Sharpening the Electromagnetic Arrow(s) of Time

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1 Introduction

The various "arrows of time" and their interrelations are the subject of a seemingly never ending discussion in the physics and the philosophy of science literature. While the discussion in recent decades has undoubtedly produced numerous advances in the details of our understanding, it is hard not to be discouraged by the overall lack of progress in reaching a consensus on key issues.[1] In addition to ruing the lack of progress, one could also make two general complaints about the arrows of time literature. First, one could complain (as several authors have) that the talk of "arrows of time" suggests that what is at issue is the directionality of time whereas what is often at issue is not directionality but temporal asymmetries. This is a defect that could be corrected by a change of terminology, but since talk of "arrows" has become well entrenched, I will continue to use it. What is not a matter of terminology, however, is whether the various asymmetries are merely asymmetries *in* time or whether they constitute asymmetries *of* time itself. That there are differences of opinion on this matter is not surprising. But what is disconcerting is the lack of agreement on how this matter is to be decided.[2] Second, it is especially vexing that the typical way of stating the puzzle about various temporal arrows or asymmetries rests on a false presupposition. The general form of the puzzle is supposed to be: "Since the fundamental laws of physics exhibit a symmetry X, why is the world we see around us so X-asymmetric?" For a continuous symmetry, such as spatial translation or spatial rotation, it is typically true that for laws as expressed in terms of differential equations, generic solutions of X-symmetric equations are X-*a*symmetric.[3] There are (as far as I know) no general results to this effect for discrete symmetries, such as time reversal invariance. Nevertheless, such results often do hold. For example, Einstein's gravitational field equations are time reversal invariant, but within the class of Friedmann-Walker-Robertson (FRW) models used in contemporary cosmology to describe the large scale features of our universe, the subclass of models that are time symmetric about a time slice $t = const$ has "measure zero" (see Castagnino et al. 2003).[4] In short, not only is it not surprising that we find ourselves in an X-asymmetric world even though the laws that govern this world are X-symmetric, it would be surprising if we *didn't* find ourselves in an X-asymmetric world! Still, some actually observed asymmetries seem so striking and/or pervasive that they call for explanation. But I am unaware of a persuasive analysis of how these privileged asymmetries are to be identified and, thus, am left with the nagging feeling that the choice of the asymmetries that get promoted to arrows of time is a matter of fashion rather than principle.

Turning from the general picture to the electromagnetic (EM) arrows in particular, several complaints could be raised about the state of the debate. First, and foremost, there is the lack of a clear and unproblematic statement of what the EM arrows are. Second, there are conflicting claims about a number of issues: the status of retarded/advanced fields; the time reversal invariance, or lack thereof, of the equations of motion of charged particles that incorporate radiation reaction; and the linkage between the EM arrow and the other arrows of time, especially the cosmological arrow. Third, there is feeling of *deja vu* all over again about the debate. The modern phase of the debate can be dated to the Einstein-Ritz controversy of 1908-1909. The predominate opinion had been that Einstein prevailed. But recently neo-Ritzian points of view has been expressed not only in the philosophy literature but the physics literature as well (see, for example, Frisch 2000, 2005 and Rohrlich 1998, 1999, 2000).

There is no hope that one review can clear up all of the unresolved issues about the EM arrow(s). But it should be possible to separate the genuine from the pseudo-problems and to put to rest the latter while sharpening the former. That is the goal of the this chapter.

In Section 2 the search for EM arrows is focused on the space of solutions of Maxwell's equations for a specified charge distribution and the initial/boundary conditions used to pick out a particular solution or a subclass of solutions. In Section 3 the focus shifts to the equations of motion of accelerating charges that radiate and experience a damping force. One has to be prepared to find that one or both of these searches may turn up different results depending upon whether the setting is Minkowski spacetime or a cosmological model whose local or global structure differs from Minkowski spacetime. Most of the discussion is devoted to classical electrodynamics, but the implications of quantum electrodynamics (QED) are examined at the end of Section 3. Conclusions are presented in Section 4.

2 Searching for the EM arrow in the solutions to Maxwell's equations

2.1 Time reversal invariance

The search for an EM arrow would immediately yield pay dirt if Albert (2000) were right and Maxwell's equations failed to be time reversal invariant.[5] However, Albert's claims rests on a non-standard and, arguably, unilluminating analysis of time reversal invariance (see Earman 2002 and Malament 2004). This matter will not be rehearsed here, and I will operate with the standard version of time reversal invariance for electromagnetism.

In covariant notation Maxwell's equations read

$$\nabla_\mu F^{\mu\nu} = J^\nu \tag{1}$$

$$\nabla_{[\sigma} F_{\mu\nu]} = 0 \tag{2}$$

where $F^{\mu\nu}$ is the Maxwell tensor, J^ν is the charge-current field, and ∇_μ is the covariant derivative operator.[6] These equations apply to curved as well as flat spacetime; but until further notice the application will be to Minkowski spacetime, in which case the covariant derivative can be replaced by ordinary differentiation with respect to inertial coordinates. In inertial coordinates the charge-current field takes the form $J^{\nu} = (\mathbf{j}, \rho)$, where $\mathbf{j}$ is the three-vector current and ρ is the charge density. Further, the contravariant components of the Maxwell tensor are related to the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, as follows[7]

$$F^{\mu\upsilon} = \begin{pmatrix} 0 & \mathbf{B}_z & \text{-}\mathbf{B}_y & \text{-}\mathbf{E}_x \\ \text{-}\mathbf{B}_z & 0 & \mathbf{B}_x & \text{-}\mathbf{E}_y \\ \mathbf{B}_y & \text{-}\mathbf{B}_x & 0 & \text{-}\mathbf{E}_z \\ \mathbf{E}_x & \mathbf{E}_y & \mathbf{E}_z & 0 \end{pmatrix} \tag{3}$$

And in three-vector notation the Maxwell equations (1)-(2) take their familiar forms

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \rho, \quad \boldsymbol{\nabla}\mathbf{x}\mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j} \tag{1'}$$

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0, \quad \boldsymbol{\nabla}\mathbf{x}\mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{2'}$$

One approach to time reversal invariance involves the literal reversal of time orientation. A time orientation is given by a continuous non-vanishing timelike vector field $\mathfrak{t}^\nu$ (or more properly by an equivalence class of such fields, where two such fields $\mathfrak{t}^\nu$ and $\mathfrak{t}'^\nu$are counted as equivalent just in case at every spacetime location x, $\mathfrak{t}^\nu(x)$ and $\mathfrak{t}'^\nu(x)$ point into the same lobe of the null cone at x).[8] Under literal time reversal (denoted by 'T'), ${}^T\mathfrak{t}^\nu = -\mathfrak{t}^\nu$. To check whether the equations of a given theory are time reversal invariant it is necessary to define the resultant action of this operation on the basic variable of the theory and verify that the solution set of the equations remains invariant under the defined action.[9] The obvious drawback of this approach to time reversal invariance is the lack of a direct connection to experiment—at least for those experimenters who do not have control of a switch by means of which they can reverse the orientation of time.

A second approach that lends itself better to experimental test works with a fixed time orientation. A model for a given theory is an assignment $x \mapsto S(x)$ to spacetime points x of a state description $S(x)$ at x appropriate for said theory. A theory is deemed to be time reversal invariant just in case whenever $x \mapsto S(x)$ satisfies the laws of the theory, so does $x \mapsto ({}^TS)(x)$ where $({}^TS)(\mathbf{x}, t) = {}^RS(\mathbf{x}, -t)$ for an inertial coordinate system $(\mathbf{x}, t)$. (Nothing depends on the choice of the inertial system since it is assumed that the laws are Poincaré invariant.) Here '${}^RS$' denotes the "reversed" state, which is supposed to describe, relative to the fixed time orientation, the analogue of the corresponding state in the world with the literally reversed time orientation. In particle mechanics the "reversed" state is standardly defined by reversing the three-velocities of the particles. For

Maxwell electromagnetism, the standard definition of the reversal operation is ${}^{R}(\mathbf{E}(\mathbf{x},t),\mathbf{B}(\mathbf{x},t),\mathbf{j}(\mathbf{x},t),\rho(\mathbf{x},t)) = (\mathbf{E}(\mathbf{x},t),-\mathbf{B}(\mathbf{x},t),-\mathbf{j}(\mathbf{x},t),\rho(\mathbf{x},t))$. It is easy to see that, whenever the history $(\mathbf{x},t)\mapsto(\mathbf{E}(\mathbf{x},t),\mathbf{B}(\mathbf{x},t),\mathbf{j}(\mathbf{x},t),\rho(\mathbf{x},t))$ satisfies (1′)-(2′), so does $(\mathbf{x},t)\mapsto {}^{T}(\mathbf{E}(\mathbf{x},t),\mathbf{B}(\mathbf{x},t),\mathbf{j}(\mathbf{x},t),\rho(\mathbf{x},t)) = (\mathbf{E}(\mathbf{x},-t),-\mathbf{B}(\mathbf{x},-t),-\mathbf{j}(\mathbf{x},-t),\rho(\mathbf{x},-t))$. In terms of the Maxwell tensor, ${}^{T}F^{4\nu}(x') = F^{4\nu}(x)$ and ${}^{T}F^{mn}(x') = -F^{mn}(x)$, where $x=(\mathbf{x},t)$, $x'=(\mathbf{x},-t)$, and $m,n=1,2,3$. The four-velocity V^{μ} transforms as ${}^{T}V^{4}=V^{4}$ and ${}^{T}V^{m} = -V^{m}$. In the alternative approach where time reversal involves the literal reversal of the time orientation, ${}^{T}F^{\mu\nu}(x) = -F^{\mu\nu}(x)$ and ${}^{T}V^{\mu} = -V^{\mu}$.

While both of the above approaches take for granted the existence of a time orientation, it is fair to ask whether it would be justified to posit such an object if all of the fundamental laws of physics were time reversal invariant. The suggestion behind the question is that, when the supposition obtains, the time reversal symmetry should be treated as a gauge symmetry in the sense that it connects equivalent descriptions of the same physical state of affairs. And the further suggestion would be that our perception of the time order of events is not due to our communion with an orientation defining vector field $\mathfrak{t}^{\nu}$ but rather to our reaction to, say, local entropy gradients. I will not attempt to tackle these issues here, and will simply assume the existence of a time orientation.[10]

2.2 Explaining electromagnetic asymmetries: the Einstein-Ritz controversy

For phenomena governed by time reversal invariant laws L (such as Maxwell's laws of electromagnetism), at least two strategies are available for explaining observed asymmetries. First, additional laws L' can be postulated such that the combined laws $L\&L'$ are not time reversal invariant. Second, the asymmetry can be attributed to the fact that, although certain time developments are allowed by the laws L, they are vastly more "improbable" than their time reversed counterparts. A familiar but still controversial example of the latter is the modern form of Boltzmannian statistical mechanics. The time reversal invariant laws of Newtonian mechanics are cast in Hamiltonian form; a measure on the phase space is adopted and is used to gauge the probability of macroscopic outcomes as identified with regions of phase space; and the tendency of (coarse-grained) entropy to increase with time is explained by the tendency of the microstate of system to evolve to regions of phase space that correspond to ever more probable macrostates. However, as is well known, the time reversal invariant character of the laws of mechanics entails that this explanation of the asymmetry of entropic behavior requires the help of a posit of special, low entropy initial conditions (see Albert 2000). Different positions can be taken with regard to the issue of whether these special initial conditions have a *de facto* or a lawlike character. In the latter case the distinction between the two strategies is somewhat blurred, but a crucial difference remains in that in the first strategy the additional laws L' are supposed to be non-probabilistic.

Commentators have interpreted the Einstein-Ritz controversy (see Ritz 1908 and 1909, Einstein 1909, and Ritz and Einstein 1909) as exemplifying the competition between these two strategies, with Ritz opting for the first strategy and Einstein for the second. Commentators typically quote the joint Einstein-Ritz declaration in which they agreed to disagree:

> [E]xperience compels one to consider the representation by means of retarded potentials as the only one possible, if one is inclined to the view that the fact of the irreversibility of radiation must already find its expression in the fundamental equations. Ritz considers the restriction to the form of retarded potentials as one of the roots of the second law [of thermodynamics], while Einstein believes that irreversibility is exclusively due to reasons of probability. (Ritz and Einstein 1909: 324)

Superficially, this quotation does seem to count as an exemplification of the competition between the two strategies. But first appearances are deceiving. Before trying to get behind the appearances, a digression is in order.

The second sentence of the joint declaration reveals a wholly implausible feature of Ritz's position, viz., the idea that the electromagnetic asymmetries explain the thermodynamic asymmetries. This explanatory linkage is hard to forge for at least two reasons. First, thermodynamics works for electrically neutral matter for which, trivially, there is no electromagnetic EM arrow. Second, the talk of retarded potentials invokes relativistic considerations, whereas thermodynamics does not cease to be valid in the Newtonian limit of velocities small in comparison with the velocity of light. Curiously, Einstein did not make these obvious points. They are made, over and over again, by commentators on the Einstein-Ritz controversy.

Of course, Ritz could be mistaken about the relation between the electromagnetic and the thermodynamic asymmetries but right about the basis of the former. But the first sentence of the joint declaration seems to reveal a confusion that Einstein noted in an earlier and more sharply critical assessment of Ritz's position. Ritz proposed to find the expression of the asymmetry of electromagnetic radiation in the fundamental equations of electromagnetism by positing that "the representation by means of retarded potentials as the only one possible." But Einstein (1909) claimed that the representation by means of retarded potentials is not more special than the representation by, say, a linear combination of retarded and advanced potentials, both being representations of the same solution. In fact, Ritz and Einstein were at cross purposes: although Einstein's claim is correct if it refers to orthodox classical electromagnetism, there is a sense in which Ritz held a scientifically respectable, if not ultimately defensible, position. This position exemplifies a third strategy, which involves changing the theory. For Ritz the fundamental equations of electrodynamics were to be formulated in the context a theory that postulates particles acting at-a-distance without the mediation of fields. For such a theory the restriction to retarded action has a natural expression, one which leads to time reversal non-invariant laws.

By contrast, the recent proposals that I label as neo-Ritzian—because they also invoke a restriction to retarded fields—are supposed to be implemented in the field theoretic setting of orthodox classical electrodynamics. As a result, they do not have, I contend, a scientifically respectable expression in terms of physical laws but require chanting incantations about "causation." In order to lay the ground work for this negative judgment, I must first explain the nomenclature of advanced and retarded representations/potentials/fields. After explaining the difference between the Ritzian and neo-Ritzian proposals, I concentrate mainly on the orthodox field-theoretic formulation of electromagnetism, but the inability of one widely discussed action-at-a-distance theory of classical electrodynamics—the Wheeler-Feynman theory—to explain the asymmetry of radiation is discussed.

2.3 The Kirchhoff representation theorem for Minkowski spacetime

In a simply connected spacetime the Maxwell equation (2) guarantees the existence of a global potential A_μ for $F_{\mu\nu}$:

$$F_{\mu\nu} = \nabla_{[\nu} A_{\mu]} \tag{4}$$

Written in terms of the potentials, Maxwell's equations do not have a well-posed initial value problem since values of the potentials and their time derivatives at a given time do not fix a unique solution, a not unexpected result since the introduction of the potentials injects gauge freedom. Fixing the gauge by imposing the Lorentz gauge condition

$$\nabla_\mu A^\mu = 0 \tag{5}$$

turns the Maxwell equation (1) into the equation

$$\Box A^\nu + R^\nu{}_\mu A^\mu \equiv g^{\alpha\beta}\nabla_\alpha\nabla_\beta(A^\nu) + R^\nu{}_\mu A^\mu = J^\nu \tag{6}$$

where $R_{\mu\nu}$ is the Ricci tensor of the spacetime metric $g_{\mu\nu}$. In Minkowski spacetime, which is the focus of this section, (6) reduces to the inhomogeneous wave equation

$$\Box A^\nu = J^\nu \tag{7}$$

In inertial coordinates (7) takes the familiar form

$$\frac{\partial^2 A^\nu}{\partial x^2} + \frac{\partial^2 A^\nu}{\partial y^2} + \frac{\partial^2 A^\nu}{\partial z^2} - \frac{\partial^2 A^\nu}{\partial t^2} = J^\nu \tag{7'}$$

which does have a well-posed initial value problem.

Consider a given electric current distribution J^ν satisfying the law of conservation of charge

$$\nabla_\nu J^\nu = 0 \tag{8}$$

which is implied by the Maxwell equation (1). Then the Kirchhoff representation theorem shows that any solution of (7) can be written in either retarded or advanced form as a sum of a volume integral and a surface integral:

$$\begin{aligned} A^\nu(x) &= \int_{\Omega^-} ret + \int_{\partial\Omega^-} ret \equiv {}_{ret}A^\nu(x) + {}_{in}A^\nu(x) & (9a) \\ &= \int_{\Omega^+} adv + \int_{\partial\Omega^+} adv \equiv {}_{adv}A^\nu(x) + {}_{out}A^\nu(x) & (9b) \end{aligned}$$

Here x is a spacetime location in a compact spacetime volume Ω with orientable boundary $\partial\Omega$, Ω^- and Ω^+ are respectively the past and future light cones of x in Ω, and $\partial\Omega^- \equiv \Omega^- \cap \partial\Omega$ and $\partial\Omega^+ \equiv \Omega^+ \cap \partial\Omega$. In *four*-dimensional Minkowski spacetime the propagation of the field is clean-cut so that the supports of the volume integrals in (9a)-(9b) are restricted to the surface of the light cones; this result does not hold if space has dimension one or two. In the retarded representation the volume integral gives the contribution of the sources in Ω^- while the surface integral gives the contribution of the incoming radiation that is either associated with sources lying outside of the chosen volume or else is truly source-free radiation that is not tied to any sources. The interpretation of the advanced representation is analogous.

To make gauge-free statements, the Maxwell tensor can be computed from the potentials in (9a)-(9b) to give the retarded and advanced representations of the field

$$F^{\mu\nu}(x) = {}_{ret}F^{\mu\nu}(x) + {}_{in}F^{\mu\nu}(x) \tag{10a}$$

$$= {}_{adv}F^{\mu\nu}(x) + {}_{out}F^{\mu\nu}(x) \tag{10b}$$

The fields ${}_{ret}F^{\mu\nu}(x)$ and ${}_{adv}F^{\mu\nu}(x)$ are solutions of the inhomogeneous Maxwell equations with the specified sources, whereas ${}_{in}F^{\mu\nu}(x)$ and ${}_{out}F^{\mu\nu}(x)$ are solutions of the homogeneous Maxwell equations ($J^{\nu} \equiv 0$). Taking advantage of the linearity of the Maxwell equations, a general solution can be written in the mixed form

$$F^{\mu\nu}(x) = \lambda[{}_{ret}F^{\mu\nu}(x) + {}_{in}F^{\mu\nu}(x)] + (1-\lambda)[{}_{adv}F^{\mu\nu}(x) + {}_{out}F^{\mu\nu}(x)] \tag{11}$$

where $0 \leq \lambda \leq 1$.

Commentators' talk of "retarded and advanced fields" and "retarded and advanced solutions" invites confusion if the following distinction is not kept in mind. The fields ${}_{ret}F^{\mu\nu}(x)$ and ${}_{adv}F^{\mu\nu}(x)$ derived from the potentials defined by the volume integrals in (9a) and (9b) respectively are generally *different solutions* of the inhomogeneous Maxwell equations—rather special time symmetric cases in which these fields are equal are the exceptions that prove the rule. By contrast, as is evident from $(10a)-(10b)$, the total retarded and total advanced fields, ${}_{ret}F^{\mu\nu}(x) + {}_{in}F^{\mu\nu}(x)$ and ${}_{adv}F^{\mu\nu}(x) + {}_{out}F^{\mu\nu}(x)$ respectively, are *not* different solutions but merely different representations of the *same solution*, as noted by Einstein (1909).

Further, it is worth noting that the distinctions between ${}_{ret}F^{\mu\nu}(x)$ and ${}_{ret}F^{\mu\nu}(x) + {}_{in}F^{\mu\nu}(x)$ on one hand and between ${}_{adv}F^{\mu\nu}(x)$ and ${}_{adv}F^{\mu\nu}(x) + {}_{out}F^{\mu\nu}(x)$ on the other are somewhat artificial since they depend on the chosen volume Ω. It is only in cases where the volume and surface integrals in (9) have well-defined limits as $\partial\Omega^-$ and $\partial\Omega^+$ "go to infinity," i.e. as Ω tends to the entire volume of spacetime, that an absolute meaning can be assigned to statements about *the* advanced and retarded fields ${}_{ret}F^{\mu\nu}(x)$ and ${}_{adv}F^{\mu\nu}(x)$ and *the* incoming and outgoing fields ${}_{in}F^{\mu\nu}(x)$ and ${}_{out}F^{\mu\nu}(x)$.

The retarded and advanced fields for the special case of a single point charge are referred to as the Liénard-Wiechert fields, the general expressions for which can be found in standard texts (see, for example, Jackson 1998: Section 14.1). The actual values of these fields have to be worked out on a case-by-case basis once the details of the motion of the charge are specified.

2.4 Retarded fields: "causality" and the "retardation condition"

Some textbooks on electromagnetism work out the expressions for the advanced Liénard-Wiechert potentials and fields only to discard them for reasons of "causality." Thus, Heald and Marion (1995) opine that

> This so-called advanced potential [the time component of the advanced four-potential] appears to have no physical significance because it corresponds to an anticipation of the charge distribution (and current distribution for the case of the vector potential [the space components of the four-potential]) at a future time. Such a potential does not satisfy the requirement that causality must be obeyed by a physical system. (260)

This requirement of "causality" must involve something in addition to satisfying Maxwell's equations, but the something more is not elucidated, much less justified, in the text.

Frisch's (2000) proposal seemed to promise an implementation of the first strategy mentioned in Section 2.2—i.e. add additional laws to the original set of time reversal invariant laws such that the augmented set of laws is time asymmetric—that bypasses issues about "causality" in favor of two neo-Ritzian posits. The first is that, in addition to satisfying Maxwell's equations, "electromagnetic fields associated with charges satisfy the retardation condition" (405); and the second is that the retardation condition is to be regarded as a law of classical electromagnetism that is just as fundamental as Maxwell's equations in that it makes no more sense to ask for an explanation of why the retardation condition obtains than it does to ask by Maxwell's equations obtain (405-406).

Unfortunately, Frisch's proposed new law reintroduces "causality" under another name: his retardation condition is the condition that each charge "physically contributes" a fully retarded component to the total field. But this proposed law does not pass master that a potential law of physics must satisfy even prior to empirical testing. In the soft sciences, where it is difficult if not impossible to find any precise and exceptionless lawlike generalizations with broad scope, it is apparently acceptable to use escape clauses—e.g. *ceteris paribus*—or to gesture to wantabe laws by using suggestive but imprecise terminology—e.g., X produces (causes, contributes, ...) Y. In physics this is not an acceptable practice. A putative fundamental law of physics must be stated as a mathematical relation without the use of escape clauses or words that require a PhD in philosophy to apply (and two other PhDs to referee the application, and a third referee to break the tie of the inevitable disagreement of the first two).

In his later book, *Inconsistency, Asymmetry, and Non-Locality* (2005), Frisch refers to the retardation condition not as a law but as a "causal constraint," which suggests that the original quest of providing additional laws to ground electromagnetic asymmetries has been abandoned in favor of a preferred interpretation of the existing laws of electromagnetism. The retardation condition as a causal constraint comes to this: the notion that a charge "physically contributes" a fully retarded component to the total field is parsed by saying that that component would be absent if the charge were absent (Frisch 2005: 153ff). The exercise of trying to divine the truth value of such counterfactual assertions, even when it is agreed at the outset what the basic laws are, is an invitation to a contest of conflicting intuitions about cotenability of conditions and the closeness of possible worlds. This is a contest that may generate many learned philosophical articles, but I am skeptical that such a contest or, more generally, the philosophical exercise of interpreting the equations of physics by performing incantations using the phrases "physically contribute," "cause," and the like will reveal an electromagnetic asymmetry that was not perceptible when the equations were allowed to speak for themselves. By "speak for themselves" I do not mean that the equations are taken as uninterpreted mathematical squiggles but rather that only minimalist interpretations are allowed, e.g. 'ρ' denotes the electric charge density, '$\mathbf{E}$' denotes the electric field strength, etc. *Until proven otherwise, my assumption is that an EM arrow worth having is one that only requires the equations of physics to speak for themselves under such a minimalist interpretation.*

It is worth understanding why Ritz himself had access to a scientifically respectable version of the retardation condition, albeit one that ultimately is not tenable by the standard criteria of theory evaluation. This is the task of the next subsection; the example given there involves action-at-a-distance electrodynamics. An example of how to achieve a scientifically respectable, time asymmetric variant of orthodox electrodynamics that does not involve the resort to incantations of "causality" will be given in Section 2.8.

2.5 The retardation condition for particle theories

Because he advocated a particle theory of electrodynamics rather than a field theory, Ritz was able to express his conviction that "experience compels one to consider the representation by means of retarded potentials as the only one possible" in the form of a law of physics requiring only minimalist interpretation and no philosophy-speak (produce, cause, contribute, ...). But because Ritz's own theory was a bit of a mess and because the claims I want to make are general conceptual claims that are independent of details of Ritz's theory, I will illustrate them using a different and much cleaner toy theory that allows the relevant points to shine forth.

Consider a pure particle theory of classical electrodynamics, by which I mean that all of the basic variables of the theory are particle variables. Electromagnetic fields may be introduced but only in an auxiliary role for purposes of calculation. The goal is to illustrate how, in this setting, Ritz's retardation condition can be implemented in terms of a Poincaré covariant form of Newton's $\mathbf{F} = m\mathbf{a}$ governing the motion of a system consisting of a finite number N of charged particles. Here is one way to proceed. For each particle j calculate its (auxiliary) retarded Liénard-Wiechert field ${}_{ret}F^{\mu\nu}_{(j)}$. Then postulate that each particle k with worldline $z^{\mu}_{(k)}(\tau_{(k)})$, parameterized by proper time $\tau_{(k)}$, obeys the equation of motion

$$m_{(k)} a^{\mu}_{(k)} = q_{(k)} \sum_{j \neq k}^{N} {}_{ret}F^{\mu\nu}_{(j)} u_{(k)\nu} \tag{12}$$

where $m_{(k)}$ is the mass of particle k, $u^{\mu}_{(k)} = \dot{z}^{\mu}_{(k)} := dz^{\mu}_{(k)}/d\tau_{(k)}$ and $a^{\mu}_{(k)} = \ddot{z}^{\mu}_{(k)} := d^2 z^{\mu}_{(k)}/d\tau^2_{(k)}$ and are respectively the four-velocity and four-acceleration of particle k, and $q_{(k)}$ is the electric charge of particle k. Once the calculation of the auxiliary fields that appear on the right hand side of (12) is completed, (12) can be restated using only particle variables.

When the equations of this theory (call it T_1) are allowed to speak for themselves, they entail an EM arrow; for these equations are not time reversal invariant. The see why, compute the (auxiliary) electromagnetic fields associated with a finite system of charges obeying the equations of motion (12). The computed fields will satisfy the inhomogeneous Maxwell equations for the prescribed sources. Thus, the Kirchhoff representation theorem applies. When the volume Ω is chosen large enough to include all of the charges, ${}_{ret}F^{\mu\nu}(x)$ in (10a) will be $\sum_{all\ j}^{N} {}_{ret}F^{\mu\nu}_{(j)}(x)$ and ${}_{in}F^{\mu\nu}(x)$ will be zero at all x. Except in very special cases where the motions of the charges are time symmetric, $\sum_{all\ j}^{N} {}_{ret}F^{\mu\nu}_{(j)}(x)$ and $\sum_{all\ j}^{N} {}_{adv}F^{\mu\nu}_{(j)}(x)$ will not be equal and, thus, ${}_{ret}F^{\mu\nu}(x)$ in (10a) will not equal ${}_{adv}F^{\mu\nu}(x)$ in (10b), which means that, except for said special cases, ${}_{out}F^{\mu\nu}$ in (10b) will not be zero everywhere. But under time reversal, ${}_{ret/adv}F^{4\nu}_{(j)}(x') \rightarrow {}_{adv/ret}F^{4\nu}_{(j)}(x)$ and ${}_{ret/adx}F^{mn}(x') \rightarrow -\ {}_{adv/ret}F^{mn}(x)$ with $x = (\mathbf{x}, t)$, $x' = (\mathbf{x}, -t)$, and and similarly for ${}_{in/out}F^{\mu\nu}$. This implies that if ${}_{in}F^{\mu\nu} \equiv 0$ in a dynamically possible history, then ${}_{out}F^{\mu\nu} \equiv 0$ in the time reversed history. Thus, a contradiction $({}_{out}F^{\mu\nu} \equiv 0$ and $\neg({}_{out}F^{\mu\nu} \equiv 0))$ would result if time reversal invariance held.

An equally time reversal non-invariant theory (T_2) would be produced by substituting advanced for retarded interactions of the charges, yielding the rival equations of motion

$$m_{(k)}a^{\mu}_{(k)} = q_{(k)} \sum_{j \neq k}^{N} {}_{adv}F^{\mu\nu}_{(j)} u_{(k)\nu} \tag{13}$$

where ${}_{adv}F^{\mu\nu}_{(j)}$ is the advanced Liénard-Wiechert field of particle j. This theory would be the embodiment of an anti-Ritz principle of advanced action.

Or, following Fokker (1929) and Wheeler and Feynman (1945, 1949), a time reversal invariant theory (T_3) can be produced by using a symmetric combination of retarded and advanced interactions of the charges, resulting in the equations of motion

$$m_{(k)}a^{\mu}_{(k)} = q_{(k)} \sum_{j \neq k}^{N} \frac{1}{2}[\ {}_{ret}F^{\mu\nu}_{(j)} \ + \ {}_{adv}F^{\mu\nu}_{(j)}]u_{(k)\nu} \tag{14}$$

$T_1 - T_3$ are distinct theories that make distinct predictions. Since they embody respectively the principles of retarded action, advanced action, and symmetric retarded-plus-advanced action, these principles are seen to have real theoretical and empirical bite in the setting of pure particle theories of electrodynamics. Choosing among these theories is not a matter to be settled by appeal to considerations of "causality" or to intuitions about counterfactual conditionals but rather by appeal to the standard criteria of theory evaluation—empirical adequacy and theoretical fruitfulness.

Unfortunately, all three of these theories are seen to be wanting by the lights of the standard criteria. One of Einstein's (1909) sharpest criticisms of Ritz's theory was that it does not uphold the validity of the "energy principle." The criticism applies to any pure particle theory in the setting of Minkowski spacetime: without a field to mediate the interactions of the particles, the conservation of energy-momentum cannot hold in the usual form as a statement about the constancy of the instantaneous values of energy-momentum. A sharp form of this negative result can be found in van Dam and Wigner (1966): compute the kinetic energy and the linear momentum of each of the particles in some inertial coordinate system $(\mathbf{x}, t)$; require that the sum of the computed energies and the sum of the computed momenta are each the same for all t; then Poincaré covariance entails that there can be no interaction among the particles, i.e., the particle world lines are geodesics of Minkowski spacetime. This is not a fatal objection to relativistic pure particle theories. In some such theories it may be possible to maintain a weakened, asymptotic version of the instantaneous form of conservation of energy-momentum as a statement about the equalities of the sums of the energies and momenta of the particles as $t \to \pm\infty$. Alternatively, conservation of energy-momentum may be expressible in a integral form rather than an asymptotic form (see van Dam and Wigner 1966).

A more troubling problem with pure particle theories is specific to electrodynamics; namely, such theories do not offer any natural explanation of the phenomenon of the radiation reaction experienced by accelerating charges. Wheeler and Feynman's (1945, 1949) heroic attempt to provide an explanation in their action-at-a-distance electrodynamics uses an argument that is of dubious validity and that relies on an "absorber condition" which may well fail for the actual universe; and even if the argument goes through, it cannot explain the observed asymmetry of radiation reaction in flat spacetime. And, finally, in curved spacetime Wheeler-Feynman action-at-a-distance electrodynamics does not produce the same radiation reaction force as standard classical electrodynamics even if the absorber condition holds. These points will be taken up in Sections 3.4 and 3.6 respectively.

A third strike against the pure particle theories comes from the criterion of theoretical fruitfulness. The best current theory we have of electrodynamics is a quantum field theory—QED—which arises as the quantization of a classical field theory—Maxwell's theory—rather than the quantization of a particle theory. Philosophers have an obsessive fascination with the Wheeler-Feynman theory. They do not balance this obsession with the remark that one of the authors of this theory—Feynman—was also a principal architect of QED. A large part of the motivation for the Wheeler-Feynman theory was the desire to avoid the infinities that arise in classical theories with a mixed ontology involving particles that create fields that act back on the particles. Wheeler and Feynman explored the escape route of eschewing fields in favor of a pure particle ontology. But the other route is to promote the field concept and demote the particle concept. Arguably, this is exactly what relativistic quantum field theory does by treating local fields as the basic entities and explaining particle-like behavior in terms of the behavior of the fields (see Wald 1994). The infinities that arise in the classical theory of electrodynamics formulated in terms of a mixed particle-field ontology are cured by the quantum field theory treatment, at least in the sense that QED is a renormalizable theory.

Despite the fact that theories $T_1 - T_3$ are found wanting, I repeat that they serve to make the conceptual point that a Ritzian retardation condition has a clear meaning and function in a pure particle theory. That this is not so for standard classical electrodynamics is emphasized in the next section.

2.6 The retardation condition for field theories

After the excursion into particle theories, I return to orthodox classical relativistic electromagnetism. How might someone who insists on trying to find some role in this context for Frisch's neo-Ritzian retardation condition—the condition that each charge "physically contributes" a fully retarded component to the total field—proceed? For a system consisting of a finite number N of charged particles, the closest analogue $\bar{T}_1$ for the retarded action-at-a-distance theory T_1 of the preceding section would consist of the conjunction of Maxwell's laws plus the posit

$$F_1^{\mu\nu}(x) = \sum_{j=1}^{N} {}_{ret}F_{(j)}^{\mu\nu}(x) + {}_{\rm hom}F_1^{\mu\nu}(x) \qquad (15)$$

where $F_1^{\mu\nu}$ stands for the (total) electromagnetic field in a physically possible history, the ${}_{ret}F_{(j)}^{\mu\nu}$ are the retarded Liénard-Wiechert fields of the particles, and ${}_{\rm hom}F_1^{\mu\nu}$ is a homogeneous solution of Maxwell's equations. Allowance for the homogeneous solution must be made on pain of restricting the range of validity of the theory.

Those who insist, contra $\bar{T}_1$, that each charge physically contributes a fully advanced component to the total field will endorse $\bar{T}_2$, consisting of Maxwell's equations plus the posit

$$F_2^{\mu\nu}(x) = \sum_{j=1}^{N} {}_{adv}F_{(j)}^{\mu\nu}(x) + {}_{\rm hom}F_2^{\mu\nu}(x) \qquad (16)$$

where the ${}_{adv}F_{(j)}^{\mu\nu}$ are the advanced Liénard-Wiechert fields of the particles. And those who insist that each charge physically contributes a half-retarded-half advanced field to the total field will endorse $\bar{T}_3$, consisting of Maxwell's equations plus the posit

$$F_3^{\mu\nu}(x) = \frac{1}{2}\sum_{j=1}^{N}[\, {}_{ret}F_{(j)}^{\mu\nu}(x) \; + \; {}_{adv}F_{(j)}^{\mu\nu}(x)] + {}_{\rm hom}F_3^{\mu\nu}(x) \qquad (17)$$

But how exactly are the proponents of these three theories $\bar{T}_1 - \bar{T}_3$ disagreeing? Measurements can in principle fix the actual value ${}_{act}F^{\mu\nu}(x)$ of the electromagnetic field at every spacetime location x, and by the Kirchoff theorem the homogeneous solutions in (16) − (18) can be chosen so that $F_1^{\mu\nu}(x) = F_2^{\mu\nu}(x) = F_3^{\mu\nu}(x) = {}_{act}F^{\mu\nu}(x)$.[11] With this choice the three theories $\bar{T}_1 - \bar{T}_3$ will agree on experimental outcomes even if we allow ourselves access to the results of thought experiments about what would happen if a hypothetical test charge were used to probe the value of the field. Indeed, in contrast to $T_1 - T_3$, which undoubtedly are distinct theories, $\bar{T}_1 - \bar{T}_3$ seem more like different modes of presentation of the same theory—Maxwell's theory written in different ways and anointed with different philosophy-speak about "physically contributes."

Nevertheless, depending on the circumstances, one mode of presentation may seem more pleasing than the others. Consider, for example, the case of a single charged particle that has been in inertial motion from time immemorial to the present and then is set into hyperbolic

[Insert Fig. 1 here]

motion by a non-electromagnetic force (see Fig. 1). [Hyperbolic motion (a.k.a. uniform acceleration) means that $\dot{a}^\mu := u^\nu \nabla_\nu a^\mu = a^\beta a_\beta u^\mu$ where u^μ is the (normed) four-velocity of the particle. Differentiating $u^\mu u_\mu = -1$ gives $a^\mu u_\mu = 0$. Using this fact, hyperbolic motion is seen to imply that $\dot{a}^\mu a_\mu = 0$ and that the magnitude of acceleration $a := (a^\mu a_\mu)^{1/2}$ is constant.] Suppose that at all spacetime locations x, ${}_{act}F^{\mu\nu}(x)$ is given by the retarded Liénard-Wiechert field of the charge. Then theory $\bar{T}_1$ provides a simple description that comes from setting ${}_{\mathrm{hom}}F_1^{\mu\nu}$ in (15) to zero, i.e. $\bar{T}_1$

simply has to be supplemented by the condition that there is no incoming source-free radiation. Consider, by contrast, the description that $\bar{T}_2$ provides. For any spacetime location x in the sectors I and IV of Fig. 1 the advanced Liénard-Wiechert field of the particle is identically zero. Thus, $\bar{T}_2$ must invoke a homogeneous solution ${}_{\text{hom}}F_2^{\mu\nu}(x)$ that, for x in sectors I and IV, exactly mimics the retarded field Liénard-Wiechert of the particle. In addition, the advanced Liénard-Wiechert field of the particle involves a delta-function field on the null surface separating sectors I and IV from II and III (see Boulware 1980), so to reproduce the hypothesized total field, ${}_{\text{hom}}F_2^{\mu\nu}$ must be arranged to cancel out this delta-function field. These features of $\bar{T}_2$'s description seem contrived. And for similar reasons $\bar{T}_3$'s description will also seem contrived.[12]

But note that in the hypothesized case no guardian angel of Maxwell's theory is needed to step in to generate the ${}_{\text{hom}}F_2^{\mu\nu}$ required by $\bar{T}_2$; for as long as we are not contemplating contra-nomological scenarios, Kirchhoff's representation theorem guarantees the existence of the required ${}_{\text{hom}}F_2^{\mu\nu}$. And again, baring magic meters, only the total field—in this case, ${}_{act}F^{\mu\nu} = {}_{ret}F^{\mu\nu} = {}_{adv}F^{\mu\nu} + {}_{\text{hom}}F_2^{\mu\nu}$—is measurable. Furthermore, the circumstances are the tail that wags the dog, for with a reversal of circumstances comes a reversal of fortunes of the theories.

[Insert Fig. 2 here]

Consider the time reversed motion of the particle in the above scenario (see Fig. 2)[13], and suppose that at all spacetime locations x, ${}_{act}F^{\mu\nu}(x)$ is given by the advanced Liénard-Wiechert field of the particle. Now $\bar{T}_2$'s description will seem natural while $\bar{T}_1$'s will seem contrived. And between the extremes of these two hypothetical cases—the one favoring $\bar{T}_1$, the other favoring $\bar{T}_2$—are all the messy cases where no one of the theories $\bar{T}_1$, $\bar{T}_2$, or $\bar{T}_3$ offers a markedly more simple description.

Moving from the hypothetical to the actual, one can ask: Is it in fact the case in the actual world that the retarded representation is always (mostly, typically, ...) simpler and more natural than the advanced or mixed representations? And if so, what is the explanation of the asymmetry? These and related issues will be taken up in the following subsection. But before turning to these issues there is some unfinished business generated by the present discussion.

The alert reader will have noticed a gap in the above analysis which concentrates exclusively on fields. When it comes to the motion of charged particles, $\bar{T}_1 - \bar{T}_3$ do seem to yield different results, at least if the Lorentz force law is operative and if the Lorentz force on a charge is computed from the fields due to the *other* charges plus the source free field. For then the electromagnetic force acting on charge k at point x on k's world line, according respectively to $\bar{T}_1 - \bar{T}_3$, is

$$q_k[\sum_{j\neq k} {}_{ret}F^{\mu\nu}_{(j)}(x) + {}_{\text{hom}}F^{\mu\nu}_1(x)]u_{(k)\nu} \tag{18}$$

$$q_k[\sum_{j\neq k} {}_{adv}F^{\mu\nu}_{(j)}(x) + {}_{\text{hom}}F^{\mu\nu}_{2}(x)]u_{(k)\nu} \tag{19}$$

$$q_k[\sum_{j\neq k} \frac{1}{2}({}_{ret}F^{\mu\nu}_{(j)}(x) \ + \ {}_{adv}F^{\mu\nu}_{(j)}(x)) + {}_{\text{hom}}F^{\mu\nu}_{3}(x)] \tag{20}$$

In general these forces are not equal and, thus, there would seem to be a decisive test to decide which of $\bar{T}_1 - \bar{T}_3$ is the true theory. However, any attempt to carry out such a crucial experiment would end in failure, for none of the equations of motion $(18) - (20)$ is empirically adequate since they all neglect the radiation damping force experienced by accelerating charges.[14]

But if they are unable to win by dint of a crucial experiment, the ever resourceful neo-Ritzians claim to find a new purchase in radiation damping since, they claim, this phenomenon requires a posit of retarded action. This matter will be taken up in Section 3.5.

2.7 Some electromagnetic arrows, real and alleged

One of the most oft cited EM arrows involves spherical electromagnetic waves: we commonly experience such waves diverging from a center, but rarely if ever do we experience such waves converging on a center. This innocent seeming dictum disguises a number of potential misunderstandings and pitfalls. In the first place it is essential to distinguish collective from individual phenomena. As an instance of the former, consider an antenna in which the electrons are induced to oscillate in unison, producing an outgoing radio signal, all parts of which are eventually absorbed. The time reverse of this process, in which the materials that played the role of absorbers in the original scenario now emit in a coordinated fashion so that the antenna receives an anti-broadcast signal, is never experienced, save for contrived situations which are hard to engineer except on small scales.

This is undoubtedly a real asymmetry, but it does not reveal anything novel about electromagnetism *per se* since analogous asymmetries are common to water waves and sound waves. The predominant view about such asymmetries is in line with that of Einstein in the Einstein-Ritz debate; namely, the asymmetries are to be traced not to a failure of time reversal invariance of a fundamental law but to statistical considerations that are of a piece with those that lie at the origin of the thermodynamic arrow and other arrows involving collective phenomena of non-charged matter. As mentioned above, the currently favored way of implementing this approach in order to explain the thermodynamic arrow is to supplement classical statistical mechanics with the posit of a low Boltzmann entropy state for the very early universe ("past hypothesis").[15] Exactly how the thermodynamic arrow links to the EM arrow under discussion is an important issue that will not be tackled here.[16]

Notice that for the asymmetry just discussed, nothing about retarded vs. advanced fields or representations need enter the discussion. The key question is simply whether the actual total field ${}_{act}F^{\mu\nu}$ is in the form of an outgoing or an incoming wave. Whether a retarded or an advanced decomposition of

${}_{act}F^{\mu\nu}$ is preferable is a side issue that might, but need not be, raised.[17]

This situation appears to change when the focus shifts from the collective phenomena associated with groups of charges to the phenomena associated with individual charges, for the typical way of describing the latter invokes retarded and advanced fields. The retarded field of a charge corresponds (it is said) to emission or a wave spreading from the charge, whereas the advanced field of the charge corresponds to absorption or a wave collapsing on the charge. Since (it is said) we experience waves spreading from a charge but not waves collapsing on a charge, the impression is left that an explanation of the asymmetry between incoming and outgoing radiation calls for a quashing of advanced fields. A number of clarifications and cautions need to be attached to these dicta if a muddle is to be avoided.

The retarded Liénard-Wiechert field for a point charge in Minkowski spacetime can be covariantly separated into a velocity (or generalized Coulomb) field ${}_{ret}F^{\mu\nu}_{Coul}$ and an acceleration (or radiation) field ${}_{ret}F^{\mu\nu}_{rad}$. The velocity field is so-called because it is independent of the acceleration of the charge. In the instantaneous rest frame of the charge, the retarded velocity field takes the familiar form of the Coulomb field found in elementary textbooks, and in a moving frame it is the Lorentz transform of the rest Coulomb field. The strength of this field falls off inversely as d_{-}^{2}, where d_{-} is the spatial distance between the field point and the retarded position of the charge as measured in the Lorentz frame in which the charge is instantaneously at rest at the retarded time. An analogous story holds for the advanced Liénard-Wiechert velocity or Coulomb field with the distance d_+ between the field point and advanced position of the charge in place of the retarded distance d_-. So far, nothing about outgoing or incoming waves.

That interpretation comes from the acceleration or radiation part of the Liénard-Wiechert field, which depends linearly and homogeneously on the acceleration of the charge. The spacetime support of the retarded radiation field of a point charge (and, thus, of the stress-energy tensor[18] calculated from this field) consists of a sequence of future null cones whose vertices lie on the world line of the charge and coincide with those points at which the charge is accelerating. The value of the retarded radiation field at a point on one of these future null cones is inversely proportional to the spatial distance d_- to the retarded position of the charge. Similarly, the advanced radiation field (and, thus, the stress-energy tensor calculated form this field) of a point charge has support on a series of past null cones whose vertices lie on the world line of the charge and coincide with those points at which the charge is accelerating, and the value of the field at point on one of these past null cones is inversely proportional to d_+.

So far so good. But what exactly is the observed asymmetry here? Let us agree to use "Observe" to indicate a liberal sense of observation that includes not only the data gathered from the immediate deliverances of our senses or measuring instruments but also inferences drawn from this data by means of Maxwell's equations and the background theories of the measuring instruments. Is it the case that we commonly observe radiation diverging from an electron but never (or hardly ever) Observe radiation collapsing on an electron? There are two senses of the latter question that need to be distinguished. The first is: Do we never (or hardly ever) Observe cases of an actual total field ${}_{act}F^{\mu\nu}$ that is collapsing on an electron? I don't know that the answer is positive, but nothing absurd results from granting that we do. The second sense of the question is: Do we never (or hardly ever) Observe the advanced Liénard-Wiechert radiation field ${}_{adv}F^{\mu\nu}_{rad}$ of an electron? No such Observation is possible under conditions where the Kirchhoff theorem is valid since otherwise the advanced representation would be invalidated. The only non-muddled message about advanced and retarded fields that could come out of a positive answer to the first question is not that advanced Liénard-Wiechert radiation fields have to quashed but only that it simpler to use the retarded representation of the actual total field ${}_{act}F^{\mu\nu}$ to describe the hypothesized Observations.

These cautionary remarks designed to ward off muddles about advanced/retarded fields do nothing positive to sharpen and explain the asymmetry of radiation reaction of accelerating charges. That job will be tackled below in Section 3. In anticipation it is worth remarking here that there is a reason not to try to quash or ignore the advanced Liénard-Wiechert field of a charge. As we will see in Section 3.4 below, the Dirac expression for the radiation damping force experienced by accelerating charges involves evaluating at the world line of the charge the difference between the retarded and advanced Liénard-Wiechert fields of the charge, offering an explanation of the origin of the radiation reaction force. A positive value for the difference indicates that more energy-momentum is radiated away by the charge than is absorbed, and energy-momentum balance requires a compensating damping force. An immediate implication—accepted as correct on all analyses of radiation reaction—is that a charged particle that is (always) uniformly accelerated does *not* experience a damping force since the difference between the advanced and retarded Liénard-Wiechert fields is zero on the world line of the charge.

With these lessons in mind in mind, let us turn to other attempts in the literature to specify some electromagnetic asymmetries.

One is posed in the form of a question by Zeh (2001: 21):

> Why does the condition ${}_{in}F^{\mu\nu} = 0$ (in contrast to ${}_{out}F^{\mu\nu} = 0$) approximately apply in most situations?[19]

Here "situation" refers to some local system, and ${}_{in}F^{\mu\nu}$ and ${}_{out}F^{\mu\nu}$ are fields corresponding respectively to the potentials ${}_{in}A^{\nu}(x)$ and ${}_{out}A^{\nu}(x)$ in (9a)-(9b) evaluated for domains of integration appropriate to the local system.

The most obvious difficulty with this formulation concerns the "most" qualification. It would seem that in a natural sense of 'most', ${}_{in}F^{\mu\nu}$ is not approximately zero in the visible part of the electromagnetic spectrum for most of the systems of which we are aware, since otherwise we would not be aware of them. And the ubiquity of the cosmic background radiation makes one think that in a natural sense of 'most', ${}_{in}F^{\mu\nu}$ is not approximately zero in the microwave spectrum for most systems, whether we are aware of them or not (see North 2003).

Seeking to preserve the idea behind Zeh's radiation asymmetry, Frisch (2005: 108) reformulates it as

> (RADASYM) There are many situations in which the total field can be represented as being approximately equal to the sum of the retarded fields associated with a small number of charges (but not as the sum of the advanced fields associated with the charges), and there are almost no situations in which the total field can be represented as being approximately equal to the sum of the advanced fields associated with a small number of charges.[20]

The counterexamples to Zeh's formulation are avoided by shifting from 'most' to the vaguer 'many' and by restricting attention to systems consisting of a small number of charges. The condition that ${}_{in}F^{\mu\nu} \approx 0$ can be assured by using an absorber to keep the incoming radiation from impinging on the system of interest and by noting that, per Frisch's stipulation, the absorber (which necessarily uses many charges) cannot be regarded as part of the system. From ${}_{in}F^{\mu\nu} \approx 0$ and Kirchhoff's representation theorem it follows that ${}_{out}F^{\mu\nu} \approx {}_{adv}F^{\mu\nu} - {}_{ret}F^{\mu\nu}$, where ${}_{ret}F^{\mu\nu}$ and ${}_{adv}F^{\mu\nu}$ are respectively the sums of the retarded and advanced Liénard-Wiechert fields associated with the charges of the system. The explanation of why ${}_{out}F^{\mu\nu}$ is typically not approximately zero in this situation (and, thus, why the total field cannot be represented purely as the sum of the advanced Liénard-Wiechert fields associated with the charges) is then quite straightforward: the result follows from the fact that typically ${}_{ret}F^{\mu\nu} - {}_{adv}F^{\mu\nu}$ is not approximately zero. This fact follows in turn from a remark in the Introduction; namely, typical solutions of time reversal invariant laws are not time symmetric. And so it is in electromagnetism: typically the motions of the charges in the system are not time symmetric, which implies that typically the retarded and advanced Liénard-Wiechert fields of the charges will not be equal. For a charge that is always and forever in hyperbolic motion (see Fig. 3), the retarded and advanced Liénard-Wiechert fields of the charge are the same near the charge and, indeed, throughout sector II to which the charge is confined (see Boulware 1980). But this is the exception that proves the rule.

[Insert Fig. 3 here]

Buried in this explanation sketch is a suppressed premise about collective behavior. The assurance that the system at issue is shielded from incoming radiation is merely probabilistic, for there is no inconsistency with the laws of electromagnetism that the material intended to function as an absorber in fact radiates into the system it was intended to shield.[21] The improbability of such behavior is presumably of a piece with the improbability of the anti-thermodynamic behavior of non-charged matter. The need to refer to such behavior is even more evident in the more general case where an absorber in not used to keep incoming radiation from impinging on the system. Here

${}_{out}F^{\mu\nu} = [{}_{adv}F^{\mu\nu} - {}_{ret}F^{\mu\nu}] + {}_{in}F^{\mu\nu}$ with the last term on the right hand side typically being non-zero. That ${}_{out}F^{\mu\nu}$ is typically not approximately zero in the general case is partly explained by Frisch's definitional move: an absorber that captures ${}_{out}F^{\mu\nu}$ cannot be considered to be part of the system which, by definition, contains only a small number of charges. But the explanation in the general case must also invoke the improbability of the incoming radiation being configured so as to cancel out ${}_{adv}F^{\mu\nu} - {}_{ret}F^{\mu\nu}$ (which, by the same argument as in the special case, is typically non-zero).

Frisch (2005) does not reject outright these explanation sketches of RADASYM but he favors an explanation in which "the brunt of the explanatory work is done by the retardation condition—the assumption that the field physically contributed by a charge is fully retarded" (152). My skepticism about explanations which do not allow the theory to speak for itself and which require philosophy-speak (causes, physically contributes, ...) can be given concrete form in the present instance: if the explanation sketches offered above are on the right track, then the philosophy-speak of the retardation condition not only is not needed but it covers up the need to fill in the details of the sketches by specifying the nature and source of the improbabilities involved.

The sorts of asymmetries to which Zeh and Frisch point have two disturbing features. First, they are vague and hedged, requiring qualifiers like 'most', 'many', 'typically', or 'approximately' to ward off counterexamples. Second, these asymmetries are formulated in terms of the quantities ${}_{ret}F^{\mu\nu}$, ${}_{in}F^{\mu\nu}$, ${}_{adv}F^{\mu\nu}_{out}$, and ${}_{out}F^{\mu\nu}$ as evaluated for local systems; and as what counts as "the system" is expanded or contracted and, thus, as the domains of integration $\Omega^{\pm}$ and $\partial\Omega^{\pm}$ implicit in these quantities change, the asymmetries can come and go. This is not to say that the vagueness and the relativity of the asymmetries means that they are not genuine asymmetries. But it does underscore the need to come to grips with an issue noted in the Introduction. Even though Maxwell's equations are time reversal invariant, one would expect that among the solutions of these equations, the subset of time symmetric solutions is "measure zero" (in a sense that needs to be made precise). Which of the unlimited number of temporal asymmetries that are present in a generic solution should be viewed as interesting and fundamental enough to merit being promoted to "arrows of time" and to merit the search for an explanatory account? The comparison with thermal physics is revealing. The asymmetries encapsulated in the Second Law of thermodynamics certainly do merit promotion to the status of thermodynamic "arrows": they are systematic and pervasive; and although there are exceptions—the use of a microscope of resolving power great enough to observe Brownian motion will reveal some of them—no Maxwell's demon that produces at will macro-scale exceptions has ever been constructed and, arguably, could ever be constructed. Of the EM asymmetries discussed in this section, the only one comes close to matching these features of the thermodynamic arrow is the non-muddled version of expanding vs. contracting waves. But this version involves collective behavior of emitters and absorbers, and as such it does not reveal a distinctively electromagnetic arrow; indeed, the most plausible line of research is to try to explain this arrow in terms of the same considerations that explain temporal asymmetries in the collective behavior of non-charged matter.

One obvious idea for finding cleaner and more robust electromagnetic asymmetries would be to look for asymmetries that are independent of the choice of the volume Ω. One safe and sure way to guarantee such independence is to formulate the asymmetries in terms of the limiting behavior of ${}_{in}F^{\mu\nu}$ and ${}_{out}F^{\mu\nu}$ as Ω is enlarged to encompass all of spacetime. Perhaps the asymmetries that emerge in this limit will reveal an arrow of time that is peculiar to electromagnetism. To return to the opening example of this section, suppose that the radio antenna broadcasts into empty space so that the outgoing radio waves are not absorbed but travel to spatial infinity. It would seem nearly miraculous if the time reverse of this scenario were realized in the form of anti-broadcast waves coming in from spatial infinity and collapsing on the antenna. The absence of such near miracles might be explained by an improbability in the coordinated behavior of incoming source free radiation from different directions in space. Or it might be explained non-probabilitsically by a prohibition against any truly source free incoming radiation. The latter is one motivation for the Sommerfeld radiation conditions.

2.8 Sommerfeld radiation conditions for Minkowski spacetime

There is no *a priori* guarantee that the volume and surface integrals in the Kirchhoff representation $(9a) - (9b)$ have well defined limits when Ω tends to the entire spacetime. But suppose for sake of discussion that the limits (denoted by a superscript ∞) exist. Then ${}^{\infty}_{in}F^{\mu\nu}$ gives the truly source free incoming radiation. The retarded Sommerfeld radiation condition is the statement that

$$ {}^{\infty}_{in}F^{\mu\nu} = 0 \tag{21} $$

If this condition is treated as an additional law of electromagnetism, then reduced set models of classical electromagnetism is not closed under time reversal. The proof is just a variant of the argument given in Section 2.5 to show that the Ritzian theory T_1 is not time reversal invariant. Here then is the promised example of how to achieve a time asymmetric variant of orthodox classical electromagnetism without using incantations about causation.

Similarly, if Maxwell's theory is augmented not by postulating (21) as an extra law but rather by postulating the advanced Sommerfeld radiation condition

$$ {}^{\infty}_{out}F^{\mu\nu} = 0 \tag{22} $$

then again the resulting theory is not time reversal invariant. If both (21) and (22) are promoted to the status of extra laws, then the resultant theory is time reversal invariant.

A motivation for promoting (21) (respectively, (22)) to law status might stem from the conviction that electromagnetic fields must have sources (respectively, must have sinks). It is hard to credit such convictions if they are supposed to stand alone. Buttressing for these convictions might come from the further conviction that electromagnetic fields are merely mathematical devices for describing what are actually direct interactions among charges. But this is a view that was rejected in Section 2.5. A further problem for motivating the promotion of (21) (or (22)) to law status is that, in conjunction with Maxwell's laws, the posit of (21) (or (22)) as a *de facto* condition that holds in the actual universe would seem to do just as much work in explaining features of the actual universe as positing (21) (or (22)) as an additional law.

Continuing this line of thought, one can wonder whether (21) or (22)—qua laws or qua *de facto* truths—can do any useful explanatory work at all with regard to the local electromagnetic asymmetries alluded to by Zeh and Frisch (Section 2.7). These asymmetries concern the behavior of systems that typically comprise small finite chunks of the universe; and for such systems asymmetries between ${}_{in}F^{\mu\nu}$ and ${}_{out}F^{\mu\nu}$ need not have anything to do with asymmetries between ${}^{\infty}_{in}F^{\mu\nu}$ and ${}^{\infty}_{out}F^{\mu\nu}$ since, for example, ${}_{in}F^{\mu\nu}$ may not represent truly source free incoming radiation but rather radiation associated with sources outside the volume Ω that is implicit in the expression for ${}_{in}F^{\mu\nu}$.

I will return below to the last point. But even if the point is well taken, there are three reasons why it is worth inquiring about ${}^{\infty}_{in}F^{\mu\nu}$ and ${}^{\infty}_{out}F^{\mu\nu}$. First, an asymmetry between ${}^{\infty}_{in}F^{\mu\nu}$ and ${}^{\infty}_{out}F^{\mu\nu}$ would give a clean and distinctively electromagnetic asymmetry; second, this asymmetry may admit linkages with cosmological arrows; and third, even if an asymmetry between ${}^{\infty}_{in}F^{\mu\nu}$ and ${}^{\infty}_{out}F^{\mu\nu}$ is irrelevant to the Zeh-Frisch asymmetries, it may be relevant to explaining the asymmetry of radiation reaction for accelerated charges. The first two reasons will be taken up in the immediately succeeding subsections, and the third will be discussed in Section 3.5.

2.9 Sommerfeld radiation conditions in cosmology

The threshold issue to be faced in discussing the Sommerfeld radiation conditions in cosmology is that the Kirchhoff representation theorem in the form $(9a) - (9b)$ for Minkowski spacetime does not generalize in any straightforward way to arbitrary cosmological models.[22] If the spacetime is not globally hyperbolic[23] the global existence and uniqueness of advanced and retarded Green's function is not assured, and without this assurance the theorem does not get off the ground. (Restricting to a globally hyperbolic neighborhood[24] does not suffice for present purposes since the Sommerfeld radiation conditions are global in nature.) Next, a condition is needed to assure that the electromagnetic field propagates cleanly without "tails" that trail along behind the wave front (as would be the case if the Green's functions do not vanish in the interior of the light cones).[25] For this assurance it is sufficient that the (four-dimensional) spacetime be conformally flat.[26] The FRW cosmological models currently used to describe the large scale structure of the actual universe provide examples of spacetimes that are both globally hyperbolic and conformally flat.

Suppose then that we are working in a cosmological model where the Kirchhoff representation of the electromagnetic field holds in its usual form, and suppose that the volume and surface integrals in the retarded and advanced Kirchhoff representations have finite limits as the volume Ω tends towards the entire spacetime of the model. Nevertheless, it may not be physically consistent to impose the retarded Sommerfeld condition (21) (respectively, the advanced Sommerfeld condition (22)) if the model has particle horizons (respectively, event horizons).[27] The standard hot big bang models (which belong to the family of FRW models) have particle horizons, regardless of whether the model contains an early inflationary epoch and regardless of whether $k = 0$ (flat space sections), $k = +1$ (space sections of constant positive curvature), or $k = -1$ (space sections of constant negative curvature). Furthermore, there is strong evidence that "dark energy" is driving the currently observed accelerated expansion of the universe (see Carroll 2004), and if this "dark energy" is due to a positive cosmological constant, the future asymptotic structure of spacetime becomes that of de Sitter spacetime, which has event horizons.

To see the difficulty with the retarded Sommerfeld radiation condition in the presence particle horizons, consider the toy example of past truncated Minkowski spacetime (see Fig. 4a) where all the spacetime points on or below some $t = const.$ hypersurface (with t an inertial time coordinate) have been deleted. Particle #2 is outside of #1's particle horizon at x, and so at x the retarded volume integral in $(10a)$ will be zero no matter how large the volume Ω is taken to be. Nevertheless, at x particle #1 will feel particle #2's Coulomb field[28] and, thus, $(10a)$ implies that ${}_{in}F^{\mu\nu}(x) \neq 0$.[29] Analogously, in future truncated Minkowski spacetime illustrated in Fig. 4b the advanced Sommerfeld radiation condition cannot be

[Insert Figs. 4a and 4b here]

satisfied. Particle #4 is outside of particle #3's event horizon at z, and so at z the advanced volume integral in $(10b)$ will be zero no matter how large the volume Ω is taken to be. Nevertheless, at z particle #3 will feel particle #4's Coulomb field and, thus, $(10b)$ implies that ${}_{out}F^{\mu\nu}(z) \neq 0$.

Aichelberg and Beig (1977) have studied the compatibility between Sommerfeld type radiation conditions and cosmological structure for an interesting toy model involving a one-dimensional, non-relativistic harmonic oscillator coupled to a scalar field Φ. Because the model is completely soluble, Aichelberg and Beig are able to determine whether there are solutions of the coupled oscillator-field equations of motion[30] with ${}^{\infty}_{in}\Phi = 0$, ${}^{\infty}_{out}\Phi = 0$, or ${}^{\infty}_{in}\Phi + {}^{\infty}_{out}\Phi = 0$ for a range of different $k = 0$ (flat space sections) FRW cosmologies. The line element in these cosmologies takes the form

$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a(t)(dx^2 + dy^2 + dz^2) - dt^2$, where $a(t)$ is the scale factor. In expanding Friedmann models for which $a(t) \sim t^n$ and $n < 1$, and also in expanding models for which $a(t) \sim t^n$ and $n \geq 1$, Aichelberg and Beig found that it consistent to impose the condition ${}^{\infty}_{in}\Phi = 0$ but not ${}^{\infty}_{out}\Phi = 0$ or ${}^{\infty}_{in}\Phi + {}^{\infty}_{out}\Phi = 0$. In the time reversed contracting versions of these cosmological models the asymmetry is reversed. But in the cases of an expanding de Sitter models with $a(t) \sim \exp(t/\alpha)$, α a positive constant, and also in Minkowski spacetime with $a(t) \sim 1$, it is not consistent to impose any of these conditions.

To return to electromagnetism, the fact that, say, ${}^{\infty}_{in}F^{\mu\nu} = 0$ is incompatible with a cosmological model does not entail that this cosmology does not enforce an interesting asymmetry between ${}^{\infty}_{in}F^{\mu\nu}$ and ${}^{\infty}_{out}F^{\mu\nu}$ but only that the asymmetry cannot be stated in the most obvious way, e.g. ${}^{\infty}_{in}F^{\mu\nu} = 0$ while ${}^{\infty}_{out}F^{\mu\nu} \neq 0$. The most dramatic such cosmologically enforced asymmetry would be a case where, say, the volume and surface integrals of the retarded representation have finite limits as the volume Ω tends to the entire spacetime of the considered cosmological model, whereas in the advanced representation the volume and/or surface integrals diverge. Sciama (1963) has worked out the details of just such a case for an expanding Einstein-de Sitter model, which is a $k = 0$ FRW model with $a(t) \sim t^{2/3}$. For a plausible distribution of sources, Sciama found that both the volume and surface integrals in the advanced Kirchhoff representation diverge as Ω tends to the entire spacetime. By contrast, the retarded volume integral converges and, thus, the demand for a finite solution means that the amount of incoming source free radiation is such that the retarded surface integral converges. Discounting the possibility that the divergences in the advanced volume and surface integrals cancel to give a finite sum, the demand for a finite solution uniquely singles out the retarded representation from the general class of representations (11). This asymmetry is reversed in the time reversed contracting Einstein-de Sitter model, emphasizing that the EM asymmetry at issue is enslaved to the cosmological arrow.

That the cosmological based electromagnetic asymmetries may well be independent from the local electromagnetic asymmetries discussed in Section 2.7 does not show that the former are not interesting nor that they are not fundamental. By way of analogy, the failure of time reversal invariance that has been demonstrated for neutral kaon decay[31] does not seem to be connected with any of the more familiar arrows of time. But arguably this failure is of fundamental importance not only in providing a fundamental lawlike asymmetry *in* time but also in providing a basis for an asymmetry *of* time by supporting the conclusion that the actual universe is time orientable and possesses a time orientation (see Earman 2002).

3 Radiation damping

3.1 From the well-defined to the not-so-well-defined

Section 2 was concerned with a well-defined situation; viz. a charge-current distribution J^{μ}, satisfying $\nabla_{\mu} J^{\mu} = 0$, is specified, and one seeks to find the corresponding solution to the Maxwell equations. One could add a second well-defined problem: an external electromagnetic field ${}_{ext}F^{\mu\nu}$ and a non-electromagnetic force ${}_{nem}F^{\mu}$ are specified, and one seeks to find the worldline $z^{\mu}(\tau)$ of a point mass carrying charge q by using the Lorentz force law and solving the resultant equation of motion

$$ma^{\mu} = q({}_{ext}F^{\mu\nu} u_{\nu}) + {}_{nem}F^{\mu} \tag{23}$$

And combining the two also leads to a harmonious situation in that (i) the coupled Maxwell-Lorentz equations have a well-posed initial value problem and (ii) the total stress-energy tensor, consisting of the sum of the stress-energy tensor of the electromagnetic field and the mechanical stress-energy tensor of the particle,[32] is conserved.

Unfortunately, equation (23) is empirically inadequate, for accelerated charges radiate and, as a result, they experience a damping force that is not reflected in (23).[33] The generally accepted equation of motion describing this phenomenon for a point charge in Minkowski spacetime is called the Lorentz-Dirac equation:

$$
\begin{aligned}
ma^\mu &= q({}_{ext}F^{\mu\nu}u_\nu) + {}_{nem}F^\mu + \frac{2}{3}q^2(\dot{a}^\mu - a^2u^\mu) \\
&= q({}_{ext}F^{\mu\nu}u_\nu) + {}_{nem}F^\mu + \frac{2}{3}q^2(\eta^{\mu\nu} + u^\mu u^\nu)\dot{a}_\nu
\end{aligned}
\tag{24}
$$

where $a := a^\mu a_\mu$ is the magnitude of the four-vector acceleration and $\eta^{\mu\nu}$ is the Minkowski metric. The second equality in (24) follows from the first by differentiating the identity $u_\mu a^\mu = 0$ and using the result to eliminate a^2. Note that the crucial radiation reaction term (the third term on the right hand side of (24)) vanishes when the acceleration of the charge is hyperbolic, suggesting that such a charge must be absorbing as much electromagnetic energy as it radiates.

Radiation reaction is observed, for example, in synchrotron radiation, but high precision tests of the predictions of (24) for the orbits of charged particles are lacking. A summary of the current experimental situation and proposed high precision tests can be found in Spohn (2004: Section 9.3).

Note that under time reversal, the spatial component of $(\dot{a}^\mu - a^2u^\mu)$ changes sign. Thus, if (24) is regarded as a fundamental law, then time reversal invariance is broken. The issue of whether radiation reaction does indeed indicate such a breaking will be approached in stages.

3.2 The reduced order equation

The Lorentz-Dirac equation is beset by several well-known difficulties. The most often mentioned is the existence of "run away" solutions in which the acceleration of the charge increases without bound (and consequently $|\mathbf{u}/c| \to 1$) even in the absence of an external force. These pathological solutions can be quashed by narrowing the class of solutions with the demand that the acceleration of the charge vanishes as the proper time along its world line approaches $+\infty$, but the price to be paid is a "pre-acceleration" effect in which the charge begins to accelerate before the external force is applied.

In response one can adopt the pragmatic attitude that these difficulties result from pushing the Lorentz-Dirac equation beyond its domain of validity. The Lorentz-Dirac equation was intended to describe the reaction of a point charge to an external force that causes the charge to accelerate and, thus, to radiate. But only a glance at (24) is needed to realize that this intent is not fulfilled since the self-interaction (q^2) term can be non-zero even when ${}_{ext}F^{\mu\nu}$ and ${}_{nem}F^{\mu}$ in (24) vanish (as emphasized by Rohrlich 2001). This suggests that the domain of validity of the Lorentz-Dirac equation is circumscribed by a value for the self-interaction term that is small in comparison with the external force (see Teitelboim et al. 1980). That the equation (24) yields pathological results when the external force is zero is then no surprise. Adding a note of optimism to the pragmatism, the Lorentz-Dirac equation can be expected to yield empirically adequate results when its domain of validity is not exceeded.

The pragmatic attitude towards the Lorentz-Dirac equation (24) sketched above may strike some as too casual. It is not unreasonable to demand that the pragmatism be justified by finding an equation of motion that is just as accurate as the Lorentz-Dirac equation within its domain of validity but which is not subject to the pathologies of run away solutions and pre-acceleration effects. A clue to finding the desired equation lies in the fact that these pathologies can be traced to the presence of the third-order time derivative in (24). This suggests that a more satisfactory equation can be found by finding a technique to reduce the highest order of the time derivatives. Such a reduction of order procedure was first proposed by Landau and Lifshitz (1975) and subsequently has been extensively discussed and refined.

Suppose that the external force is due to the external electromagnetic field ${}_{ext}F^{\mu\nu}$. Since the self-interaction term in (24) is supposed to be small compared to the external force, the acceleration a^{ν} appearing in the $\dot{a}^{\nu}$ term on the right hand side of the second equality of (24) can be approximated by ${}_{ext}F^{\nu\beta}u_{\beta}/m$. When the proper time derivative of this expression is taken and substituted for $\dot{a}^{\nu}$ the resulting equation of motion takes the form

$$\begin{aligned} ma^{\mu} &= q({}_{ext}F^{\mu\nu}u_{\nu}) + \\ & \frac{2}{3}\frac{q^3}{m}\left[u^{\beta}\nabla_{\beta}({}_{ext}F^{\mu\nu})u_{\nu} + \frac{q}{m}{}_{ext}F^{\mu\nu}{}_{ext}F_{\nu\beta}u^{\beta} + \frac{q}{m}u^{\mu}{}_{ext}F^{\alpha\beta}{}_{ext}F_{\beta\gamma}u_{\alpha}u^{\gamma}\right] \end{aligned} \tag{25}$$

This heuristic derivation of the reduced order equation can be replaced by a more rigorous argument, as given in detail by Spohn (2000, 2004).[34]

Like the Lorentz-Dirac equation in the form (24), the reduced order equation (25) is not time reversal invariant (*pace* Rohrlich 2001). To see this it suffices to focus on a special case that reduces the formidable clutter of super- and sub-scripts in (25). Work in an inertial system and take the case where (in the chosen system) the external magnetic field vanishes and the external electrostatic potential $\phi(\mathbf{r})$ varies only along, say, the y-axis. Setting $\mathbf{r} = (0, y, 0)$, $\mathbf{u} = (0, \dot{y}, 0)$, and $\phi(\mathbf{r}) = V(y)$, (25) reduces to

$$m\frac{d}{dt}(\gamma\dot{y}) = -V'(y) - \frac{2}{3}\frac{q^3}{m}V''(y)\gamma\dot{y} \tag{26}$$

where $\gamma := \sqrt{1 - \mathbf{u}\cdot\mathbf{u}}$ and the prime and the over-dot denote respectively differentiation with respect to y and with respect to inertial time (see Spohn 2000). When $V''(y) > 0$ the charge experiences a frictional damping force. But under time reversal, there is a flip in the sign of the term describing this force and, thus, in the time reversed history the charge experience an anti-damping force.

Because equation (25) avoids the pathologies of the Lorentz-Dirac equation and is just as accurate as the Lorentz-Dirac equation within the domain of validity of the latter, Rohrlich (2001) has dubbed equation (25) as “the exact” classical equation of motion covering radiation reaction of a point charge. But these virtues of (25) do not constitute a proof that (25) is anything more than a phenomenological equation that encapsulates the observed asymmetry of radiation—that accelerating charges radiate and experience a damping force rather than absorbing radiation and experiencing an anti-damping force—but not does provide an explanation of the origins of this asymmetry. Before turning to possible explanations of the observed asymmetry of radiation, it will be helpful to look at another form of the Lorentz-Dirac equation that, unlike (24) and (25), is time reversal invariant.

3.3 The Dirac expression for radiation reaction

The retarded and advanced Liénard-Wiechert fields of a point charge are singular on the world line of the charge; but since both solutions have the same singularity structure near the world line of the charge, the *difference* between the two is non-singular on the world line. Following Dirac (1938), postulate that the radiation reaction force Γ^μ experienced by a point particle with charge q is given by

$$\Gamma^\mu := \frac{q}{2}[{}_{ret}F^{\mu\nu} - {}_{adv}F^{\mu\nu}]u_\nu \tag{27}$$

where ${}_{ret}F^{\mu\nu}$ and ${}_{adv}F^{\mu\nu}$ are respectively the retarded and advanced Liénard-Wiechert fields of charge in question. By performing a power series expansion in the proper time of the worldline of the charge and dropping terms of third and higher order, Dirac (1938) showed that Γ^μ is equal to the radiation reaction term in (24).[35] As far as phenomenology goes, (27) has just as good a claim to providing a correct description of radiation reaction as the expression in (24).

Now consider a closed system consisting of a finite number N of point charges subject to no external forces. Using the retarded representation, the external electromagnetic force acting on one of the charges will be sum of the retarded Liénard-Wiechert fields of the other charges plus the incident source free radiation. Using the Dirac expression (27) for the radiation reaction and setting ${}_{nem}F^{\mu}$ in (24) to zero, the equation of motion of the kth charge is

$$ma^{\mu}_{(k)} = q_k \sum_{j \neq k}^{N} {}_{ret}F^{\mu\nu}_{(j)} u_{(k)\nu} + q_k ({}_{in}F^{\mu\nu} u_{(k)\nu}) + \frac{q_k}{2} [{}_{ret}F^{\mu\nu}_{(k)} - {}_{adv}F^{\mu\nu}_{(k)}] u_{(k)\nu} \quad (28)$$

To get the equation of motion describing the time reverse of the history described by (28), replace each of the quantities in (28) by its time reverse counterpart (per Section 2.1). The result is

$$ma^{\mu}_{(k)} = q_k \sum_{j \neq k}^{N} {}_{adv}F^{\mu\nu}_{(j)} u_{(k)\nu} + q_k ({}_{out}F^{\mu\nu} u_{(k)\nu}) + \frac{q_k}{2} [{}_{adv}F^{\mu\nu}_{(k)} - {}_{ret}F^{\mu\nu}_{(k)}] u_{(k)\nu} \quad (29)$$

where the quantities in (28) are evaluated the proper times $\tau_{(k)}$ and the corresponding quantities (29) are evaluated at the proper times $\tau'_{(k)} = -\tau_{(k)}$. At this juncture appeal can be made to the Kirchhoff theorem in the form

$$\sum_{j\neq k}^{N} {}_{adv}F^{\mu\nu}_{(j)} + {}_{out}F^{\mu\nu} = \sum_{j\neq k}^{N} {}_{ret}F^{\mu\nu}_{(j)} + {}_{in}F^{\mu\nu} + [{}_{ret}F^{\mu\nu}_{(k)} - {}_{adv}F^{\mu\nu}_{(k)}] \tag{30}$$

Substituting (30) into (29) shows that the equation describing the time reversed history can be brought into the form (28) for the original history but with the terms evaluated at the proper times $\tau'_k = -\tau_k$ appropriate to the time reversed history.

The conclusion that the Lorentz-Dirac equation in the form (28) is time reversal invariant might seem to break down in the special case where ${}_{in}F^{\mu\nu} = 0$ and $N = 1$. But taking into account the inadequacies of the Lorentz-Dirac equation (recall Sections 3.2 and 3.3), we can reason informally as follows. Since the impressed force acting on the lone charge is zero by hypothesis, the acceleration of the charge is zero; and since the radiation reaction force arises only when a charge is accelerating, it too is zero in the present case (see Rohrlich 1990: 251).

However, it does seem that the failure of time reversal invariance announces itself in the $N = 1$ case when there is a *non*-electromagnetic force that drags along the lone charge. But postulating by brute force a ${}_{nem}F^{\mu}$ term to be added to the right hand side of (28) amounts to treating the system as an open system and to allowing ourselves to play God by "poking" the system with an arbitrary force. The point remains that for a closed electromagnetic system that is fully described by the Lorentz-Dirac equation in the form (28), no violation of time reversal invariance can be detected. It remains to investigate the time reversal invariance properties of a closed system that couples electromagnetism to another force, once the source equations of that additional force field are specified.

That the Lorentz-Dirac equation in the form (28) is time reversal invariant seems to generate a puzzle since, taken at face value, (28) seems to entail the temporally asymmetric consequence that an accelerating charge experiences a damping rather than an anti-damping force. The latter impression is incorrect.

3.4 Attempts to explain the asymmetry of radiation reaction

The Lorentz-Dirac equation in the form (28) cannot explain the observed asymmetry of radiation reaction. Rather than applying the time reversal transformation to (28) to get the equation describing the time reversed history, simply rewrite (28) using the advanced representation. The result is an equation (29′) with the same form as (29) but with the terms evaluated at the same proper times τ_k as in (28). Thus, if (28) licenses the inference that an accelerating charge experiences a damping force (because the last term on the right hand side of this equation represents a damping force), then (29′) licences the inference that an accelerating charge experiences an anti-damping force (because the last term on the right hand side of this equation represents an anti-damping force). But since (28) and (29′) are equivalent, neither inference can be correct (unless (28)-(29′) is self-contradictory) since otherwise an accelerating charge would experience both a damping and an anti-damping force. Thus, why it is that we observe radiation damping rather than radiation anti-damping remains to be explained.

Here is where cosmological considerations may be relevant. With the volume Ω implicit in the expressions ${}_{in}F^{\mu\nu}$ and ${}_{out}F^{\mu\nu}$ pushed to the limit, these expressions become ${}^{\infty}_{in}F^{\mu\nu}$ and ${}^{\infty}_{out}F^{\mu\nu}$ respectively. Suppose then that the Sommerfeld radiation condition ${}^{\infty}_{in}F^{\mu\nu} = 0$ obtains. Then as a consequence of the Kirchoff theorem, ${}^{\infty}_{out}F^{\mu\nu} = \sum_{all\ j}^{N} [{}_{ret}F^{\mu\nu}_{(j)} - {}_{adv}F^{\mu\nu}_{(j)}]$. Substituting this latter relation into (29′) shows that (29′) reduces to (28) (with the domains of integration chosen so that ${}_{in}F^{\mu\nu} = {}^{\infty}_{in}F^{\mu\nu}$ and ${}_{out}F^{\mu\nu} = {}^{\infty}_{out}F^{\mu\nu}$), which might be seen as an explanation of why we observe radiation damping rather than anti-damping.

However, we know from Section 2.9 the Sommerfeld radiation condition is inconsistent with various cosmologies; and even when the Sommerfeld condition obtains, it may also be the case that ${}^{\infty}_{out}F^{\mu\nu} = 0$, in which case symmetry is restored and the proffered explanation fails. In addition, even in cases where ${}^{\infty}_{in}F^{\mu\nu} = 0$ but ${}^{\infty}_{out}F^{\mu\nu} \neq 0$ it is not clear that the proffered explanation is cogent. How does showing that (29′) reduces to (28) license the inference that radiation damping rather than radiation anti-damping is to be expected? After all, one could equally well argue that the Sommerfeld condition ${}^{\infty}_{in}F^{\mu\nu} = 0$ shows that (28) reduces to (29′). The best good hope for using cosmology to explain the asymmetry of radiation reaction in standard classical electrodynamics appears to lie in cases where the cosmology is such that only the retarded representation leads to a finite solution (see Section 2.9).

The attempt to explain the asymmetry of radiation reaction does not fare any better in the Wheeler-Feynman action-at-a-distance version of classical electrodynamics. Their basic equation of motion for a point charge is (14). It is postulated that the universe consists of an island of matter containing N charges surrounded by empty space. Because of destructive interference effects Wheeler and Feynman conclude that in empty space

$$\sum_{all\ j}^{N} \left[{}_{ret}F^{\mu\nu}_{(j)} + {}_{adv}F^{\mu\nu}_{(j)}\right] = 0 \tag{31}$$

Then by means of a not entirely convincing argument, they conclude that the absorber condition

$$F^{\mu\nu}_{rad,tot} =: \sum_{all\ j}^{N} \left[{}_{ret}F^{\mu\nu}_{(j)} - {}_{adv}F^{\mu\nu}_{(j)}\right] = 0 \tag{32}$$

holds everywhere, in which case their equation of motion (14) reduces to the Lorentz-Dirac equation in the form (28). [The Wheeler-Feynman definition of the total radiation field $F^{\mu\nu}_{rad,tot}$ differs from the sum of the retarded radiation fields $\sum_{all\ j}^{N} {}_{ret}F^{\mu\nu}_{rad(j)}$, with ${}_{ret}F^{\mu\nu}_{rad(j)}$ as defined above in Section 2.7. At a field point x sufficiently far from the charges and at a time sufficiently long after the charges have ceased to accelerate, the two expressions for the total radiation field will agree because the Coulomb part of ${}_{ret}F^{\mu\nu}_{(j)}(x)$ will be approximately zero as will ${}_{adv}F^{\mu\nu}_{(j)}(x)$. But one could complain that the explanation of radiation reaction is not concerned solely with such locations.]

Of course, the actual universe is not an island universe. This inconvenient fact can be overcome if the universe is opaque to electromagnetic radiation, but there is evidence that it is not (see Partridge 1973).[36] But the main point for the present discussion is that, even if all of the qualms about the Wheeler-Feynman argument are waived, their theory, supplemented by the absorber condition (32), does not explain the observed asymmetry of radiation reaction since (32) equally well shows that their equation of motion (14) reduces to (29′) (as emphasized by Zeh 2001: 35).

It seems that conjuring with the time reversal invariant form of the Lorentz-Dirac equation (28) or with the time reversal invariant Wheeler-Feynman direct particle interaction equations is not going to lead to a satisfactory explanation of the observed asymmetry of radiation. Non-time reversal invariant forms of the equation, such as the reduced order equation (25), would provide the basis for an explanation if they stood for fundamental laws rather than phenomenological descriptions of what is be explained, but affirming the 'if' seems highly dubious.

3.5 Neo-Ritzian explanations of the asymmetry of radiation

The rather dreary accounting in the preceding subsection of the failures in explaining the observed asymmetry of radiation and radiation reaction might tempt one to listen again to the Siren song of neo-Ritzian posits. For those who are seduced by this song, one way to proceed is to reject the Dirac analysis of radiation reaction for point charges and to provide an alternative analysis that appeals only to retarded fields. A conceptually clear and elegant derivation along these lines of the Lorentz-Dirac equation in the form (24) has been given by Teitelboim (1970, 1971). As mentioned above, the retarded Liénard-Wiechert field for a point charge in Minkowski spacetime can be covariantly separated into a velocity (or generalized Coulomb) field ${}_{ret}F^{\mu\nu}_{Coul}$ and an acceleration (or radiation) field ${}_{ret}F^{\mu\nu}_{rad}$. This splitting of the field induces a splitting of the stress-energy tensor $T^{\mu\nu}_{em}$ of the field into a "bound part" $T^{\mu\nu}_{Coul}$ and an "emitted part" $T^{\mu\nu}_{rad}$, each of which is separately conserved off the world line of the charge. The Lorentz-Dirac equation (24) is then obtained as a consequence of the assumption of energy-momentum balance in the form

$$\nabla_\nu T^{\mu\nu}_{bare} + \nabla_\nu T^{\mu\nu}_{Coul} = -\nabla_\nu T^{\mu\nu}_{rad} - {}_{ext}f^\mu \qquad (33)$$

where ${}_{ext}f^\mu$ is the force density associated with the external force and $T^{\mu\nu}_{bare}$ is the mechanical stress-energy tensor for a point particle of "bare" mass

m_b. The observed mass m of the particle is assumed to result from the "renormalization" of the total mass $m_b + m_{em}$ where the electromagnetic mass m_{em} is the self-energy $\frac{q^2}{\varepsilon}$ of the charge, with ε being a parameter that is zero for a point particle. Thus, a subtraction of an infinity is needed to produce a finite value for m. This is is not a blemish on the Teitelboim derivation since mass renormalization is a feature of any derivation of (24) from energy-momentum balance considerations.[37]

Such a derivation using only retarded fields is not sufficient to explain the asymmetry of radiation reaction. The explanation could be completed by using a neo-Ritzian invocation of "causality" to block any competing analysis of the radiation reaction that appeals to advanced fields. Rohrlich, who has heeded the Siren song (see Rohrlich 1998, 1999, 2000), is not opposed to this form of explanation; but because he finds a point charge to be an unacceptable idealization in classical electrodynamics, his analysis starts with the equations of motion for charges of finite size. His own neo-Ritzian position is based on two claims. The first is that the correct classical equation of motion for a finite sized charge is the Caldirola-Yaghjian equation (see Yaghjian 1992) and that this equation is not time reversal invariant. The latter part of this first claim has been disputed by Zeh (1999) and Rovelli (2004). I will not enter this dispute because I want to concentrate on Rohrlich's second claim which provides a diagnosis of the (alleged) failure of time reversal invariance for the equations of motion for radiating charges of finite size. The diagnosis goes as follows: the self-interaction of a finite sized charge involves the interaction of one element of the charge on another; this interaction "takes place by the first element emitting an electromagnetic field, propagating along the *future* light cone and then interacting with the other element of charge" (2000: 9); the choice of the future light cone over the past light cone is dictated by "causality" which "requires *retarded* rather than advanced self-interaction" (2000: 1); in sum, it is causality which is "ultimately responsible for the [EM] arrow of time" (ibid.).

Several comments are in order. 1. Rohrlich thinks that it follows from his analysis that in the limit of a point particle, the equation of motion ought to be time reversal invariant:

> In the point limit, the retarded and advanced actions can no longer be distinguished because the interaction distance between the charge-mass elements shrinks to zero. Therefore, in that limit the equations of motion are time reversal invariant (Rohrlich 1999: 5).

Thus, he is committed to the (I think) incorrect position that the reduced order form of the Lorentz-Dirac equation for a point charge is time reversal invariant (see Rohrlich 2005). 2. It also seems to follow from Rohrlich's analysis that the behavior of the equations of motion under time reversal is irrelevant to the asymmetry of radiation reaction; for even if the equations of motion are time reversal invariant (as Rohrlich wrongly says is the case with reduced order form (25) of the Lorentz-Dirac equation (24) and rightly says is the case for Lorentz-Dirac equation in form (28)), the appeal to "causality" still provides a basis for the asymmetry of radiation—which is a consequence that Rohrlich (2005) embraces. 3. I have already railed enough against the invocation of "causality" in lieu of genuine scientific theorizing, and at this juncture I only want to note that if Rohrlich's causality based account of the origins of the arrow of electromagnetic radiation is correct, then this arrow is non-contingent in that it does not depend on initial/boundary conditions.[38] One persuasive reason for resisting this consequence—and thus for rejecting causality based accounts of the asymmetry of radiation—derives from the facts that QED is widely accepted as the correct theory of electrodynamics and that the basic laws of this theory are time reversal invariant. So either taking classical limit of QED introduces an temporal asymmetry (implausible) or else asymmetry must drive from contingency of initial/boundary conditions.

This last thought suggests that a fruitful strategy for finding an explanation of the asymmetry of radiation is to study how the Lorentz-Dirac expression for radiation reaction emerges from QED in an appropriate classical limit. But before pursuing this line of inquiry, it is worth making a final comment about the relevance of cosmology for the asymmetry of radiation reaction in classical electrodynamics.

3.6 The equation of motion of a point charge in a cosmological setting

The above discussion of radiation reaction in classical electrodynamics assumed a flat spacetime background. The generalization from flat to curved spacetime of the Lorentz-Dirac equation in the form (24) replaces a^μ, $\dot{a}^\mu$, and a^2 by their counterparts that use the covariant path derivative $D/D\tau$ along the world line of the charge in place of $d/d\tau$. However, a more important modification is necessitated by the presence of spacetime curvature since the interaction between the electromagnetic field of a charge and the curvature produces additional terms in the expression for the radiation reaction. Specifically, in a conformally flat but non-Ricci flat spacetime (such as a FRW spacetime), the addition is

$$-\frac{q^2}{3}(R^\mu{}_\beta u^\beta + u^\mu R_{\beta\gamma} u^\beta u^\gamma) \tag{34}$$

(see Hobbs 1968 and Quinn and Wald 1997). Since odd powers of the four-velocity are involved in (34), the presence of the Ricci curvature seems to give rise to additional time asymmetry that is distinct from the asymmetry of radiation reaction in flat spacetime.

This curvature based asymmetry has received virtually no attention in the philosophical literature on the arrows of time. But it deserves attention if for no other reason than that it provides a decisive (in-principle) test of standard classical electrodynamics vs. Wheeler-Feynman action-at-a-distance electrodynamics since in the natural generalization of the latter to curved spacetimes, the Ricci curvature term (34) is absent even when the Wheeler-Feynman absorber condition imposed (see Unruh 1976).

3.7 Radiation reaction in QED

It is high time to conduct the search for the origins of asymmetry of radiation reaction by appealing to the theory that is currently thought to be the correct theory of electrodynamics—QED—and to the assumption that the valid core of classical electrodynamics is to be identified by what emerges from QED in an appropriate classical limit.

The threshold question is whether the classical limit of QED for a point charge reproduces the Lorentz-Dirac equation. Unfortunately, there is no clean cut answer because of technical issues concerning the implementation of the classical limit (see Higuchi 2002). I will suppress these technicalities as far as possible, and will outline the approach of Higuchi and Martin (2004, 2005) which studies the radiation reaction of a wave packet of a charged scalar field moving in an external potential.[39]

In the classical model, a point particle of charge q is linearly accelerated by an external potential $V = V(z)$ such that $V(z) = V_o = const > 0$ for $z < Z_1$ and $V(z) = 0$ for $z > -Z_2$, where $Z_1 < Z_2$ are positive constants. Assume that the particle initially moves in the positive z-direction and that at $t = 0$ it has passed through the region $(-Z_1, -Z_2)$ where it is accelerated. Let z_o denote the position the particle would have at $t = 0$ if there there were no radiation reaction force. And let z be the actual position at $t = 0$ when the radiation reaction force—per the Lorentz-Dirac equation—is acting. The classical position shift due to radiation reaction is then $\delta z_C := z - z_o$.

The goal is to compare the classical position shift δz_C to the position shift δz_{QED} calculated for a charged scalar field $\hat{\varphi}$ coupled to the electromagnetic field and subject to the same external potential V as in the classical model.[40] Towards this end, define the charge density operator $\hat{\varrho}$ per usual as

$$\hat{\varrho}(x) := \frac{i}{\hbar} : \hat{\varphi}^{\dagger}\partial_t\hat{\varphi} - \partial_t\hat{\varphi}^{\dagger} \cdot \hat{\varphi} : \tag{35}$$

where : : indicates normal ordering. And define the expectation value of the position of the particle by

$$< \hat{z} > := \int z < \hat{\varrho}(\mathbf{x}, t) > d^3\mathbf{x} \tag{36}$$

Using these definitions, calculate the expectation value $< \hat{z} >^{off}$ of position at $t = 0$ with the electromagnetic field turned off using an initial state $|i\rangle$ in which the momentum of the particle is strongly peaked about a value pointing in the positive z-direction:

$$< \hat{z} >^{off} \quad = \int z\langle i|\hat{\varrho}(\mathbf{x}, t)|i\rangle d^3\mathbf{x} \tag{37}$$

Next, use the WKB approximation to calculate to lowest non-trivial order in q the expectation value of position $< \hat{z} >^{on}$ at $t = 0$ with the electromagnetic field turned on. To this order, time dependent perturbation theory give a final state $|f\rangle$ of of the form

$$|f\rangle = |1\varphi, 0\gamma\rangle + |1\varphi, 1\gamma\rangle \tag{38}$$

where the elements of the superposition are interpreted as follows. The first element $|1\varphi, 0\gamma\rangle$ is a state with one scalar particle and no photon, and is equal to $|i\rangle + |s\rangle$, where $|s\rangle$ arises from the the forward scattering of the wave packet. The second element $|1\varphi, 1\gamma\rangle$ is a state with one scalar particle and one photon. The expectation value $< \hat{z} >^{on}$ calculated from $|f\rangle$ is the sum of three terms $< \hat{z} >^{off} + < \hat{z} >^{(s)} + < \hat{z} >^{(1)}$, where the last two terms are respectively the contributions of the forward scattering and the one photon state. Since the former contribution arises without photon emission, Higuchi and Martin deem it to be irrelevant to radiation reaction, and they define the QED position shift by $\delta z_{QED} := < \hat{z} >^{on} - < \hat{z} >^{(s)} - < \hat{z} >^{off} = < \hat{z} >^{(1)}$. (If subtracting off the $< \hat{z} >^{(s)}$ term seems like hocus-pocus, two responses can be given. First, one could impose additional conditions as part of the classical limit to assure that $< \hat{z} >^{(s)}$ is small in comparison with the other terms. Second, one could take the attitude that classical electrodynamics is wrong because it does not include the effect codified in $< \hat{z} >^{(s)}$, which is of essentially quantum origins.) Higuchi and Martin (2004, 2005) show that in the $\hbar \to 0$ limit, $\delta z_{QED} = \delta z_C$. The same result is demonstrated for a time-dependent but position-independent potential.

What light does this derivation cast on the time asymmetry of radiation reaction? QED is a time reversal invariant theory (see Atkinson 2006). So if an initial state $|I\rangle$ evolves to the (exact) final state $|F\rangle$ over Δt, then ${}^R|F\rangle$ evolves to ${}^R|I\rangle$ over the same Δt. In the above model calculation $|I\rangle = |i\rangle$, but the state $|f\rangle$ calculated from time dependent perturbation theory differs from the exact final state $|F\rangle$, and so ${}^R|f\rangle$ won't evolve to ${}^R|i\rangle$.[41] But to the extent that approximation procedure outlined above is to be trusted, calculating the QED position shift from the exact final state $|F\rangle$ which $|f\rangle$ approximates should not change the conclusion that in the $\hbar \to 0$ limit $\delta z_{QED} = \delta z_C$. Thus, from the perspective of QED the observed asymmetry of radiation and radiation reaction is to be traced to the fact that, in the circumstances we find ourselves, it is overwhelmingly more "probable" that $|I\rangle$ $(= |i\rangle)$ type states will be realized than it is that ${}^R|F\rangle$ type states will be realized. But if circumstances were different and $|I\rangle$ type and ${}^R|F\rangle$ type states are equally "probable" then the observed asymmetry would disappear. Scare quotes were used because the the relevant sense of probability is not supplied by QED in particular or quantum field theory in general. The mystery of the asymmetry of radiation reaction is thus kicked upstairs to quantum statistical mechanics. Reducing one mystery to another does not count as a solution of the first, but progress has been made in the sense that, despite first appearances gained from classical electrodynamics, the time asymmetry of radiation and radiation reaction is shown not to be of different in kind from other time asymmetries having a statistical origin.

As for the Ricci curvature asymmetry, I can only offer opinion and conjecture. As with radiation reaction, I would maintain that the curvature effect given by equation (34) is valid only to the extent that it emerges in the $\hbar \rightarrow 0$ limit of QED done on the background of a globally hyperbolic, conformally flat, but not Ricci flat spacetime. Linear quantum field theory on a curved, globally hyperbolic spacetime is well understood, at least for stationary spacetimes (see Wald 1994).[42] Presumably QED can be generalized to this setting, and presumably this generalization is time reversal invariant. If the presumptions hold, then as with radiation reaction, the time asymmetry of the Ricci curvature effect (if valid) has to be due to the asymmetry of probabilities of realization of initial and reversed final states.[43]

There are uncomfortably many promissory notes left to be redeemed. But if the suggested line of analysis is on the right track, then the neo-Ritzianism can find no purchase in the observed asymmetry of radiation and radiation reaction.

4 Conclusion

One overarching conclusion that emerges from the above discussion is that the siren song of neo-Ritzian posits to supplement classical relativistic electrodynamics should not be heeded. These posits are not needed to explain the classical EM asymmetries. Furthermore, in the setting of a pure particle theory of electrodynamics—the type of theory Ritz hankered after—a Ritzian "retardation condition" makes sense as a scientific hypothesis that speaks for itself; but in the setting of orthodox classical relativistic electrodynamics, such a condition requires philosophy-speak (cause, produce, contribute) to gain any traction. And such traction as is gained not only does not produce any genuine scientific explanation, but by offering soothing words in place of scientific theorizing, it retards the search for scientific understanding.

The task of tracing the origins of EM asymmetries would ideally start with a clear formulation of the asymmetries. But some the formulations that are found in the literature are vitiated by muddles about retarded and advanced solutions/representations. Others have a distressingly vague and hedged character, requiring the use of qualifiers like 'most', 'many', 'typically', and well as 'approximately'. Additionally some of the asymmetries that are formulated in terms of the quantities ${}_{ret}F^{\mu\nu}$, ${}_{in}F^{\mu\nu}$, ${}_{adv}F^{\mu\nu}$, and ${}_{out}F^{\mu\nu}$ can come or go as the domains of integration implicit in these quantities change. It is far from clear which of the asymmetries exhibiting these characteristics of vagueness and the relativity deserve to be promoted to the status arrows of time.

A clean EM asymmetry worthy of promotion to an arrow of time may emerge when the domains of integration implicit in the quantities ${}_{ret}F^{\mu\nu}$, ${}_{in}F^{\mu\nu}$, ${}_{adv}F^{\mu\nu}_{out}$, and ${}_{out}F^{\mu\nu}$ are pushed to their limits and cosmological considerations are brought into play. The large scale structure of the spacetime, together with the distribution of the sources, may allow one but not another of the conditions ${}^{\infty}_{in}F^{\mu\nu} = 0$, ${}^{\infty}_{out}F^{\mu\nu} = 0$, or ${}^{\infty}_{in}F^{\mu\nu} + {}^{\infty}_{out}F^{\mu\nu} = 0$ on incoming and outgoing radiation. Additionally, the demand for a finite solution may, for example, uniquely single out the retarded as opposed to the advanced Kirchhoff representation. But although clean, the resultant EM arrow is clearly enslaved to the cosmological arrow.

The case of radiation reaction of a point charge appears to offer an EM asymmetry that is pervasive enough and unequivocal enough to be promoted to an arrow of time and that is distinctively electromagnetic in origin, being neither enslaved to the cosmological arrow nor due to the probability considerations that underlie the temporal asymmetries of collective phenomena of non-charged matter. However, when the investigation is carried into QED the initial impression about the status of this arrow changes: the arrow of radiation reaction is of a piece with other arrows that derive from asymmetries of the probabilities of initial and reversed final states. It was conjectured that a similar conclusion will hold for the Ricci curvature asymmetry that arises for point charges moving in a conformally flat but not Ricci flat spacetime. But confirming this conjecture will involve difficult calculations in quantum field theory on curved spacetime.

I will mention a more general conjecture: any EM asymmetry that is clean and pervasive enough to merit promotion to an arrow of time is enslaved to either the cosmological arrow or the same source that grounds thermodynamic arrow (or a combination of both). But much more work would be needed before I would be willing to make this conjecture with any confidence.

Notes

[1]The reader may find it instructive to consider five bench marks for recent decades: Davies (1976), Zeh (1989, 2001), Savitt (1995), and Savitt (2006).

[2]For remarks on the considerations that go into such a decision, see Sklar (1993, pp. 378-384).

[3]The technical result here assumes that the symmetry is codified as a Lie group of transformations.

[4]Here t is the time coordinate for which the FRW line element takes the form $ds^2 = a(t)d\sigma^2 - dt^2$. Here $a(t)$ is called the scale factor, and the spatial line element $d\sigma^2$ can take one of three forms corresponding to $t = const$ slices which have zero curvature, constant positive curvature, or constant negative curvature.

[5] "[C]lassical electrodynamics is *not* invariant under time-reversal" (Albert 2000: 20).

[6]Greek indices and Latin indices run respectively from 1 to 4 and 1 to 3. The signature convention for the spacetime metric is $(+++-)$. Units are chosen so that $c \equiv 1$.

[7]More generally, the electric and magnetic fields as measured by an observer whose (normed) four-velocity is V^μ are defined respectively by $E^\mu := F^{\mu\nu}V_\nu$ and $B^\mu := \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}V_\nu F_{\alpha\beta}$, where $\epsilon^{\mu\nu\alpha\beta} = \epsilon^{[\mu\nu\alpha\beta]}$ is the volume element of the spacetime. Note that the electric and magnetic fields measured by the observer V^μ are spatial vectors in that they lie in the spacelike plane

orthogonal to V^{μ}.

[8]Any simply connected spacetime admits a time orientation. Thus, if a given spacetime is not time orientable, one can obtain a time orientable spacetime that is locally the same as the given spacetime by passing to a covering spacetime. It is assumed that all of the spacetimes at issue here are time orientable.

[9]See Malament (2004) for an elegant application of this approach to classical electromagnetism.

[10]For a discussion of these issues, see Earman (2002).

[11]This also follows from the facts that $\sum_{j=1}^{N} {}_{ret}F^{\mu\nu}_{(j)}(x)$ and $\sum_{j=1}^{N} {}_{adv}F^{\mu\nu}_{(j)}(x)$ are solutions to the inhomogeneous Maxwell equations and that any two such solutions differ only by a homogeneous solution.

[12]I take it that North (2003) is proposing that the judgment of what electromagnetic field is produced by a charged source is to be formed relative to the representation that has the most natural source-free field. I do not object to this as long as 'produce' means just this. I *do* object if 'produce' has a metaphysically charged meaning.

[13]Again, it is supposed that the acceleration of the particle is due to non-electromagnetic forces.

[14]Note that the difference between any two of the forces $(18)-(20)$ is proportional to $q_k({}_{ret}F^{\mu\nu}_{(k)} - {}_{adv}F^{\mu\nu}_{(k)})u_{(k)\nu}$. According to Dirac's (1938) analysis, the value of the radiation reaction force experienced by charge k is $\frac{q_k}{2}({}_{ret}F^{\mu\nu}_{(k)}$

$- {}_{adv}F^{\mu\nu}_{(k)})u_{(k)\nu}$ (see Section 3.3).

[15]For some skepticism about the ability of the past hypothesis to explain the thermodynamic arrow, see Weisberg (2004) and Earman (2006); for skepticism about the ability of the past hypothesis to explain EM arrows, see Frisch (2006). For sake of completeness it should also be acknowledged that there are alternative approaches, such as that championed by Penrose and Percival (1962) who posit a time asymmetric statistical law. Their proposed law, called the "law of conditional independence," is supposed to explain several of the arrows of time, including the thermodynamic arrow and the EM arrow. This law is inconsistent with distant correlations between relatively spacelike regions in a cosmology that has particle horizons; see Section 2.9 below.

[16]Price (2006) sees the linkage forged in the following way: the observed asymmetry at issue depends on the contrast between a few large outgoing waves vs. many small incoming waves; that contrast is explained by the thermodynamics of the environment which derives large additions of energy but few large subtractions; and the second contrast is explained by the low entropy past. I find the first two links plausible but am suspicious of the third.

[17]Thus, I agree with Price (2006) that the asymmetry at issue is not captured by the condition that ${}_{in}F^{\mu v} = 0$ leading to a purely retarded description ${}_{act}F^{\mu\nu} = {}_{ret}F^{\mu\nu}$. However, unlike Price I do not identify the Sommerfield radiation condition with ${}_{in}F^{\mu v} = 0$ for local systems (see below).

[18]The stress-energy tensor $T^{\mu\nu}_{em}$ associated with the the electromagnetic field $F^{\mu\nu}$ is defined by $T^{\mu\nu}_{em} := \frac{1}{4\pi}(F^{\mu\beta}F^{\nu}_{\beta} - \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta})$.

[19]I have replaced the potentials in Zeh's formulation with the Maxwell field tensor in order to assure gauge independence.

[20]A somewhat different version (labeled (R)) is given in Frisch (2006: 546).

[21]As Price (2006) puts it, unless we posit an EM arrow, we have no right to assume that ordinary matter will act as an absorber.

[22]For an overview of the problems encountered, see Ellis and Sciama (1972).

[23]See Wald (1984, pp. 210-209) for various equivalent definitions, one of which is the existence of a Cauchy surface, a spacelike hypersurface that is intersected exactly once by every endless timelike curve.

[24]Let $\mathcal{M}, g_{\mu\nu}$ be an arbitrary relativistic spacetime. Then for any $p \in \mathcal{M}$ there is a neighborhood $\mathcal{N}(p)$ of p such that the spacetime $\mathcal{N}, g_{\mu\nu}|_{\mathcal{N}}$ is globally hyperbolic.

[25]In the non-conformally flat case, the retarded (respectively, advanced) representation contains an additional "tail" term consisting of an integral over the interior as well as the surface of the past (respectively, future) light cone of the field point.

[26]A metric $g_{\mu\nu}$ is conformally flat just in case there is a scalar field ϕ such that at all points x of the spacetime manifold, $g_{\mu\nu}(x) = \phi^2(x)\eta_{\mu\nu}$ where $\eta_{\mu\nu}$ is the Minkowski metric.

[27]For an introduction to horizons in cosmology, see Ellis and Rothman (1993).

[28] This is a result of the fact that two of the Maxwell equations are constraint equations—in the case of Minkowski spacetime, these equations are the first of the equations in (1′)-(2′). These elliptic equations constrain the joint values of the fields and the charge-current on a spacelike hypersurface. See Penrose (1964) for a more precise presentation of this point.

[29] An exception occurs when the charges are symmetrically arranged around the point x so that their Coulomb fields cancel out.

[30] The Aichelberg-Beig model assumes that the scalar field Φ obeys a conformally invariant wave equation. Thus, they use a conformal coupling to the spacetime, and Φ satisfies

$$(\Box_g + \tfrac{1}{6}R)\Phi(\mathbf{x}, t) = \lambda \frac{\delta^3(\mathbf{x})}{a^3(t)} Q(t), \text{ where } \Box_g := \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu}(\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu}),$$

R is the Ricci curvature scalar, and Q is the oscillator amplitude obeying the equation of motion

$$\ddot{Q}(t) + \omega_o^2 Q(t) = \lambda \Phi(0,0,0,t), \text{ where } \omega_o \text{ is the spring constant.}$$

[31] The experimental results reported in Angelopoulos et al. (1998) give a direct demonstration of time reversal invariance in the sense that they do not (as earlier results did) appeal to observed CP violation and the CPT theorem.

[32] $T^{\mu\nu}_{mech} := m \int d\tau u^\mu u^\nu \delta(x - z)$.

[33] An exception to the rule will be mentioned shortly.

[34] When the external force is due to a non-electromagnetic force ${}_{nem}F^\mu$, a reduced order equation can be produced by approximating the $\dot{a}^\nu$ term on the right hand side of the second equality of (21) by $\frac{1}{m} d({}_{nem}F^\nu)/d\tau$.

[35]For a user friendly presentation of this result, see Poisson (1999).

[36]Detailed critical discussions on the Wheeler-Feynman absorber theory of radiation can be found in Davies 1972; Davies and Twamley 1993; Price 1966; and Zeh 2001).

[37]It might be noted, however, that the Teitelboim derivation also relies on the assumption that the worldline of the charge tends to a straight line as the proper time along the world line approaches $-\infty$, an assumption that limits the domain of validity of (24) and introduces a temporal asymmetry.

[38]Unless, of course "causality" itself depends on initial/boundary conditions, a notion which the neo-Ritzians seem to reject.

[39]For other approaches, see Sharp and Munitz (1977) and Johnson and Hu (2002).

[40]The Lagrangian density for the QED model is taken to be

$$\mathcal{L} = [(D_\mu + iqA_\mu)\varphi]^\dagger[(D_\mu + iqA_\mu)\varphi] - (m/\hbar)^2\varphi^\dagger\varphi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\mu A^\mu)^2,$$

where $D_\mu := \partial_\mu + iV_\mu/\hbar$ and $V_\mu := V(z)\delta_{\mu 0}$.

[41]Suppose that $|f\rangle$ and $|F\rangle$ are close in the Hilbert space norm. Since the norm is preserved by the reversal operation R and by a unitary transformation, it follows that the unitary time evolutes of ${}^R|f\rangle$ and ${}^R|R\rangle$ remain close in the Hilbert space norm.

[42]Intuitively, a stationary spacetime is one whose metric $g_{\mu\nu}$ is time independent. The precise mathematical statement is that there exists a timelike Killing vector field V^μ, which implies that $\nabla_{(\alpha}V_{\beta)} = 0$.

[43]When the background spacetime is not stationary, even linear quantum

field theory becomes problematic due to lack of a natural way to separate positive and negative frequencies and to define a vacuum state.

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Figure Captions

Figure 1 A charge that is hyperbolically accelerated in the future

Figure 2 A charge that is hyperbolically accelerated in the past

Figure 3 A charge that is hyperbolically accelerated for all times

Figure 4a Past truncated Minkowski spacetime

Figure 4b Future truncated Minkowski spacetime