

On a fallacy in the Kaldor-Hicks efficiency-equity analysis

David Ellerman
University of California at Riverside

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Abstract

This paper shows that implicit assumptions about the numeraire good in the Kaldor-Hicks efficiency-equity analysis involve a "same-yardstick" fallacy (a fallacy pointed out by Paul Samuelson in another context). These results have negative implications for cost-benefit analysis, the wealth-maximization approach to law and economics, and other parts of applied welfare economics—as well as for the whole vision of economics based on the "production and distribution of social wealth."

JEL: D6, K

Key Words Kaldor-Hicks criterion, wealth maximization, efficiency, equity, law and economics, cost-benefit analysis

1 Introduction: Two visions of economics

John R. Hicks (12) has usefully juxtaposed two visions or schools of economics.

- One school goes from the classical economists through Marshall and Pigou to Kaldor and Hicks (e.g., Kaldor-Hicks or KH principle and the rehabilitation of consumer's surplus) and through Keynes to macroeconomics. It emphasized the production and distribution of the social product (Pigou's "national dividend"), so it might be called the *production and distribution school* of economics.
- The other school of economics was ushered in by the marginalist revolution of the Lausanne and Austrian schools, and it emphasized mutually beneficial exchange and it had no overarching notion of social wealth. It might be called the *exchange or catallactics school* of economics.

This paper explains a crucial methodological error in the principal applications of the production and distribution school, particularly those derived from the KH principle.

At first the marginalist revolution and the notion of Pareto optimality displaced the older welfare economics that analyzed the production and distribution of social wealth. But since the notion of a Pareto superior change (a Pareto improvement) was considered rather theoretical and impractical, there was a counter-reformation led by Hicks and Kaldor to rehabilitate in more modern terms the

older vision based on the production and distribution of social wealth. The austere criterion of a Pareto superior change in social state was to be replaced by the Kaldor-Hicks principle (13), (10).

For any proposed change in social state, the gains of the gainers and the losses of the losers were to be measured in terms of the numeraire and added up. If the total was positive, then the change would represent an increase in social wealth, a *Kaldor-Hicks improvement*, where, in principle, the gainers could compensate the losers so that there were no post-compensation losers. The change in social state, i.e., the KH improvement, plus the compensations—if actually made—would constitute a Pareto improvement. Having thus parsed the total Pareto superior change into the wealth-increasing "production" part (the KH improvement) and the constant-wealth "distribution" part, the Kaldor-Hicks principle would recommend the wealth-increasing KH improvement on efficiency grounds since efficiency is the professional concern of economics—while leaving to one side the equity question involved in the constant-social-wealth redistributive compensations part.

This argument lends justification to the procedure, adopted by Professor Pigou in *The Economics of Welfare*, of dividing "welfare economics" into two parts: the first relating to production, and the second to distribution. (13, p. 551)

Tibor Scitovsky quickly showed that under certain circumstances, a change recommended by the KH principle could change the situation in such a way that the KH principle would then recommend reversing the change (20). Then modifications were suggested and a whole literature of detailed analysis ensued [see one summary in: (17)]. This paper identifies a more fundamental problem, namely the division of a Pareto improvement into the "efficiency" part and the "equity" part depends crucially on the unit of account or numeraire. The resulting same-yardstick fallacy applies to all cases in the KH analysis, not just to special Scitovsky-like examples.

The Marshall–Pigou production and distribution tradition was thus modernized by Kaldor and Hicks and the seemingly austere Paretian notion of efficiency was broadened in the "Kaldor–Hicks (wealth maximization. . .) concept of efficiency"(16, p. 317) Today any project or change in social state that increases the "social wealth" according to the Kaldor–Hicks criterion (i.e., a KH improvement) is routinely interpreted as an "increase in efficiency" particularly in the law and economics literature, cost–benefit analysis, policy analysis, and other parts of applied welfare economics.¹

Another staple in the modern production-and-distribution school is Alfred Marshall's notion of consumer's surplus (and the related notion of producer's or supplier's surplus) which had a similar trajectory. Initially the concept of consumer's surplus came in for much criticism from the neoclassical school [summarized in: (18)] but then it too was rehabilitated by Hicks (11) and more recently by Harberger (9) and Willig (21) among others. Here again, in spite of the stringent assumptions needed to theoretically justify the construction, it continues to be used broadly in applied welfare economics and in the textbooks.

¹The change in state could include many projects in one time period or an extended program over many time periods. Also the state changes could be stated in probabilistic terms so that the expected changes in social wealth of the winners would outweigh the expected decreases in social wealth of the losers in the "multi-change hypothetical criterion".(15, p. 420) But that KH hypothetical criterion suffers from the same methodological fallacy in the hypothetical compensation criterion.

Today, much of the work in applied welfare economics, e.g., in the law and economics literature and in cost-benefit analysis, is ultimately based on the Kaldor-Hicks principle and uses "without apology" the notion of consumer's surplus. In Robin Boadway and Neil Bruce's monograph on welfare economics, they evoke the Kaldor-Hicks parsing of a total (Pareto superior) change into the efficiency and equity parts.

The purpose of considering hypothetical redistributions is to try and separate the *efficiency* and *equity* aspects of the policy change under consideration. It is argued that whether or not the redistribution is actually carried out is an important but *separate* decision. The mere fact that it is possible to create potential Pareto improving redistribution possibilities is enough to rank one state above another on efficiency grounds. (3, p. 97)

Boadway and Bruce as well as others (1) have noted that general equilibrium considerations may frustrate the clean separation of the efficiency and equity parts, but here again this criticism concerns higher-order effects with little or no impact in applied welfare economics.

Richard Posner makes the same parsing in the context of law and economics as well as cost-benefit analysis using the standard metaphor for the efficiency question—make the pie larger—and the equity question—the relative shares of the pie.

But to the extent that distributive justice can be shown to be the proper business of some other branch of government or policy instrument. . . , it is possible to set distributive considerations to one side and use the Kaldor-Hicks approach with a good conscience. This assumes, . . . , that efficiency in the Kaldor-Hicks sense—making the pie larger without worrying about how the relative size of the slices changes—is a social value. (16, pp. 318-9)

The purpose of this note is to point out a basic methodological error in the reasoning about the Kaldor-Hicks principle and the consumer's surplus concept that seems to have gone unnoticed in the debate—and yet the error vitiates the use of those tools in applied welfare economics such as in law and economics and cost-benefit analysis.

2 The blind-spot in any measurement system

Empirical statements often involve the use of some coordinate or measurement system. Non-trivial statements like "The intersection of the supply and demand curves has the coordinates (3,40)" will use the coordinate system. But there are other statements which are superficially similar like "The origin has the coordinates (0,0)" which are just trivial statements about the coordinate system itself and have no empirical content beyond that. The empirically empty statements that only reiterate an aspect of the coordinate or measurement system can be considered 'blind-spots' of the system (like the spot in the eye that transmit no visual information to the brain). "John values apples at one per dollar" is an empirical statement but "John values dollars at one per dollar" is a blind-spot statement that only reiterates that dollars are the numeraire. Fallacious arguments can be easily constructed which use blind-spot arguments as if they had empirical content. Here are some examples.

2.1 The Same-Yardstick Fallacy: A "proof" that yardsticks cannot change

Perhaps a yardstick has expanded or contracted and is thus not accurate. In order to determine if it still measures a true yard, we mark off its distance on, say, a table. Then we measure the distance using the (same) yardstick and conclude that the distance is indeed a yard long. But the conclusion that the distance on the table was a yard long (aside from negligible measurement error) was a blind-spot statement giving no new information since the distance was measured by the same yardstick used to mark off the distance. Hence the argument that the yardstick had not changed was fallacious. All the fallacies described below might also be called *same-yardstick fallacies*.

2.2 A "proof" that inflation is impossible

In the year, say, 1900, a dollar would buy a dollar's worth of goods. Would a dollar buy more or less in the year 2000? We find that a dollar still buys a dollar's worth of goods in 2000 so we conclude that a dollar has the same value at these two times. The same argument works for any two times so we must conclude that inflation is impossible. But the statements that a dollar buys a dollar's worth of goods is a blind-spot statement with no empirical content so the argument is another same-yardstick fallacy.

2.3 A "proof" that the Earth does not move

Let $S(t)$ be the coordinates of the Sun and $E(t)$ the coordinates of the Earth at time t where we are using the geocentric coordinate system. We observe that the coordinates of the Sun change over time while the coordinates of the Earth always stay the same (at the origin of the geocentric coordinate system). Hence we conclude that the position of the Earth does not change so the Church is vindicated and Galileo refuted. But the statement that the Earth does not move in geocentric coordinates is a blind-spot statement with no empirical content unlike a seemingly similar statement that a satellite has stationary coordinates (i.e., is a geostationary satellite). Hence this empirical conclusion that "the Earth does not move" is another example of the same-yardstick fallacy.

2.4 A "proof" that the Marginal Utility of Income is constant

The marginal utility of income is the marginal rate of change of utility with respect to a change in the consumer's income. Let $U(Q)$ be the utility level that results from a consumer maximizing utility at given prices and income. Since any monotonic transform of a utility function is equally acceptable as a utility function, we consider the money-metric utility function $E(P, U(Q))$ which is the minimum expenditure necessary to reach the level of utility $U(Q)$ at the given prices and income. Then we consider the marginal change in the money-metric utility $E(P, U(Q))$ with respect to a change in income. We find that the minimum expenditure necessary to reach the level of utility $U(Q)$ reached with, say, a dollar increase in income is exactly a dollar, so we conclude that the marginal utility of income is in fact constant (with value unity).

What is wrong with this "proof" that the marginal utility of income is constant? Instead of being an empirical statement about the marginal utility of income, it is only a mathematical consequence

of the use of the money-metric utility function to measure the marginal utility of income.

[T]he money-metric marginal utility of income is constant at unity. For how could it be otherwise? If you are measuring utility by money, it must remain constant with respect to money: a yardstick cannot change in terms of itself.(19, p. 1264)

Indeed, "a yardstick cannot change in terms of itself" is a good statement of the general same-yardstick fallacy.

2.5 A "proof" that an apple has the same value to all consumers

If Mary had an apple, then it would be worth one apple to Mary (using apples as the numeraire). If John had the apple, then it would also be worth one apple to John. An apple's an apple for all that. Since the same argument could be repeated for any potential consumer, we must conclude that an apple has the same value to any consumer so any exchange involving an apple would be pointless. But the statement that an apple is worth one apple to any consumer using apples as the numeraire is only a blind-spot statement with no empirical content. One needs a different yardstick of value, i.e., a different numeraire, to meaningfully determine if the apple had different values for Mary and John.

2.6 A "proof" that compensation payments do not change social wealth

While the above argument using the apple may seem obvious, our point is that exactly the same sort of blind-spot statement is involved in the 'argument' that the hypothetical compensation payments (which would turn a potential Pareto improvement, i.e., a "Kaldor-Hicks improvement" where the gains to the winners outweigh the losses to the losers all measured in the numeraire, into an actual Pareto improvement) do not change social wealth and thus are a question of equity rather than efficiency. Any unit of the numeraire will have the same value to anyone in terms of the same numeraire. Any argument based on the statement that transfers in the numeraire (compensation payments) are only redistributive (when their value is measured in terms of the same numeraire) is only an illusion—which might be called *numeraire illusion* (6), and is the principal example of the same-yardstick fallacy used here.

The empty blind-spot statements can only be avoided when the numeraire is not involved in the whole proposed change including the compensation payments. Consider a KH improvement where the compensation payments are actually made in some non-numeraire good so those who would have been losers—absent the compensations—are now winners (or are indifferent). But then the compensation payments will in general change the *size* of social wealth (measured using a non-involved numeraire) so the whole Kaldor-Hicks parsing of the total change into efficiency and equity parts breaks down.²

The Kaldor-Hicks methodology was based on numeraire illusion—using the blind-spot statement that the potential compensation payments in the numeraire were only redistributive (i.e., did not

²Or keep the compensation payments in money but evaluate all the changes using a non-involved good such as apples. Then the compensation payments like the other changes will change the *size* of the apple pie ("social wealth" measured in apples). See (6) for the algebraic proof.

change social wealth measured in terms of the same numeraire) to set that part of the total Pareto improvement to one side and to recommend the other part on 'efficiency' grounds. When a non-involved numeraire is used, then one cannot in general set aside any part of the Pareto improvement as being merely redistributive so all that can be recommended on efficiency grounds is the total Pareto improvement. Then we are back to the Pareto criterion and the attempt to establish a different notion of "Kaldor-Hicks efficiency" is seen to be based on the methodological error of the same-yardstick fallacy.

3 An example in the literature

The methodological error can be illustrated in a number of ways. As just noted, one can use a non-involved numeraire to evaluate the total proposed change including compensations in non-numeraire goods and then the total change becomes the efficiency part (in the case of a Pareto improvement) and the "redistributive part" (transfers in the numeraire) is empty since the numeraire is, by assumption, not involved in either the KH improvement or the compensations.

Another way to illustrate the problem is to switch to another involved numeraire and see that the recommendations of the KH principle will change so they do not pass the rather minimum requirement of being stable under changes of the numeraire or unit of account used for the "social wealth" calculation.

In the simple case of a single commodity transfer used in the textbooks (including the textbook use of consumer's and producer's surplus to illustrate an equilibrium in a single-commodity market), one even has the dramatic result of the efficiency and equity parts being interchanged.

The John and Mary apple example was used in David Friedman's law and economics text: *Law's Order: What Economics has to do with Law and why it matters.*(8) Mary has an apple which she values at fifty cents while John values an apple at one dollar. There might be a voluntary exchange where Mary sold the apple to John for, say, seventy-five cents. There are two changes in that Pareto improvement: the transfer of the apple from Mary to John and the transfer of seventy-five cents from John to Mary.

If we apply the Kaldor-Hicks reasoning to the apple transfer using money as the numeraire, then the apple was worth fifty cents to Mary and a dollar to John, so social wealth would be increased by fifty cents by the apple transfer from Mary to John. That is the KH improvement presented as an increase in efficiency. The other change, the transfer of seventy-five cents from John to Mary, is a question of distribution or equity. Social wealth (measured in dollars and cents) would be unchanged by the mere transfer of seventy-five cents from one person to another.

It would still be an improvement, and by the same amount, if John stole the apple—price zero—or it Mary lost it and John found it. Mary is fifty cents worse off, John is a dollar better off, net gain fifty cents. All of these represent the same efficient allocation of the apple: to John, who values it more than Mary. They differ in the associated distribution of income: how much money John and Mary each end up with.

Since we are measuring value in dollars it is easy to confuse "gaining value" with

"getting money." But consider our example. The total amount of money never changes; we are simply shifting it from one person to another. The total quantity of goods never changes either, since we are cutting off our analysis after John gets the apple but before he eats it. Yet total value increases by fifty cents. It increases because the same apple is worth more to John than to Mary. Shifting money around does not change total value. One dollar is worth the same number of dollars to everyone: one. (8, p. 20)

Now we change the numeraire from the involved good of money to the involved good of apples. "Changing the numeraire" does *not* mean just recomputing the values at some fixed public exchange rate. It means going back to each involved person and recomputing their gain or loss using their own rate of substitution. For John the (marginal) rate of substitution of dollars for apples was one so John would value the loss of the three-quarters of a dollar at three-quarters of an apple. The acquired apple would be worth one apple so the total change is worth one-quarter apple to John.

Mary's rate of substitution of dollars for apples was one-half dollar per apple so the reciprocal rate of substitution of apples for dollars is two apples per dollar. Hence she values the acquired three-quarters of a dollar at $\frac{3}{4} \times 2 = 1.5$ apples. Since the loss of her apple is worth one apple to her, she also has a positive net gain (half an apple) from the exchange.

But now the total "pie" of social wealth is an apple pie. When we evaluate the social wealth consequences of the money transfer and the apple transfer, we find that the money transfer of three-quarters of a dollar increased the size of the social apple pie by $\frac{3}{2} - \frac{3}{4} = \frac{3}{4}$ of an apple. The apple transfer had no effect on the size of the apple pie—an apple's an apple for all that—so that transfer was merely redistributive. One apple is worth the same number of apples to everyone: one.

That blind-spot statement might be contrasted with a similar-sounding but empirically significant statement. Suppose we use a non-involved numeraire of nuts, and that John and another person Tom both happen to value an apple at five nuts. The statement that "John and Tom assign the same value to an apple in terms of nuts" has empirical significance whereas the similar statement with "apples" substituted for "nuts" is a blind-spot statement with no empirical content and only restates the fact that apples were the numeraire.

Hence by merely describing the exact same transfers with a reversed numeraire, we reverse the recommendations of the Kaldor-Hicks principle. Now it is the transfer of three-quarters of a dollar that increases the size of the pie, and the transfer of the apple is merely redistributive. Hence, in their professional role, economists can with clear conscience recommend the efficiency-increasing transfer of the three-quarters of a dollar while leaving the redistributive transfer of the apple aside as a separate question outside the scientific purview of economics.

	Normal Description	Inverted Description
Transfer that increased "pie"	1 apple	\$0.75
Transfer that redistributed "pie"	\$0.75	1 apple

Table 1: Same transfers with normal and reversed descriptions

Of course, the KH principle is usually applied in cost-benefit analysis and law and economics to much more complex changes than a simple exchange. But the logic is exactly the same. For instance,

Friedman's blind-spot statement "One dollar is worth the same number of dollars to everyone: one" in the context of a simple transaction is mirrored in the much more complex case of the economic evaluation of projects in cost-benefit analysis:

It should be emphasized that pure transfers of purchasing power from one household or firm to another per se should be typically attributed no value.(2, p. 30)

Although the context is more complex, that does not change the underlying logic or the methodological error. Change the numeraire to another involved good, and the recommendations of the KH principle will change—although perhaps not as dramatically as the reversal seen in the case of a simple exchange. Change the numeraire to an uninvolved good to avoid the same-yardstick fallacy, and the whole project *including compensations* (in non-numeraire goods) will then become the "efficiency part" and will be recommended (assuming the total change in "social wealth" is positive) but then we are back to the Pareto improvement criterion. Thus the whole efficiency-equity parsing and the KH principle result from the same-yardstick fallacy of taking the compensation payments as being in the same numeraire used to measure "social wealth."

Where the KH criterion differs from the Pareto criterion due to the use of an involved numeraire and the resulting blind-spot statements (mistaken as empirically significant numeraire-invariant statements), then the KH argument breaks down in giving different and perhaps even reversed recommendations under a mere redescription using a different numeraire. When the same-yardstick fallacy is avoided by using a non-involved numeraire, then the KH principle collapses back to the Pareto criterion.

4 The same-yardstick fallacy in consumer's and producer's surplus analysis

Analysis using consumer's and producer's surpluses is also a staple in the Marshall-Pigou-Kaldor-Hicks production and distribution tradition which parses total changes into a part that changes social wealth and a part that is allegedly "redistributive" (transfers in the numeraire). Instead of analyzing a set of discrete transfers, the surplus analysis is based on an integration that has been the focus of the controversy. But the surplus analysis involves exactly the same methodological error as in the analysis of discrete transfers, and the error has nothing to do with the controversial assumptions involved in the integration (e.g., constancy of the marginal utility of income).

Consider the standard textbook analysis of the consumer's surplus (the upper shaded triangle "*cs*" in the diagram below) and producer's surplus (the lower shaded triangle "*ps*") at equilibrium in the market for a single commodity *Q*.

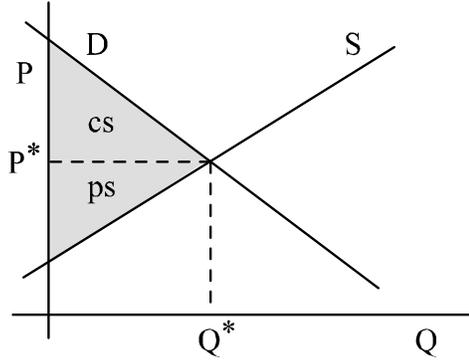


Figure 1: Consumer's and producer's surpluses at equilibrium

Although Marshall's surplus analysis is usually presented independently of the KH principle (and, of course, precedes the KH principle in time), it involves the same methodological error of counting the increase in social wealth as resulting solely from the increase in Q from 0 to the equilibrium value Q^* . The implicit blind-spot statement is that the payments, say, P^* per unit Q , *play no role* in the increase in social wealth represented by the sum of the surplus areas: $cs + ps$. Marshall and the later users of the surplus concepts may not explicitly draw the blind-spot conclusion that the payments do not affect the change in social wealth represented by the sum of the surpluses. But a theory must be judged by its implications, not by whether or not the proponents of a theory personally abstain from drawing certain troublesome conclusions from their theory.

There is a corollary for the history of economic thought. Since the positive recommendation based on the consumer's and producer's surpluses is actually based solely on the quantity increase from 0 to Q^* with the payment P^*Q^* *playing no role*, the later Kaldor-Hicks principle only made explicit what was implicit all along in the Marshall-Pigou methodology.

Moreover, it is again easy enough to apply the same numeraire-reversal applied above to the apple-money example to see that the surplus analysis then gives the opposite result that it is the payments, say P^*Q^* , that give the surpluses which increase social wealth while the quantity transfers Q^* have no effect.

Suppose the demand curve is $Q_d = D(P)$ and the supply curve is $Q_s = S(P)$ and that as indicated in the above diagram, there is an equilibrium at: $D(P^*) = Q^* = S(P^*)$. In an equilibrium market transaction, the equilibrium quantity Q^* of the good is exchanged for the equilibrium quantity P^*Q^* of revenue or money.

Now we give the inverted or reverse description of exactly the same situation except that the good Q becomes the numeraire and money-to-be-spent-on- Q , symbolized by R , becomes the good demanded and supplied. In the inverted description, the Q -demanders become R -suppliers (i.e., those who will supply a certain amount of R in return for a certain amount of Q), and the Q -suppliers become R -demanders (i.e., those who demand a certain amount of R in return for their Q). In the inverted description, the price is not the R per unit Q price P but the reciprocal Q per unit R price $P' = \frac{1}{P}$. The revenue demand function is obtained by reinterpreting Q -supply as R -demand:

$$\text{Revenue demand function: } R_d(P') = S(P)P = S(1/P')/P'.$$

Similarly, the Q -demand function is reinterpreted as R -supply:

$$\text{Revenue supply function: } R_s(P') = D(P)P = D(1/P')/P'.$$

As usual, an equilibrium occurs at a price P'^* that equates the quantity supplied and demanded:

$$\text{Equilibrium at price } P'^*: R_s(P'^*) = R^* = R_d(P'^*).$$

Plugging in the definitions, we have: $R_s(P'^*) = D(1/P'^*)/P'^* = S(1/P'^*)/P'^* = R_d(P'^*)$ so we see that an equilibrium occurs at $P'^* = 1/P^*$ and that the equilibrium quantity is: $D(1/P'^*)/P'^* = D(P^*)P^* = R^* = S(P^*)P^* = S(1/P'^*)/P'^*$ so that $R^* = P^*Q^*$ as in the original description.

Thus the trivial redescription inverting the roles of the quantities and dollars supplied and demanded trivially gives us exactly the same equilibrium.

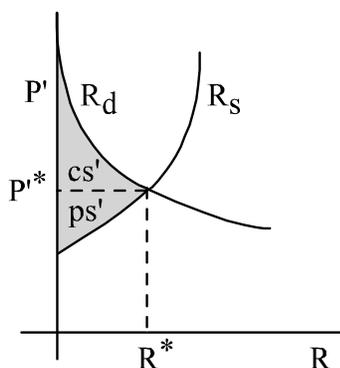


Figure 2: R -demander's surplus (cs') and R -supplier's surplus (ps') in inverted description

But the point of the exercise is now if we apply the consumer's and producer's surplus analysis using the inverted description, then we have the reverse result that the surpluses (cs' and ps' in the above diagram) attach to the increase in the quantity R from 0 to R^* and the "payments" Q^* play no role in accounting for the surpluses.

	Normal Description	Inverted Description
Transfer that increased "pie"	Q^*	$R^* = P^*Q^*$
Transfer that redistributed "pie"	$R^* = P^*Q^*$	Q^*

Table 2: Same equilibrium transfers with normal and inverted descriptions

5 Conclusions

We have seen that the analyses based on the KH principle (and the surpluses) involve a rather simple methodological error, the same-yardstick fallacy, that vitiates the conclusions since the conclusions can be changed and even reversed simply by redescribing using a different numeraire. This renders as rather moot the fine-grained technical controversies such as the Scitovsky-like special cases or the higher-order general equilibrium effects (or the background assumptions needed for the integrals involved in the surpluses).

The methodological point made here challenges the whole way of thinking about the "production and distribution of social wealth" that descends from the classical economists to Marshall and Pigou, and that Kaldor, Hicks, and many others have tried to rehabilitate after the Paretian and marginalist revolutions which established the alternative exchange or catallactics vision of the Lausanne and Austrian schools. These results are consistent with the exchange-catallactics school including a rights-based approach to normative economics which treats individuals as ends-in-themselves and thus has no overarching notions of "social welfare" or "social wealth" [(5) and (7)].

For instance, it is congruent with the Wicksell-Buchanan perspective in constitutional political economy (4). Instead of using KH reasoning to supply an "efficiency" gloss to a planning process based on cost-benefit analysis, it is the job of democratic politics to ultimately work out changes that are mutually voluntary on the part of those whose rights are affected.

The Paretian precepts are preached in the cathedrals of high theory, but in the trenches of macroeconomic policy and applied welfare economics (e.g., cost-benefit analysis and much of law and economics), the old thought patterns of the production-distribution school persist. A thorough appreciation of the same-yardstick fallacy in those old patterns of thought may have more costs than benefits for many economists.

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