UNSTRUCTURED PURITY

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Abstract

Purity is the principle that fundamental facts only have fundamental constituents. In recent years, it has played a significant role in metaphysical theorizing—but its logical foundations are underdeveloped. I argue that recent advances in higher-order logic reveal a subtle ambiguity regarding Purity's interpretation; there are stronger and weaker versions of that principle. The arguments for Purity only support the weaker interpretation, but arguments that employ it only succeed if the stronger interpretation is true. As a result, nearly every metaphysician who has appealed to Purity has made a mistake—in that the inferences that they make are not justified by the arguments that they provide.

Introduction

The notion of fundamentality holds a special place in the hearts of metaphysicians. To be sure, many debates concern derivative phenomena. The literatures on personal identity and causation are expansive—and the metaphysics of race and gender has arguably received more attention in the past few decades than at any other point in history. But few topics have captured metaphysicians' singular focus in the manner that fundamentality has. We dream of a final theory—a complete description of the ultimate foundations of the world: the basis from which the entirety of reality arises.

Not that we seem particularly close to realizing this dream. Fundamentality is as vexing as it is tantalizing. Long gone are the days when we thought we could provide a final theory from the armchair; some questions fall within the purview of the empirical sciences. Even within philosophy, there are a number of intractable puzzles. What does it take for a fact to be fundamental?[2] Is there a fundamental basis at all, or are there infinite chains of dependence?[3] How does the derivative depend upon the fundamental?[4]

[1]My thanks to David Builes and Isaac Wilhelm for their extremely enlightening discussions of material that occurs within this paper.

[2]There is a lively debate over the definition of fundamentality. Independence Theorists (like Schaffer (2009) and Bennett (2017)) hold that for something to be fundamental is for it to not depend on anything else. Minimal Foundationalists (like Tahko (2018)) hold that for something to be fundamental is for it to belong to the minimal basis on which everything depends. Truthmaking Foundationalists (like Heil (2003) and Cameron (2008)) hold that for something to be fundamental is for it to belong to the class of truthmakers for all truths. Pragmatists (like Carnap (1950) and Thomasson (2015)) hold that for something to be fundamental is for it to answer to certain pragmatic needs. And Primitivists (like Fine (2001) and Wilson (2014, Forthcoming)) hold that the notion of fundamentality is primitive and unanalyzable.

[3]For discussions about infinite chains of dependence, see Dixon (2016) and Raven (2016).

[4]There are numerous accounts of how the derivative depends upon the fundamental. Superinternalists

One principle that often governs how metaphysicians reason about fundamentality is Purity—according to which fundamental facts only have fundamental constituents.[5] For example, if the fact [Electron e is spin-up] is fundamental, then both *electron e* and the property of *being spin-up* are fundamental. Typically, philosophers appeal to Purity when motivating their positive accounts. For a given fact $[F]$, a metaphysician argues that $[F]$ contains a derivative entity, and cites Purity as a reason to deny that $[F]$ is fundamental—before providing a theory of what $[F]$ depends upon.[6]

I maintain that arguments of this structure are flawed. Recent developments in higher-order logic reveal a subtle ambiguity regarding Purity's interpretation; there are stronger and weaker versions of that principle. The arguments for Purity only support the weaker interpretation—but the arguments that rely upon it only succeed if the stronger interpretation is true. As a result, metaphysicians who appeal to Purity have almost universally made a mistake; their arguments do not justify their inferences. To be clear, I do not claim that the strong interpretation *could not* be justified—but rather that it *is not* (at least at present). As much as anything, this paper is a call to action. Those who would rely to Purity ought to provide a reason to think that the strong interpretation is true.

A brief remark on the formalisms that I employ. When I began this project, I expected to reason within the confines of a standard, simply-typed λ-calculus. However, it quickly became clear that this language lacks the expressive power that I need. At various points, I require the ability not only to quantify over terms of arbitrary type, but to quantify over the types themselves. I will begin by operating with a simply-typed language, but will shift to pure-type theory (a language which has the appropriate quantifiers) when needed. I presuppose general familiarity with simply-typed systems—and so will not dedicate space to discussing how they function—but will provide a brief overview of pure-type theory at the appropriate time.[7]

(like Bennett (2011) and deRosset (2013, 2023)) hold that if a fundamental fact $[F_1]$ grounds $[F_2]$, then $[F_1]$ grounds the fact that it grounds $[F_2]$. Grounding Essentialists (like Dasgupta (2014*b*)) hold that, in this case, the essences of the constituents of $[F_2]$ ground the fact that $[F_1]$ grounds $[F_2]$. Bridge Principlists (like Schaffer (2017)) hold that there are principles akin to metaphysical laws that ground the fact that $[F_1]$ grounds $[F_2]$. And Grounding Disunionists (like Sider (2020)) hold that different grounding facts are grounded in different ways.

[5] Arguably, the most canonical discussion of Purity occurs in Sider (2011). However, see Fine (2010); Rosen (2010); deRosset (2013); Dasgupta (2014*a*); Raven (2016) and Litland (2017) for other defenses of Purity. For arguments against Purity, see Merricks (2013) and Barker (2023).

[6] I will primarily address two instances of this form of argument: as motivations for theories of iterated ground and identification. Other instances involve the grounds of negative facts, nongrounding facts, and certain modal facts.

[7] I direct those seeking an introduction to simply-typed languages to texts dedicated to that purpose—e.g., Dorr, Hawthorne and Yli-Vakkuri (2021); Bacon (2023*a*); Bacon and Dorr (2024); Dorr (Forthcoming) and Goodman (2024). Suffice it to say that I adopt a functional, higher-order language with two basic types: a type e for entities and a type t for sentences. For any types τ_1 and τ_2, there is a type $(\tau_1 \rightarrow \tau_2)$—which is to be interpreted as a function from terms of type τ_1 to terms of type τ_2. There are infinitely many constants and variables of every type, as well as λ terms that serve to bind these variables. The logical operators are identified with constants in the standard way. I omit types when they are contextually evident, or when

Higher-Order Structure

It is not unreasonable to identify the beginning of the analytic tradition with the development of type-theory. The insights of Frege (1884), Russell (1903, 1908) and Church (1940) served not only to precisify philosophical argumentation, but to do so within the framework of the rigid, hierarchical structure that type-theory provides. But attention to these systems waned by the second half of the 20th century. Quine (1970)'s insistence on the primacy of first-order logic—and denigration of type-theory as "set theory in sheep's clothing" (pg. 66) left little use for higher-order reasoning. Fortunately, matters have changed in recent years; type-theory has become nearly compulsory in much of contemporary metaphysics.[8]

One of the most significant programs of higher-order metaphysics has been a sustained attack on structured propositions.[9] The idea underlying structuralism is that propositions are 'built' from worldly items—in much the way that sentences are built from words.[10] For example, the proposition *Socrates is wise* is constructed from Socrates and the property of *being wise*, and the proposition *Napoleon is short* is constructed from Napoleon and the property of *being short*. Propositions built from different components are distinct, in virtue of their differing compositions. So, if propositions are identical, they must have the same objects and properties contained within them; they must have been built from the same materials. Structuralists are thus committed to the Principle of Singular Extraction (the PSE) according to which if $Fa = Gb$, then $F = G$ and $a = b$.

The PSE radically conflicts with an orthodox principle of higher-order logic: β-equivalent terms co-refer.[11] (For example, β-identification entails that $Fa = \lambda x.Fx(a)$). Jointly, the PSE and β-identification entail *higher-order monism*: there is but a single referent for every type—for each type τ, all constants of type τ denote the same thing. This can be established as follows:

i.	$\lambda x.(x = x)(a) = \lambda x.(x = a)(a)$	β-identification
ii.	$\lambda x.(x = x) = \lambda x.(x = a)$	*i*, PSE
iii.	$\forall \lambda x.(x = x)$	Classical Logic
iv.	$\forall \lambda x.(x = a)$	*ii*, *iii*, Leibniz's Law
v.	$\exists \lambda y.\forall \lambda x.(x = y)$	*iv*, Classical Logic

formula are to be interpreted as schema with applications in every type.

[8]For significant works that employ higher-order inferences, see, e.g., Williamson (2003); Dorr (2016); Bacon and Russell (2019); Caie, Goodman and Lederman (2020); Dorr, Hawthorne and Yli-Vakkuri (2021) and Fritz and Jones (2024).

[9]See, e.g., Dorr (2016); Goodman (2017); Fritz (2022) and Elgin (2024*b*).

[10]An alternate conception of structured propositions is advanced by Bacon (2023*b*), building off of Dixon (2018), according to which propositions have pictorial, rather than syntactic structure.

[11]This inconsistency was first noted in Dorr (2016)—though this particular proof is from the simplified derivation in Elgin (2024*b*).

This derivation is to be interpreted as a schema with applications in every type. It follows that there is but a single object, but a single property, but a single sentential operator and, mostly worryingly, but a single proposition; an argument of this structure establishes that all propositions are identical. For this reason $p = \neg p$—and the two therefore have the same truth-value. Higher-order monism is not only unintuitive, but inconsistent.

I myself interpret this as a reason to reject the PSE—and, by extension, structured theories of propositions. But the inconsistency could be avoided by rejecting β-identification instead.[12] This argument establishes that the PSE and β-identification are incompatible, but does not determine which principle ought to be rejected.

But there is another, more serious problem for structured propositions: one first discovered by Russell (1903) and noted (apparently independently) by Myhill (1958). The problem concerns how many propositions there are: what the cardinality of the set of propositions is.[13] Structuralists maintain that syntactically distinct sentences express different propositions; because 'Grass is green' differs syntactically from 'Grass is not not green,' the two sentences do not express the same thing. The problem is that, for every collection of propositions, a sentence can be constructed asserting that their conjunction is true. These sentences all differ syntactically from one another; each is a conjunction of different conjuncts. Because these sentences differ syntactically, structuralists maintain that they express different propositions. For this reason, there is an injection from the powerset of propositions to the set of propositions; each element of the powerset is mapped to the sentence conjoining those propositions, and each of these sentences is then mapped to a unique proposition. But Cantor's Theorem entails that there is no such mapping. There cannot exist an injection from the powerset of a set S to set S. So, the claim that all syntactically distinct sentences express different propositions is false.

These arguments differ in their details, but their implications are the same. The same proposition can be constructed in multiple ways; given a proposition p, we cannot determine the unique components from which p was constructed. For example, the proposition Raa could be constructed from $\lambda x.Rxx$, $\lambda x.Rxa$ or $\lambda x.Rax$. All of these could be seen as figuring within Raa; there is no fact of the matter as to which is 'the' property contained therein. Accounts that depend upon singular construction—like the structured view—are false. While much of this debate has centered on propositions, similar arguments apply to facts.[14] Just as the same proposition could be constructed from different material, so too the same fact could be constructed from different material; when theorizing about facts, we cannot presuppose a unique method of construction.

[12] To the best of my knowledge, the only philosophers who have independently advocated for rejecting β-identification are Rosen (2010) and Fine (2012), who argue that terms are grounded in, rather than identical to, their β-reductions.

[13] My reference to set theory here is purely expository—it is possible to generate this problem without reference to sets.

[14] An application of this problem to facts occurs in Fritz (2022).

The Interpretation of Purity

If the same fact can be built from different components, then Purity—the principle that the constituents of fundamental facts are themselves fundamental—could be interpreted in one of two ways. On the stronger interpretation, if a fact is fundamental then *every* method of construction relies on purely fundamental constituents, while, on the weaker interpretation, if a fact is fundamental then *at least one* method of construction relies on purely fundamental constituents.

An example might help highlight the distinction between the strong and weak interpretations. Suppose that $[Fa] = [Gb]$, that F, a and $[Fa]$ are all fundamental, but that G and b are derivative. This case falsifies Strong Purity, but verifies Weak Purity. There is one way to construct $[Fa]$—namely, by predicating F of a—that relies upon fundamental constituents, but there is another way to construct $[Fa]$—by predicating G of b—that relies upon derivative constituents.[15]

It is valuable to formalize the two interpretations of Purity in order to facilitate rigorous argumentation; those who would formalize arguments that appeal to Purity have a need to state the principle that they appeal to. For every type τ, let us introduce a predicate *Fundamental* of type $(\tau \to t)$, with the intended interpretation that '$Fundamental(A)$' means that A is fundamental.[16] At a first pass, we might suggest the following:

Strong Purity$_1$:
$\forall p^t \forall X^{e \to t} \forall x^e ((Fundamental(p) \wedge Xx = p) \to (Fundamental(X) \wedge Fundamental(x))$

Weak Purity $_1$:
$\forall p^t (Fundamental(p) \to \exists X^{e \to t} \exists x^e (Xx = p \wedge Fundamental(X) \wedge Fundamental(x))$

According to Strong Purity$_1$, if a fact $[p]$ is fundamental, then every property F and object a such that $[Fa] = [p]$ are fundamental. According to Weak Purity$_1$, if a fact $[p]$ is fundamental, then there exists at least one fundamental property F and object a such that $[Fa] = [p]$ (for each fundamental $[p]$).

The limitations of these principles ought to be apparent. They only apply to fundamental facts of a particular syntactic structure: those involving monadic, first-order predication. Strong Purity$_1$ lacks its intended scope. For example, it could be that $[\forall x Fx]$ is a fundamental fact, but that property F is derivative. While there is a method of constructing $[\forall x Fx]$ that relies upon derivative constituents (by predicating $\forall$ of F), this does not conflict with Strong Purity$_1$, as this construction does not involve monadic, first-order predication. Correspondingly, Weak Purity$_1$ is too restrictive. The intended principle was that there

[15] Note that the weak interpretation is not the claim that at least one constituent of $[Fa]$ is fundamental, but rather that there is at least one method of construction where all of the components are fundamental.

[16] Note that introducing these predicates does not itself guarantee that, for every type τ, there is a fundamental term of that type—it could be that some of these predicates have empty extensions.

is some-method-or-other of constructing each fundamental fact from fundamentalia—not that this construction need involve first-order predicates.

We can generalize Strong Purity$_1$ into a schema with applications in every type:

Strong Purity$_2$:
$\forall p^t \forall X^{\tau \to t} \forall x^\tau ((Fundamental(p) \wedge Xx = p) \to (Fundamental(X) \wedge Fundamental(x))$

Strong Purity$_2$ effectively universally quantifies over the types; for every type τ a principle of this form holds. This quantification can only be made explicit in the meta-language. It is impossible, within the simply typed λ-calculus, to quantify over terms of all types whatsoever. Quantifiers are constants that occur within the hierarchy of types: ones that quantify over terms lower on that hierarchy than themselves. Any quantifier stated within the object-language would omit instances of Strong Purity that fall higher on the hierarchy than itself.

But how are we to modify Weak Purity$_1$? As with the generalization of Strong Purity$_1$, we might attempt a schematic interpretation:

Weak Purity$_2$:
$\forall p^t (Fundamental(p) \to \exists X^{\tau \to t} \exists x^\tau (Xx = p \wedge Fundamental(X) \wedge Fundamental(x))$

Weak Purity$_2$ is far too demanding for our purposes (indeed, even more demanding than Weak Purity$_1$). It does not state that there is some-way-or-other to construct each fundamental fact, but rather that there is a method of fundamental construction *for every type*. It holds that a fundamental fact $[p]$ can be constructed from a fundamental first-order property, from a fundamental second-order property, from a fundamental sentential operator, and, indeed from a fundamental term of each of the infinitely many types that there are. The problem is that the schematic approach effectively *universally* quantifies over the types—but we require existential quantification.

We might attempt to express the existential interpretation in various ways. For example, we could appeal to an infinitely long disjunction; if a fact $[p]$ is fundamental, then either $[p]$ can be constructed from a fundamental first-order predicate, or a fundamental second-order predicate, ... But this is less than ideal.[17] Such a principle could never be stated, so its utility for philosophical arguments is severely limited. Moreover, the standard syntax of λ precludes infinitely long disjunctions—so we would need to revise our syntax for this interpretation to even be grammatical.

We confront the expressive limitations of simply-typed languages. In disambiguating Purity, we attempt to quantify over the types themselves. For a fundamental fact $[p]$, we would like to distinguish the claim that terms of *every* type that construct $[p]$ are fundamental from the claim that terms of *some* type that construct $[p]$ are fundamental. These quantifiers cannot be stated in simply-typed languages. It seems to me that the right thing to do, in this case, is to shift to a language where they can be expressed.

[17]For a more detailed discussion of limitations of this sort, see Wilhelm (Forthcoming).

The Calculus of Constructions

One language that allows for type-quantification is the Calculus of Constructions. The Calculus contains all of the grammatical categories of simply-typed languages, and many more besides. In the simply-typed λ-calculus, the types serve to mark the syntactic categories of constants and variables. There are no operations that can be performed on the types themselves—aside from functional relations that serve to generate further types. By contrast, the Calculus of Construction allows for claims to be made explicitly about types. To that end, new grammatical categories, called 'sorts,' are introduced. Two sorts are especially significant: $\star$, which is the grammatical category of types, and $\square$, which is the grammatical category of grammatical categories. Thus, e, t and all functional constructions of them are of category $\star$, while $\star$ itself is of category $\square$.

The process for constructing additional grammatical categories is complex, but—as in simply-typed languages—builds complex categories out of simpler ones. For nearly any grammatical categories A and B, there is a category:

$$\prod x : A.B$$

We can break down the interpretation of this formalism. Here, '$\prod$' is a device for variable binding; it binds occurrences of 'x'—which would otherwise occur free within 'B'. The substring '$x : A$' asserts that term x has grammatical category A. For example, the expression '$p : t$' asserts that the term 'p' has the grammatical category t, and the expression '$t : \star$' asserts that t has the grammatical category $\star$.[18] As these examples indicate, types could appear on either side of ':'. When they occur on the right, they perform a similar function that they do within λ—indicating the grammatical category of the term on the left. However, when they occur on the left, they figure within the 'object language'; claims are made about the types themselves. The symbol '.' performs the same role here as in λ; it indicates a function from expressions on its left to ones on the right. So, collectively, '$\prod x : A.B$' is a grammatical category of expressions that combines with expressions of category A in order to generate expressions of category B.

More precisely, the language of the Calculus of Constructions is given by the following:

1. $\star$ and $\square$ are constants where $\star$ has grammatical category $\square$.
2. e and t are constants that both have grammatical category $\star$.
3. For any string of symbols A and B such that A is of category s and B is of category s', for every variable x of category A, the string '$\prod x : A.B$' is of category s'.
4. For any string of symbols A of grammatical category s for some sort s, there are infinitely many constants and variables of category A.

[18] In a sense, the string '$p : t$' expresses two claims simultaneously. Not only does it assert that 'p' has category t, but also that p is true.

A quick point about syntax. In line 4, note the restriction to an *s* that is a *sort*. This prevents terms like '*p*' and '*a*' from functioning as grammatical categories themselves, as the string '*p*' is of grammatical category *t*, which is a type (and not a sort).

We can introduce the standard logical constants—though their grammatical categories are somewhat more complicated than in simply typed languages:

1. $\neg$ is of category $(\prod x : t.t)$
2. $\wedge, \vee, \rightarrow, \leftrightarrow$ are of category $(\prod x : t.(\prod y : t.t))$
3. $=$ is of category $(\prod \alpha : \star.(\prod x : \alpha.(\prod y : \alpha.t)))$
4. $\approx$ is of category $(\prod \alpha : \star.(\prod \beta : \star.t))$
5. $\forall$ and $\exists$ are of category $(\prod \alpha : \star.(\prod .y : (\prod x : \alpha.t).t))$
6. $\mathbb{\forall}$ and $\mathbb{\exists}$ are of category $(\prod x : (\prod \alpha : \star.t))$

Truth-functional connectives operate analogously in pure-type theory as in simply-typed languages. In each, they take sequences of sentences as their inputs—and have a sentence as their outputs. However, the identity sign '=' operates somewhat differently. In simply-typed languages, we introduce infinitely many identity signs; for every type τ there is a constant of type $(\tau \rightarrow (\tau \rightarrow t))$ used to express the claim that terms of type τ are identical. By contrast, the Calculus of Constructions contains a single identity predicate for terms of every type. It is a function with a sequence of three inputs, rather than two; the first input is a type, and the subsequent two inputs are terms of that type. The output of this function is a sentence—intuitively, the sentence asserting that the second two inputs are identical.

Nevertheless, it would be inaccurate to claim that there is only one identity sign within this language. While there is but a single identity predicate for terms of arbitrary type, there is a second identity predicate—$\approx$—used to express the identity of the types themselves.[19] The introduction of this second identity predicate is no accident, but reflects an underlying limitation of the Calculus of Constructions; while it is possible to quantify over the types within this language, it is impossible to introduce variables that range over both types and ordinary terms. Any identity sign '=' that takes terms as its inputs cannot also take types as its inputs.

Some might fear that this language ventures toward inconsistency. Quantification over all terms—regardless of type—has historically led to problems. Quantifiers of absolute generality fall within their own extension. Once we allow properties to apply to themselves, self-referential paradox is immanent.

In some respects, consistency concerns are premature. Thus far, I have informally characterized the syntax of the Calculus of Constructions. I have not yet introduced any axioms of inferential rules. Without a proof-theory, we lack the ability to prove

[19] Similarly, there is but a single universal and existential quantifier for terms of every type (one which takes a type α as its first input, and a term typed as a function from α to t as its second)—as well as a universal and existential quantifier for the types themselves.

anything—much less the power to prove everything. But there are natural ways to introduce this proof-theory. We might stipulate that the standard logical constants operate classically—that quantifiers both over terms and types obey higher-order analogs of first-order inferences (so that $p : t$ entails both $\exists x : t.x$ and $\mathbb{E}x : \star.p : x$)—and that there inferences that resemble β-reduction.

As it turns out, the resulting language is provably consistent—though it is a matter of some interest why.[20] Inconsistency sometimes arises due to the manner in which absolute quantification is introduced. These quantifiers are often introduced with the use of comprehension principles—principles that entail that a quantifier falls within its own extension. Typically, philosophers respond to the ensuing paradox by retreating to languages like the simply-typed λ-calculus, that lack this quantification.[21] But in the Calculus of Constructions, no terms fall within their own extensions. Rather, quantifiers over types are introduced—but these quantifiers lack types themselves. In effect, we 'push' the types into the object language without the use of comprehension principles. This, we can do consistently.

Purity Revisited

In the simply-typed λ calculus, we required infinitely many predicates for fundamentality—one for each of the infinitely many types. By contrast, within the Calculus of Constructions we can operate with a single fundamentality predicate: one that applies to terms of every type. Just as identity is function whose first input is a type, and whose subsequent inputs are terms of that type, so too 'fundamental' is a function whose first input is a type and whose subsequent input is a term of that type. More precisely, let us introduce the following fundamentality predicate:[22]

$$\mathcal{F} : (\prod \alpha : \star.(\prod x : \alpha.t))$$

With the use of this predicate, we restate Strong Purity in a manner that makes the universal quantification over types explicit:

Strong Purity$_3$:
$\forall p^t \mathbb{V}\alpha \forall X^{\prod w : \alpha.t} \forall x^\alpha.((\mathcal{F}(p) \wedge p = Xx) \rightarrow (\mathcal{F}(X) \wedge \mathcal{F}(x)))$

[20]For proof, see Barendregt (1984).

[21]The extent to which philosophers theorize in simply-typed languages 'because' of these paradoxes is somewhat debated; Goodman (2024) argues that the restrictions of type theory are motivated by independent restrictions on grammatical expressions within our language—rather than the need to avoid paradox. For what it's worth, the first type-theories of Church (1932) were inconsistent. See Kleene and Rosser (1935) for the original derivation of inconsistency.

[22]Note that this predicate cannot assert that a type itself is fundamental—though we could introduce an alternate predicate $\mathcal{F}_\star$ of category $\prod \alpha : \star.t$ for this purpose.

Weak Purity can then be formalized in the obvious way:

Weak Purity$_3$:
$\forall p^t(\mathcal{F}(p) \rightarrow \exists\!\!\exists\alpha \exists X^{\Pi w : \alpha . t} \exists x^{\alpha}.(\mathcal{F}(X) \wedge \mathcal{F}(x) \wedge (p = Xx)))$

A metaphysician who appeals to either principle ought to be concerned with its consistency. As languages expand their expressive power, the risk of contradiction grows. In order to prove that Strong and Weak Purity$_3$ are consistent, much work would be required. We would need to introduce a model theory for the Calculus of Constructions—one that is notoriously arduous—and establish the existence of a model that validates each principle. I will not discuss this model theory here, and so am unable to prove that these principles are consistent.

Nevertheless, I think that we ought to be extremely confident that both Strong and Weak Purity$_3$ are consistent—perhaps as confident as we could possibly be in the absence of proof. Deviant interpretations of our predicate $\mathcal{F}$ are sure to validate both principles. While my intended use of '$\mathcal{F}$' is to denote fundamentality, from a mathematical perspective it could denote any property whatsoever. Suppose we interpret it to mean self-distinctness—so that $\mathcal{F}(x^t) := \lambda x^t . x \neq x$. On this interpretation, both Strong and Weak Purity$_3$ are vacuously true; each is a conditional with a false antecedent, since every proposition is self-identical. [23] Thus, if we can consistently assert that no proposition is distinct from itself, then both Weak and Strong Purity$_3$ are consistent.

Weak Purity$_3$ strikes me as the most faithful interpretation yet. But perhaps some maintain that even this formulation is too strong. It requires that, for a fundamental fact $[F]$, there exist *immediate* fundamental terms that generate $[F]$. There must exist two fundamental terms of some-type-or-other, such that predicating the first of the second is identical to $[F]$. But perhaps $[F]$ is constructed from fundamentalia 'down the line.' That is, it could be that the immediate terms that generate $[F]$ are derivative, but these derivate terms are themselves constructible from purely fundamental terms.[24]

[23]There are also almost surely non-vacuous models for these principles. We could, alternatively, interpret $\mathcal{F}$ to mean 'is self identical'–so that $\mathcal{F}(x) := \lambda x^{\tau} . x = x$. This interpretation nearly validates Strong and Weak Purity$_3$ itself; the only further requirement is that every proposition is identical to some-instance-of-predication-or-other. If the Calculus of Constructions permits β-identification—so that we can consistently state that $\lambda x . Fx(a) = Fa$—this will suffice. Note, however, that for the vacuous interpretation we only needed the assumption that all propositions are not distinct from themselves; for the nonvacuous interpretation we need the assumption that terms of arbitrary type are self-identical.

[24]Given additional assumptions, this sort of case could not arise. If the result of predicating one fundamental term of another always results in something fundamental (e.g., if $(\mathcal{F}(F) \wedge \mathcal{F}(a)) \rightarrow \mathcal{F}(Fa)$), fundamentality would never 'leapfrog' over terms in this way. However, the claim that combining fundamental terms always results in something fundamental is not particularly plausible. Even if we grant that [Electron e is negatively charged], [Proton p is positively charged] and disjunction are all fundamental, we might deny that [Either electron e is negatively charged or proton p is positively charged] is fundamental. (This sort of case is taken from Raven (2016)). Quite generally, it seems that we will be able to generate gruesome combinations of fundamental facts—combinations that are not plausibly fundamental themselves.

Considering a particular example might help to make this thought concrete. Of course, any particular instance of fundamentality is bound to be controversial—as there is currently no consensus over what the fundamental facts are—but it can serve to illustrate the structure that I have in mind. Suppose that both electron e and the relation of *having the same charge as* are fundamental—and let us denote these with 'e' and '$\lambda x, y.SCxy$' respectively. It is at least somewhat plausible that the property '$\lambda y.SCey$'—the property of *having the same charge as e*—is derivative. Many objects bear this property due to facts about their microphysical structures; the reason an object has the same charge as an electron is due to the fact that it contains one more electron than proton. Because the bearing of this property is typically explained in virtue of the distribution of various particles, the property is not itself fundamental. Nevertheless, we might maintain that $[SCee]$ is a fundamental fact; nothing explains why e has the same charge as itself. If this is so, then $[SCee]$ is a fundamental fact whose immediate constituents are derivative, that can nevertheless be constructed from fundamentalia.[25]

We can modify Weak Purity$_3$ to allow for mediate—and not merely immediate—fundamental construction. Interestingly, this modification can be stated in simply-typed languages; the schematic approach used to express Strong Purity$_2$ applies to this principle as well. The obvious way formalize mediate fundamental construction, in the present context, is via recursion.

For every type τ, let us introduce a new predicate FC of type $(\tau \to t)$ with the intended interpretation that '$FC(\phi)$' asserts that ϕ is fundamentally constructible. For any terms ϕ of type $(\tau_1 \to \tau_2)$ and ψ of type τ_1, the conditions of fundamental constructibility are given as follows:[26]

i. $(Fundamental(\phi) \wedge Fundamental(\psi)) \to FC(\phi(\psi))$
ii. $(Fundamental(\phi) \wedge FC(\psi)) \to FC(\phi(\psi))$
iii. $(FC(\phi) \wedge Fundamental(\psi)) \to FC(\phi(\psi))$
iv. $(FC(\phi) \wedge FC(\psi)) \to FC(\phi(\psi))$

With this new predicate, Weak Purity can be formalized simply as:

Weak Purity$_4$:
$\forall p^t(Fundamental(p) \to FC(p))$

[25] This example could be resisted in various ways. Aside from denying that either e or $\lambda x.y.SCxy$ are fundamental, a philosopher might claim that $[SCee]$ is itself derivative; perhaps *charge* is a gradable property—and $[SCee]$ is grounded in the fact that every object bears the same gradable property as itself. Alternatively, it could be that there is some other way of constructing this fact from other fundamental properties; perhaps $\lambda x.SCxx$ is fundamental (my thanks to Isaac Wilhelm for this suggestion).

[26] In the obvious way, we could express this in the Calculus of Constructions by introducing FC as a relation between types and terms—in much the way that we introduced a single predicate for fundamentality within this language.

If $[F]$ is fundamental, then $[F]$ is fundamentally constructible.

Thus far, we have engaged in a process of progressively weakening the interpretation of Purity. However, Weak Purity$_4$ may have overshot the mark. Given plausible assumptions about fundamentality and granularity, this principle is trivial. Moreover, there are numerous paths toward this triviality; several combinations of plausible assumptions entail that Weak Purity$_4$ is true.

Suppose, as seems plausible, that the boolean connectives are fundamental; $\neg$, $\vee$, $\wedge$ and all the rest are fundamental operators.[27] Many accounts of propositional identity license the principle Involution, according to which $p = \neg\neg p$.[28] If this is so, then Weak Purity$_4$ is true. For an arbitrary fundamental p, line *i* entails that $\neg p$ is fundamentally constructible—and line *ii* then entails that $\neg\neg p$ is fundamentally constructible as well. Given Involution, $\neg\neg p = p$; so, by Leibniz's Law, p is fundamentally constructible. Because the selection of p was arbitrary, all fundamental facts are fundamentally constructible.

The appeal to Involution can be avoided. For analogous reasons, philosophers who endorse Idempotence—the claims that $p = p \wedge p$ and $p = p \vee p$—are committed to Weak Purity$_4$ as well. Nor is it necessary to appeal to fundamental boolean connectives. If the operator $\lambda x^t.x$ is fundamental—and if β-equivalent terms co-refer—Weak Purity$_4$ is trivially true.[29] In each of these cases, a fundamental fact $[F]$ can be constructed from fundamentalia—but *it itself* is one of the fundamental constituents that it is built from. Insofar as we consider construction to be a process whereby some terms are constructed from others, we might want to preclude cases where $[F]$ constructs itself.

It is straightforward to modify the definition of fundamental construction to rule out these cases. For each fundamental p, we recursively introduce a property FC_{-p}:

i. $(Fundamental(\phi) \wedge Fundamental(\psi) \wedge \phi \neq p \wedge \psi \neq p) \rightarrow FC_{-p}(\phi(\psi))$
ii. $(Fundamental(\phi) \wedge FC_{-p}(\psi) \wedge \phi \neq p) \rightarrow FC_{-p}(\phi(\psi))$
iii. $(FC_{-p}(\phi) \wedge Fundamental(\psi) \wedge \psi \neq p) \rightarrow FC_{-p}(\phi(\psi))$
iv. $(FC_{-p}(\phi) \wedge FC_{-p}(\psi)) \rightarrow FC_{-p}(\phi(\psi))$

[27]There are various reasons why philosophers might deny that these connectives are fundamental. Some, like Lewis (1986), maintain that fundamental reality is non-redundant; there are no fundamental facts $[F_1]$ and $[F_2]$ such that $[F_1]$ entails $[F_2]$. The thought underlying non-redundancy requirements is that, when God was creating the world, she was lazy—and did not want to do more work than was needed. Fundamental logical operators would be redundant, as facts about many combinations of them entail facts about the others. For discussions of this point, see Sider (2011).

[28]Coarse-grained theories (according to which necessarily equivalent propositions are identical) license involution—see e.g., Lewis (1986). Relatedly, classicist theories, which maintain that provably equivalent propositions are identical, also license Involution—see e.g., Bacon and Dorr (2024). Many fine-grained theories of propositional identity also license this principle—see, e.g., Fine (2017*a*,*b*). For an explicit argument that propositions are their double negations (along the lines that languages whose syntax ensured involution would not seem to be missing anything about the world) see Ramsey (1927).

[29]This last suggestion threatens to trivialize not only Weak Purity$_4$, but Weak Purity$_3$ as well. Those who hold that $\lambda x^t.x$ is fundamental ought to index Weak Purity$_3$ in a similar manner to the indexing of Weak Purity$_4$.

The only further complication is that the type of $=$ must be generalized from $(\tau \rightarrow (\tau \rightarrow t))$ (for every τ) to $(\tau_1 \rightarrow (\tau_2 \rightarrow t))$ (for every τ_1 and τ_2) to ensure that expressions like '$F \neq p$' are grammatical. I make the (hopefully uncontroversial) assumption that, in the case where ϕ and ψ have different types, they are distinct from one another; that is, for τ_1 and $\tau_2 \neq \tau_1$, the extension of $=^{(\tau_1 \rightarrow (\tau_2 \rightarrow t))}$ is empty.

As before, Weak Purity can then be interpreted as:

Weak Purity$_5$:
$\forall p^t(Fundamental(p) \rightarrow FC_{-p}(p))$

Every fundamental fact can be constructed from fundamental terms other than itself.

I think that Weak Purity$_5$ is the most faithful interpretation yet. It reflects the thought that every fundamental fact can be fundamentally constructed—while precluding trivial self-construction. Strong Purity$_2$ and Strong Purity$_3$ strike me as the most faithful versions of the universal interpretation; they reflect the thought that every method of constructing fundamental facts relies upon fundamental constituents. Ultimately, this discussion reveals that Purity might mean a number of different things; the principle that fundamental facts have fundamental constituents allows for stronger and weaker interpretations. Given this ambiguity, it is incumbent to determine how Purity figures in philosophical argumentation.

Appeals to Purity

Purity has broad scope; it applies to all facts, regardless of their content. It could be (and has been) used in any number of contexts, but arguments that appeal to it often have a similar structure. Here, I address two of the most significant applications of Purity: in theories of iterated ground and in the grounds of identification. While these cases are far from exhaustive, they illustrate the structure that I have in mind.

Suppose that Chris is in pain, and that the fact that Chris is in pain is grounded in the fact that his C-fibers are firing.[30] Why is Chris's pain grounded in this way? Why does phenomenal pain correspond to firing C-fibers, rather than some other neurological configuration? More generally, what is it in virtue of that the grounding relation obtains in the cases it does? What grounds facts about grounding? While these questions might appear academic, they impact traditional philosophical debates. Philosophers armed with a theory of iterated ground have the resources to explain, in a deep sense, what makes their theories true. For example, a normative naturalist who understands that grounds of ground can explain not only the natural foundations of normative facts—but also why normative facts are grounded in the way that they are.

[30]Some philosophers in this area do not interpret fundamentality in terms of grounding. For ease of prose, I will describe these cases in terms of ground, but this locution could easily be switched for any other that a metaphysician prefers.

Some might maintain that the answer is 'nothing.' Grounding facts seem to be natural stopping points for metaphysical explanation. If the grounding relation itself is primitive, there may be no reason why it obtains in some cases but not others. Nevertheless, numerous philosophers deny that grounding facts are fundamental. Most appeal to Purity.[31] Take the following:

> "Here is a truth: there exists a city. Since the notion of a city is not fundamental, purity says that this truth is not fundamental...this truth holds in virtue of some fundamental truth *T*–perhaps some truth of microphysics. So we have:
>
> (1) There is a city in virtue of the fact that *T*
>
> ...But now consider (1) itself. Just like 'There are cities,' (1) is a truth involving the notion of a city. And so, given purity, it cannot be a fundamental truth...Purity...requires facts about the *relationship* between the fundamental and the nonfundamental to be themselves nonfundamental. Thus purity brings a heavy explanatory burden: it requires there to be facts in virtue of which in-virtue-of-facts hold." (Sider, 2011, pg. 107)

It is straightforward to reconstruct this argument. Let us denote the grounds of cityhood—whatever those may be—as 'the fact that *T*'. There is, then, a fact: [There is a city in virtue of the fact that *T*]. We can construct this fact from the property of *being a city*; in particular, this fact results from predicating $\lambda X^{e \to t}$*.there is an X in virtue of the fact that T* of *is a city*. According to Purity, fundamental facts only have fundamental constituents. Since we can construct this fact from *being a city*, the property of *being a city* is one of its constituents—and is therefore fundamental if [There is a city in virtue of the fact that *T*] is fundamental. But the property of *being a city* is *not* fundamental. Therefore, the fact [There is a city in virtue of the fact that *T*] is not fundamental.

Take another example: the grounds of identity. There is a fact: [Hesperus is identical to Phosphorus]. What is it in virtue of that this identification obtains? Some philosophers think that the answer is: nothing; identifications are metaphysically fundamental. As (Dorr, 2016, pg. 41) says, "Identities are excellent stopping places for explanation; they do not cry out for explanation in their own right. Indeed, there is something odd about questions like 'Why is Hesperus Phosphorus?'. Unless this is understood as a request to be reminded of the reasons for believing that Hesperus is Phosphorus, it is hard to know what would count as a satisfying answer."[32]

[31]Philosophers who appeal to this argument include Sider (2011, 2020); Bennett (2011); deRosset (2013, 2023); Dasgupta (2014*b*). This is not the only motivation for theories of iterated ground. Another—which appeals to principles of free modal recombination, rather than Purity, occurs in Schaffer (2010); Bennett (2011).

[32]Other philosophers who deny that identity can be metaphysically explained include Lewis (1986); Salmon (1987); Horsten (2010).

Nevertheless, a growing number of philosophers deny that identifications are fundamental, many citing Purity.[33] Take the following:

> "The identity problem arises when we look to metaphysically explain the identity facts involving concrete objects, facts like The Rock of Gibraltar = The Rock of Gibraltar, and The Original McDonalds Big Mac = The Original McDonalds Big Mac...many such identity facts strike us as non-fundamental: they often involve non-fundamental objects like Big Macs and giant rocks, and it is doubtful that fundamental facts should involve non-fundamental objects." (Shumener, 2019, pg. 2074)

This argument can also be reconstructed. Take an identification—for example, [The Rock of Gibraltar = the Rock of Gibraltar]. We can construct this fact from *The Rock of Gibraltar*; in particular, it results from predicating $\lambda x^e.x = x$ of *The Rock of Gibraltar.* According to Purity, fundamental facts only have fundamental constituents. Because we can construct this fact from *The Rock of Gibraltar*, the rock is one of its constituents—and is therefore fundamental if [The Rock of Gibraltar = the Rock of Gibraltar] is fundamental. But *The Rock of Gibraltar* is *not* fundamental. Therefore, the fact [The Rock of Gibraltar = the Rock of Gibraltar] is not fundamental.

Analogous arguments crop up in other areas. Structurally similar appeals to Purity occur in discussions of the grounds of certain modal facts—e.g., [Necessarily, all cities are cities]—the grounds of negative facts—e.g., [It is not the case that there is a direct flight between Australia and Spain]—and the grounds of nongrounding facts—e.g., [The fact that Socrates is wise is not grounded in the fact that {Socrates} contains someone wise].[34]

The structure of all of these arguments is as follows. We take a fact $[F]$—some instance of a fact in our domain of interest. We note that fact $[F]$ can be constructed from an entity e, before citing Purity: the fact that fundamental facts only have fundamental constituents. Because e is a constituent of $[F]$, we conclude that $[F]$ is fundamental only if e is fundamental. But entity e is not plausibly fundamental. Therefore, $[F]$ is not fundamental.

Arguments of this structure only succeed on Strong interpretations of Purity. If every method of constructing a fundamental $[F]$ involves fundamental constituents—and if one method of construction involves entity e, then e must be fundamental; if we deny that e is fundamental, we ought to deny that $[F]$ is fundamental as well. By contrast,

[33] Aside from Shumener (2019), examples of philosophers who appeal to an argument of this sort include Litland (2023); Rubenstein (2023); Elgin (2024*a*). A philosopher who hints at—but does not explicitly endorse—this argument is Wilhelm (2020). Not all philosophers who maintain that identifications are derivative appeal to this argument—e.g., Fine (2016) argues that identifications are zero-grounded (but may have substantive grounds) without appeal to Purity.

[34] For discussions of modal and negative facts, see Sider (2011)—for a discussion of nongrounding facts see Elgin (Forthcoming).

Weak Purity—in its various indexes—does not justify this inference. If there is some-method-or-other of constructing $[F]$ from fundamental constituents, and if one method of construction involves entity e, this does not entail that e is fundamental. The method of fundamental construction could involve entities other than e. So, this style of argument depends upon Strong Purity, and not merely Weak Purity. This is not to say that these arguments universally succeed if Strong Purity is true—there are any number of other places that they might fail. But for us to be justified in appealing to them, we need a reason to think that Strong Purity holds. Weak Purity is not enough.

Arguments for Purity

I think that we ought to be skeptical of Strong Purity. There are two reasons for my skepticism. Plausible metaphysical positions are compatible with Weak, but not Strong Purity. Moreover, the positive arguments given for Purity (limited though they are) only justify the Weak interpretation.

A dominant theory of propositional identity is *classicism*, according to which provably equivalent terms are identical: if $\vdash p \leftrightarrow q$, then $p = q$.[35] Classicism is a simple and elegant theory of identification. Moreover, if follows from coarse-grained accounts, which hold that necessarily equivalent propositions are identical.[36]

Classicists who endorse Strong Purity must maintain that everything is fundamental.[37] Select a fundamental fact $[F]$ and an arbitrary object a. Provably, $[F] \leftrightarrow [\lambda x.(F \vee (Gx \wedge \neg Gx)(a)]$; given Classicism, it follows that $[F] = [\lambda x.(F \vee (Gx \wedge \neg Gx)(a)]$. Therefore, fundamental $[F]$ can be constructed from a. If Strong Purity is true, it follows that a is fundamental—and, since the selection of a was arbitrary, that absolutely everything is fundamental. This does not hold for Weak Purity. While a fundamental $[F]$ can be constructed from an arbitrary a, this need not entail that a is fundamental, so long as there is some way to construct $[F]$ from fundamentalia other than a. Classicists who would avoid universal fundamentality ought to reject Strong Purity, but need not reject Weak Purity.

Another metaphysical view that precludes Strong Purity takes a stand on the types of fundamental things. Some maintain that fundamental reality consists of objects and properties that those objects bear. The foundational facts consist of what objects there are, and how those objects are. On this view (which sometimes goes by the label 'truth supervenes on being'), fundamental facts contain terms for fundamental object and fundamental properties, but there are no fundamental terms of other types. This sort of position

[35]See Bacon and Dorr (2024).

[36]See Lewis (1986). Note that this entailment requires the use of the Necessitation Rule, according to which $\vdash p$ entails $\vdash \Box p$.

[37]More precisely, they must maintain that either everything is fundamental or nothing is—but I set aside the possibility that nothing is fundamental for the purposes of this paper.

has been used to argue against the fundamentality of counterfactuals, totality facts and negative facts.[38]

Arguments for this view would take us far afield. For my purposes, what matters is that *if this is true*, then nearly all conceptions of propositional identity are incompatible with Strong Purity. As mentioned previously, many license the principle of Involution: $[F] = [\neg\neg F]$. For this reason, a fundamental fact $[F]$ can be constructed from negation—$[F] = [\lambda x^{t \to t}.(\neg F)(\neg)]$. If Strong Purity is true, it follows that negation is fundamental. But, on the Truth Supervenes on Being view, negation is *not* fundamental. There are no fundamental sentential operators on this view, and negation is a sentential operator. So, this view rules out Strong Purity (on the assumption that involution is true).[39] Notably, it does not rule out Weak Purity. So long as $[F]$ is constructible from other fundamental terms, negation need not be fundamental.

To be clear, I do not commit either to Classicism or the claim that truth supervenes on being here. They strike me as plausible—yet controversial—metaphysical commitments. But I find it telling that eminently defensible views rule out Strong Purity, without ruling out Weak Purity.

What about positive arguments for Purity? Do they justify the Strong or Weak claims? Admittedly, some philosophers do not provide an argument for Purity, but rather treat it as a starting point for inquiry.[40] But we can consider the arguments that there are.

The most common argument depends upon the theoretical role that the fundamental plays. While fundamental facts may stand in grounding relations, fundamental things serve to construct fundamental facts. As such, it ought to be possible to describe fundamental reality without reference to derivative things. While this characterization (in terms of reference) is given semantically, the underlying point is worldly. As Sider (2011) says, "When God was creating the world, she was not required to think in terms of nonfundamental notions like city, smile, or candy." (pg. 106).

This argument only supports Weak Purity. It is possible to refer to fundamental facts without reference to derivative entities—as our terms may denote the fundamentalia that witness the existential claim. Metaphorically, God could have constructed the fundamental facts purely from fundamental consituents, by not appealing to the methods of construction involving the derivative. More generally, the fundamental can serve its intended purpose by serving as the building blocks for fundamental facts even if Strong Purity is false; Weak Purity allows for every fundamental fact to be constructed from purely fundamental constituents, which is what Purity was intended to require.

[38]For a discussion of this kind of view, see Sider (2011).

[39]In an analogous way, this view is incompatible with Idempotence—$p = p \wedge p$ and $p = p \vee p$. This result reflects a more general feature of Strong Purity. If it holds, then on nearly every account of propositional identity $\neg$, $\wedge$ and $\vee$ are fundamental.

[40]Other philosophers simply define the notion of 'fundamental' so that Purity is true. Take, e.g., "Say that a fact is fundamental (or brute) if it does not obtain in virtue of other facts, and that a thing is fundamental if it is a constituent of a fundamental fact." (Rosen, 2010, pg. 112). I will not respond to argument by definition here—aside to say that one cannot define one's theory into truth.

Conclusion

The principle of Purity—according to which fundamental facts only have fundamental constituents—contains a subtle ambiguity. It could either be interpreted as the claim that every method of constructing a fundamental fact relies upon fundamental constituents or, alternatively, that there exists at least one way of constructing each fundamental fact from fundamental constituents. There are various ways of formalizing these principles, some of which can be expressed in simply typed languages—and others of which require more expressive power. The arguments that metaphysicians typically use involving Purity only succeed if Strong Purity is true; Weak Purity is not sufficient for their purposes. Nevertheless, it strikes me that Strong Purity is open to doubt. Various defensible philosophical commitments are compatible with Weak—but not Strong—Purity, and the argument functional argument for Purity can be correct even if only Weak Purity is true. As such, metaphysicians who would appeal to Strong Purity owe an argument: a reason to think that the stronger version holds as well.

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