Group Field Theories and Phase Transitions: Revisiting the Problem of Spacetime Emergence

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draft

Abstract

With the present paper I maintain that the group field theory (GFT) approach to quantum gravity can help us clarify and distinguish the problems of spacetime emergence from the questions about the nature of the quanta of space. I will show that the mechanism of phase transition suggests a form of indifference between scales (or phases) and that such an indifference allows us to black-box questions about the nature of the ontology of the fundamental levels of the theory. I consider the GFT approach to quantum gravity which derives spacetime as an emergent property from abstract non-geometrical tetrahedra through a processes of phase transition. Because the physical interpretation of the tetrahedra is problematic in view of their (non)-spatiotemporal nature, I will apply the considerations about phase transitions to GFT and conclude that the fundamental ontology of the theory and the mechanism of spacetime emergence can and should be treated separately.

1 Introduction

Many approaches to quantum gravity (QG) seem to agree that spacetime is not a fundamental entity and, as such, it should emerge from a different non-spatiotemporal structure —for a general overview of many such approaches, see for example: (Oriti 2009)[1] The immediate consequence is that the fundamental ontology of QG seems to be non-spatiotemporal in nature, and this raises several issues to the working philosophers. For example: is the emergence of spacetime an inter-theoretic property (Bain 2013b), or is it an ontological property that requires a metaphysical account (Huggett and Wüthrich 2013), (Lam and Wüthrich 2018)? Instead of making spacetime emergent, can we relinquish either space or time separately (Smolin 2020), (Gomes 2016)? What account of emergence should we expect (Oriti 2021a)?

Perhaps, for the sake of simplicity, we can divide some of the technical and philosophical problems of quantum gravity into two broad categories: on the one hand we have the challenge of accounting for the emergence of spacetime, that is, we need to provide a mechanism (mathematical or physical) that describes how non-spatiotemporal and pre-geometric entities can approximate the geometry of spacetime. On the other hand, we have to face the problem of accounting for the existence of fundamental entities (of a theory of QG) that are pre-geometric and that do not live in spacetime. In what follows, I will focus on the group field theory approach and use the analogy with the physics of phase transitions to show that we can treat the two classes of problems as independent of one another.

I will use the physics of phase transitions to show that the use of approximation methods (such as mean field theory and renormalization group techniques) place a metaphorical wedge between the fundamental entities of the theory and the macroscopic dynamics. At the occurrence of a phase transition, the properties of the macroscopic system are studied in terms of the collective behavior of the component parts, and the macroscopic quantities appear to be partly independent of the properties of the individual components. I will use the Ising model, mean field theory, and the hydrodynamic description of many-body systems to show and describe this partial independence. What will follow is that: if a macroscopic system is indeed (at least partly) independent of its component parts, then the problem of addressing the properties of the individual components can be separated from the problem of addressing the macroscopic properties of the system. With respect to our theory of quantum gravity: if we can demonstrate that the emergence

[1]Notably, there are also non-emergent approaches such as string theory, and emergent approaches that posit fundamental entities that have spatiotemporal entities different form those described by general relativity (for example: (Volovik 2006)). In this contribution I will focus on emergentist approaches to quantum gravity that posit non-spatiotemporal fundamental entities.

of spacetime is (partly) indifferent to the dynamics of the individual quanta of space, we can separate the problems related to spacetime emergence from those related to the fundamental ontology of the theory. Even further, we can maintain that the two problems are not only separated, but also independent of one another.

It follows that the reason as to why I focus on the group field theory approach is the immediate connection with the physics of phase transitions. As a matter of fact, the key idea in the GFT approach —which also led to the derivation of cosmological models (at this stage: homogeneous and isotropic)— is to interpret the collective behavior of the quanta of space as a form of Bose-Einstein condensate (see: (Gielen, Oriti, and Sindoni 2014), (Gielen, Sindoni, et al. 2016), (Oriti 2017), (Pithis and Sakellariadou 2019), (Gielen and Polaczek 2020), (Gabbanelli and De Bianchi 2021)). In other words, the individual quanta of space of the theory are taken to be living in a pre-geometric phase. By studying their collective behavior (the thermodynamic limit) in the mean field approximation, one obtains the equations of a condensate from which it is possible to extract some geometric quantities (for example, the volume of the condensate).[2]

In Section 2 and 3, I present the philosophical problems of quantum gravity that I will be discussing in this paper. I will clarify notions such as emergence and reduction and review some of the recent literature. In section 4 I will provide a brief discussion of the Ising model to the purpose of showing how phase transitions offer us a case of indifference of the macroscopic properties of a given system from the dynamics of its individual components. Section 5 briefly introduces the group field theory approach to quantum gravity and applies the results of Section 4 to the emergence of spacetime interpreted as a phase transition. Section 6 offers some concluding remarks.

[2]One of the main results of the approach is that the equation of motion of the condensate is analogous to the Gross-Pitaevskii equation in canonical condensed matter (Gielen and Polaczek 2020). The geometric quantities defined by the condensate can be interpreted as the Friedmann equations for the GFT condensate and they can be shown to be consistent with their canonical counterpart (see: (Oriti, Sindoni, and Wilson-Ewing 2016), (Oriti, Sindoni, and Wilson-Ewing 2017)).

2 Setting the Stage: Levels of Emergence

Two recent papers, (Oriti 2021a) and (Oriti 2021b), set out a multilevel-ontology account of spacetime emergence in the context of some approaches to quantum gravity. The starting point of Oriti's argument is the necessity (and corresponding difficulties) of identifying the fundamental degrees of freedom of a theory that aims at justifying the emergence of spacetime from entities that are fundamentally non-spatiotemporal. The problem, which stems from the common understanding of observables to be living in spacetime, has even led to doubts about the very possibility of verifying a theory of quantum gravity in the first place (see: (Huggett and Wüthrich 2013)). The atoms of space, the fundamental degrees of freedom of the theory, shall define not only a quantum dynamics, but also show how at some (continuum) limit general relativity (GR) becomes a good effective description of spacetime. In other words: one of the challenges faced by the community working on quantum gravity is to show how GR spacetime emerges from non-spatiotemporal fundamental entities.

The instance of emergence, in this case, is taken to be one that justifies novel properties that are missing from the properties of the underlying entities. See, for example: (Butterfield and Bouatta 2012), and (Butterfield 2011a). In this sense: "[e]mergence is understood to be the appearance, in a certain description of a physical system, of properties that are novel with respect to a different (more 'fundamental') description of the same system, robust, and thus stable enough to represent a characterization of the new description and to form part of new predictions stemming from it" (Oriti 2021a, p. 17). The emergence of such new phenomena from some underlying entities often requires the use of limiting procedures and approximations. These provide a new description of the system *via* novel quantities that, as we shall see below, are partially indifferent to the dynamics of the more fundamental levels.

In his account, Oriti defines four non-sequential levels of spacetime emergence, to be interpreted as increasing levels of complexity of the emergence of spacetime from the atoms of space of a theory of quantum gravity. The first level (listed as 'Level 0') emphasizes how general relativity already implies a disappearance of a notion of absolute space and time. In GR the ingredients that form the notion of spacetime, i.e., the matter and metric fields, are silent with respect to any preferred directions of space or time. Similarly, at the quantum level, the theory does not select any preferred time directions or coordinate frames. A possible solution is to define some internal degrees of freedom to act as (imperfect) internal rods and clocks, for example, (Gambini and Porto 2001). This strategy "amounts to identifying internal degrees of freedom of the complete system composed of metric and matter fields that can be used to approximate rods and clocks to parametrize the spatial relations and temporal evolution of the remaining degrees of freedom" (Oriti 2021a, p. 22). However, it remains that we would not be able to recover the idealized clocks and rods provided by a coordinate system.

A second level of spacetime emergence implies the existence of quanta of space (or atoms of space) as non-spatiotemporal entities. Such new degrees of freedom constitute the theory's new fundamental building blocks from which space, time, and geometry, are supposed to emerge in some continuum limit. But, since atoms of space are non-spatiotemporal in themselves, how can they carry spatiotemporal properties in the first place?[3] A possible response is that the collective behavior of individual entities can lead to novel properties that are not possessed by the underlying components. Yet, some theoretical, if not ontological, bridging between the underlying and emergent components should be accounted for: "if spacetime has to be reconstructed at all, the more fundamental theory should allow for a dictionary, mapping its basic entities and some of their properties into continuum fields including those defining spatiotemporal notions" (Oriti 2021a, p. 25).

There is an important distinction that should be accounted for in the previous quote: whether the dictionary is to be taken as translating concepts from one theory to another, or whether the ontology tracks the respective theories. The first possibility implies a more timid perspective in that it would require an account of inter-theoretic reduction between the atoms of space of quantum gravity and the spacetime of general relativity. On the other hand, a stronger claim (and one that requires some metaphysical finessing) is that not only a dictionary between theories is possible, but also that the ontology of the respective theories follows such a reduction. With respect to these possibilities, (Oriti 2021b) emphasizes how the physical entities we endow with ontological status are defined within the contexts of either the theory, or models thereof. This is true especially for theories operating at scales beyond immediate sensory experience, such as quantum mechanics, quantum

[3]In (Oriti 2021b), it is argued that the quantization of gravity starting from the relational clocks constitutes another level of emergence, one that does not directly involve the emergence of spacetime for the basic entities remain the same in quantum and classical general relativity.

field theory, and, consequently, quantum gravity. It is therefore hard to imagine how the ontologies proper of each theory could be independent from the corresponding mathematical framework. This amounts to following a cautious scientific realism, and I emphasize the term 'cautious' because, while a complete separation between formal apparatus and ontology is unlikely, it is also too strong of a claim to say that all theoretical objects partake into the ontology of the theory. However, to provide a detailed analysis of how to separate the ontological wheat from the mathematical chaff goes far beyond the scope of the present contribution.

In addition, one could argue that I am confusing emergence and reduction. In the context that I will be discussing in this paper, i.e., that of phase transitions (and group field theory), to determine which concept should apply is a matter open to debate. For example, (Butterfield 2011b) and (Butterfield and Bouatta 2012) maintain that phase transitions combine a form of inter-theoretic reduction and emergence at the thermodynamic limit, (Batterman 2011) and (Morrison 2012) maintain that some approximations and limit procedures imply the emergence of new phenomena. Alternatively, (Palacios 2019) argues for a sophisticated notion of inter-theoretic reduction between thermodynamics and statistical mechanics that involves logical deduction between theories and a form of limiting reduction (which makes use of approximations and idealizations). The bottom line is that notions such as reduction and emergence need not be incompatible if applied to different contexts. Indeed we can have inter-theoretic reduction via limiting procedures and emergent properties stemming from the collective behavior of fundamental entities.

The second and third level of emergence —as indicated in (Oriti 2021a)— focus on the atoms of space in terms of collective behavior. While Oriti (2021a) seems to suggest a realist attitude about the fundamental entities postulated by the theory, Oriti (2021b) is more cautious about such an ontological commitment. Nonetheless, he maintains that ontological emergence might follow from the inter-theoretic one: "This intertheoretic (or epistemic) emergence amounts in fact to a relation between mathematical and conceptual models of the world, from which we imply a relation between natural phenomena described by those theories" (Oriti 2021b, p. 2). In what follows, I will focus mostly on these two levels and on the ontological problems they naturally imply. More specifically, I will emphasize the distinction between the problem of spacetime emergence —that is, how non-spatiotemporal entities can approximate spacetime structures— and the problem of the nature of non-spatiotemporal entities —that is, the apparent difficulty of formulating a theory whose fundamental ontology does not live in spacetime.

3 The twofold problem

Having shed some light on the subtleties of concepts such as emergence, reduction, and the difference between emergence of theories *vs* emergence of phenomena, we can now move back to the original problem I discussed here: the justification of spacetime emergence from non-spatiotemporal entities. The problem is twofold, and thus requires separate considerations. On the one hand, we want an account of how spacetime emerges from, or reduces to, more fundamental entities. On the other hand, we want a precise account, possibly endowed with a plausible physical interpretation, of the kinematics and dynamics of such fundamental entities (the atoms of space). The latter problem, as far as the current research goes, presents itself with the demand for the atoms of space to be fundamentally non-spatiotemporal. How to conceive of some fundamental physical entities to be non-spatiotemporal is a philosophical conundrum, yet one that ought not to be confused with the question of how such entities can approximate spacetime.

Notably, that spacetime is not the sort of fixed background that allows us to absolutely identify objects and events is already questioned in general relativity. However, one of the challenges posed by quantum gravity is that even the individual rods and clocks used by GR to keep track of the dynamics between entities and events seem to vanish. Perhaps, one could relinquish the idea that being-in-spacetime is a necessary condition for existence.[4] Alternatively, one can opt for an instrumentalist view and consider fields as the only truly physical entities, while the non-spatiotemporal basic structures of the theory are conceived of as mathematical artifacts.

In what follows, I will discuss how emergent properties in the context of phase transitions can be considered to be independent from their fundamental constituents. Afterward, I will review the analogy between GFT and hydrodynamics models (Kadanoff and Martin 1963) (Marchetti et al. 2022a), (Marchetti et al. 2022b), (Volovik 2006) and emphasize how GFT condensate offers a mechanism for spacetime emergence that relies on the independence

[4]Notably, this is not necessarily a new view. Mathematical and abstract objects do not exist in spacetime and yet they can be (for the most part) well-defined and individuated. See: (Linnebo 2018).

of spacetime from the dynamics of the underlying physics.

4 Ising Model and Indifference

In this section I will briefly present the Ising model, and some approximation methods that allow us to define salient mesoscale quantities. The relevant feature of these quantities is that they are indifferent to the micro-dynamics of the more fundamental levels. This indifference will turn out to be central for our discussion on quantum gravity, since it will allow us to 'black-box' the questions about the nature of the atoms of space and focus on the emergence of spacetime.

The Ising model is a simplified lattice model that can be used to describe the total magnetization of a system composed of many individual atomic spins. In conjunction with the Ising model, I will introduce the mean field theory approximation, which derives an effective field by averaging over the behavior of the individual atomic spins. As a result, the effective field 'ignores' the interactions between individual spins and allows us to derive the thermodynamic properties of the macroscopic system (for example, its magnetization). While I will emphasize that a change in the orientation of some of the individual atomic spins would not be relevant to the overall magnetization, it would be too strong of a claim to characterize such an indifference as a complete autonomy. Indeed, while we could modify some of the orientations of the individual spins without affecting the total magnetization, a change in the orientation of all spins would inevitably change the total magnetization.[5] It is precisely because of this partial indifference that we have the emergence of novel (and as we will see, robust) properties at the macroscopic phase.

4.1 Ising Model and Indifference

The Ising model (Onsager 1944), (Stanley 1971) is a mathematical model that can be used to represent the ferromagnetic behavior of a collection of atomic spins on a lattice. Each spin σ_i has values ± 1 and interacts with the neighbor sites on the lattice, and with an external magnetic field h. The

[5] I leave the discussion about how many atomic spins we can change before affecting the macroscopic system to later works.

Hamiltonian of the system reads:

$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - h \sum_i \sigma_i \tag{1}$$

The first sum on the right hand side of the equation is taken over all the sites of the lattice $\langle i, j \rangle$ close to σ_i, and J is the coupling factor between spins. The exact solution of the two-dimensional Ising model was presented by (Onsager 1944), but higher dimensionality leads to untractable terms. As a consequence, scientists rely on approximation methods to derive the macroscopic properties of the modeled systems. One of these approximation methods is the mean field theory (MFT), which derives the macroscopic properties of the system by assuming that the microscopic interactions (in this case between atomic spins) can be ignored.[6]

In our case, we can express the Hamiltonian of a given spin σ_i in terms of its neighbors:

$$H_i = -J \sum_{\delta} \sigma_i \sigma_j - h\sigma_i = -(J \sum_j \sigma_j + h)\sigma_i \tag{2}$$

The idea behind MTF is that instead of accounting for all the interactions of the individual atomic spins, we can average their behavior and treat them as a single effective field.[7] To do so, we replace the fluctuating spins by their mean value and obtain the effective field:

$$h_{eff} = J \sum_j \langle \sigma_j \rangle + h \tag{3}$$

Now we can calculate the mean value of an isolated spin subject to a field as $\langle \sigma \rangle = \tanh{(h/k_\beta T)}$ and we incorporate the generic field h in our effective

[6]Mean field theory is but one method that involves forms of coarse-graining. Another example is represented by renormalization group techniques (or, renormalization group theory (RG), (Wilson 1975)). As pointed out in (Batterman 2013, p. 8): "In a mean field theory, the order parameter M is defined to be the magnetic moment felt at a lattice site due to the average over all the spins on the lattice. This averaging ignores any large-scale fluctuations that might (and, in fact, are) present in systems near their critical points. The RG corrects this by showing how to incorporate fluctuations at all length scales, from atomic to the macro, that play a role in determining the macroscopic behavior [...] of the system near criticality".

[7]This amounts to approximating the average product to the product of the averages: $< \sigma_i \sigma_j > \approx < \sigma_i >< \sigma_j >$

field to obtain the effect of h_{eff} onto a single atomic spin:

$$\langle \sigma_i \rangle = \tanh\left(\frac{h_{eff}}{K_\beta T}\right) = \tanh\left(\frac{1}{K_\beta T}\left(h + J\sum_j \langle \sigma_j \rangle\right)\right) \tag{4}$$

Because the expectation values of the effective field are indifferent to the sites on the lattice, i.e., our system is invariant under translation, we can say that:

$$m = \frac{1}{N}\sum_j \langle \sigma_j \rangle = \langle \sigma_i \rangle\,, \quad \forall i$$

We can thus write equation (4) as:

$$m = \tanh\left(\beta h + \beta J z m\right) \tag{5}$$

which is the mean field equation for the magnetization of the system, where $\beta = 1/K_\beta T$, and z is the number of neighbors of σ_i (e.g., four in 2-d, eight in 3-d, and so on.) The equation means that the particles do not interact with each other, but only with the external field h and with the effective field Jzm. By solving the equation, one obtains that the description of the thermodynamic properties of the system is independent from the dynamics of the microscopic quantities (the orientation of the individual spins):[8]

$$m(T, h=0) = \begin{cases} \pm\left(1 - \frac{T}{T_c}\right)^{1/2} & T \to T_c^- \\ 0 \quad T \to T_c^+ \end{cases} \tag{6}$$

The result is that, starting from a system composed of (infinitely) many atomic spins, it is possible to average over the degrees of freedom using some statistical methods and obtain a new quantity that was not present at the scale of the individual atoms. This new quantity can be used to describe the macroscopic properties of the system. The crucial point is that the new defined quantity (the total magnetization of the system) seems to be independent from the orientation of the individual atomic spins, where such an independence is warranted by the approximation provided by mean field theory.

[8]The mathematical details of the solution can be found in, for example: (Selinger 2016), (Kadanoff 2000), (Goldenfeld 2018).

Another approach that testifies the irrelevance of the microscopic degrees of freedom to the order parameter M is the hydrodynamic description of many-body systems. Let us consider, again, the Ising model and let us define $M(r,t)$ the magnetization of the system at site r and time t. At equilibrium, the variation over time of the magnetization will be conserved: $\partial/\partial t \int M(r,t)dr = 0$. If we add a perturbation, the system will relax back to equilibrium state, thereby determining a flux $\left\langle j^M \right\rangle = -D\nabla \langle M(r,t)\rangle$ and a diffusion equation that uses averages to describe the diffusion of the long-lived disturbances to the system by the external field:[9]

$$\frac{\partial}{\partial t}\langle M(r,t)\rangle - D\nabla^2 \langle M(r,t)\rangle \tag{7}$$

where M indicates the magnetization, the brackets indicate averages and thus not individual spins, and D the transport coefficient (or spin-diffusion coefficient).[10]

Then, Kadanoff and Martin (1963, p. 419) point out how the hydrodynamic description is equivalent to that provided by correlation functions, which means that one can use equilibrium statistical mechanics to describe non-equilibrium behaviors. For instance, let us characterize an external field acting on the atomic spins as: $\langle M(r)\rangle = \chi h(r)$ where $\chi = \partial M/\partial h$ is the magnetic susceptibility. The relaxation back to equilibrium after the macroscopic disturbance by the external field h "[...] follows the same laws as the regression of microscopic fluctuations at equilibrium. These fluctuations are represented by correlation functions" (Batterman 2021, p. 59). This means that correlation functions such as $\langle C_{ij}\rangle = \langle \sigma_i\sigma_j\rangle - \langle\sigma_i\rangle\langle\sigma_j\rangle$ lead to thermodynamic information. For example, one can show that magnetic susceptibility can be expressed in terms of a sum of correlation functions over all sites of the lattice:[11]

$$\chi(T,H) = N\frac{m^2}{k_\beta T}\sum_i C_i(T,H) \tag{8}$$

From magnetic susceptibility one can obtain the thermodynamic properties of the system by using the free energy and appropriate variational principles,

[9] Batterman (2021) notes that the averages do not indicate individual spins nor continuum systems, and therefore they are to be considered as mesoscale quantities.

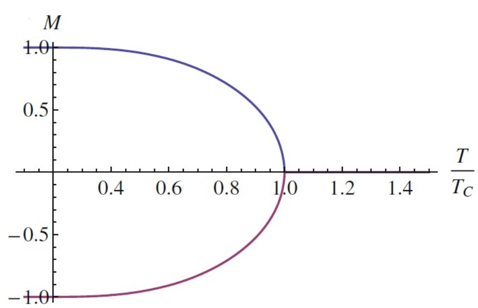
[10] Because magnetization is expressed in terms of averages, (Batterman 2021) maintains that the equation describes mesoscale quantities.

[11] A rigorous derivation of the magnetic susceptibility from hydrodynamic equations is offered in (Kadanoff and Martin 1963)

see: (Solé 2011), (Selinger 2016), (Kadanoff 2000), (Goldenfeld 2018).

The example, which I have reported here in a simplified form, shows that there is a connection between the linear response of a system to an external 'macroscopic push', and the internal fluctuations of a system in equilibrium —I shall say more on this in the next section. Such a connection allows us to use hydrodynamic equations (such as the diffusion equations) to describe the behavior of a many-body system without needing a description of the microscopic behavior. For example, to calculate the thermodynamic properties of the system we do not need the fine details of the individual spins —e.g., we do not need the interaction coupling constant between spins to calculate the spontaneous magnetization below critical temperature. This is made evident graphically in Figure 1: below the critical temperature, the system undergoes a symmetry-breaking corresponding to the ferromagnetic phase.

Figure 1: The parameter M (magnetization) as a function of parametrized temperature T/T_c (Selinger 2016, p. 18).



4.2 Neither Realism nor Instrumentalism

Models such as the Ising model have the capacity of representing correlations that are statistically representative at different scales. In this sense: "equilibrium statistical mechanics *itself* has the means to describe the non-equilibrium behavior of the transport properties in the slow, linear regime" (Batterman 2021, p. 58). This is considered by Batterman as a consequence of the fluctuation-dissipation theorem in (Kubo 1966, p. 256), for which there is: "a general relationship between the response of a given system to an external disturbance and internal fluctuation of the system in the absence of the disturbance. Such a response is characterized by a response function or equivalently by an admittance, or an impedance. The internal fluctuation is characterized by a correlation function of relevant physical quantities of the system fluctuating in thermal equilibrium, or equivalently by their fluctuation spectra". In other words, the fluctuation-dissipation theorem proves a connection between the relaxation back to equilibrium of correlated atoms (or molecules) with some external perturbation of the system. This justifies the use of hydrodynamic equations, as shown in (Kadanoff and Martin 1963, p. 800):

> The response of a system to an external disturbance can always be expressed in terms of time dependent correlation functions of the undisturbed system. More particularly the linear response of a system disturbed slightly from equilibrium is characterized by the expectation value in the equilibrium ensemble, of a product of two space -and time- dependent operators. When a disturbance leads to a very slow variation in space and time of all physical quantities, the response may alternatively be described by the linearized hydrodynamic equations.

Thus, as we have seen earlier for the Ising model, the hydrodynamics approach emphasizes the autonomy of the mesoscale from the (more) fundamental micro-dynamics. Because of this autonomy, (Batterman 2021) considers the correlation variables as 'natural variables' of the system at a given scale. What gives credibility to this claim is the robustness in terms of applicability to different systems, i.e., what is oftentimes called: universality. For example, the discontinuity of thermodynamic quantities (indicating critical behavior) is the same for all systems at $(T - T_c)^{1/2}$.[12]

There are cases, though, where the suppressed fluctuations at all scales have indeed an effect on the macro dynamics of the system. Renormalization Group Theory (RG) solves this problem by iterating the process of averaging over different length scales: "Instead of using the ensemble to calculate an average, as in SM [statistical mechanics], we use RG to transform one ensemble into another one with different couplings. Each transformation increases

[12] While the relationship between critical behavior, phase transitions, universality, and autonomy is not the central focus of this paper, the topic is well-discussed in the literature. Some examples: (Butterfield and Bouatta 2012), (Butterfield 2011b), (Batterman 2000), (Palacios 2022), and others.

the length scale so that the transformation eventually extends to information about the parts of the system that are infinitely far away" (Morrison 2014). This way, while the system loses information about the microscopic structure, it displays the new macroscopic correlations. With respect to our purposes, the crucial point remains: one can describe macroscopic properties using an appropriately defined mesoscale which is (partly) indifferent to the behavior of the more fundamental entities.

At this point, one might question whether the mesoscale level is to be considered as merely an instrumental tool for calculation purposes. For example, (Williams 2019) advocates a form of effective realism that includes entities derived from approximation methods such as renormalization group techniques: "focusing exclusively on fundamental ontology [...] leaves one with an interpretation unequipped to support the theory in the performance of its explanatory duties [...] many explanatory affirmations made in the theory simply cannot be made true by including in one's ontology only those entities at the fundamental scale" (Williams 2019, p. 19). Alternatively, one could argue that the indifference of the mesoscales from the fundamental ontology will one day be explained by a complete physical theory. The argument calls for a form of strong reductionism that attempts to 'build the universe from fundamental entities' (see: (Anderson 1972)). Other forms of reductionism set a less stringent requirement (Bain 2013a), but they still do not invalidate the fact that the explanatory power of approximate models is provided by quantities that lie at the mesoscale level: densities and gradients in flowing contexts, geometrical properties and topological features in static cases, effective fields in magnetic phenomena. The (more) fundamental levels of the theory remain irrelevant to the explanatory power of those models.

This is very clear in the case of Putnam's pegs and board ("Philosophy and our mental life" 1975). Suppose we have a wooden board with two holes drilled on it. The first hole is circular with diameter 1cm and the second is squared with each side being 1cm long. Direct experience tells us that a cubical peg that perfectly fits the squared hole will not fit the circular one. We can offer two types of explanations to this fact. On the one hand, we can adopt a bottom-up approach and attempt to derive an explanation starting from the microscopic structure of the system. On the other hand, we can rely on the geometrical and topological structure of both pegs and holes, since the area of a circle with diameter d is smaller than the area of a square whose sides are of the same length as the diameter of the circle. The geometrical explanation is indifferent to the microscopic structure of the board (or of the cubical peg).

The crucial point is that independently of the philosophical attitude we assume towards geometric properties and microscopic structure, the (partial) irrelevance of the latter to the former seems to remain a brute fact when it comes to explaining why the square peg does not find the round hole. This fact can tell us something about the tension that I mentioned in the first section between realism of non-spatiotemporal entities and instrumentalism towards the atoms of space.

The discussion about the Ising model and approximation methods leaves us with the conclusion that we can separate the problem of accounting for spacetime emergence in QG from the discussion over the ontology of the corresponding fundamental entities. In the next section I will review a theory of quantum gravity that interprets the emergence of spacetime in terms of phase transitions, thereby rendering the problem of the ontology of the atoms of space, at least partly, irrelevant.

5 GFT and Phase Transition

Group field theories (GFT) are theories that attempt to derive spacetime from combinatorial structures endowed with additional geometric and pre-geometric data, which are represented by parallel transports of gravitational connections along the edges of a graph.[13] In brief, these theories assume some fundamental building blocks (so-called atoms of space) that get combined to form all sorts of geometry and topology. At the perturbative level, spacetime is "the discrete (virtual) history of creation/annihilation of these fundamental atoms; it has no real existence, at least no more real existence in itself than each of the infinite possible interaction processes corresponding to individual Feynman diagrams in any field theory" (Oriti 2009, p. 328).

Originally, a theory of quantum fields of geometry was developed in the context of global quantum cosmology (see: (Giddings and Strominger 1989), (Banks 1988), and (Oriti 2009) for a review). The theory would construct a sum over possible topologies where each topology would correspond to a Feynman graph and corresponding quantum amplitude. However, the approach presented interpretative and mathematical problems which could be partially eased by adopting a local framework that generalizes dynamically to the whole universe (Oriti 2006). Thus, a partial solution is to use a simplicial

[13]See for example: (Oriti 2006), and (Oriti 2011).

description of spacetime obtained by gluing together many fundamental discrete building blocks. The complex simplicial structures would be expressed by the tensor product of the wave functions associated to each individual block, where the geometry of each building block of space is described in terms of group and representation variables (see: (Oriti 2016) and (Oriti 2012)).

The theory defines a complex scalar field $\varphi : G^{\times d} \rightarrow \mathbb{C}$ on a group manifold G which is usually taken to be either the Lorentz group $SO(3)$ or the rotation group $SU(2)$. The many wave functions are then promoted to operators and the field theory is thus "specified by a choice of action and by the definition of the quantum partition function expressed perturbatively in terms of Feynman Diagrams" (Oriti 2009, p. 311). The action is chosen so that the perturbative expansion of the partition function equals the discretized path integrals for quantum gravity of the form:

$$Z = \int \mathcal{D}\varphi \mathcal{D}\varphi^* e^{-S(\varphi,\varphi^*)} \tag{9}$$

Then, from the path integrals form, one can couple a scalar field to provide the dynamics for the structure of the tetrahedra: "in particular, we are interested in adding degrees of freedom that can be interpreted as discretized scalar matter, just like the group-theoretic variables can be interpreted as discrete geometric data" (Oriti 2021b, p. 10). The initial field defined on the $SU(2)$ group will assume the form: $\varphi(g_I, \phi^J) : SU(2)^4 \times \mathbb{R} \rightarrow \mathbb{C}$. The newly added free, masless, real-valued field ϕ will act as a relational clock, i.e., as an internal time variable with respect to which the other variables evolve.[14]

At this stage, the individual tetrahedra are analogous to the individual atomic spins of the Ising model I described above. They do not carry any spatiotemporal information in the sense of General Relativity, similarly to how the orientation of the individual spins does not inform us about the overall magnetization. Then, how does spacetime emerge in such a context?

In general, in quantum field theory, the evaluation of the full partition function incorporates all dynamical degrees of freedom and thus the continuum limit of the theory as well. In the GFT context, this amounts to: "[...] resumming the full perturbation series, thus the sum over triangulations weighted by a discrete gravity path integral [...] including infinitely refined

[14] The strategy of adding a field to play the role of relational clock is not new, see: (Dittrich 2006) and (Brown and Kuchař 1995).

lattices. In physical terms, this means being able to control the full collective quantum dynamics of the QG atoms, looking for regimes in which the discrete picture can (and should) be replaced by one in terms of continuum spatiotemporal fields" (Oriti 2021b, p. 11).

That is, from the perspective of spacetime emergence, one needs to move from the atoms of space to the continuum phase, but this requires the control of the dynamics of the theory at all scales and regimes. Yet, the mathematical control of the theory does not discern between physical and mathematical phases. Therefore, one needs to identify which continuum phases allowed by the theory can be rewritten in terms of spatiotemporal fields and dynamics of general relativity. One can employ approximation methods (such as renormalization group techniques and mean field theory) to obtain a picture of the continuum phases from which to extract some physical insights:

> If the emergence of space and time takes place due to the collective dynamics of the QG atoms, we need approximation schemes that capture such collective dynamics, that correspond to some form of coarse-graining of the fundamental 'atomic' dynamics, and that maintain visible the quantum nature of the same atoms (since the continuum limit is distinct from the classical one, and it could well be that quantum properties of the QG atoms are in fact responsible for key aspects of the spatiotemporal physics we want to reproduce) (Oriti 2021b, p. 12)

Oriti (2021b) uses mean field theory to approximate the full theory with quantum states expressed in terms of excitations of the Fock vacuum $|\sigma\rangle = \exp(\hat{\sigma})\,|0\rangle$ that are simplified with respect to the initial tetrahedra, for they now do not encode correlation information or quantum entanglement. The use of such a simplification shifts the theory to a new level of description: "we are then moving from the QG atoms to the full continuum description of quantum gravity, but within a specific regime of approximation, which remains quantum and focused on the collective properties of the same QG atoms, rather than their individual, pre-geometric features" (Oriti 2021b, p. 12). This way, the approximation methods allow us to obtain simplified states (in a quantum superposition) associated with a wave function σ of infinitely many degrees of freedom; the new wave function describes the collective behavior of infinitely many atoms of space.

The point is then to obtain from such collective behavior an (effective) dynamics that can be understood as quantum general relativity. One possibility is to individuate a phase of the quantum gravity system that resembles a condensate phase, and to treat the dynamics of the fundamental atoms of space in terms of hydrodynamics regime. Granted the feasibility of such a strategy, what kind of spacetime physics can we expect? The most supported answer (see, for example: (Oriti and Sindoni 2011)) is cosmological dynamics; that's because the focus on macroscopic variables and maximal coarse-graining limit us to a dynamics close to equilibrium. In addition, it is only collectively that one can talk about geometries, since the atoms of space are, strictly speaking, non-geometrical. Consequently, a notion of local geometric behavior is not available. In addition, the condensate wave function and the mean field can only be treated statistically due to the coarse-graining, even though the fundamental degrees of freedom are treated quantum mechanically.

5.1 GFT Condensate

The idea of using some forms of coarse-graining to model relativistic effects in quantum theories is not new. For example, (Volovik 2006) studied the similarity of relations between classical and quantum hydrodynamics, and quantum hydrodynamics and quantum gravity. Originally, (Landau 1941) discussed the derivation of quantum hydrodynamics from its classical counterpart by expressing the quantum Hamiltonian as the energy of a liquid where the classical velocity $\mathbf{v}$ and density ρ are replaced by the corresponding quantum operators $\hat{\mathbf{v}}$ and $\hat{\rho}$. The classical Hamiltonian reads (Volovik 2006, p. 2):

$$H_{hydro}(\rho, \mathbf{v}) = \int d^3 \left(\frac{1}{2}\rho v^2 + \tilde{\epsilon}(\rho) \right) \tag{10}$$

where $\tilde{\epsilon}(\rho) = \epsilon\rho - \mu\rho$ and $\epsilon(\rho)$ is the energy of the static liquid which will be related to the vacuum energy state (assumed that the temperature is $T = 0$) and μ is the constant chemical potential. The relation $P = -\tilde{\epsilon}$ between pressure and energy can be taken to be as the equation of state for the vacuum of any system, and it does not depend on underlying physics of the vacuum state. Similarly, one can obtain the hydrodynamics equations using Poisson brackets, which depend on symmetry conditions of the system and not on its microscopic physics. As argued in (Volovik 2006), Poisson brackets can be obtained from the commutation relations of the quantum operators $\hat{v}$ and ρ, which were originally derived by Landau from the microscopic physics.

By looking at cases in which quantum hydrodynamics and quantum gravity can be used to obtain fine corrections to classic hydrodynamics and general relativity, (Volovik 2006) concludes that such cases are not generalizable. The route from classical to quantum hydrodynamics does not lead to a theory that is completely faithful to the microscopic theory, and this can be considered as an instance of emergence in physics:

> One can quantize sound waves in hydrodynamics to obtain quanta of sound waves —phonons. Similarly, one can quantize gravitational waves in general relativity to obtain gravitons. But one should not use the low-energy quantization for calculation of the radiative corrections which contain Feynman diagrams with integration over high momenta. In particular, the effective field theory is not appropriate for calculations of the vacuum energy in terms of the zero-point energy of quantum fields (Volovik 2006, p. 18).

In a similar fashion, some approaches to GFT aim at using the condensate analogy to model relativistic behavior of quantum systems. Indeed, a great advantage of the GFT formalism is that one can use quantum field theory (QFT) methods for treating many degrees of freedom. Instead of N-particle states and perturbations around the Fock vacuum state (like, for example, in loop quantum gravity (Rovelli and Vidotto 2014), (Ashtekar 2013)), we can use QFT methods to study the collective behavior of many interacting atoms or molecules: "condensation of many atoms into a common ground state can be viewed as a transition from a perturbative phase around the Fock vacuum (of zero atoms) into a condensed phase, with associated symmetry breaking of the U(1) symmetry of the theory" (Gielen, Sindoni, et al. 2016, p. 2). The aim is to approximate 3-d geometries and cosmological evolution in terms of some specific condensate states in the formalism of GFT. These states should come from the macroscopic quantum dynamics, in a way inspired by phase transitions.

The construction of the condensate state is analogous to the construction of the effective field we have seen for the Ising model. That is, one can coarse grain the many degrees of freedom of the theory represented by N-excitations of the Fock vacuum and define a new state (which now plays the role of order parameter) as a superposition of one-particle wave functions (Gielen, Sindoni, et al. 2016, p. 19):

$$|\sigma\rangle := \mathcal{N}(\sigma) \exp\left(\int dg\sigma(g_I)\varphi^{\dagger}(g_I)\right)|0\rangle \tag{11}$$

where $\mathcal{N}$ is a normalization factor, $\int dg$ is the integral over the local gauge group. The state $|\sigma\rangle$ corresponds to a single particle condensate state which is an eigenstate of the field operator $\hat{\varphi}(g_I)|\sigma\rangle = \sigma(g_I)|\sigma\rangle$ with non vanishing expectation values $\langle\sigma|\hat{\phi}(g_I)|\sigma\rangle \neq 0$, unlike for the Fock vacuum where $\langle 0|\hat{\phi}(g_I)|0\rangle = 0$. The condensate wave function, together with the massless scalar field ϕ^J can be interpreted as a continuum spacetime geometry (Gielen, Sindoni, et al. 2016) in a way analogous to how magnetization was defined over the effective field in the Ising model. This is because the condensate wave function ignores the fluctuations between individual quanta due to the mean field theory approximation which assumes that: "the system exhibits a separation of scales which allows to average over the microscopic details. [...] This leads to a model which only involves scales which extend from the mesoscale to the macroscale. The field variable is an averaged quantity (the order parameter) which only reflects general features of the system such as symmetries and the dimensionality of the domain" (Marchetti et al. 2022b, p. 5). Then, the thermodynamic limit corresponds to having $N \to \infty$ and it is described by states that are no longer in the GFT Fock space: "this is standard in quantum field theory, where in the limit corresponding to a phase transition one needs to change representation to a different, unitarily inequivalent, Hilbert space" (Gielen, Sindoni, et al. 2016, p. 19).[15]

The condensate approach to GFT rests on the fact that low-energy scale physics can be (at least partly) independent from its high-energy counterpart, and that one can study quantum gravity models in terms of collective behavior of fundamental entities. However, how should we obtain geometric quantities from a condensate function obtained as the thermodynamic limit of N-many non-geometric atoms of space? To answer the question, even though I will skimp on the details, we can consider (Oriti 2021b), (Gielen, Oriti, and Sindoni 2014) and use the simplest case of homogeneous and isotropic cosmology. To do so, explains (Oriti 2021b, p. 16), "we define the relational observables that we expect to be relevant for describing homogeneous cosmological evolution [...] the universe volume [the operator $\hat{V}$] (constructed from the matrix elements of the 1st quantized tetrahedra, with eigenvalues

[15]Notably, $\sigma(g_I)$ is not an ordinary wave function: it is a superposition of states $\psi(g_I)$ but it is not linear $|\sigma\rangle + |\sigma\rangle' \neq |\sigma + \sigma'\rangle$.

V_j, convoluted with field operators) [...] the operator adds the individual volume contributions from the GFT quanta populating the state:"

$$V(\chi_0) \equiv \left\langle \hat{V} \right\rangle_{\sigma;\chi_0,\pi_0} = \sum_j V_j \rho_j^2(\chi_0) \tag{12}$$

where $\rho_j(\chi)$ is the density of the fluid which is obtained from the decomposition of the condensate wave function in hydrodynamics variables. From the volume observable (and others that are calculated in (Oriti 2021b)), and from the description of the evolution of the condensate with respect to the relational clock χ_0, (Oriti 2021b, p. 17) obtains the generalized Friedmann equations (in χ_0): "that our quantum gravity model gives for the emergent spacetime in the homogeneous case".

Philosophically, the point I raised for the Ising model applies quite naturally to the case of group field theory and condensate models. The geometric properties of the condensate are indifferent to the individual tetrahedra, and that is warranted by the use of approximation methods such as the mean field theory.[16] Group field theory is silent with respect to the ontology of its fundamental entities and it would require some philosophical (heavy) work to justify a bottom-up approach to spacetime emergence, especially in view of the fact that the atoms of space do not live in spacetime. However, GFT defines a mechanism of spacetime emergence based on the analogy with phase transitions and via approximation techniques.

I have looked into the case of the Ising model to show that hydrodynamic description and mesoscale quantities are indifferent to the microscopic dynamics of the fundamental entities. This allows us to discuss higher-order properties with a moderate indifference with respect to their more fundamental counterparts. I added the term 'moderate' here because, although magnetization, spacetime, and geometric properties can be considered independently of their constituent parts, the question about the ontology of fundamental entities remains unattended. However, such an indifference allows us to separate between the previous question and that of spacetime emergence. In this sense, 'how does spacetime emerge' and 'what does spacetime emerge from' become separate problems that require separate analysis. For example, the former calls for further investigations about inter-theoretic reduction and the possibility of interpreting phase transitions as actual physical

[16] Even further, the vacuum state of the condensate lives on a unitarily inequivalent Hilbert space from the Hilbert space of the tetrahedra (which is a common feature of phase transitions on quantum field theory).

processes. The latter, looks at the very possibility of having experimental verifications of non-spatiotemporal entities, or at the possibility of relinquishing the property of 'being-in-spacetime' as necessary requirement for existence.

6 Conclusion

In this contribution I clarified some philosophical aspects about the problem of spacetime emergence in quantum gravity. More specifically, after discussing some of the literature on reduction, emergence, and corresponding philosophical problems, I divided the problem of spacetime emergence into two sub-problems: (i) to account for the ontology of non-spatiotemporal fundamental entities, and (ii) to provide a mechanism for the emergence of spacetime from such entities. Afterward, I argued that such a division is especially helpful in the context of the group field theory approach to quantum gravity and in the analogy with the physics of phase transitions. I presented the Ising model to show how the total magnetization of the system is indifferent to the microscopic dynamics of the individual atomic spins. This means that the macroscopic thermodynamic properties of the system can be studied independently of the corresponding microscopic dynamics. Similarly, we can study the thermodynamical properties such as total magnetization while ignoring the ontological properties of the atomic spins. This conclusion is less interesting for the case of the Ising model, in that we already know the properties of the individual atomic spins. However, the same conclusion becomes relevant to the discussion on quantum gravity. Because group field theory treats the collective behavior of the individual atoms of space as a type of phase transition (geometrogenesis), we can warrant the same type of partial indifference that we argued for in the Ising model. It follows that the problem of how to account for the ontology of fundamental entities that do not live in spacetime is independent from the problem of spacetime emergence, at least in the context of group field theory.

Notably, what this paper leads to is a form of 'black-boxing' of the debate on the ontological properties of the fundamental entities. This warrants a form of independence between the issue of researching a mechanism of spacetime emergence and the question about the fundamental degrees of freedom of the theory. Nonetheless, such an independence does not diminish the need for a serious philosophical account of the fundamental ontology of quantum gravity, nor it dispenses philosophers from having to investigate the problem.

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