Actuality and The A Priori

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Abstract

We consider a natural-language sentence that cannot be formally represented in a first-order language for epistemic two-dimensional semantics. We also prove this claim in the appendix. It turns out, however, that the most natural ways to repair the expressive inadequacy of the first-order language render moot the original philosophical motivation of formalizing a priori knowability as necessity along the diagonal.

In this paper we investigate some questions concerning the expressive power of a first-order modal language with two-dimensional operators. In particular, a language endowed with a two-dimensional semantics intended to provide a logical analysis of the discourse involving a priori knowledge. We consider a natural-language sentence that cannot be formally represented in such a language. This was firstly conjectured in Lampert (manuscript), but here we present a proof. It turns out, however, that the most natural ways to repair this expressive inadequacy render moot the original philosophical motivation of formalizing a priori knowability as necessity along the diagonal.

In what follows, first we present the basic principles involved in a quantified semantics containing the operators for necessity, actuality, and apriority. In the second part of the paper we argue that any attempt to formalize the relevant natural-language sentence fails, and a proof that there is actually no sentence of the language with equivalent truth conditions can be found in the appendix. Finally, we show how adding a new operator or plural quantifiers to the language are not available options for proponents of this kind of two-dimensional semantics, for in both cases the models validate intuitively false principles concerning the a priori.

1 Quantified Two-Dimensional Semantics

By and large, two-dimensional semantics involve evaluating sentences with respect to a pair of possible worlds, states, points, or whatever.[1] In terms of possible worlds, for example, this means that a sentence will be considered true at a world *relative to* or *with respect to* another possible world, although not necessarily a different one. Two-dimensional semantics have been developed in order to investigate the semantics of "now" and "then" in tense

[1]For more on this, and different characterizations of two-dimensionality, see Humberstone (2004).

logics (Kamp (1971), Vlach (1973)), as well as the semantics of "actually" in modal logics (Crossley and Humberstone (1977), Davies and Humberstone (1980), Cresswell (1990)).[2] In particular, it has also been illuminating in investigations concerning a priori knowledge (Davies and Humberstone (1980), Kaplan (1989), Davies (2004), Chalmers (1996, 2004)). More recently, formal systems have been defined by Restall (2012) and Fritz (2013, 2014) containing primitive a priori knowability operators; in particular, the logic defined by Fritz is explicitly motivated by Chalmers' epistemic two-dimensional semantics.

There are several important differences regarding not only the motivations involved in such proposals, but also with respect to many formal details. Since our concern, however, involves the notion of a priori knowability, we shall pass over those and concentrate on what is common to the logical systems defined by Davies and Humberstone, Restall, Fritz, as well as Chalmers' version of two-dimensional semantics.[3] Broadly speaking, what is common amongst them is a semantics involving the so-called metaphysical modality expressed in terms of $\Box$-modalization, a priori knowability, which is taken as necessity along the diagonal, as well as an indexical interpretation of actuality formalized by the actuality operator. The pertinent question then is how should we expand this kind of semantics to the first-order case with identity? Quantified modal semantics already brings up too many philosophical problems. But, in effect, we can naturally extend propositional two-dimensional modal semantics with quantifiers in a way that is very similar to the one-dimensional case.

In the usual Kripke semantics, first-order quantifiers can be defined as ranging over the domains of each world of evaluation or even the entire domain of all possible worlds. The former gives us variable domains, and the latter gives us a constant domain semantics for the quantifiers. In each case, formulas can also be interpreted in a two-dimensional manner by adding an extra world parameter to the semantic evaluation clauses, namely, the distinguished or actual world of the models.[4] Since the basic modal semantics make no explicit use of such world, except, maybe, in defining logical properties such as validity, this simple addition will not cause any difference in the resulting logics. Basic modal semantics can be taken without harm as containing such a hidden parameter for actuality. By making it explicit, we get the simplest two-dimensional modal logic. Thus, a quantified two-dimensional semantics may consider the range of the quantifiers to be similar to the one-dimensional case, with the exception that the actual world now varies, whereupon the extensions of the individual constants and predicate symbols of the language will be determined by each pair of possible worlds.

In what follows we give a more formal treatment of a quantified two-dimensional semantics along these lines. We use a constant domain semantics for the quantifiers, although a variable domain semantics can also be easily defined.[5] We discuss the semantics for identity before

[2]Such a framework proved to be very useful also for the purposes of investigating context-dependent terms (Montague (1968), Lewis (1970), Kaplan (1989)), the pragmatics of assertion (Stalnaker (1978)), as well as conceptual analysis and reductive explanation (Jackson (1998), Chalmers and Jackson (2001)).

[3]Perhaps the main point can be generalized for other cases as well. What is important, as we shall make explicit, is a semantics considering necessity and actuality interpreted by the corresponding modal operators, as well as apriority taken as truth on the diagonal.

[4]Where $\mathcal{M}$ denotes a Kripke model and $w*$ is the distinguished element of a set W, this can be done by rewriting $\mathcal{M}, w \vDash p$ as $\mathcal{M}, \langle w*, w \rangle \vDash p$.

[5]We purposefully avoid single quotation marks in order to not clutter the presentation. The context should make use-mention distinctions clear.

closing this section.

Definition 1.1 (First-order Language) For our language, $\mathcal{L}$, let $\{c_1, c_2, ...\}$ be a set of *constant symbols*, $\{x_1, x_2, ...\}$ a set of *individual variables*, and $\{P_1^n, P_2^n, ...\}$ a set of n-place *predicate symbols* for each $n \in \mathbb{N}$. The terms t and formulas φ are recursively generated by the following grammar ($i, k \in \mathbb{N}$):

$$t ::= c_i \mid x_i$$

$$\varphi ::= P_i^n(t_1, ..., t_k) \mid t = t' \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid \mathcal{A}\varphi \mid \mathcal{D}\varphi \mid \exists x_i \varphi$$

We define $\forall x_i \varphi$, $\Diamond$, and the other Boolean connectives as usual. Moreover, we define $\mathcal{C}$, the dual of $\mathcal{D}$, as $\neg\mathcal{D}\neg$. In what follows we occasionally drop the subscripts involved in terms, writing $a, b, c, ...$ for constant symbols and $x, y, z, ...$ for individual variables. Next we define models equipped with accessibility relations for both $\Box$ and $\mathcal{D}$ formulas.[6] Since our set W of possible worlds will only contain pairs, its distinguished element will be a pair as well :

Definition 1.2 (2D-centered models) A *constant domain 2D-centered model* is a tuple, $\mathcal{M} = \langle W, \langle w*, w* \rangle, \mathcal{R}_\Box, \mathcal{R}_\mathcal{D}, \mathscr{D}, V \rangle$, such that

- $W = Z \times Z$ for some set Z,
- $\langle w*, w* \rangle$ is a distinguished element of W,
- $\mathcal{R}_\Box \subseteq W \times W$, the $\Box$-accessibility relation, is the least relation such that for every $v, w, z \in Z$, $\langle v, w \rangle \mathcal{R}_\Box \langle v, z \rangle$,
- $\mathcal{R}_\mathcal{D} \subseteq W \times W$, the $\mathcal{D}$-accessibility relation, is the least relation such that for every $v, w, z \in Z$, $\langle v, w \rangle \mathcal{R}_\mathcal{D} \langle z, z \rangle$,
- $\mathscr{D}$ is a non-empty domain of quantification, and
- V is a function assigning to each constant c_i of $\mathcal{L}$ and $\langle v, w \rangle \in W$, an object $V(c_i, \langle v, w \rangle) \in \mathscr{D}$, and to each n-place predicate symbol P_i^n and $\langle v, w \rangle \in W$, a set $V(P_i^n, \langle v, w \rangle) \subseteq \mathscr{D}^n$.

Moreover, for any constant symbol c_i and $v, w, z \in Z$, let $V(c_i, \langle v, w \rangle) = V(c_i, \langle v, z \rangle)$. We call this the *rigidity condition*.[7]

Definition 1.3 (Truth) We define 'φ is true at w relative to v in $\mathcal{M}$', written $\mathcal{M}_w^v \vDash \varphi$,[8] by

[6] The accessibility relations $\mathcal{R}_\Box$ and $\mathcal{R}_\mathcal{D}$ are just the ones in Fritz (2013, p. 1758), except that he also adds an accessibility relation for $\mathcal{A}$ formulas, which would be useful for us if we were investigating properties of the logic corresponding to how the frames are defined.

[7] Here we follow Holliday and Perry (2014) in their quantified two-dimensional semantics. Even though their semantics is defined to deal with the Hintikka-Kripke problem in the context of an epistemic logic, their rigidity condition is useful for our purposes. More on this below.

[8] About notation: Davies and Humberstone (1980, p. 4) write $\mathcal{M} \vDash_w^v \varphi$, appending the superscripts and subscripts to the right side of the turnstile. We simply prefer having the world variables on the left side, which, in its present form, is intended as a two-dimensional version of $\mathcal{M}, \langle v, w \rangle \vDash \varphi$.

recursion on φ. For a pair $\langle v, w\rangle \in W$, and a valuation V in $\mathcal{M}$,

$$
\begin{array}{lll}
\mathcal{M}^v_w \vDash P^n_i(t_1,...,t_n) & \text{iff} & \langle V(t_1,\langle v,w\rangle),...,V(t_n,\langle v,w\rangle)\rangle \in V(P^n_i,\langle v,w\rangle);\\
\mathcal{M}^v_w \vDash t = t' & \text{iff} & V(t,\langle v,w\rangle) = V(t',\langle v,w\rangle);\\
\mathcal{M}^v_w \vDash \neg\varphi & \text{iff} & \mathcal{M}^v_w \nvDash \varphi\\
\mathcal{M}^v_w \vDash \varphi \wedge \psi & \text{iff} & \mathcal{M}^v_w \vDash \varphi \text{ and } \mathcal{M}^v_w \vDash \psi;\\
\mathcal{M}^v_w \vDash \Box\varphi & \text{iff} & \text{for every } \langle v,z\rangle \in W \text{ such that } \langle v,w\rangle\mathcal{R}_\Box\langle v,z\rangle,\ \mathcal{M}^v_z \vDash \varphi;\\
\mathcal{M}^v_w \vDash \mathcal{A}\varphi & \text{iff} & \mathcal{M}^v_v \vDash \varphi;\\
\mathcal{M}^v_w \vDash \mathcal{D}\varphi & \text{iff} & \text{for every } \langle z,z\rangle \in W \text{ such that } \langle v,w\rangle\mathcal{R}_\mathcal{D}\langle z,z\rangle,\ \mathcal{M}^z_z \vDash \varphi;\\
\mathcal{M}^v_w \vDash \exists x_i\varphi & \text{iff} & \text{for some } x_i\text{-variant } V' \text{ of } V, \mathcal{M}^v_w \vDash \varphi[c_i/x_i];
\end{array}
$$

where the notion of an x-variant is understood as usual.

We say that a sentence φ is *true* in $\mathcal{M}$, written $\mathcal{M} \vDash \varphi$, if and only if φ is true at $\langle w*, w*\rangle$ in $\mathcal{M}$ (i.e. $\mathcal{M}^{w*}_{w*} \vDash \varphi$), and a sentence φ is a *logical consequence* of a set of sentences Γ if and only if for every $\mathcal{M}$, if $\mathcal{M} \vDash \gamma$ for all $\gamma \in \Gamma$, then $\mathcal{M} \vDash \varphi$. The notion of validity generalizes logical consequence as usual, and we can define *local validity* for a sentence as truth in every model, *general validity* as truth at every pair $\langle v, w\rangle$ in every model, and *diagonal validity* as truth at every coincident pair $\langle w, w\rangle$ in every model.

The notion of validity for logics containing actuality operators is notoriously controversial, and a satisfactory philosophical defense of either local or general validity is not our main objective.[9] Nonetheless, there is something to be said about the notion of validity in logics containing diagonal operators. Amongst the ones mentioned above, the ones in vogue comprise either general or diagonal validity, and not local validity as we have defined it. Davies and Humberstone's logic of fixedly actually, or $\mathbf{S5}\mathcal{AF}$, is defined with respect to general validity, and the two-dimensional logics developed by Restall and Fritz both use diagonal validity.[10] In particular, both define a set of distinguished elements rather than fixing a specific point in the models as we did. This set in turn comprises every diagonal point in a frame, which makes sense vis-à-vis the fact that the first coordinate of each pair is intuitively an actual world. Logical truth, consequently, turns out to be truth at every diagonal point in every model. And this is plausible on philosophical grounds since it counts $\mathcal{D}p \supset p$ as valid — also, $\mathcal{R}_\mathcal{D}$ can be immediately seen to be reflexive on the diagonal points of our models. Presumably, if it is a priori that p, then p is true, since apriority is usually taken to be factive.

However, by inspection of the semantics defined above, it is obvious that a sentence is diagonally valid if and only if it is locally valid.[11] Suppose that a sentence, say, φ, is diagonally valid. Then φ holds at every diagonal point of every model, in which case it is obviously locally valid as well. On the other hand, suppose φ is not diagonally valid, in which

[9]See, for example, Crossley and Humberstone (1977, p. 14–15) and Hanson (2006, 2014) for a defense of general validity, and Zalta (1988), Nelson and Zalta (2012) for arguments in favour of local validity.

[10]This is also the case in Kocurek (forthcoming).

[11]Local validity and general validity coincide for basic one-dimensional modal logics without actuality operators since there is no use for a distinguished element in the models. Similarly, there is no use for a distinguished point on the diagonal of a 2D frame since the actuality operator is not fixed to any particular point but to the first coordinate of every pair of possible worlds.

case it fails to hold at some diagonal point in some model. Since there is a model in which that diagonal point is the distinguished one, φ also fails to be locally valid. Consequently, by adopting local validity we have not changed the set of validities in comparison to the logics of both Restall and Fritz insofar as the propositional portion is concerned.[12]

There are philosophical motivations, however, which we take to speak in favour of local validity. Informally, every actual world is alternative to *this* one, and this idea is better captured by fixing a single point in the models as distinguished. By the same token, Restall calls every diagonal point an indicative alternative, in contrast to the subjunctive alternatives delivered by horizontal, or $\mathcal{R}_\Box$-related points.[13] But not having a single distinguished point in the models makes it unclear in relation to what those points are ultimately supposed to be alternatives to. All of this is less plausible, on the other hand, if we take the first coordinates of every pair to be epistemic scenarios, for there is no apparent reason to have a single privileged scenario in a class of models. Since, however, the resulting logic is the same whether we single out a point or every diagonal point as distinguished, at least momentarily, such remarks matter only to the extent of a more informal level of philosophical motivation. It is only in §1.2 that we make an important use of a distinguished point given the new actuality operator therein defined, and so we have found it useful to define our models in this manner from the outset.

Informally, we can read the semantic clauses for $\Box$, $\mathcal{A}$, and $\mathcal{D}$, respectively, as saying that 'Necessarily φ' is true at $\langle v, w\rangle$ if and only if for every $\mathcal{R}_\Box$-related pair of possible worlds $\langle v, z\rangle$, φ is true at $\langle v, z\rangle$; 'Actually φ' is true at $\langle v, w\rangle$ if and only if φ is true at $\langle v, v\rangle$; and 'It is a priori that φ' is true at $\langle v, w\rangle$ if and only if for every $\mathcal{R}_\mathcal{D}$-related pair of possible worlds $\langle z, z\rangle$, φ is true at $\langle z, z\rangle$. Hence, the apriority operator, $\mathcal{D}$, delivers truth along the diagonal. This is easy to see by displaying a 2D matrix, where the worlds arranged on the vertical represent scenarios or actual worlds, and the worlds on the horizontal are the worlds of evaluation:

$$\begin{pmatrix} & v & w \\ v & - & - \\ w & - & - \end{pmatrix}$$

If a sentence is a priori knowable, or $\mathcal{D}$-true, it receives the value T in the following manner:

$$\begin{pmatrix} & v & w \\ v & T & - \\ w & - & T \end{pmatrix}$$

[12] Some terminological remarks are in order. It is usual to say 'real-world' instead of 'local' validity, in accordance with the terminology used in Crossley and Humberstone (1977). We find it odd, however, to call real-world validity truth at the distinguished *pair* in every model, otherwise we have no reason to prefer a different nomenclature.

[13] See p. 1611.

Apart from several notational variances, the semantics just defined conservatively extends the semantics presented by both Restall and Fritz. With respect to Davies and Humberstone's case, there are more significant differences besides the fact that $\mathbf{S5}\mathcal{AF}$ is defined only as a propositional logic. First, they define a fixedly operator, $\mathcal{F}$, rather than a primitive apriority operator, where $\mathcal{F}\varphi$ is true at $\langle v, w\rangle$ if and only if for every pair of possible worlds $\langle z, w\rangle \in W$, φ is true at $\langle z, w\rangle$.[14] Thus, it is the concatenation $\mathcal{FA}$ that defines diagonal necessity in $\mathbf{S5}\mathcal{AF}$.[15] Davies and Humberstone claim that $\mathcal{FA}$ corresponds to Evans' (1979) notion of deep necessity, identified by the latter with what is a priori knowable. Such a notion was defined in contrast to superficial necessity, which is just the necessity delivered by the modal operator $\Box$, and it was used by Evans to argue that Kripke's (1980) examples of the contingent a priori involved cases of superficial contingency but deep necessity, where a sentence is said to be superficially contingent just in case its $\Box$-modalization and the $\Box$-modalization of its negation are both false. Davies and Humberstone claim they have not found counterexamples in the language of $\mathbf{S5}\mathcal{AF}$ of sentences that are a priori but not deeply necessary, or necessary along the diagonal, even though the identification of diagonal necessity with the a priori is made with some reservations (p. 10). Finally, the truth of propositional letters in $\mathbf{S5}\mathcal{AF}$ is sensitive only to the world of evaluation, and not to a pair of worlds. This feature transforms the actual world in the evaluation of an atomic formula into a free parameter, resulting in an equivalence for $\Box$ and $\mathcal{D}$-formulas not containing $\mathcal{A}$.[16] Yet, neither $\Box p \supset \mathcal{D}p$ nor $\mathcal{D}p \supset \Box p$ are valid in the semantics defined here, in sharp contrast to Davies and Humberstone's $\mathbf{S5}\mathcal{AF}$.[17]

Now it is not difficult to see that our semantics is also very much in agreement with the semantics developed by Chalmers, where expressions are evaluated with respect to pairs of scenarios and possible worlds. Chalmers take scenarios to be epistemically possible worlds, i.e. "ways things might be that cannot be ruled out a priori." (2004, p. 211) For the purposes of a more formal treatment, however, he allows those to be considered just as possible worlds. (idem) The same simplification is made by Fritz (2013, p. 1757) in his formal system. Thus, we might just as well take the Cartesian product $Z \times Z$ in a 2D-centered frame as intuitively representing scenarios and possible worlds, respectively. Although this is a somewhat implausible assumption as scenarios give rise to distinct, epistemic modalities, it does not affect our main points in any substantial way.[18] Furthermore, despite the fact that Chalmers usually takes semantic values in terms of primary, secondary, and two-dimensional intensions in order to ascribe meanings to expressions, his semantics for necessity, actuality, and apriority correspond exactly to our operators $\Box$, $\mathcal{A}$, and $\mathcal{D}$, respectively.[19]

[14]We have adapted, of course, the semantic clause of $\mathcal{F}$ for pairs of worlds.

[15]Davies (2004, p. 89) makes it clear that they did not intend to formalize anything like an epistemic logic, although the resulting system does give rise to a priori truths. Moreover, in response to Evans' criticisms to the fixedly operator, he also considered the possibility of adding a primitive operator $\mathcal{D}$ for diagonal necessity (cf. p. 92).

[16]Axiom $\mathcal{F}6$, for example, of $\mathbf{S5}\mathcal{AF}$, reads $\Box\varphi \leftrightarrow \mathcal{FA}\varphi$ for $\mathcal{A}$-free φ. See Davies and Humberstone (1980, p. 4).

[17]For more on this, see §3. Fritz (2013, p. 1761) makes similar observations.

[18]Both Restall and Fritz constrain their frames in such a way that there are at least as many horizontal as vertical worlds. See Restall (2012, p. 1618), and Fritz (2013, p. 1761). Without loss of generality, we can define 2D frames in a similar manner.

[19]See, for example, Chalmers (2014, p. 212), although in the same paper he recognizes difficulties for the

The more interesting aspects of our quantified two-dimensional semantics, however, involve identity and proper names. In modal logics, the former is usually defined irrespective of the world of evaluation, consequently validating the *necessity of identity*, viz., the claim that if two things are identical, they are necessarily identical. In contrast, an unrestricted semantics for identity in a two-dimensional framework would result in the models validating that it is a priori that two things are the same whenever two things are the same.[20] But this is certainly not intended by Chalmers' version of two-dimensional semantics, or any formal treatment designed to capture what is intuitively taken to be a priori knowable. For even though 'Hesperus is Phosphorus' is a true identity claim, and hence necessary, it is cognitively significant, whence two-dimensionalists take it to be only a posteriori knowable.

Following Kripke (1980), we take proper names to be rigid designators, where a proper name designates rigidly just in case it has the same extension in every possible world. In a two-dimensional framework this means that a proper name has the same extension in every possible world w relative to a world v taken as actual. Following Chalmers' terminology, the secondary or counterfactual intensions of both 'Hesperus' and 'Phosphorus' are the same, which accounts for the necessity of 'Hesperus is Phosphorus.' This is captured by the rigidity condition assumed in 2D-centered models, which in turn gives us $V(Hesperus, \langle v, w\rangle) = V(Phosphorus, \langle v, w\rangle)$ for all $w \in Z$, whence this identity is $\mathcal{R}_\Box$ necessary.[21] In contrast, we want the primary or actual intensions of both 'Hesperus' and 'Phosphorus' to not designate rigidly, for they pick out something more related to a Fregean descriptive sense like the last bright object in the morning sky and the first bright object in the evening sky, respectively. And in some scenarios the first bright object in the evening sky is not Venus, whence 'Hesperus is Phosphorus' is only a posteriori knowable.[22] If we let this scenario be some $z \in Z$, then we can have for some $\langle v, w\rangle \mathcal{R}_\mathcal{D} \langle z, z\rangle$, $V(Hesperus, \langle z, z\rangle) \neq V(Phosphorus, \langle z, z\rangle)$. In fact, the 2D matrix below illustrates a 2D model in which 'Hesperus is Phosphorus' is necessary and yet a posteriori:

$$\begin{pmatrix} & v & w \\ v & H = P & H = P \\ w & H \neq P & H \neq P \end{pmatrix}$$

Since, 'Hesperus is Phosphorus' is not true along the diagonal, it is not a priori as desired. There are, of course, more subtleties involved in the relationship between names, designation, and a two-dimensional framework, but this is enough for the present purposes.

semantics of the apriority operator caused by the nesting problem.

[20]Proof: Suppose $\mathcal{M} \vDash x = y$ for an assignment V. By the unrestricted semantics of identity terms, $V(x) = V(y)$. Let $\langle w, w\rangle$ be any pair of possible worlds in W such that $\langle w*, w*\rangle \mathcal{R}_\mathcal{D} \langle w, w\rangle$. Then, given $V(x) = V(y)$, it follows that $\mathcal{M}^w_w \vDash x = y$, whence $\mathcal{M} \vDash \mathcal{D}x = y$, therefore $\mathcal{M} \vDash x = y \supset \mathcal{D}x = y$. An analogous result is available for Davis and Humberstone's $\mathbf{S5}\mathcal{AF}$, the only difference being that it involves the compound operator $\mathcal{FA}$ rather than $\mathcal{D}$.

[21]We thereby assimilate proper names to individual constants in $\mathcal{L}$. This follows closely the presentation in Holliday and Perry (2014, §4.5).

[22]Cf. Chalmers (2004, pp. 158–162, 2014, p. 212).

2 Expressive incompleteness

It is well-known that there are sentences that cannot be formalized in a language for quantified modal logic **S5**. A classic example from Crossley and Humberstone (1977) is

(1) It is possible for everything that is actually red to be shiny.

The problem is that (1) intends to quantify over the actual red things, except that the quantifier occurs within the scope of a possibility operator, whereby it can only quantify over the entities at that possible world. As a solution, Crossley and Humberstone suggest adding an actuality operator to the language designed to block the effect of any occurrences of modal operators to its left.[23] This can be seen in the semantics defined above for $\mathcal{A}$, except that Crossley and Humberstone's logic, called $\mathbf{S5}\mathcal{A}$, only extends **S5** with $\mathcal{A}$, and not with apriority or diagonal operators, in which case there is no variation in the actual world of the models.[24] Thus, in $\mathbf{S5}\mathcal{A}$ the actuality operator rigidifies the evaluation of formulas in its scope to the distinguished element of the models. (1) can then be formalized as follows:

(2) $\Diamond\forall x(\mathcal{A}Rx \supset Sx)$

It might be thought, however, that a language for a two-dimensional modal logic enriched with actuality and diagonal operators is expressive enough so that similar problems do not arise. But consider the following:

(3) It is not a priori knowable that something that is actually red is shiny.

There is a reading of this sentence that resists formalization in the language $\mathcal{L}$ for quantified two-dimensional semantics, namely:

(3a) $\neg\forall\langle w, w\rangle\exists x(Rx\text{-at-}\langle w*, w*\rangle \wedge Sx\text{-at-}\langle w, w\rangle)$.

Or, equivalently,

(3b) $\exists\langle w, w\rangle\forall x(Rx\text{-at-}\langle w*, w*\rangle \supset \neg Sx\text{-at-}\langle w, w\rangle)$.

Notice that in Davies and Humberstone's $\mathbf{S5}\mathcal{AF}$, (3) can also be stated as

(3c) It is not deeply necessary that something that is actually red is shiny.[25]

Moreover, if one takes *conceivability* as the dual of apriority, another rendition of (3) might be

(3d) It is conceivable that everything that is actually red fails to be shiny.[26]

[23]The same strategy was employed in tense logics by Kamp (1971) for the "now" operator.

[24]In fact, in $\mathbf{S5}\mathcal{A}$ there is no need at all of evaluating formulas with respect to a pair of worlds.

[25]This is similar to how the sentence appears in Lampert (manuscript). The only difference being that (3*) was formulated in terms of *deep possibility*, the dual of deep necessity. Admittedly, not much of a natural-language statement, since the notion of deep necessity seems to be even more philosophically loaded in comparison to apriority. Yet, in a language intended to formalize it, this is exactly the kind of sentence expected to be expressible.

[26]Chalmers (2004, p. 219) motivates conceivability as the dual of a priori knowability. However, in Chalmers (2011) he argues that this is problematic.

The most natural attempt to formalize such sentences would be as follows:

(4) $\neg\mathcal{D}\exists x(\mathcal{A}Rx \wedge Sx)$

This will not do, however, for it says that for some $\langle z, z\rangle \in W$ such that $\langle w*, w*\rangle\mathcal{R}_{\mathcal{D}}\langle z, z\rangle$, something that is red at $\langle z, z\rangle$ is shiny at $\langle z, z\rangle$. In contrast, the reading of (3) we are interested in says that something at $\langle z, z\rangle$ that is *in fact* red, i.e. red at $\langle w*, w*\rangle$, is shiny at $\langle z, z\rangle$. And this cannot be formalized by using the actuality operator as suggested in (4). The problem occurs because the actuality operator is designed to inhibit any outlying occurrences of the modal operators $\Box$ and $\Diamond$, but it does not scope out of apriority operators. When occurring within the scope of $\mathcal{D}$, the actuality operator merely reproduces, as it were, the actual world (or scenario) already introduced by $\mathcal{D}$. Since, however, (3) clearly belongs to the discourse intended to be captured by the language in question, $\mathcal{L}$ is expressively incomplete. Moreover, the expressive deficit in this case is exhibited by an analogue of the quantified **S5** case without an actuality operator, indicating iterations of the same problem as we add different operators to the language.[27] In the appendix we show that there is in fact no sentence of $\mathcal{L}$ with the same truth conditions as (3).[28]

3 The ***distinguished actuality*** operator

The first and most natural approach to the expressive incompleteness of $\mathcal{L}$ would be to revise it by adding yet another actuality operator, which in turn will be designed to protect any formula in its scope from modal or apriority operators affixed to its left. In Lampert (manuscript) we suggested adding an operator called *distinguishedly*, symbolized by $\odot$, with the following semantics:

$$\mathcal{M}_w^v \models \odot\varphi \quad \text{iff} \quad \mathcal{M}_w^{w*} \models \varphi$$

The distinguishedly operator was designed to protect the formulas in its scope from occurrences of the fixedly operator in a logic resembling Davies and Humberstone's $\mathbf{S5}\mathcal{AF}$.[29] If we affix $\odot$ to the immediate left of $\mathcal{A}$ in (4), we get in effect the desired truth conditions. However, we can also just define a new operator, @, pronounced *distinguished actuality*, which takes any pair of worlds to the distinguished element of a 2D-centered model:

$$\mathcal{M}_w^v \models @\varphi \quad \text{iff} \quad \mathcal{M}_{w*}^{w*} \models \varphi$$

This suffices in order to correctly formalize (3), for we will just need to replace the actuality operator in (4) with @, whereby we have the following:

(5) $\neg\mathcal{D}\exists x(@Rx \wedge Sx)$

[27] Notice that a rendering similar to (4) is available for Davies and Humberstone's $\mathbf{S5}\mathcal{AF}$, the only difference being that $\mathcal{FA}$ occupies the place of $\mathcal{D}$.

[28] Hodes (1984, p. 25, Theorem 15) proved that a sentence resembling (1), but restricted to a single predicate letter, is not representable in **S5**, although similar inexpressibility results were previously conjectured by Hazen (1976). Wehmeier (2001) offers an elegant simplification of Hodes' argument. More recently, Kocurek (forthcoming) presents a thorough investigation of several inexpressibility results using bisimulations.

[29] See Lampert (manuscript).

It should be clear why we call @ distinguished actuality, for this is in contrast with the behavior of $\mathcal{A}$ in a two-dimensional modal semantics. Rather than evaluating the formulas in its scope relative to the distinguished element of the models, $\mathcal{A}$ just 'copies down' whatever possible world is momentarily taken as actual. @, on the other hand, behaves similarly to $\mathcal{A}$ in a standard one-dimensional modal language. Thus, we might just as well call $\mathcal{A}$, in a two-dimensional framework, *relative actuality*.

The actual truth conditions for any sentences φ, $\mathcal{A}\varphi$, and $@\varphi$ are the same, but they differ under both subjunctive and indicative alternatives. In a one-dimensional modal language it makes no difference if we add either $\mathcal{A}$ or @, for $\mathcal{A}\varphi$ and $@\varphi$ hold at a possible world w just in case φ holds at the actual world. It is only in a two-dimensional framework that the two come apart — and, in effect, that @ is needed. This also suggests that $\mathcal{A}$ does not correspond to the English adverb 'actually' after all. Hazen (1976, p. 40), for instance, claims that $\mathcal{A}$ is "quite well attested in ordinary English." And his suggestion seems to be that just like $\Box$ and $\Diamond$ correspond to the English adverbs 'necessarily' and 'possibly,' $\mathcal{A}$ stands for 'actually.' However, as Wehmeier (2004) also observes, an occurrence of 'actually' in a sentence seems to be neither necessary nor sufficient for its logical form to contain $\mathcal{A}$.[30] As a matter of fact, it is plausible that both (1) and (3) above could have been written without the word 'actually' while retaining the intended reading where $\mathcal{A}$ and @ were needed. For 'actually' seems to have an emphatic role in both (1) and (3), and we could have used different locutions like 'in fact' while preserving the same meaning.[31] On the other hand, Wehmeier uses an example from Kripke (1980, p. 124) as evidence that occurrences of 'actually' are also not sufficient for a sentence to contain an actuality operator.

> Consider a counterfactual situation in which, let us say, fool's gold or iron pyrites was actually found in various mountains in the United States, or in areas of South Africa and the Soviet Union.

The above does not look like a case where "found" is evaluated with respect to the actual world, but rather to a counterfactual one. Another example comes from Wehmeier (2005, fn. 6):

(6) Under certain circumstances, no-one would believe in aliens, although there would actually be aliens.

Again, the claim is that 'actually' does not force evaluation with respect to the actual world. But despite the fact that in those cases 'actually' does not force evaluation with respect to *the* actual world, it is also plausible that it forces evaluation relative to an alternative actual world. If this is true, both examples involve not counterfactual possibilities, but rather diagonal alternatives, in which case an actuality operator, although unnecessary, could be used in their formalization. In those cases, a two-dimensional semantics can be illuminating. Of course, this is not a decisive argument for the claim that those examples involve diagonal possibilities, but it is plausible that they do. In any event, that 'actually' is not sufficient for the logical form of a sentence to contain $\mathcal{A}$ is shown in two-dimensional contexts where the English adverb requires a distinguishedly actually operator.

[30] This is also acknowledged by Humberstone. See his (1982, fn. 16).

[31] This contrasts with a purely rhetorical use of 'actually', as pointed out by Crossley and Humberstone (1977, p. 11).

Now, this solution to the expressive inadequacy of $\mathcal{L}$ seems simple and well motivated, since it comes down to a simple generalization of the analogous fix in **S5**. Moreover, it makes sense to have @ contraposing apriority operators in the same way $\mathcal{A}$ relates to the modal operators $\Box$ and $\Diamond$. One consequence, however, of adding $\mathcal{A}$ to **S5** is that the semantics validates the following:

(7) $\mathcal{A}\varphi \supset \Box\mathcal{A}\varphi$

Even though (7) is valid in **S5**$\mathcal{A}$, there is a certain intuitive sense in which it seems false. For take any sentence that is actually true such as 'grass is green'. Then, according to (7), 'grass is actually green' is a necessary truth, even though it is contingent whether grass is actually green. This feature of $\mathcal{A}$ is, in effect, what motivated Crossley and Humberstone to add a fixedly operator to **S5**$\mathcal{A}$, which in turn has the effect of varying the actual world of the models, thereby making it a contingent matter which world turns out to be actual.

A sentence resembling (7) is obtained when we add @ to $\mathcal{L}$, for the following now turns out to be valid:

(8) $@\varphi \supset \mathcal{D}@\varphi$

Notwithstanding its validity, (8) is intuitively false. For it says of anything that is distinguishedly actually true, or *d*-actual, that it is a priori that it is *d*-actual. Thus, given that water is *d*-actually H_20, it follows that it is a priori knowable that water is *d*-actually H_20. However, this is clearly not the kind of thing we can know a priori. Furthermore, since the converse of (8) is obviously valid, the semantics ends up licensing the following material equivalence:

(9) $@\varphi \leftrightarrow \mathcal{D}@\varphi$.

This, moreover, is also relevant to Davies and Humberstone's claim about not having noticed "any examples of truths expressed in terms of '$\Box$', '$\mathcal{F}$', and '$\mathcal{A}$' which are a priori knowable and also not deeply necessary." (p. 10) But adding @ to $\mathcal{L}$ immediately results in one:

(10) $@\varphi \supset \varphi$

Despite the fact that 'water is *d*-actually H_20' is not a priori knowable, the conditional 'if water is *d*-actually H_20, then water is H_20' seems to be a priori knowable. But in order to see that this does not hold along the diagonal, just let $\langle w*, w*\rangle \in V(p)$ and $\langle v, v\rangle \notin V(p)$, in which case $\mathcal{M}_v^v \models @p$ but $\mathcal{M}_v^v \not\models p$, resulting in $\mathcal{M}_v^v \not\models @p \supset p$. True, this is not expressible just in terms of $\Box$, $\mathcal{F}$, and $\mathcal{A}$, but neither is (3*). Hence, this leaves us with a dilemma: either one ignores the expressive deficit of $\mathcal{L}$, or one enriches it with a distinguished actuality operator. In any case, the language turns out to be inadequate as a formal rendering of the a priori.

Another interesting issue about (10) concerns its status as a logical truth. As we have already pointed out, there are philosophical motivations to endorse a local account of validity. In modal semantics, such is, in fact, the orthodox view, for this is the definition of validity appearing in Kripke's (1963) development of modal semantics. Even though there is no important distinction to be made between local and general accounts of validity with respect to basic modal languages, since those turn out to be equivalent, it is well-known that once we add an actuality operator this equivalence is severed. Zalta (1988) argues that there are contingent logical truths, since $\mathcal{A}\varphi \supset \varphi$ is locally but not generally valid, and he favours a local account of validity. We do not find contingent logical truths particularly appealing. However, by admitting local validity, we have contingent logical truths, viz. (10), that are not a priori knowable, since $\mathcal{D}(@\varphi \supset \varphi)$ is not locally valid. Of course, one could take logic to be an a posteriori enterprise altogether, but we confess being unsure whether this is an esteemed company for two-dimensional semantics. In any case, we shall refrain from announcing the discovery of a posteriori logical truths.

4 Plural quantification

Is there any other way to formalize (3) without the need of a distinguished actuality operator? Bricker (1989) suggested using plural quantification in order to formalize sentences such as (1) in first-order **S5**.[32] This seems to be the best approach so far avoiding the addition of new operators into basic modal languages, whence it is pertinent to investigate whether it can be generalized to a two-dimensional modal language. A two-dimensional modal language with plural quantifiers, $\mathcal{L}_{pl}$, will be just like $\mathcal{L}$, except that it is two-sorted, for now we add *plural variables* $\{x_1x_1, x_2x_2, ...\}$ and *plural constant symbols* $\{c_1c_1, c_2c_2, ...\}$, both of which we call *plural terms*, denoted by tt, as well as a two-place predicate, $\prec$, relating a single term to a plural one. Quantification over plural variables, $\exists x_ix_i\varphi$, is read as *there are some things such that...*,[33] and $y \prec xx$ is read as *y is one of xx's.*

The model theory should also be modified appropriately. A constant domain 2D-centered model for plural quantification is a tuple, $\mathcal{M} = \langle W, \langle w*, w*\rangle, \mathcal{R}_{\Box}, \mathcal{R}_{\mathcal{D}}, \mathscr{D}, \mathscr{D}*, V\rangle$ where everything is just as in Definition 1.2 except that we add another domain, $\mathscr{D}*$, for the plural variables, where $\mathscr{D}*$ is a set of non-empty subsets of $\mathscr{D}$. Moreover, V also assigns pluralities to each plural constant cc of $\mathcal{L}_{pl}$, $V(cc) \in \mathscr{D}*$. Regarding the semantics, we just need to consider the new atomic formula $y \prec xx$ and the plural quantifiers:

$$\mathcal{M}^v_w \vDash t \prec tt' \quad \text{iff} \quad V(t) \in V(tt');$$
$$\mathcal{M}^v_w \vDash \exists x_ix_i\varphi \quad \text{iff} \quad \text{for some } x_ix_i\text{-variant } V' \text{ of } V, \mathcal{M}^v_w \vDash \varphi[cc/x_ix_i];$$

where V' is an x_ix_i-variant of V if they disagree at most on what is assigned to x_ix_i. Finally, we define $\forall x_ix_i\varphi$ as $\neg\exists\neg x_ix_i\varphi$. A treatment of plural quantification along these lines added to a basic modal language allows us to regiment (1) as follows:

(11) $\exists xx(\forall y(y \prec xx \leftrightarrow Ry) \wedge \Diamond\forall y(y \prec xx \supset Sy))$

Accordingly, the plural quantifier $\exists xx$ in (11) should be taken as ranging "neither over sets, nor classes, nor properties; it ranges in an irreducibly plural way over the [red things] themselves." (Bricker (1989, p. 389)) And it is true that (11) delivers the correct truth conditions

[32] A similar proposal can be found in Forbes (1989, pp. 93–102). Nonetheless, the first time a proposal like this one appeared in the literature seems to be in Humberstone (1982, p. 2), with the only difference that he used explicit quantification over sets.

[33] This interpretation comes from Boolos (1984).

for (1).[34] Now, it is straightforward to generalize this proposal for the two-dimensional language, for the apparatus of plural quantification seems to give the correct result for (3) as well:

(12) $\exists xx(\forall y(y \prec xx \leftrightarrow Ry) \wedge \neg\mathcal{D}\exists y(y \prec xx \wedge Sy))$

Since the existential plural quantifier does not occur within the scope of $\mathcal{D}$ or any modal operator, it is evaluated with respect to the distinguished element of the model, viz., $\langle w*, w*\rangle$. Consequently, any witness of the single quantifier $\exists y$ is an object that is red at $\langle w*, w*\rangle$, which is exactly what we would get had we use @ rather than plural quantification. This indicates just how powerful plural quantification can be when dealing with scope issues in a modal language. Also, it suggests a certain elegance to the original proposal of adding plural quantifiers to modal languages. After all, (3) can be seen as mere iteration in a two-dimensional framework of the expressibility problem illustrated by (1) in **S5**, while the very same solution is available for the two cases.

The first difficulty with this proposal, however, is the move from seemingly unproblematic *de dicto* attributions of a priori knowledge to *de re* attributions. For (12) in effect says that

(13) There are red things such that it is not a priori that one of them is shiny,

and so the apriority operator occurs within the scope of a plural quantifier, whereby the plural version of (3) is *de re*, in contrast with the *de dicto* occurrence of $\mathcal{D}$ in (3). Thus, although (12) might get the desired truth conditions, it might nevertheless be unsatisfying as a formalization of (3) given its commitment to an apparent *de re* ascription of a priori knowledge. On the face of it, however, both (13) and (3) involve no attributions of a priori knowledge at all, but the lack thereof, whence this objection can be dismissed as a non-starter.[35] Nevertheless, a constant domain for the single quantifiers validates a two-dimensional version of the Barcan Formula and its converse, which does involve *de re* a priori knowledge ascriptions. In particular, the *2D Converse Barcan Formula* is as follows:

(14) $\mathcal{D}\forall x\varphi \supset \forall x\mathcal{D}\varphi$

It is easy to check that any instance of (14) is valid in 2D-centered models.[36] However, cases of *de re* knowledge involve, presumably, a relation of acquaintance between the knower and

[34] We should point out that Bricker's formalization depends on his assumption that $y \prec xx$ may hold at world at which the objects assigned to the plural variables do not exist. (See p. 387) We run along with Bricker on this, but only to illustrate how plural quantifiers do not solve the problem for the two-dimensional case even on such controversial grounds.

[35] The move from *de dicto* to *de re* knowledge ascriptions has been defended by Soames (2004) on the basis of the following exportation principle — understood relative to an assignment function and pair of possible worlds:

(E) For any name n and predicate F, if 'a knows/believes that n is F' is true, then 'a knows/believes that $[x/n]$ is F' is true.

Soames takes the above principle to be "intuitively compelling," (p. 261) which seems to be the only support offered in favour of (E). Yet, some two-dimensionalists — Chalmers, in particular — deny that (E) has any plausibility whatsoever. See Chalmers (2011, pp. 630–633) for counterexamples to (E) based on knowledge attributions. A more recent defense of *de re* a priori knowledge can nevertheless be found in Dorr (2011).

[36] Ditto for the corresponding *2D-Barcan formula*.

the object of knowledge. But it is far from clear how this can be made consistent with cases of a priori knowledge. Nonetheless, (14) shows that *de re* a priori knowledge ascriptions arise independently of plural quantification. The natural solution in order to avoid this — at least in this case — would be to assume a variable domain semantics for the quantifiers, but this will also cause problems for sentences like (12), for varying the domain of the quantifiers might incur loss of objects while changing from world to world. In (12), we need the objects at the pair of worlds introduced by the apriority operator to be red at the distinguished point of the model. But there will be no guarantee that there will be objects in the domains of both the distinguished point and the relevant pair of possible worlds in a varying domain semantics. This is a reason for why a constant domain semantics was always preferable in order to formalize sentences such as (1) and (3).[37]

A word is in order, too, with respect to the nature of the semantics specified for the plural quantifiers. As defined above, the truth of an atomic formula $y \prec xx$ relative to an assignment is given independently of pairs of possible worlds. This is just the usual model theory for plural quantification in the modal case,[38] whereby plural terms are given a purely extensional interpretation.[39] In **S5** endowed with plural quantification, this will validate the following two principles:

$(\Box \prec)$ $\forall y \forall xx(y \prec xx \leftrightarrow \Box y \prec xx)$

$(\Box \not\prec)$ $\forall y \forall xx(\neg y \prec xx \leftrightarrow \Box \neg y \prec xx)$

For any objects, if something is one of those, then it is necessarily one of those. And since our two-dimensional modal logic extends **S5**, such principles are also valid in the two-dimensional case. Their validity is easy to check given the extensional character of plural terms — they are interpreted rigidly in the same manner as set membership, and so the truth of $y \prec xx$ becomes "world-invariant."[40] This yields the non-contingency of $\prec$:[41]

(NC$\prec$) $\forall y \forall xx(\Box y \prec xx \vee \Box \neg y \prec xx)$,[42]

For any objects, something is either necessarily one of them or necessarily not one of them. Now, whether the principles $(\Box \prec)$ and $(\Box \not\prec)$ should be validated by one's favourite modal logic is something which we shall not discuss, although this has been defended on philosophical grounds.[43] However, in a two-dimensional framework, the following two principles are also valid:

$(\mathcal{D} \prec)$ $\forall y \forall xx(y \prec xx \leftrightarrow \mathcal{D} y \prec xx)$

$(\mathcal{D} \not\prec)$ $\forall y \forall xx(\neg y \prec xx \leftrightarrow \mathcal{D} \neg y \prec xx)$

[37] For example, see Humberstone (1982, p. 13).

[38] See, for example, Uzquiano (2011, p. 225) and Williamson (2003, pp. 456–7).

[39] It would be the same if we were quantifying over sets rather than plurally.

[40] See Uzquiano (2011, p. 225).

[41] This is also defended by Bricker (1989, p. 387) and Forbes (1985, p. 109).

[42] See also Williamson (2013, p. 248) for similar remarks.

[43] For recent discussion on this, see Uzquiano (2011), Williamson (2003, 2010, 2013, pp. 241–254), and Linnebo (2016).

In our two-dimensional modal language, the evaluation of the open formula $y \prec xx$ is not just world-invariant, it is also pairwise world-invariant, whereby ($\mathcal{D} \prec$) and ($\mathcal{D} \not\prec$) are valid in the corresponding models. If $y \prec xx$ holds at any point in a 2D-centered model, then it holds on the diagonal as well.[44] But even though we might want both ($\Box \prec$) and ($\Box \not\prec$), ($\mathcal{D} \prec$) and ($\mathcal{D} \not\prec$) seem to deliver intuitively false principles. First of all, those are clear cases involving *de re* a priori knowledge ascriptions. Moreover, ($\mathcal{D} \prec$) says of any objects that if something is one of those, it is a priori that it is one of those. But why should we think that? George is one of the Cheerios lovers, but this is not a priori knowable even on a *de dicto* reading. Additionally, ($\mathcal{D} \prec$) and ($\mathcal{D} \not\prec$) also give us the a priori analogue of (NC$\prec$), whence the relation denoted by $\prec$ is not *a posteriori*:

(NP$\prec$) $\forall y \forall xx(\mathcal{D}y \prec xx \vee \mathcal{D}\neg y \prec xx)$

But it is not a priori of Frege that he is one of the philosophers, and it is not a priori of him that he is not. Alternatively, one could read (NP$\prec$) as attributing *de re* a priori knowledge of the plurality, but it is also not a priori of the philosophers that Frege is (or is not) one of them. A solution might be to develop an alternative model theory in which the truth of the atomic formula $y \prec xx$ is relativized to a pair of possible worlds just like the other atomic cases in the semantics. Consequently, we would be able to invalidate principles such as ($\mathcal{D} \prec$) and ($\mathcal{D} \not\prec$). The only semantic clause we would need to reconsider is the following:

$$\mathcal{M}^v_w \vDash t \prec tt' \quad \text{iff} \quad V(t, \langle v, w\rangle) \in V(tt', \langle v, w\rangle)$$

It makes sense to restrict, too, the valuation of plural constants to pairs of possible worlds, whereupon it imitates the valuation of the single case: $V(cc, \langle v, w\rangle) \in \mathscr{D}*$. This is sufficient to invalidate ($\mathcal{D} \prec$) and ($\mathcal{D} \not\prec$), since a counter-model for the former can be constructed by letting $V(c, \langle w*, w*\rangle) \in V(cc, \langle w*, w*\rangle)$ and $V(c, \langle w, w\rangle) \notin V(cc, \langle w, w\rangle)$, for every single and plural constants c and cc, respectively, where $\langle w, w\rangle$ is any pair on the diagonal different from the distinguished one — a similar argument is available to falsify ($\mathcal{D} \not\prec$). This model can be illustrated by the 2D matrix below:

$$\begin{pmatrix} & w* & w \\ w* & c \prec cc & c \prec cc \\ w & c \not\prec cc & c \not\prec cc \end{pmatrix}$$

It should be noted that we still have the correct truth conditions for (12) in this restricted semantics, as well as both ($\Box \prec$) and ($\Box \not\prec$) given the rigidity condition for $\mathcal{R}_\Box$-related worlds.

Would this be a good move for the two-dimensionalist? Even though it invalidates the problematic principles ($\mathcal{D} \prec$) and ($\mathcal{D} \not\prec$), restricting the truth conditions of $\prec$ formulas to

[44] The proof is similar to the necessity of identity. For suppose that $\mathcal{M} \vDash y \prec xx$ for an assignment V. Where $\langle w, w\rangle$ is any pair of possible worlds in W such that $\langle w*, w*\rangle \mathcal{R}_\mathcal{D} \langle w, w\rangle$, since $V(y) \in V(xx)$, it follows that $\mathcal{M}^w_w \vDash y \prec xx$, whence $\mathcal{M} \vDash \mathcal{D}y \prec xx$. The other direction is obvious, and the argument for ($\mathcal{D} \not\prec$) is analogous.

a pair of worlds does nothing in order to avoid explicit *de re* a priori knowledge ascriptions delivered by the single quantifiers.[45] Yet, plural quantification brings with it even more of those cases. Consider, for instance, the uncontroversial principle that for any objects, something is one of them:

(15) $\forall xx \exists y\, y \prec xx$

One can check that (15) is valid in any constant domain 2D-centered model with plural quantifiers since the domains $\mathscr{D}$ and $\mathscr{D}*$ are non-empty. However, if (15) is locally valid, i.e. true at the distinguished point of every model, then it is diagonally valid as well, since local and diagonal validity are equivalent for $\mathcal{L}$ and $\mathcal{L}_{pl}$. Therefore, the following is also valid:

(16) $\mathcal{D}\forall xx \exists y\, y \prec xx$

This is an instance of a two-dimensional analogue of the rule of *necessitation* in basic modal logics:

$$\text{Diagonalization} \quad \frac{\varphi}{\mathcal{D}\varphi}$$

If a sentence, φ, is valid, then $\mathcal{D}\varphi$ is valid too.[46] But given the plural version of the 2D Converse Barcan Formula, which is valid notwithstanding the restriction on the relation $\prec$, one can deduce (17) by truth-functional reasoning:

(17) $\forall xx \mathcal{D} \exists y\, y \prec xx$

However, (17) seems to be quite objectionable. For it is not a priori of someone that he or she is amongst the Cheerios lovers, or of the Cheerios lovers that someone is one of them. We take this to be a high price for two-dimensionalists to pay. Unless they can show such principles to be reasonable, plural quantifiers do not seem to offer an adequate solution for the expressive deficit of the language. Quantification seems to be, again, the Achilles' heel of modal semantics.

[45] Another possible move is to use full quantification over sets, rather than pluralities. This is the strategy adopted by Meyer (2013) in order to eliminate the actuality operator in quantified modal logic. However, the same issues arise concerning what is validated by the models, regardless of whether we assume sets or pluralities in the semantics.

[46] Again, this inference rule is valid for any sentence unless @ is in the language.

A Proof of the Expressive Incompleteness of $\mathcal{L}$

Since bisimulation implies model equivalence, we show that there is a bisimulation between two constant domain 2D-centered models differing with respect to sentence (3). But first some notation is in order. Let $\vec{c}$ denote a sequence of objects or terms of $\mathcal{L}$, where its length is denoted by $\|\vec{c}\|$. The nth-member of $\vec{c}$ is denoted by $\vec{c}_n$. Furthermore, let $\varphi|\vec{x}|$ be a formula whose free variables are all in $\vec{x}$, and whenever $\|\vec{c}\| = \|\vec{x}\|$, let $\varphi|\vec{c}|$ be the result of appropriately substituting constants in $\vec{c}$ for variables in $\vec{x}$. Also, we use $\langle v, w\rangle$ for members of W_1 and $\langle y, z\rangle$ for members of W_2, as well as c for members of $\mathscr{D}_1$ and d for members of $\mathscr{D}_2$. We omit any mention of $\mathcal{M}$ in the evaluation clauses by writing ${}^{v}_{w} \vDash_1 \varphi$ whenever φ holds at $\langle v, w\rangle$ in $\mathcal{M}_1$ — similarly for $\mathcal{M}_2$.

Next we define the general notion of a world-object bisimulation for constant domain 2D-centered models:

Definition A.1 (World-Object Bisimulation) Let $\mathcal{N}_1 = \langle W_1, \langle w*, w*\rangle, \mathcal{R}_{\Box 1}, \mathcal{R}_{\mathcal{D}1}, \mathscr{D}_1, V_1\rangle$ and $\mathcal{N}_2 = \langle W_2, \langle v*, v*\rangle, \mathcal{R}_{\Box 2}, \mathcal{R}_{\mathcal{D}2}, \mathscr{D}_2, V_2\rangle$ be two constant domain 2D-centered models. A *world-object bisimulation* between $\mathcal{N}_1$ and $\mathcal{N}_2$ is a non-empty relation $\cong \subseteq (W_1 \times \mathscr{D}_1) \times (W_2 \times \mathscr{D}_2)$ such that $\langle w*, w*\rangle\vec{c} \cong \langle w*, w*\rangle\vec{d}$, satisfying the following conditions:

1. $(\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \Rightarrow ({}^{v}_{w}\vDash_1 P^n_i\vec{c} \Leftrightarrow {}^{y}_{z} \vDash_2 P^n_i\vec{d})$, for every predicate symbol P^n_i (the *atomic* condition);

2. $(\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \Rightarrow \forall n, m \leq \|\vec{c}\|({}^{v}_{w}\vDash_1 \vec{c}_n = \vec{c}_m \Leftrightarrow {}^{y}_{z} \vDash_2 \vec{d}_n = \vec{d}_m)$ (the *identity* condition);

3. $((\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \wedge \langle y, z\rangle\mathcal{R}_{\Box 2}\langle y, z'\rangle) \Rightarrow (\exists\langle v, w'\rangle \in W_1 : \langle v, w\rangle\mathcal{R}_{\Box 1}\langle v, w'\rangle \wedge (\langle v, w'\rangle\vec{c} \cong \langle y, z'\rangle\vec{d}))$ (the $\mathcal{R}_\Box$-forth condition);

4. $((\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \wedge \langle v, w\rangle\mathcal{R}_{\Box 1}\langle v, w'\rangle) \Rightarrow (\exists\langle y, z'\rangle \in W_2 : \langle y, z\rangle\mathcal{R}_{\Box 2}\langle y, z'\rangle \wedge (\langle v, w'\rangle\vec{c} \cong \langle y, z'\rangle\vec{d}))$ (the $\mathcal{R}_\Box$-back condition);

5. $(\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \Rightarrow (\langle v, v\rangle\vec{c} \cong \langle y, y\rangle\vec{d})$ (the *actuality* condition);

6. $((\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \wedge \langle y, z\rangle\mathcal{R}_{\mathcal{D}2}\langle y', y'\rangle) \Rightarrow (\exists\langle v', v'\rangle \in W_1 : \langle v, w\rangle\mathcal{R}_{\mathcal{D}1}\langle v', v'\rangle \wedge (\langle v', v'\rangle\vec{c} \cong \langle y', y'\rangle\vec{d}))$ (the $\mathcal{R}_{\mathcal{D}}$-forth condition);

7. $((\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \wedge \langle v, w\rangle\mathcal{R}_{\mathcal{D}1}\langle v', v'\rangle) \Rightarrow (\exists\langle y', y'\rangle \in W_2 : \langle y, z\rangle\mathcal{R}_{\mathcal{D}2}\langle y, y'\rangle \wedge (\langle v', v'\rangle\vec{c} \cong \langle y', y'\rangle\vec{d}))$ (the $\mathcal{R}_{\mathcal{D}}$-back condition);

8. $((\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \wedge d \in \mathscr{D}_2) \Rightarrow (\exists c \in \mathscr{D}_1 : \langle v, w\rangle\vec{c}, c \cong \langle y, z\rangle\vec{d}, d)$ (the *quantifier*-forth condition);

9. $((\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \wedge c \in \mathscr{D}_1) \Rightarrow (\exists d \in \mathscr{D}_2 : \langle v, w\rangle\vec{c}, c \cong \langle y, z\rangle\vec{d}, d)$ (the *quantifier*-back condition);

Conditions 1, 3, and 4 are standard for both atomic and modal formulas, although they appear here under a two-dimensional framework. Similarly, 8 and 9 are usual for constant domain quantifiers.[47] One can find conditions 2 and 5 for (unrestricted) identity and actuality in Kocurek (forthcoming), which also presents conditions for the fixedly operator. Our conditions 6 and 7 for $\mathcal{R}_{\mathcal{D}}$ are defined in order to handle the $\mathcal{D}$ operator.

Lemma A.1 (Invariance) *For any constant domain 2D-centered models $\mathcal{N}_1$ and $\mathcal{N}_2$, if $\cong$ is a world-object bisimulation between $\mathcal{N}_1$ and $\mathcal{N}_2$, $\langle v, w\rangle \in W_1$, $\langle y, z\rangle \in W_2$, and $\varphi|\vec{x}|$ is a formula,*

$$(\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}) \Rightarrow ({}^{v}_{w}\vDash_1 \varphi|\vec{c}| \Leftrightarrow {}^{y}_{z} \vDash_2 \varphi|\vec{d}|).$$

Proof. By induction on $\varphi|\vec{x}|$. The atomic cases including identity hold by construction given conditions 1 and 2, while the truth-functional cases are straightforward.

Let $\varphi|\vec{x}|$ be $\Box\varphi|\vec{x}|$. Suppose that $\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}$ and ${}^{v}_{w} \vDash_1 \Box\varphi|\vec{c}|$. If $\langle y, z\rangle\mathcal{R}_{\Box 2}\langle y, z'\rangle$, then $\exists\langle v, w'\rangle \in W_1$ such that $\langle v, w\rangle\mathcal{R}_{\Box 1}\langle v, w'\rangle$ and $\langle v, w'\rangle\vec{c} \cong \langle y, z'\rangle\vec{d}$, by condition 3. Thus, ${}^{v}_{w'} \vDash_1 \varphi|\vec{c}|$. By the induction hypothesis, ${}^{y}_{z'} \vDash_2 \varphi|\vec{d}|$. Therefore, ${}^{y}_{z} \vDash_2 \Box\varphi|\vec{d}|$. The other direction is analogous given condition 6.

Let $\varphi|\vec{x}|$ be $\mathcal{A}\varphi|\vec{x}|$. Suppose that $\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}$ and ${}^{v}_{w} \vDash_1 \mathcal{A}\varphi|\vec{c}|$. Thus, ${}^{v}_{v} \vDash_1 \varphi|\vec{c}|$, by the semantics of $\mathcal{A}$. By condition 5, $\langle v, v\rangle\vec{c} \cong \langle y, y\rangle\vec{d}$, whence ${}^{y}_{y} \vDash_2 \varphi|\vec{d}|$, by the induction hypothesis. Therefore, ${}^{y}_{z} \vDash_2 \mathcal{A}\varphi|\vec{d}|$, by the semantics of $\mathcal{A}$. The other direction is analogous.

Let $\varphi|\vec{x}|$ be $\mathcal{D}\varphi|\vec{x}|$. Suppose that $\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}$ and ${}^{v}_{w} \vDash_1 \mathcal{D}\varphi|\vec{c}|$. If $\langle y, z\rangle\mathcal{R}_{\mathcal{D}2}\langle y', y'\rangle$, then $\exists\langle v', v'\rangle \in W_1$ such that $\langle v, w\rangle\mathcal{R}_{\mathcal{D}1}\langle v', v'\rangle$ and $\langle v', v'\rangle\vec{c} \cong \langle y', y'\rangle\vec{d}$, by condition 6. Thus, ${}^{v'}_{v'} \vDash_1 \varphi|\vec{c}|$. By the induction hypothesis, ${}^{y'}_{y'} \vDash_2 \varphi|\vec{d}|$. Therefore, ${}^{y}_{z} \vDash_2 \mathcal{D}\varphi|\vec{d}|$. The other direction is analogous given condition 7.

Let $\varphi|\vec{x}|$ be $\exists y\varphi|\vec{x}, y|$. Suppose that $\langle v, w\rangle\vec{c} \cong \langle y, z\rangle\vec{d}$ and ${}^{v}_{w} \vDash_1 \exists y\varphi|\vec{c}, y|$. Thus, $\exists c \in \mathscr{D}_1$ such that ${}^{v}_{w} \vDash_1 \varphi|\vec{c}, c|$. By condition 9, $\exists d \in \mathscr{D}_2$ such that $\langle v, w\rangle\vec{c}, c \cong \langle y, z\rangle\vec{d}, d$. By the induction hypothesis, ${}^{y}_{z} \vDash_2 \varphi|\vec{d}, d|$, whence ${}^{y}_{z} \vDash_2 \exists y\varphi|\vec{d}, y|$. The other direction is analogous given condition 8. □

We define our two models as follows.[48] Let $\mathcal{M}_1 = \langle W_1, \langle w*, w*\rangle, \mathcal{R}_{\Box 1}, \mathcal{R}_{\mathcal{D}1}, \mathscr{D}_1, V_1\rangle$ be such that W_1 is the set of all pairs of subsets of $\mathbb{N}$ that are both infinite and coinfinite containing at least one odd number, $\langle w*, w*\rangle$ is the pair $\langle 2\mathbb{N}+1, 2\mathbb{N}+1\rangle$ of sets of odd numbers, $\mathscr{D}_1$ is $\mathbb{N}$, for each $\langle i, j\rangle \in W_1$ let $V_1(R, \langle i, j\rangle) = V_1(S, \langle i, j\rangle) = i \cup j$, and let the accessibility relations be defined just as in Definition 1.2. The extension of every other predicate symbol is empty. On the other hand, our second model, $\mathcal{M}_2 = \langle W_2, \langle w*, w*\rangle, \mathcal{R}_{\Box 2}, \mathcal{R}_{\mathcal{D}2}, \mathscr{D}_2, V_2\rangle$, is just like $\mathcal{M}_1$ except for $W_2 = W_1 \cup \{\langle 2\mathbb{N}, 2\mathbb{N}\rangle\}$, where $2\mathbb{N}$ is the set of even numbers, and $V_2(R, \langle 2\mathbb{N}, 2\mathbb{N}\rangle) = V_2(S, \langle 2\mathbb{N}, 2\mathbb{N}\rangle) = 2\mathbb{N} \cup 2\mathbb{N}$. No other predicate has an extension defined on $\langle 2\mathbb{N}, 2\mathbb{N}\rangle$.

In order to prove that $\mathcal{M}_1$ and $\mathcal{M}_2$ are bisimilar, let $\langle v, w\rangle \in W_1$ and $\langle v', w'\rangle \in W_2$ be corresponding pairs of worlds in the models $\mathcal{M}_1$ and $\mathcal{M}_2$. For any $\langle v, w\rangle \in W_1$ and $\langle v', w'\rangle \in W_2 - \{\langle 2\mathbb{N}, 2\mathbb{N}\rangle\}$, say that the mapping ρ from W_1 onto $W_2 - \{\langle 2\mathbb{N}, 2\mathbb{N}\rangle\}$ is an *isomorphism* between $\mathcal{M}_1$ and the submodel of $\mathcal{M}_2$ defined on $W_2 - \{\langle 2\mathbb{N}, 2\mathbb{N}\rangle\}$ such that $\rho(c) = d$, in which case for $\langle v, w\rangle \in W_1$ and $\langle v', w'\rangle \in W_2$ other than $\langle 2\mathbb{N}, 2\mathbb{N}\rangle$, and for any non-empty predicate symbol P^n_i, we have

[47] See, for instance, van Benthem (2010) and, more recently, Urquhart (2016).

[48] The models are constructed resembling the ones in Wehmeier (2001), where he proves a similar result for a first-order modal language.

$$(c \in V_1(P_i^n, \langle v, w\rangle)) \Leftrightarrow (\rho(c) \in V_2(P_i^n, \langle v', w'\rangle)).$$

Moreover, we define the following relations between $\mathcal{M}_1$ and $\mathcal{M}_2$. For any $\langle v, w\rangle \in W_1$ and $\langle v', w'\rangle \in W_2$ other than $\langle 2\mathbb{N}, 2\mathbb{N}\rangle$, set

$$(\langle v, w\rangle \vec{c} \cong \langle v', w'\rangle \vec{d}) \Leftrightarrow (\rho(\vec{c}) = \vec{d}).$$

For the actuality condition, let

$$(\langle v, w\rangle \vec{c} \cong \langle v', w'\rangle \vec{d}) \Leftrightarrow (\langle v, v\rangle \vec{c} \cong \langle v', v'\rangle \rho(\vec{c})).$$

In the case of the extra pair of worlds $\langle 2\mathbb{N}, 2\mathbb{N}\rangle \in W_2$, we define

$$(\langle v, v\rangle \vec{c} \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}) \Leftrightarrow \forall n \leq \|\vec{c}\|(\vec{c}_n \in (V_1(S, \langle v, v\rangle)) \Leftrightarrow (\vec{d}_n \in V_2(S, \langle 2\mathbb{N}, 2\mathbb{N}\rangle))),$$

and

$$(\langle v, v\rangle \vec{c} \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}) \Leftrightarrow \forall n \leq \|\vec{c}\|(\vec{c}_n \in (V_1(R, \langle v, v\rangle)) \Leftrightarrow (\vec{d}_n \in V_2(R, \langle 2\mathbb{N}, 2\mathbb{N}\rangle))).$$

Finally, set

$$(\langle v, w\rangle \vec{c} \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}) \Leftrightarrow (\langle v, v\rangle \vec{c} \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}).$$

Lemma A.2 $\cong$ *is a bisimulation between* $\mathcal{M}_1$ *and* $\mathcal{M}_2$.

Proof. Conditions 1, 2, and 5 hold by construction. Since $\langle 2\mathbb{N}, 2\mathbb{N}\rangle$ is not $\mathcal{R}_\Box$-accessible with respect to any pair of worlds, conditions 3 and 4 are easily seen to be met as well.

For conditions 6 and 7, the only cases we need to consider involve $\langle 2\mathbb{N}, 2\mathbb{N}\rangle$. Suppose $\langle v, w\rangle \vec{c} \cong \langle v', w'\rangle \vec{d}$ and that $\langle v', w'\rangle \mathcal{R}_{\mathcal{D}2} \langle 2\mathbb{N}, 2\mathbb{N}\rangle$. Since $\vec{c}$ contains a single element, we have only two cases. If $\vec{c}$ is even, choose a pair $\langle v, w\rangle \mathcal{R}_{\mathcal{D}1} \langle y, y\rangle$ such that $\vec{c} \in V_1(S, \langle y, y\rangle)$. By the definition of $\mathcal{M}_1$, we know that there will be such a pair, in which case we have $\langle y, y\rangle \vec{c} \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}$, by construction. If $\vec{c}$ is odd, choose a pair $\langle v, w\rangle \mathcal{R}_{\mathcal{D}1} \langle z, z\rangle$ such that $\vec{c} \notin V_1(S, \langle z, z\rangle)$, whence $\vec{d} \notin V_2(S, \langle 2\mathbb{N}, 2\mathbb{N}\rangle)$. Therefore, $\langle z, z\rangle \vec{c} \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}$. Condition 7 is analogous.

With respect to conditions 8 and 9, again, we only need to check the cases involving the extra pair $\langle 2\mathbb{N}, 2\mathbb{N}\rangle$. Suppose that $\langle v, w\rangle \vec{c} \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}$ and that $c \in \mathscr{D}_1$. We only have cases involving the predicates S and R, since all the other predicates are empty. If $c \in V_1(S, \langle v, v\rangle)$, then $\langle v, w\rangle \vec{c}, c \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}, d$, by construction, since $V_2(S, \langle 2\mathbb{N}, 2\mathbb{N}\rangle)$ is not empty. If $c \notin V_1(S, \langle v, v\rangle)$, then choose any $d \in V_2(S, \langle w*, w*\rangle)$, in which case $d \notin V_2(S, \langle 2\mathbb{N}, 2\mathbb{N}\rangle)$, and then we have $\langle v, w\rangle \vec{c}, c \cong \langle 2\mathbb{N}, 2\mathbb{N}\rangle \vec{d}, d$. The argument for R is very similar. Condition 8 is analogous. □

Theorem A.1 *There is no sentence* φ *of* $\mathcal{L}$ *such that for every constant domain 2D-centered model* $\mathcal{M} = \langle \mathrm{W}, \langle w*, w*\rangle, \mathcal{R}_\Box, \mathcal{R}_\mathcal{D}, \mathscr{D}, V\rangle$, $\mathcal{M} \vDash \varphi$ *if and only if there is a pair of possible worlds* $\langle w, w\rangle \in W$, *such that for two predicate symbols* R *and* S, $V(R, \langle w*, w*\rangle) \cap V(S, \langle w, w\rangle) = \varnothing$.

Proof. By construction of the models, in $\mathcal{M}_2$ there is a pair $\langle 2\mathbb{N}, 2\mathbb{N}\rangle \in W_2$ such that $V_2(R, \langle w*, w*\rangle) \cap V_2(S, \langle 2\mathbb{N}, 2\mathbb{N}\rangle) = \varnothing$, but in $\mathcal{M}_1$ there is no pair of worlds $\langle w, w\rangle \in W_1$ such that $V(R, \langle w*, w*\rangle) \cap V(S, \langle w, w\rangle) = \varnothing$, since $V_1(R, \langle w*, w*\rangle)$ contains only odd numbers, and for any $\langle w, w\rangle \in W_1$, $V_1(S, \langle w, w\rangle)$ contains at least one odd number for each coordinate of the pair $\langle w, w\rangle$. □

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