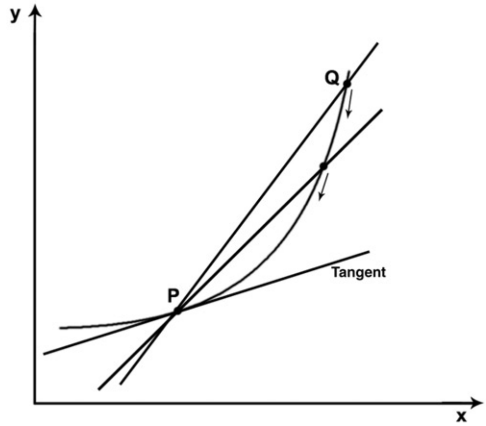

Fig. 3.

newton's method of fluxions and infinite series

Newton began thinking of the rate of change, or fluxion, of continuously varying quantities, which he called fluents such as lengths, areas, volumes, distances, temperatures, in 1665, which pre-dates Leibniz by about ten years. Newton regards his variables as generated by the continuous motion of points, lines, and planes, and offers an account of the fundamental problem of the calculus as follows: "Given a relation between two fluents, find the relation between their fluxions, and conversely."[12] Newton thinks of the two variables whose relation is given as changing with time, and, although he does point out that this is useful rather than necessary, it remains a defining feature of his approach and is exemplified in the geometrical reasoning about limits, which Newton was the first to come up with.[13] Put simply, to determine the tangent to a curve at a specified point, a second point on the curve is selected, and the gradient of the line that runs through both of these points is calculated. As the second point approaches the point of tangency, the gradient of the line between the two points approaches the gradient of the tangent. The gradient of the tangent is, therefore, the limit of the gradient of the line between the two points as the points become increasingly close to one another.

He conceptualized the tangent geometrically, as the limit of a sequence of lines between two points, P and Q, on a curve, which is a secant (see Fig. 3). As the distance between the points approached zero, the secants became progressively smaller; however, they always retained "a real length." The secant therefore approached the tangent without reaching it. When this distance "got arbitrarily small (but remained a real number)"[14] it was considered insignificant for practical purposes, and was ignored. What is different in Leibniz's method is that he "hypothesized infinitely small numbers – infinitesimals – to designate the size of infinitely small intervals" (Lakoff and Núñez 224) (see Fig. 1). For Newton, on the contrary, these intervals remained only small, and therefore real. When performing calculations, however, both approaches yielded the same results. But they differed ontologically, because Leibniz had hypothesized a new kind of number, a number Newton did not need, since "his secants always had a real length, while Leibniz's had an infinitesimal length" (Lakoff and Núñez 224). Leibniz's symbolism also treats quantities independently of their genesis, rather than as the product of an explicit functional relation. Deleuze uses this distinction between the methods of Leibniz and Newton to characterize the mind–body distinction in Leibniz's account of the monad, the details of which will be returned to later in the paper.

Both Newton and Leibniz are credited with developing the calculus as a new and general method, and with having appreciated that the operations in the new analysis are applicable to infinite series as well as to finite algebraic expressions. However, neither of them clearly understood nor rigorously defined their fundamental concepts. Newton thought his underlying methods were natural extensions of pure geometry, while Leibniz felt that the ultimate justification of his procedures lay in their effectiveness. For the next two hundred years, various attempts were made to find a rigorous arithmetic foundation for the calculus, one that relied on neither the mathematical intuition of geometry, with its tangents and secants – which was perceived as imprecise because its conception of limits was not properly understood – nor the vagaries of the infinitesimal, which cannot be justified either from the point of view of classical algebra or from the point of view of arithmetic, and therefore made many mathematicians wary, so much so that they refused the hypothesis outright despite the fact that Leibniz "could do calculus using arithmetic without geometry – by using infinitesimal numbers" (Lakoff and Núñez 224–25).

the emergence of the concept of the function

Seventeenth-century analysis was a corpus of analytical tools for the study of geometric objects, the most fundamental object of which, thanks to the development of a curvilinear mathematical physics by Christiaan Huygens (1629–95), was the curve, or curvilinear figures generally, which were understood to embody relations between several variable geometrical quantities defined with respect to a variable point on the curve. The variables of geometric analysis referred to geometric quantities, which were conceived not as real numbers but rather as having a dimension: for example, "the dimension of a line (e.g., ordinate, arc length, subtangent), of an area (e.g., the area between curve and axis) or of a solid (e.g., the solid of revolution)."[15] The relations between these variables were expressed by means of equations. Leibniz actually referred to these variable geometric quantities as the *functiones* of a curve,[16] and thereby introduced the term "function" into mathematics. However, it is important to note the absence of the fully developed concept of function in the context of algebraic relations between variables for Leibniz. Today, a function is understood to be a relation that uniquely associates members of one set with members of another set. Neither the equations nor the variables are functions; rather, the relation between x and y was considered to be one entity. Thus the curve was not seen as a graph of a function but rather as "a figure embodying the relation between x and y."[17] In the first half of the eighteenth century a shift of focus occurred from the curve and the geometric quantities themselves to the formulas which expressed the relations among these quantities, thanks in large part to the symbols introduced by Leibniz. The analytical expressions involving numbers and letters, rather than the geometric objects for which they stood, became the focus of interest. It was this change of focus towards the formula that made the emergence of the concept of function possible. In this process, the differential underwent a corresponding change; it lost its initial geometric connotations and came to be treated as a concept connected with formulas rather than with figures.

With the emergence of the concept of the function, the differential was replaced by the derivative, which is the expression of the differential relation as a function, first developed in the work of Euler (1707–83). One significant difference, reflecting the transition from a geometric analysis to an analysis of functions and formulas, is that the infinitesimal sequences are no longer induced by an infinitangular polygon standing for a curve, according to the law of continuity as reflected in the infinitesimal calculus, but by a function, defined as a set of ordered pairs of real numbers.

subsequent developments in mathematics: the problem of rigour

The concept of the function, however, did not immediately resolve the problem of rigour in the calculus. It was not until the late nineteenth century that an adequate solution to this problem was posed. It was Karl Weierstrass (1815–97) who "developed a pure nongeometric arithmetization for Newtonian calculus" (Lakoff and Núñez 230), which provided the rigour that had been lacking. The Weierstrassian program determined that the fate of calculus need not be tied to infinitesimals, and could rather be given a rigorous status from the point of view of finite representations. Weierstrass's theory was an updated version of an earlier account by Augustin Cauchy (1789–1857), which had also experienced problems conceptualizing limits.

It was Cauchy who first insisted on specific tests for the convergence of series, so that divergent series could henceforth be excluded from being used to try to solve problems of integration because of their propensity to lead to false results.[18] By extending sums to an infinite number of terms, problems began to emerge if the series did not converge, since the sum or limit of an infinite series is determinable only if the series converges. It was considered that reckoning with divergent series, which have no sum, would therefore lead to false results.

Weierstrass considered Cauchy to have actually begged the question of the concept of limit in his proof.[19] In order to overcome this problem of conceptualizing limits, Weierstrass "sought to eliminate all geometry from the study of... derivatives and integrals in calculus" (Lakoff and Núñez 309). In order to characterize calculus purely in terms of arithmetic it was necessary for the idea of a curve in the Cartesian plane defined in terms of the motion of a point to be completely replaced with the idea of a function. The geometric idea of "approaching a limit" had to be replaced by an arithmetized concept of limit that relied on static logical constraints on numbers alone. This approach is commonly referred to as the epsilon-delta method.[20] The calculus was thereby reformulated without either geometric secants and tangents or infinitesimals; only the real numbers were used.

Because there is no reference to infinitesimals in this Weierstrassian definition of the calculus, the designation "the infinitesimal calculus" was considered to be "inappropriate."[21] Weierstrass's work not only effectively removed any remnants of geometry from what was now referred to as the differential calculus, but it eliminated the use of the Leibnizian-inspired infinitesimals in doing the calculus for over half a century. It was not until the late 1960s, with the development of the controversial axioms of non-standard analysis by Abraham Robinson (1918–74), that the infinitesimal was given a rigorous foundation,[22] thus allowing the inconsistencies to be removed from the Leibnizian infinitesimal calculus without removing the infinitesimals themselves.[23] Leibniz's ideas have therefore been be "fully vindicated,"[24] as Newton's had been thanks to Weierstrass.[25]

In response to these developments, Deleuze brings renewed scrutiny to the relationship between the developments in the history of mathematics and the metaphysics associated with these developments, which were marginalized as a result of efforts to determine the rigorous foundations of the calculus. This is a part of Deleuze's broader project of constructing an alternative lineage in the history of philosophy that tracks the development of a series of metaphysical schemes that respond to and attempt to deploy the concept of the infinitesimal. The aim of the project is to construct a philosophy of difference as an alternative speculative logic that subverts a number of the commitments of the Hegelian dialectical logic which supported the elimination of the infinitesimal in favour of the operation of negation, the procedure of which postulates the synthesis of a series of contradictions in the determination of concepts.[26]

the theory of singularities

Another development in mathematics, the rudiments of which are in the work of Leibniz, is the theory of singularities. A singularity or singular point is a mathematical concept that appears with the development of the theory of functions, which historians of mathematics consider to be one of the first major mathematical concepts upon which the development of modern mathematics depends. Even though the theory of functions doesn't actually take shape until later in the eighteenth century, it is in fact Leibniz who contributes greatly to this development.

Indeed, it was Leibniz who developed the first theory of singularities in mathematics, and, Deleuze argues, it is with Leibniz that the concept of singularity becomes a mathematico-philosophical concept.[27] However, before explaining what is philosophical in the concept of singularity for Leibniz, it is necessary to offer an account of what he considers singularities to be in mathematics, and of how this concept was subsequently developed in the theory of analytic functions, which is important for Deleuze's account of (in)compossibility in Leibniz, despite its not being developed until long after Leibniz's death.

The great mathematical discovery that Deleuze refers to is that singularity is no longer thought of in relation to the universal, but rather in relation to the ordinary or the regular.[28] In classical logic, the singular was thought of with reference to the universal; however, that doesn't necessarily exhaust the concept since in mathematics the singular is distinct from or exceeds the ordinary or regular. Mathematics refers to the singular and the ordinary in terms of the points of a curve, or more generally concerning complex curves or figures. A curve, a curvilinear surface, or a figure includes singular points and others that are regular or ordinary. Therefore, the relation between singular and ordinary or regular points is a function of curvilinear problems which can be determined by means of the Leibnizian infinitesimal calculus.

The differential relation is used to determine the overall shape of a curve primarily by determining the number and distribution of its singular points or singularities, which are defined as points of articulation where the shape of the curve changes or alters its behaviour. For example, when the differential relation is equal to zero, the gradient of the tangent at that point is horizontal, indicating that the curve peaks or dips, determining, therefore, a maximum or minimum at that point. These singular points are known as stationary or turning points.

The differential relation characterizes not only the singular points which it determines but also the nature of the regular points in the immediate neighbourhood of these points, that is, the shape of the branches of the curve on either side of each singular point.[29] Where the differential relation gives the value of the gradient at the singular point, the value of the second-order differential relation, that is if the differential relation is itself differentiated and which is now referred to as the second derivative, indicates the rate at which the gradient is changing at that point. This allows a more accurate approximation of the shape of the curve in the neighbourhood of that point.

Leibniz referred to the stationary points as *maxima* and *minima* depending on whether the curve was concave up or down, respectively. A curve is concave up where the second-order differential relation is positive and concave down where the second-order differential relation is negative. The points on a curve that mark a transition between a region where the curve is concave up and one where it is concave down are points of inflection. The second-order differential relation will be zero at an inflection point. Deleuze distinguishes a point of inflection, as an intrinsic singularity, from the *maxima* and *minima*, as extrinsic singularities, on the grounds that the former "does not refer to coordinates" but rather "corresponds" to what Leibniz calls an "ambiguous sign,"[30] that is, where concavity changes, the sign of the second-order differential relation changes from $+$ to $-$, or vice versa.

The value of the third-order differential relation indicates the rate at which the second-order differential relation is changing at that point. In fact, the more successive orders of the differential relation that can be evaluated at the singular point, the more accurate the approximation of the shape of the curve in the "immediate" neighbourhood of that point. Leibniz even provided a formula for the nth-order differential relation, as n approaches infinity ($n \rightarrow \infty$). The nth-order differential relation at the point of inflection would determine the continuity of the variable curvature in the immediate neighbourhood of the inflection with the curve. Because the point of inflection is where the tangent crosses the curve (see Fig. 4.) and the point where the nth-order differential relation as $n \rightarrow \infty$ is continuous with the curve, Deleuze characterizes the point of inflection as a *point-fold*, which is the trope that unifies a number of the themes and elements of *The Fold*.

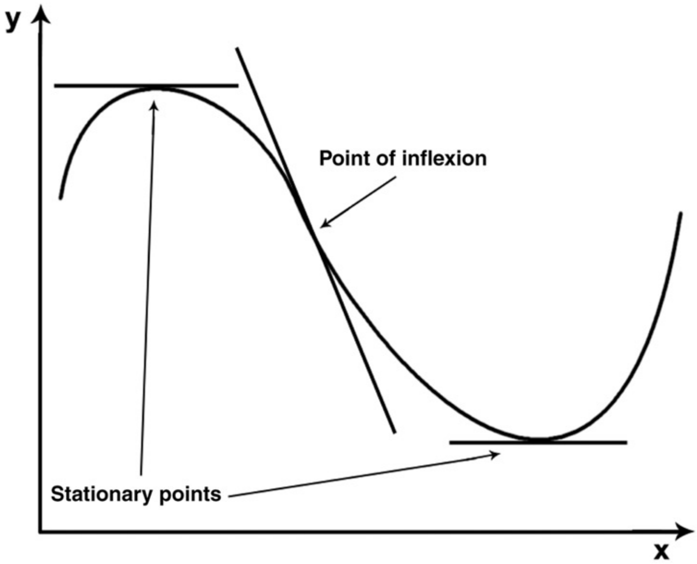

Fig. 4.

the characteristics of a point-fold as reflected in the point of inflection

Deleuze considers Baroque mathematics to have been born with Leibniz, and he gives two examples of how infinite variables emerge as the object that defines the discipline of this period, and in both cases Deleuze remarks on the presence of a curved element that he characterizes as a *point-fold*.

(1) The first is the irrational number and the corresponding serial calculus. An irrational number cannot be written as a fraction, and has decimal expansions that neither terminate nor become periodic. Pythagoras believed that all things could be measured by the discrete natural numbers (1, 2, 3, . . .) and their ratios (ordinary fractions, or the rational numbers). This belief was shaken, however, by the discovery that the hypotenuse of a right isosceles triangle (that is, diagonal of a unit square) cannot be expressed as a rational number. This discovery was brought about by what is now referred to as Pythagoras's theorem,[31] which establishes that the square of the hypotenuse of a right isosceles triangle is equal to the sum of the squares of the other two sides, $c^2 = a^2 + b^2$. In a unit square, the diagonal is the hypotenuse of a right isosceles triangle, with sides $a = b = 1$, hence $c^2 = 2$, and $c = \sqrt{2}$, or "the square root of 2." Thus there exists a line segment whose length is equal to $\sqrt{2}$, which is an irrational number. Against the intentions of Pythagoras, it had thereby been shown that rational numbers did not suffice for measuring even simple geometric objects.

Another example of a simple irrational number is π, which is determined by the relation between the circumference, c, of a circle relative to its diameter, d (where $\pi = c/d$). Leibniz was the first to find the infinite series $(1 - 1/3 + 1/5 - 1/7 + \cdots)$ of which $\pi/4$ was the limit. Leibniz only gave the formula of this series, and it was not until the end of the eighteenth century that this formula was demonstrated to be an infinite convergent series by the mathematician Johann Heinrich Lambert (1728–77).

Irrational numbers can therefore remain in surd form, as for example $\sqrt{2}$, or they may be represented by an infinite series. Deleuze defines the irrational number as "the common limit of two convergent series, of which one has no maximum and the other no minimum"

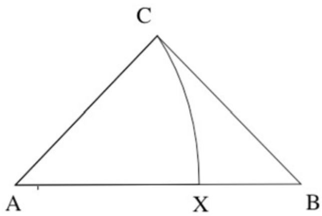

Fig. 5.

(*The Fold* 17), thus any irrational number is the limit of the sequence of its rational approximations, which can be represented as follows: increasing series $\rightarrow$ irrational number $\leftarrow$ decreasing series. The diagram that Deleuze provides on page 17 is of a right isosceles triangle, the sides of which are in the ratio $1{:}1{:}\sqrt{2}$ (see Fig. 5).

It functions as a graphical representation of the ratio of the sides of AC:AB (where $AC = AX) = 1{:}\sqrt{2}$. The point X is the irrational number, $\sqrt{2}$, which represents the meeting point of the arc of the circle, of radius AC, inscribed from point C to X, and the straight line AB representing the rational number line. The arc of the circle produces a *point-fold* at X. The "straight line of rational points" is therefore exposed "as a false infinite, a simple indefinite that includes the lacunae" of each irrational number $\sqrt{n}$, as $n \rightarrow \infty$. The rational number line should therefore be understood to be interrupted by these curves such as that represented by $\sqrt{2}$ in the given example. Deleuze considers these to be events of the line, and then generalizes this example to include all straight lines as intermingled with curves, *point-folds* or events of this kind.

(2) The second example is the differential relation and differential calculus. Here Deleuze argues that the diagram from Leibniz's account of the calculus in "Justification of the Infinitesimal Calculus by That of Ordinary Algebra" (see Fig. 2) can be correlated with a *point-fold* by mapping the hypotenuse of the virtual triangle onto a side of the infinitangular polygon, which, if prolonged, forms a tangent line to the curve. Once the virtual triangle vanishes or becomes unassigned, the relation dy/dx, and therefore the unassigned virtual triangle, is retained by point F, just as the differential relation designates the gradient of a tangent to the curve at point F, which can therefore be characterized as a *point-fold*.

Deleuze maps these characteristics of a *point-fold* onto the inflection and identifies it as "the pure Event of the line or of the point, the Virtual, ideality par excellence" (*The Fold* 15).

The inflection is therefore deployed throughout *The Fold* as the abstract figure of the event, and any event is considered to be a concrete case of inflection. By means of explanation, Deleuze offers three examples, drawn from the work of Bernard Cache,[32] of the kind of virtual or continuous transformation that the inflection can be understood to be characteristic of.

(1) The first set of transformations are "vectorial, or operate by symmetry, with an orthogonal or tangent plane of reflection" (*The Fold* 15). The example that Deleuze offers is drawn from Baroque architecture, according to which an inflection serves to hide or round out the right angle. This is figured in the Gothic arch which has the geometrical shape of an ogive.

(2) The second set of transformations is characterized as "projective." The example that Deleuze gives is the transformations of René Thom (1923–2002) which refer "to a morphology of living matter." Thom developed catastrophe theory, which is a branch of geometry that attempts to model the effect of the continuous variation of one or more variables of a system that produce abrupt and discontinuous transformations in the system. The results are representable as curves or functions on surfaces that depict "seven elementary events: the fold; the crease; the dovetail; the butterfly; the hyperbolic, elliptical, and parabolic umbilicus" (*The Fold* 16). The problem of the conceptualization of matter in Leibniz and the role of projective methods in its conceptualization, specifically those of Desargues, will be addressed later in the paper.

(3) The third set of transformations "cannot be separated from an infinite variation or an infinitely variable curve" (*The Fold* 17). The example that Deleuze gives is the Koch curve, demonstrated by Helge von Koch (1870–1924)

in 1904. The method of constructing the Koch curve is to take an equilateral triangle and trisect each of its sides. On the external side of each middle segment, construct equilateral triangles and delete the above-mentioned middle segment. This first iteration resembles a Star of David composed of six small triangles. Repeat the previous process on the two outer sides of each small triangle. This basic construction is then iterated infinitely. With each order of iteration, the length of any side of a triangle is 4/3 times longer than the previous order. As the order of iteration approaches infinity, so too then does the length of the curve. The result is a curve of infinite length surrounding a finite area. The Koch curve is an example of a non-differentiable curve, that is, a continuous curve that does not have a tangent at any of its points. More generalized Koch or fractal curves can be obtained by replacing the equilateral triangle with a regular *n*-gon, and/or the "trisection" of each side with other equipartitioning schemes.[33] In this example, the line effectively and continuously defers inflection by means of the method of construction of the folds of its sides. The Koch curve is therefore "obtained by means of rounding angles, according to Baroque requirements" (*The Fold* 16).

deleuze's "leibnizian" interpretation of the theory of compossibility

What, then, does Deleuze mean by claiming that Leibniz determines the singularity in the domain of mathematics as a philosophical concept? A crucial test for Deleuze's mathematical reconstruction of Leibniz's metaphysics is how to deal with his subject–predicate logic. Deleuze maintains that Leibniz's mathematical account of continuity is reconcilable with the relation between the concept of a subject and its predicates. The solution that Deleuze proposes involves demonstrating that the continuity characteristic of the infinitesimal calculus is isomorphic to the series of predicates contained in the concept of a subject. An explanation of this isomorphism requires an explication of Deleuze's understanding of Leibniz's account of predication as determined by the principle of sufficient reason.

For Leibniz, every proposition can be expressed in subject–predicate form. The subject of any proposition is a complete individual substance that is a simple, indivisible, dimensionless metaphysical point or monad.[34] Of this subject it can be said that "every analytic proposition is true," where an analytical proposition is one in which the meaning of the predicate is contained in that of the subject. Deleuze suggests that if this definition is reversed, such that it reads: "every true proposition is necessarily analytic," then this amounts to a formulation of Leibniz's principle of sufficient reason,[35] according to which each time a true proposition is formulated it must be understood to be analytic, that is, every true proposition is a statement of analyticity whose predicate is wholly contained in its subject. It follows that if a proposition is true, then the predicate must be contained in the concept of the subject. That is, everything that happens to, everything that can be attributed to, everything that is predicated of a subject – past, present and future – must be contained in the concept of the subject. So for Leibniz, all predicates, that is, the predicates that express all of the states of the world, are contained in the concept of each and every particular or singular subject.

There are, however, grounds to distinguish truths of reason or essence, from truths of fact or existence. An example of a truth of essence would be the proposition $2+2=4$, which is *analytic*; however, it is analytic in a stronger sense than a truth of fact or existence. In this instance, there is an *identity* of the predicate, $2+2$, with the subject, 4. This can be proved by analysis, that is, in a finite or limited number of quite determinate operations, it can be demonstrated that 4, by virtue of its definition, and $2+2$, by virtue of their definition, are identical. So, the identity of the predicate with the subject in an analytic proposition can be demonstrated in a finite series of determinate operations. While $2+2=4$ occurs in all time and in all places, and is therefore a necessary truth, the proposition that "Adam sinned" is specifically dated, that is, Adam will sin in a particular place at a particular time. It is therefore a truth of existence and, as we shall see, a contingent truth. According to the principle of sufficient reason, the proposition "Adam sinned" must be analytic. If we pass from one predicate to another to retrace all the causes and follow up all the effects, this would involve the entire series of predicates contained in the subject Adam, that is, the analysis would extend to infinity. So, in order to demonstrate the inclusion of "sinner" in the concept of "Adam," an infinite series of operations is required. However, we aren't capable of completing such an analysis to infinity.

While Leibniz was committed to the idea of potential ("syncategorematic") infinity, that is, to infinite pluralities such as the terms of an infinite series which are indefinite or unlimited, Leibniz ultimately accepted that, in the realm of quantity, infinity could in no way be construed as a unified whole by us. As Bassler clearly explains:

> So if we ask how many terms there are in an infinite series, the answer is not: an infinite number (if we take this either to mean a magnitude which is infinitely larger than a finite magnitude or a largest magnitude) but rather: more than any given finite magnitude.[36]

The performance of such an analysis is indefinite both for us, as finite human beings, because our understanding is limited, and for God, since there is no end of the analysis, that is, it is unlimited. However, all the elements of the analysis are given to God in an actual infinity. We can't grasp the actual infinite, nor reach it via an indefinite intuitive process. It is only accessible for us via finite systems of symbols that approximate it. The differential calculus provides us with an "artifice" to operate a well-founded approximation of what happens in God's understanding. We can approach God's understanding thanks to the operation of differential calculus, without ever actually reaching it. While Leibniz always distinguished philosophical truths and mathematical truths, Deleuze maintains that the idea of infinite analysis in metaphysics has "certain echoes" in the calculus of infinitesimal analysis in mathematics. The infinite analysis that we perform as human beings in which sinner is contained in the concept of Adam is an indefinite analysis, just as if the terms of the series that includes sinner were isometric with $1/2 + 1/4 + 1/8$, etc., to infinity. In truths of essence the analysis is finite, whereas in truths of existence the analysis is infinite under the above-mentioned conditions of a well-determined finitude.

So what distinguishes truths of essence from truths of existence is that a truth of essence is such that its contrary is contradictory and therefore impossible, that is, it is impossible for 2 and 2 not to equal 4. Just as the identity of 4 and $2+2$ can be proved in a series of finite procedures, so too can the contrary, $2+2$ not equalling 4, be proved to be contradictory and therefore impossible. While it is impossible to think what $2+2$ not equalling 4 or a squared circle may be, it is possible to think of an Adam who might not have sinned. Truths of existence are therefore contingent truths. A world in which Adam might not have sinned is a logically possible world, that is, the contrary is not necessarily contradictory. While the relation between Adam sinner and Adam non-sinner is a relation of contradiction since it is impossible that Adam is both sinner and non-sinner, Adam non-sinner is not contradictory with the world where Adam sinned, it is rather incompossible with such a world. Deleuze argues that to be incompossible is therefore not the same as to be contradictory; it is another kind of relation that exceeds the contradiction.[37] Deleuze characterizes the relation of incompossibility as "a difference and not a negation" (*The Fold* 150). Incompossibility conserves a very classical principle of disjunction: it's either this world or some other one. So, when analysis extends to infinity, the type or mode of inclusion of the predicate in the subject is compossiblity. What interests Leibniz at the level of truths of existence is not the identity of the predicate and the subject but rather the process of passing from one predicate to another from the point of view of an infinite analysis, and it is this process that is characterized by Leibniz as having the maximum of continuity. While truths of essence are governed by the principle of identity, truths of existence are governed by the law of continuity.

Rather than discovering the identical at the end or limit of a finite series, infinite analysis substitutes the point of view of continuity for that of identity. There is continuity when the extrinsic case – for example the circle, the unique triangle or the predicate – can be considered as included in the concept of the intrinsic case, that is, the infinitangular polygon, the virtual triangle, or the concept of the subject. The domain of (in)compossibility is therefore a different domain to that of identity/contradiction. There is no logical identity between sinner and Adam, but there is a continuity. Two elements are in continuity when an infinitely small or vanishing difference is able to be assigned between these two elements. Here Deleuze shows in what way truths of existence are reducible to mathematical truths.

Deleuze offers a "Leibnizian" interpretation of the difference between compossibility and incompossibility "based only on divergence or convergence of series" (*The Fold* 150). He proposes the hypothesis that there is compossibility between two singularities

> when series of ordinaries converge, series of regular points that derive from two singularities and when their values coincide, otherwise there is discontinuity. In one case, you have the definition of compossibility, in the other case, the definition of incompossibility.[38]

If the series of ordinary or regular points that derive from singularities diverge, then you have a discontinuity. When the series diverge, when you can no longer compose the continuity of this world with the continuity of this other world, then it can no longer belong to the same world. There are therefore as many worlds as divergences. All worlds are possible, but they are incompossibles with each other. God conceives an infinity of possible worlds that are not compossible with each other, from which He chooses the best of possible worlds, which happens to be the world in which Adam sinned. A world is therefore defined by its continuity. What separates two incompossible worlds is the fact that there is discontinuity between the two worlds. It is in this way that Deleuze maintains that compossibility and incompossibility are the direct consequences of the theory of singularities.

projective geometry and point of view

While each concept of the subject contains the infinite series of predicates that express the infinite series of states of the world, each particular subject in fact only expresses clearly a small finite portion of it from a certain point of view. In any proposition, the predicate is contained in the subject; however, Deleuze contends that it is contained either actually or virtually. Indeed, any term of analysis remains virtual prior to the analytic procedure of its actualization. What distinguishes subjects is that although they all contain the same virtual world, they don't express the same clear and distinct or actualized portion of it. No two individual substances have the same point of view or exactly the same clear and distinct zone of expression.

Deleuze considers the explanation of point of view to be mathematical or geometrical, rather than psychological. In order to characterize the point of view of the monad Deleuze draws upon the projective geometry of Desargues (1591–1661). Desargues extends the work of Apollonius (262–190 BC) and Kepler (1571–1630) by introducing new methods for proving theorems about conics. He introduced a method of proof called projection and section that unified the approach to the several types of conics that had previously been treated separately. Conic sections are curves formed by the intersection of a plane with the surface of a cone, that is, two right circular cones placed apex to apex. The principles of projection and section can be understood according to the example of a flashlight that projects a circular patch of light on a wall. The flashlight is regarded as a point or apex, the lines of light from the flashlight to the circle are said to constitute a projection, and the wall itself is the plane that is said to contain a section of that projection. The circle on the wall would therefore be understood mathematically as the section that is projected on a plane passing through the projection at 90 degrees. This problem is extended mathematically if we suppose that a different section of this same projection is made by a different plane that cuts the projection at a different angle. For example, if the flashlight were held at an angle to the wall it would project an ellipse. To project a figure from some point and then take a section of that projection is to transform the figure to a new one. Shapes and sizes change according to the plane of incidence that cuts the cone of the projection, but certain properties remain the same throughout such changes, or remain invariant under the transformation, and it is these properties that Desargues studied. Conic sections, including the parabola, ellipse, hyperbola, and circle, can be obtained by continuously varying the inclination of the plane that makes the section, which means that they may be transformed into one another by suitable projections and are therefore continuously derivable from each other.

It is possible that Leibniz had read or at least knew of Desargues's work through the work of Pascal (1623–62) and La Hire (1640–1718), which Leibniz had became acquainted with during the years in which he was working on his early papers on situational analysis. Desargues, Pascal and La Hire first proved properties of the circle and then carried these properties over to the other conic sections by projection and section on the basis that since it is true of the circle it must by projection and section be true of all conics.

The commitment to algebra was so strong by Descartes and his followers that projective geometry went almost unnoticed at the time. The realization that a new branch of geometry was implicit in their work did not come about until the nineteenth century, by which time new developments in mathematics allowed mathematicians to bring to fruition the ideas still dormant in projective geometry.[39]

The summit of a cone is a point of view because, according to projective geometry, it "is the condition under which we apprehend the group of varied forms or the series of curves" (*The Fold* 24), for example the circle, ellipse, parabola and hyperbole, that are derivable from one another by projection and section. It is in this way that a continuity has been constructed by means of the projective properties of the conic sections, and it is this very continuity that Deleuze maps onto the variable curvature represented by the point of inflection to determine the point of view of a monad. However, rather than being mappable onto the entire series of inflections that make up the curve included in a monad, point of view only projects onto the neighbourhood of a single inflection or singularity. This does not yet account for the fact that each individual subject only expresses clearly a partial zone or subdivision of the infinite series of predicates or states of the world included in the monad, but is only the first step of the explanation that Deleuze offers.

The next step draws upon Leibniz's distinction between three kinds of points: the physical, the mathematical and the metaphysical. Leibniz draws a clear distinction between the world of mathematical entities (lines, surfaces, numbers), and the world of concrete things, which is reflected in the distinction between mathematical points and physical points. For him a physical point is a centre of radiation of force which cannot be further contracted. The physical point is what traces the lines of inflection that are extended up to the neighbourhood of other singularities, and which is characterized by Deleuze as the point of inflection. The mathematical point is a position, a site, a focus, or location that Deleuze characterizes as the point of view. And the metaphysical point is simple, indivisible, and dimensionless, it is the soul or the subject that "must be placed in the body where its point of view is located,"[40] or, as Deleuze maintains, it "is what occupies the point of view, it is what is projected in point of view" (*The Fold* 23).[41]

While the inflection is a section of the projection of a point of view, what comes to occupy this point of view is a soul, a substance, a subject, a concept of the individual, designated by a proper name (see *The Fold* 12, 19). The point of view is therefore the mode of individuation of the individual subject. Because the subject occupies the point of view, point of view pre-exists the subject which is placed there. Singularities are therefore pre-individual (see *The Fold* 64). The finite portion of the world that the individual subject expresses clearly is actually constituted by a small number of the points of view of convergent inflections that represent the principal singularities or primary predicates of the monad. For example, Deleuze defines the individual "Adam," which Leibniz, in the letters to Arnauld, describes as the "first man," as the first singularity; the "garden" as the second singularity; "having a woman born of his own side" as the third singularity; and as having "succumbed to temptation" as the fourth singularity.

Deleuze's hypothesis is that the individual subject is a condensation of such compossible, or convergent, singularities, and he draws upon Leibniz's distinction between the three kinds of points for his explanation. Leibniz maintains that it is possible for mathematical points to coincide; for example, given an infinite number of triangles, it is possible to make their summits coincide in the one point, "as the different summits of separate triangles coincide at the common summit of a pyramid" (*The Fold* 63). This is why the mathematical points are not constituent parts, or physical points of extension. The condensation of singularities in an individual subject therefore means that the summits of the triangles that represent the mathematical points characteristic of point of view coincide in this way in a metaphysical point. So Deleuze maintains that the individual subject is a point, but a metaphysical point, and the metaphysical point is the "concentration, accumulation, coincidence of a certain number of converging preindividual singularities" (*The Fold* 63).

the theory of the differential unconscious and the body as phenomenal

The number of mathematical points, or points of view, coincident in the individual subject at any one time corresponds to the proportion of the world that is expressed clearly and distinctly by that individual, in relation to the rest of the world that is expressed obscurely and confusedly. The explanation as to why each monad only expresses clearly a limited subdomain of the world that it contains pertains to Leibniz's distinction between "perception, which is the inner state of the monad representing external things, and apperception, which is consciousness or the reflective knowledge of this inner state itself."[42] The infinite series of predicates or states of the world is in each monad in the form of minute perception. These are infinitely tiny perceptions, which Deleuze characterizes as "unconscious perceptions" (*The Fold* 89), or as the "differentials of consciousness" (93). Each monad expresses every one of them, but only obscurely or confusedly, like a clamour. Leibniz therefore distinguishes conscious perception as apperception from minute perception, which is not given in consciousness.

When Leibniz mentions that conscious perceptions "arise by degrees from" minute perceptions,[43] Deleuze claims that what Leibniz indeed means is that conscious perception "derives from" minute perceptions. It is in this way that Deleuze links unconscious perception to infinitesimal analysis. Just as there are differentials for a curve, there are differentials for consciousness.

When the series of minute perceptions is extended into the neighbourhood of a singular point, or point of inflection, that perception becomes conscious. Conscious perception, just like the mathematical curve, is therefore subject to a law of continuity, that is, an indefinite continuity of the differentials of consciousness. We pass from minute perception to conscious perception when the series of ordinaries reaches the neighbourhood of a singularity. In this way, the infinitesimal calculus operates as the unconscious psychic mechanism of perception. Deleuze understands the subdomain that each monad expresses clearly in terms of the constraints that the principle of continuity places on a theory of consciousness. "At the limit, then, all monads possess an infinity of compossible minute perceptions, but have differential relations that will select certain ones in order to yield clear perceptions proper to each" (*The Fold* 90).

Before addressing Leibniz's understanding of the phenomenal nature of a monad's body, his account of matter, and Deleuze's characterization of it, requires explication. At the most basic level, Leibniz identified extended matter with primitive passive force that includes both impenetrability and resistance.[44] In addition to this, Leibniz considered nature to be infinitely divisible such that "the smallest particle should be considered as a world full of an infinity of creatures."[45]

He also maintained that "The division of the continuous must not be taken as of sand dividing into grains, but as that of a sheet of paper or of a tunic in folds, in such a way that an infinite number of folds can be produced, some smaller than others, but without the body ever dissolving into points or minima" (*The Fold* 6).[46]

Deleuze takes this trope of "a tunic in folds" to characterize Leibnizian matter as "solid pleats" that "resemble the curves of conical forms," that is, the actual surface of the projection from apex to a curve of the cone of a conic section "sometimes ending in a circle or an ellipse, sometimes stretching into a hyperbola or a parabola." Deleuze then proposes *origami*, the Japanese art of folding paper, as the model for the sciences of Leibnizian matter (*The Fold* 6). This accounts for the first type of fold that characterizes the pleats of matter, which are then organized according to a second type of fold, which Deleuze characterizes mathematically by means of Albrecht Dürer's (1471–1528) projective method for the treatment of solids. Dürer, in his work on the shadow of a cube, devised a

> proto-topological method of developing [solids] on the plane surface in such a way that the facets form a coherent "net" which, when cut out of paper and properly folded where the two facets adjoin, will form an actual, three-dimensional model of the solid in question.[47]

What, then, does this mean for bodies? Bodies are extended in so far as geometry is projected in this proto-topological way onto them. In a metaphysical sense, what is really there is force. In his notes on Foucher, Leibniz explains that "Extension or space and the surfaces, lines, and points one can conceive in it are only relations of order or orders of coexistence."[48] The extensionality of bodies is therefore phenomenal in so far as it results from the projection of geometrical concepts onto the "tunic in folds" of matter. What to each monad is its everyday reality is to Leibniz a phenomenal projection, which is rendered intelligible only when it is understood to reflect the intelligible, mathematical order that determines the structure of Leibniz's metaphysics.[49]

So there is a projection of structure from the mathematico-metaphysical onto the phenomenal, which Deleuze distinguishes according to the distinction canvassed earlier between the functional definition of the Newtonian fluxion and the Leibnizian infinitesimal as a concept. "The physical mechanism of bodies (fluxion) is not identical to the psychic mechanism of perception (differentials), but the latter resembles the former" (*The Fold* 98). So Deleuze maintains that "Leibniz's calculus is adequate to psychic mechanics where Newton's is operative for physical mechanics" (98), and here again draws from the mathematics of Leibniz's contemporaries to determine a distinction between the mind and body of a monad in Leibniz's metaphysics.

How, then, does this relate to the body that belongs to each monad? In so far as each monad clearly expresses a small region of the world, what is expressed clearly is related to the monad's body. Deleuze maintains that "I have a body because I have a clear and distinguished zone of expression" (*The Fold* 98).

What is expressed clearly and distinctly is what relates to the biological body of each monad, that is, each monad has a body that is in constant interaction with other bodies, and these other bodies affect its body. So what determines such a relation is precisely a relationship between the physical elements of other bodies and the monad's biological body, each of which is characterized as a series of microperceptions which are the differentials of consciousness. Deleuze assimilates the relation between these two series to the differential relation. Microperceptions are brought to consciousness by differentiating between the monad's own biological body and the physical affects of its relations with other physical elements or bodies. This results in the apperception of the relation between the body of the monad and the world it inhabits. However, the reality of the body is the realization of the phenomena of the body by means of projection, since the monad draws all perceptive traces from itself. The monad acts as if these bodies were acting upon it and were causing its perceptions. However, among monads there is no direct communication. Instead, each individual subject is harmonized in such a way that what it expresses forms a common compossible world that is continuous and converges with what is expressed by the other monads. So it is necessary that the monads are in harmony with one another; in fact the world is nothing other than the pre-established harmony amongst monads. The pre-established harmony is, on the one hand, the harmony amongst monads, and on the other the harmony of souls with the body, that is, the bodies themselves are realized as phenomenal projections which puts them in harmony with the interiority of souls.

The reconstruction of Leibniz's metaphysics that Deleuze provides in *The Fold* draws upon not only the mathematics developed by Leibniz but also upon developments in mathematics made by a number of Leibniz's contemporaries and a number of subsequent developments in mathematics, the rudiments of which can be more or less located in Leibniz's own work. Deleuze then retrospectively maps these developments back onto the structure of Leibniz's metaphysics in order to bring together the different aspects of Leibniz's metaphysics with the variety of mathematical themes that run throughout his work. The result is a thoroughly mathematical explication of Leibniz's metaphysics, and it is this account that subtends the entire text of *The Fold*.

notes

I am grateful to the reviewer René Guitart for his constructive suggestions.

1 See, for example, Duffy, "Leibniz, Mathematics and the Monad."

2 Panofsky 256.

3 Transcendental in this mathematical context refers to those curves that were not able to be studied using the algebraic methods introduced by Descartes.

4 A concept that was already in circulation in the work of Fermat and Descartes. Leibniz, *Mathematische Schriften* V: 126.

5 See ibid. 223.

6 Leibniz, *Mathematische Schriften* VII: 222–23.

7 Leibniz, *Philosophical Papers and Letters* 545.

8 The lettering has been changed to more directly reflect the isomorphism between this algebraic example and Leibniz's notation for the infinitesimal calculus.

9 This example presents a variation of the infinitesimal or "characteristic" triangle that Leibniz was familiar with from the work of Pascal. See Leibniz, "Letter to Tschirnhaus (1680)" in *The Early Mathematical Manuscripts*; and Pascal, "Traité des sinus du quart de cercle (1659)" in *Œuvres Mathématiques*.

10 Deleuze, *Sur Leibniz*, 22 Apr.

11 Leibniz, *Mathematische Schriften* V: 220–26.

12 Newton, *Method of Fluxions and Infinite Series*.

13 Newton's reasoning about geometrical limits is based more on physical insights rather than mathematical procedures. In "Geometria Curvilinea," Newton develops the synthetic method of fluxions which involves visualizing the limit to which the ratio between vanishing geometrical quantities tends.

14 Lakoff and Núñez 224.

15 Bos 6.

16 Leibniz, *Methodus tangentium inversa*; see Katz 199.

17 See Bos 6.

18 See Boyer 287. While Leibniz had already envisaged the convergence of alternating series, and by the end of the seventeenth century the convergence of most useful concrete examples of series, which were of limited quantity, if not finite, was able to be shown, it was Cauchy who provided the first extensive and significant treatment of the convergence of series. See Kline 963.

19 For an account of this problem with limits in Cauchy, see Potter 85–86.

20 See Potter 85. While the epsilon-delta method is due to Weierstrass, the definition of limits that it enshrines was actually first proved by Bernard Bolzano (1741–1848) in 1817 using different terminology (Ewald 225–48); however, it remained unknown until 1881 when a number of his articles and manuscripts were rediscovered and published.

21 Boyer 287.

22 See Bell.

23 The infinitesimal is now considered to be a hyperreal number that exists in a cloud of other infinitesimals or hyperreals floating infinitesimally close to each real number on the hyperreal number line (Bell 262). The development of non-standard analysis, however, has not broken the stranglehold of classical analysis to any significant extent, but this seems to be more a matter of taste and practical utility rather than of necessity (Potter 85).

24 Robinson 2.

25 Non-standard analysis allows "interesting reformulations, more elegant proofs and new results in, for instance, differential geometry, topology, calculus of variations, in the theories of functions of a complex variable, of normed linear spaces, and of topological groups" (Bos 81).

26 For a more extensive discussion of this aspect of Deleuze's project, see Duffy, *The Logic of Expression*.

27 Deleuze, *Sur Leibniz*, 29 Apr.

28 Ibid.

29 The concept of neighbourhood, in mathematics, which is very different from contiguity, is a key concept in the whole domain of topology.

30 Deleuze, *The Fold* 15.

31 Which was actually known to the Babylonians one thousand years earlier, although Pythagoras is considered to be the first to have proved it.

32 Cache 34–41, 48–51, 70–71, 84–85.

33 See Lakhtakia et al. 3538.

34 Leibniz's distinction between the three kinds of points – physical, mathematical, and metaphysical – will be returned to in the following section.

35 Deleuze, *Sur Leibniz*, 15 Apr.

36 Bassler 870.

37 And that Deleuze characterizes as "vice-diction" (*The Fold* 59).

38 Deleuze, *Sur Leibniz*, 29 Apr.

39 It did not achieve prominence as a field of mathematics until the early nineteenth century through the work of Poncelet (1788–1867), Gergonne (1771–1859), Steiner (1796–1863), von Staudt (1798–1867) and Plücker (1801–68). One of the leading themes in Poncelet's work is the "principle of continuity" which he coined and in a broad philosophical sense goes back to the law that Leibniz used in connection with the calculus. However, Poncelet advanced it as an absolute truth and applied it to prove many new theorems of projective geometry. See Kline 843.

40 "Letter to Lady Masham (1704)" in Leibniz, *Philosophical Essays* 290.

41 Leibniz provides a mathematical representation of the metaphysical points in his ontological proof of God as $\infty/1$. If the infinite is the set of all possibilities, and if the set of all possibilities is possible, then there exists a singular individual who corresponds to it, and this singular individual is God represented mathematically by $\infty/1$. From God to the monad is to go from the infinite to the individual unit that includes an infinity of predicates. The metaphysical point that occupies the position of a monad's point of view is the inverse of the position occupied by God, and is represented mathematically by $1/\infty$. There is an infinity of $1/\infty$ (monads), and one all-inclusive $\infty/1$ (God). "For Leibniz the monad is. . . the inverse, reciprocal, harmonic number. It is the mirror of the world because it is the inverted image of God" (*The Fold* 129).

42 "Principles of Nature and Grace (1714)" in Leibniz, *Philosophical Papers and Letters* §13.

43 In the preface to *New Essays on Human Understanding*, Leibniz says that "noticeable perceptions arise by degrees from ones which are too minute to be noticed" (56).

44 Leibniz, *Philosophical Essays* 120.

45 "Letter to Simon Foucher (1693)" in Leibniz, *Die philosophischen Schriften* I: 415–16.

46 *Pacidus Philalethi* in Leibniz, *Opuscules et fragments* 614–15.

47 Panofsky 259. This method was systematized by Gaspard Monge (1746–1818) in what he called "descriptive geometry."

48 Leibniz, *Philosophical Essays* 146. See Garber 34–40.

49 See Grene and Ravetz 141. Deleuze also poses the question of whether this topological account can be extended to Leibniz's concept of the vinculum (*The Fold* 111). If so, the topology of the vinculum would have to be isomorphic to that of matter; however, it would be so within each

monad, and would be complicated by itself being a phenomenal projection. For further discussion of the vinculum in Leibniz see Look.

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