Field equations, quantum mechanics and geotropism.
A study in theoretical biochemistry.

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Abstract
The biochemistry of geotropism in plants and gravisensing in e.g. cyanobacteria or paramacia is still not well understood today [1]. Perhaps there are more ways than one for organisms to sense gravity. The two best known relatively old explanations for gravity sensing are sensing through the redistribution of cellular starch statoliths and sensing through redistribution of auxin. The starch containing statoliths in a gravity field produce pressure on the endoplasmic reticulum of the cell. This enables the cell to sense direction. Alternatively, there is the redistribution of auxin under the action of gravity. This is known as the Cholodny-Went hypothesis [2], [3]. The latter redistribution coincides with a redistribution of electrical charge in the cell. With the present study the aim is to add a mathematical unified field explanation to gravisensing.

Keywords: Einstein gravity field, Maxwell electromagnetic field, relativistic quantum Dirac equation, gravitropism.

1 Introduction
The biochemical and biophysical aspects of geotropism remain largely unknown. The mechanisms that result in geotropism are poorly understood and the signalling pathways remain elusive [4]. Despite of a considerable historical body of experimental knowledge there is still no clear idea what needs to be researched experimentally [5]. However, we do know that the molecular mechanism underlying gravity perception and signal transduction which controls assymetric plant growth in response to gravity are likely to be linked to the plant hormones cytokinins, auxins and the gibberellins [6]. It is generally acknowledged that auxin and/or gibberellins are involved in ‘tropic’ behavior of plants. The inclusion of gibberellins in gravitropism is stated by e.g. Vandebussche et al. They note that a gibberellin signalling system may have its
evolutionary molecular onset in *Physcomitrella patens*, where gibberellins at relative high concentrations affect gravitropism [7]. Additionally, amyloplasts are widely considered to act as statoliths and are thought to be present e.g. in the rootcap cell [8].

In addition a bio-geoelectric effect in plants has been established following gravity stimulation [9]. However the question is whether this bio-geoelectric effect explains gravisensing or is just a side effect of it. In 1987, Björkman and Leopold demonstrated an electrical current associated to gravity sensing [10]. A more recent study showed changes in cytosolic pH [11], denoted by pH$_c$, after gravistimulation for cells that are probably involved in graviperception [12]. It is noted that during the initial stages of their life cycle, rootcap cells can perceive gravity and cause orthogravitropic growth [13]. Other studies suggest other places in addition to the rootcap for gravisensing [14]. Movement of auxin in response to gravistimulation was already demonstrated [15]. Changes in pH$_c$ also occur in response to auxin stimulation [16]. In addition, proton, H$_3$O$^+$ efflux seems to mediate the action of auxin [17] while root growth strongly depends on pH. This evidence suggests a causal role of efflux of H$_3$O$^+$ in gravisensing [18].

## 2 Unified field theory

### 2.1 Bio-geoelectric response

The problem with gravisensing and a cellular electric response is that there is no well established theoretical physics connection between gravity and a possible cellular response. Of course the electric field can be explained by mechano-reception through statoliths but there seem to be organisms that do not contain starch statoliths for gravisensing organelles. The latter include, *Phycomyces*, *Neurospora* and starch-free *Arabidopsis* mutants [19]. Perhaps that other cell structures play the role of statolith. Perhaps other mechanisms without statolith-like reception are involved. In the latter category one can find e.g. neurobiological explanations of graviperception (see ref. [1]), or of a link between actine cytoskeleton and gravisensing [20].

Older studies indicate that gibberellins are able to promote growth in wheat coleoptiles by stimulating the breakdown of starch into sugar. The starch freed plants responded relatively normal to gravistimulation [21]. Moreover, a maize mutant lacking amyloplasts expressed geotropism [22], see also [23]. This draws the attention back to the Cholodny - Went hypothesis with an electrical field guided by or guiding the auxin redistribution in gravisensing. The author adds that an explanation of auxin redistribution and one with statoliths are not by
necessity mutual exclusive.

In any case the conclusion seems justified that a deeper understanding of geotropism would benefit from an understanding of a mathematical description of a possible relation between gravity and electromagnetism.

2.2 Gravisensing and cellular phytochrome

Adding to the already mentioned explanations there also exists a relation between phytochrome response in plants to red and far-red light and the geotropic response. For instance the effect of irradiation with red-light on the capacity of *Avena* coleoptiles to respond to geotropic stimulation has three phases [24] (also [26]). The first is a period of increase in geotropic responsiveness. The second phase shows no difference between irradiated and not radiated coleoptiles and the third is characterized by less responsiveness to gravistimulation in the irradiated coleoptiles. Moreover, Wilkins et al also found that geotropic responsiveness of the *Zea* coleoptile was different than from *Avena*. In *Zea* a decreased responsiveness developed after 8 hours [25]. Another indication for red-light or phytochrome influencing gravitropism is the fact that hypocotyls of dark grown *Arabidopsis* seedlings exhibit strong gravitropism whereas in red-light gravitropism is strongly reduced [27]. Light has been shown to modulate the gravitropic response of root or stem through the action of phytochrome [28].

Let us for this moment stop at the lumino biological side of gravity sensing and turn to the mathematical physics of gravity sensing.

2.3 Gravity electromagnetic unification

In the course of his studies in the foundation of physics, Einstein stated the importance of unifying the gravity field with the electromagnetic field [29]. In the present paragraphs this form of unification is accomplished in case of a weak gravity field. The unification runs over a mathematical form which is Dirac’s relativistic quantum mechanical (DRQM) equation. By demonstrating that the DRQM is intrinsic in weak gravity Einstein field equations, the relation with the electromagnetic field is laid with the use of the author’s previous results [30]. Hence, establishing this fact implies the unification of the gravity and the electromagnetic fields. For our present bio-sensing geotropism application it is important that gravity can be treated as a form of electromagnetic radiation.
3 Equations

In the previous sections the context for the search was given. The next paragraphs are devoted to the mathematics.

3.1 Preliminaries

Before embarking, let us first define the employed Minkowski metric tensor, \( \eta_{\mu,\nu} = \eta^{\mu,\nu} = \text{diag}(-1, 1, 1, 1) \), or, \( \eta_{1,1} = \eta_{2,2} = \eta_{3,3} = -\eta_{0,0} = 1 \), and, \( \eta_{\mu,\nu} = 0 \), when, \( \mu \neq \nu \), with, \( \mu, \nu = 0, 1, 2, 3 \). Secondly, raising or lowering an index is performed with contraction using the Minkowski metric. For example, suppose, \( a_\mu \) is a tensor, then 'raising the index' is done with, \( a^\lambda_\mu = \eta^{\lambda,\mu} a_\mu \). Thirdly the to be investigated derivation is based on 'weak distortion' of the Minkowski metric. This means that

\[
g_{\mu,\nu}(x) = \eta_{\mu,\nu} + \sqrt{\epsilon} \varphi_{\mu,\nu}(x)
\]

with \( \varphi_{\mu,\nu}(x) \sim O(\sqrt{\epsilon}) \), terms of \( O(\epsilon^2) \) will be suppressed and \( x = (x_0, x_1, x_2, x_3) \) spacetime coordinates. Note that \( O(\epsilon) \) is Landau’s ordering symbol.

3.2 Metric assumptions

The to be developed analysis is fairly general. However, below a specific example is given to show that the conditions can be met. Let us furthermore for notational convenience leave out the separating comma if there can be no mistake in reading the indices.

In the derivation we will need the following facts. Firstly, \( g^{\mu\nu} g_{\mu\nu} = \eta^{\mu\nu} \eta_{\mu\nu} = 4 \). From equation (1) it then follows that \( \eta^{\mu\nu} \varphi_{\mu\nu}(x) = 0 \) and from the epsilonics it follows that \( \epsilon \varphi_{\mu\nu}(x) \varphi_{\mu\nu}(x) \sim O(\epsilon^2) \) and can be suppressed. Secondly, we aim to have \( g = -\det(g_{\mu\nu}) = 1 \). Let us suppose, for example, that \( (\varphi_{\mu\nu}) = \text{diag}(\varphi_0, \varphi_1, \varphi_2, \varphi_3) \), with, to be even more specific, \( \varphi_0(x) = \varphi_1(x) = f(x) \) and \( \varphi_2 = -h(x), \varphi_3 = h(x) \) and \( f \) and \( h \) two real functions. Then, the determinant equals

\[
-g = \det(g_{\mu\nu}) = (-1 + \sqrt{\epsilon} \varphi_0)(1 + \sqrt{\epsilon} \varphi_1)(1 + \sqrt{\epsilon} \varphi_2)(1 + \sqrt{\epsilon} \varphi_3)
\]

(2)

We have, \( \epsilon \varphi_\mu \varphi_\nu = O(\epsilon^2) \). Because \( \varphi_2 + \varphi_3 = 0 \) and \( \varphi_0 = \varphi_1 = f(x) \) we have \( g^{\mu\nu} g_{\mu\nu} = 4 \). The diagonal structure of \( \varphi_{\mu\nu} \) permits to have, \( \eta^{\lambda,\mu} \varphi_{\mu,\nu} = 0 \). Moreover, suppressing \( O(\epsilon^2) \)

\[
-g = (-1 + \sqrt{\epsilon}(\varphi_0 - \varphi_1))(1 + \sqrt{\epsilon}(\varphi_2 + \varphi_3))
\]

(3)

it follows, \( g = 1 \). The reason for \( g = 1 \) will become clear later on. This example shows that there is a genuine metric \( g_{\mu\nu}(x) \) that differs from the Minkowski
metric and has the necessary characteristics. An additional assumption will be that \( \partial \lambda \phi^\lambda = a_\nu \), with \( a_\nu \) absolute constant. Now it should be noted that for, \((\phi_\mu \nu) = \text{diag}(\phi_0, \phi_1, \phi_2, \phi_3)\), we have \( \partial \lambda \eta^{\lambda \sigma} \phi_{\sigma \nu} = a_\nu \) and hence, \(-\partial_0 \phi_0 = a_0 \) and \( \partial_k \phi_k = a_k \), with \( k = 1, 2, 3 \). This further restricts the example metric \( g_{\mu \nu} \) to
\[
f(x) = a_1 x_1 - a_0 x_0 + f_{23}(x_2, x_3)
\]
and, \( f_{23}(x_2, x_3) \) containing the remainder in of space-time dependence of \( f(x) \). Further,
\[
h(x) = a_3 x_3 - a_2 x_2 + h_{01}(x_0, x_1)
\]
and similarly \( h_{01}(x_0, x_1) \) the remainder in of space-time dependence of \( h(x) \). Note that e.g. \( \phi_2(x) = -h(x) \) and that in the previous further specification of \( h(x) \) it is ensured that: \( \partial_2 \phi_2 = a_2 \).

### 3.3 Field equations

As is well known, Einstein’s field equations, relating the Ricci tensor, \( R_{\mu \nu}(x) \), the stress-energy \( T_{\mu \nu}(x) \) and the metric tensor, \( g_{\mu \nu}(x) \), can be written as
\[
R_{\mu \nu} = 8\pi G \left( T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T \right)
\]
Note, \( T = T(x) = g^{\mu \nu}(x) T_{\mu \nu}(x) \) and we use the notation \( T \) for \( \eta^{\mu \nu}(x) T_{\mu \nu}(x) \).
In equation (6) \( G \) is the gravitation constant, while it is assumed that \( c = \hbar = 1 \). The field equations are rewritten slightly for the convenience of the analysis. If \( \kappa \sqrt{\epsilon} = 8\pi G \), then,
\[
R_{\mu \nu} = \kappa \sqrt{\epsilon} \left( T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T \right)
\]
The Ricci tensor in equation (7) can be decomposed into \( R_{\mu \nu} = r_{\mu \nu} + s_{\mu \nu} \). In terms of the affine connections the components of the Ricci tensor can be rewritten. For completeness:
\[
\Gamma^\lambda_{\mu \nu} = \frac{1}{2} g^{\lambda \sigma} ( \partial_\nu g_{\sigma \mu} + \partial_\mu g_{\sigma \nu} - \partial_\sigma g_{\mu \nu} )
\]
The constituents of the Ricci tensor can subsequently be written in terms of the affine connections. For \( r_{\mu \nu} \)
\[
r_{\mu \nu} = -\partial_\lambda \Gamma^\lambda_{\mu \nu} + \Gamma^{\sigma}_{\mu \alpha} \Gamma^\alpha_{\nu \sigma}
\]
and \( s_{\mu \nu} \)
\[
s_{\mu \nu} = \partial_\nu u_\mu - \Gamma^{\sigma}_{\mu \mu} u_\sigma
\]
Here \( u_\mu \) is defined as a contraction on the affine connection related to \( g = -\det(g_{\mu \nu}) \).
\[
u_\mu = \Gamma^\lambda_{\mu \lambda} = \partial_\mu ln \left( \sqrt{g} \right)
\]
We suppose that \( g = 1 \) and in a previous section (section 3.2) we saw that this is a genuine possibility among our other assumptions. In case \( g = 1 \), we have \( u_\mu = 0 \) and hence, \( s_{\mu\nu} = 0 \). The field equations can then be rewritten as

\[
    r_{\mu\nu} = \kappa \sqrt{\epsilon} \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) \tag{12}
\]

### 3.4 Relation with QM

The basic equations for a derivation of Dirac’s relativistic quantum equation will be given below. Because \( R_{\mu\nu} \) is, using \( g = 1 \), replaced by \( r_{\mu\nu} \) in the field equations, according to equation (9) we need to inspect forms like \( \partial_\lambda \Gamma^\lambda_{\mu\nu} \).

Remembering the general form of the metric tensor in equation (1) we obtain the following

\[
    \partial_\lambda \Gamma^\lambda_{\mu\nu} = \frac{\sqrt{\epsilon}}{2} \gamma^\lambda_{\mu\nu} \left( \partial^2_{\lambda\mu} \varphi_{\sigma\nu} + \partial^2_{\lambda\nu} \varphi_{\sigma\mu} - \partial^2_{\lambda\sigma} \varphi_{\mu\nu} \right) \tag{13}
\]

Note that the term \( \epsilon \partial_\lambda Q^\lambda_{\mu\nu} \) contained in \( \partial_\lambda \Gamma^\lambda_{\mu\nu} \) is \( O(\epsilon^2) \) because

\[
    2Q^\lambda_{\mu\nu} = \varphi^{\lambda\sigma} \left( \partial_\mu \varphi_{\sigma\nu} + \partial_\nu \varphi_{\sigma\mu} - \partial_\sigma \varphi_{\mu\nu} \right) \tag{14}
\]

and contractions like \( \varphi^{\lambda\sigma} \partial_\mu \varphi_{\sigma\nu} \) are \( O(\epsilon) \) because \( \partial_\mu \varphi_{\sigma\nu} \sim O(\sqrt{\epsilon}) \). Because of the additional assumption \( \partial_\lambda \varphi^\lambda_{\mu\nu} = a_\nu \) and \( a_\nu \) absolute constant suppressing \( O(\epsilon^2) \)

\[
    \partial_\lambda \Gamma^\lambda_{\mu\nu} = -\frac{\sqrt{\epsilon}}{2} \left( \nabla^2 - \partial^2_0 \right) \varphi_{\mu\nu} \tag{15}
\]

From equation (8) and the definition of the metric in equation (1) we see that \( \Gamma^\lambda_{\mu\nu} \) is \( O(\epsilon) \). Hence, the product term of affine connections in the expression for \( r_{\mu\nu} \) in equation (9) is of order \( O(\epsilon^2) \) and can be suppressed. This leads to

\[
    r_{\mu\nu} = \frac{\sqrt{\epsilon}}{2} \left( \nabla^2 - \partial^2_0 \right) \varphi_{\mu\nu} \tag{16}
\]

From the field equations it then follows that

\[
    \Box^2 \varphi_{\mu\nu} = 2\kappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) = k_{\mu\nu} \tag{17}
\]

with \( \Box^2 = (\nabla^2 - \partial^2_0) \) the D’Alembertian. From the previous equation, with \( b^\nu \), possibly complex absolute constants, let us derive a vector \( \phi_\mu = b^\nu \varphi_{\mu\nu} \) and \( k_\mu = b^\nu k_{\mu\nu} \). With those two vectors \( \phi_\mu \) and \( k_\mu \) in the relation \( \Box^2 \phi_\mu = k_\mu \), the present author derived a DRQM [30],

\[
    \gamma^\mu \left( \partial_\mu - G_\mu(x) \right) \Psi(x) = \gamma^\mu D_\mu \Psi(x) = 0. \tag{18}
\]
$G_\mu$ are gauge functions and $\partial_\mu = \frac{\partial}{\partial x_\mu}$. The 4x4 matrices $\gamma^\mu$ obey a Clifford algebra with $(k = 1, 2, 3)$

$$\gamma^k = \begin{pmatrix} 0 & -i\sigma^k \\ i\sigma^k & 0 \end{pmatrix}$$

(19)

and

$$\sigma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$  

(20)

together with $\gamma^4 = \text{diag}(1, 1, -1, -1)$. $G_\mu(x)$ is related to U(1) gauge and $\Psi(x)$ is a complex four vector.

4 Meaning & discussion

4.1 Physics consequences

Deriving a relativistic quantum equation from the (weak) gravity field equations is from a physical point of view remarkable. In the first place because relativistic quantum theory can be derived from classical gravity. Secondly and because of a previous established relation between Maxwell’s classical electromagnetic field equations, we now have obtained a theoretical relation between classical gravity and electromagnetic fields. The obtained relation is not similar to e.g. gravity lensing known from astronomy, because of the weak gravity field we study.

Concludingly, with the present formalism a gravity field can be transformed into an electromagnetic field and vice versa. The connection with the quantum mechanical Dirac form shows this to be relevant for understanding of the graviton. With the transformation an Einsteinian unification in field theory has been accomplished [29].

5 Connection with a biochemical context

The metric gravity disturbance $\delta g_{\mu\nu}(x)$ can be written as $\sqrt{\epsilon} \phi_{\mu\nu}(x)$. If we also define $\vec{F} = \vec{E} + i\vec{B}$ then the Maxwell equations without net charge ($q = 0, \vec{j} = \vec{0}$) can be written as

$$\nabla \times \vec{F} = i \frac{\partial}{\partial t} \vec{F}.$$  

(21)

In the previous we have demonstrated that for $\phi_\mu = b^\nu \phi_{\mu\nu}$ and $k_\mu = b^\nu k_{\mu\nu}$, with $b^\nu$ a constant array, that

$$\Box^2 \phi_\mu = k_\mu.$$  

(22)
From reference [30] it follows that the $\vec{F}$ from equation (21) is related to $\phi_\mu$ via the equation
\[
\nabla \phi(x) = \vec{F}(x) - i \frac{\partial \vec{C}(x)}{\partial t} - i \vec{J}(x)
\] (23)
with, generally, the current density, $\vec{J}$ equal to $\nabla \times \vec{J}$ and $\phi = \phi_0 + \phi_1 + \phi_2 + \phi_3$. With $\vec{F} = \nabla \times \vec{C}$ noticing $\vec{J} = 0$, (21) follows from (23).

In [30] the relation $\nabla \cdot \vec{C}(x) = i \frac{\partial}{\partial t} \phi(x)$ enables a connection between the electromagnetic field and functions derivable from the metric gravity disturbance $\delta g_{\mu\nu}(x)$. Let us suppose $\vec{C}$ depends on spacetime $(x_0, \vec{x})$, with $x_0 = ct$, through the radius $r = \sqrt{x^2 + y^2 + z^2}$. If we assume that $\vec{C}(t, \vec{x})^T = (1, 1, 1)C(t, r)$, in 'natural units ($c = \hbar = 1$') with $T$ the transposed, it then follows that
\[
\nabla \cdot \vec{C}(x) = \left(\frac{x + y + z}{r}\right) \frac{\partial C(t, r)}{\partial r}.
\] (24)

The relation with the gravity disturbance functions $\phi_\lambda$ is
\[
C(t, r) = i \int_0^r dr' \frac{r' \dot{\phi}}{x' + y' + z'}
\] (25)
with $\dot{\phi}$ the time derivative of $\phi$. Hence, from (25) and $\vec{F} = \nabla \times \vec{C}$ it follows that
\[
\vec{F} = i\vec{x} \times (\hat{e}_1 + \hat{e}_2 + \hat{e}_3) \frac{\dot{\phi}}{x + y + z}
\] (26)
with $\hat{e}_k$ unit vector of spatial direction $k = 1, 2, 3$. The expression (26) leads to the electric vector
\[
\vec{E} = -\vec{x} \times (\hat{e}_1 + \hat{e}_2 + \hat{e}_3) \frac{Im \dot{\phi}}{x + y + z}
\] (27)
and magnetic vector
\[
\vec{B} = \vec{x} \times (\hat{e}_1 + \hat{e}_2 + \hat{e}_3) \frac{Re \dot{\phi}}{x + y + z}.
\] (28)

Note that $|\vec{E}| = c|\vec{B}|$ in 'natural units ($c = \hbar = 1$') means $Im(\dot{\phi}) = \pm Re(\dot{\phi})$. Hence, we see that for the angle between $\vec{E}, \vec{B}$ the following
\[
\cos \angle \left( \vec{E}, \vec{B} \right) = \frac{\vec{E} \cdot \vec{B}}{|\vec{E}| \cdot |\vec{B}|} = -\text{sgn} \left( Im(\dot{\phi}) Re(\dot{\phi}) \right)
\] (29)
obtains.
From this we may conclude that a photon field can be obtained from the gravity disturbance whereby the electric and magnetic vectors are either opposite or parallel.

The relation between the electric and magnetic field vectors of the generated photon field to the metric tensor from gravity can be made more clear if we write $\delta g$ for the diagonal array in $\delta g_{\mu\nu}$. With this notation we may write

$$\phi = \frac{1}{\sqrt{\epsilon}} \text{Tr} \{ \text{diag}(b) \text{diag}(\delta g) \}$$

and $\delta g$ the gravity induced disturbance of the Minkovski metric. For completeness, $\text{diag}(\delta g) = \sqrt{\epsilon} \text{diag}(\varphi_0, \varphi_1, \varphi_2, \varphi_3)$. Explicitly this leads to the photon electrical and magnetic field vectors

$$\vec{E}_{\delta g} = -\hat{x} \times (\hat{e}_1 + \hat{e}_2 + \hat{e}_3) \text{Im} \left( \frac{\text{Tr} \{ \text{diag}(b) \text{diag}(\delta g) \}}{(x + y + z)\sqrt{\epsilon}} \right)$$

and magnetic vector

$$\vec{B}_{\delta g} = \hat{x} \times (\hat{e}_1 + \hat{e}_2 + \hat{e}_3) \text{Re} \left( \frac{\text{Tr} \{ \text{diag}(b) \text{diag}(\delta g) \}}{(x + y + z)\sqrt{\epsilon}} \right).$$

$\vec{E} = \vec{E}_{\delta g}$ and $\vec{B} = \vec{B}_{\delta g}$ are the gravity to photon transformed electrical and magnetic field vectors.

### 5.1 Photonic gravity field and its hypothetical influence on phytochrome

The gravity induced photon field (equations (31) and (32)) has a vanishing Poynting vector. Hence, the effect of the constant presence of the gravity induced photic field is different from the influence of red or far-red light because in the latter case there will be a non-zero Poynting vector leading to exchange of energy between the photonic field and the phytochrome molecule. In case of the gravitophotic field the photon will only be supposed to have a disturbing influence of the distribution of electrons over the molecule. In the present theory the gauge invariance of the underlying Dirac equation is associated to the distribution of charge. Recall that, in experiments, charge re-distribution is demonstrated to be part of the geotropic response of the plant.

From the Dirac equation [30], associated to the Maxwell electromagnetic field equations related to (31) and (32) we have

$$\gamma^\mu D_\mu \Psi = \gamma^\mu \left( \frac{\partial}{\partial x_\mu} - G_\mu \right) \Psi = 0$$

(33)
with $\gamma^\mu$, $4 \times 4$ matrices satisfying the Clifford algebra $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu}$ whereas the $G_\mu$ are gauge function. Let us study the $U(1)$ gauge transformation $\Psi \rightarrow e^{iR}\Psi = \Psi'$. If the gauge functions transform like

$$G_\mu \rightarrow G_\mu + i\frac{\partial R}{\partial x_\mu}$$

the Dirac equation (33) remains valid for $\Psi'$. As was demonstrated by the present author [30], this leads to a re-distribution of net charge

$$q'' = \Re(q') + \nabla \cdot \left\{ \Im \left[ e^{iR} \left( J - i\vec{C}\frac{\partial R}{\partial t} \right) \right] \right\}$$

with $R$ the wave function transformation function, $\vec{J}$ the vector of functions leading to $\vec{j} = \nabla \times \vec{J} = \vec{0}$ and $\vec{C}$ such that, in our present example, $\vec{C}^T = (1, 1, 1)C(t, r)$ and here the function $C(t, r)$ as in (25). The term $q'$ in (35) derives from $q' = \nabla \cdot \vec{Q}'$. From [30] (formula (48)) it follows that $\vec{Q}'$ is equal to

$$\vec{Q}' = i e^{iR} \left( \phi \nabla R - (\nabla R) \times \vec{C} \right)$$

where $\vec{Q} = \vec{0}$ in accordance with the original 'no net charge' $q = 0$. Hence, $\vec{Q}'$ arises exclusively from the transformation of the wave function. Hence, gravity induced electromagnetic field such as in e.g. (31) and (32) has gauges that show a redistribution of net charge, i.e. a $q \rightarrow q''$ transformation following a $\Psi \rightarrow e^{iR}\Psi = \Psi'$ transformation plus restructuring into Maxwell’s e.m. field equations without 'magnetic charge'.

Of course a discussion can be started whether the previous transformation of a gravity metric disturbance into electromagnetism needs statolith, or similar, transducers or not. The author claims that the gravity disturbance of the Minkovsky metric is intrinsically already an electromagnetic field and not in need of any transduction of gravity to electromagnetism. In the latter case it can be that the direct gravity influence on molecules part of the Cholodny - Went hypothesis is true and that statoliths only amplify the already existent primordial intrinsic signal and co-prepare the cells for the geotropic response.

5.2 Resonance structures related to $\vec{E}_{\delta g}$ and $\vec{B}_{\delta g}$

In this section the attention is turned to the way the $\vec{E}_{\delta g}$ and $\vec{B}_{\delta g}$ can interfere with the structure of the phytochrome. We start with $P_{fr}$, in (1), and show that $\cos \angle \left( \vec{E}_{\delta g}, \vec{B}_{\delta g} \right) = \pm 1$ enhances $P_r$ in (8).
Fig. 1. Electron resonance in ring D (upper) and C (lower) of the planar tetrapyrrole chromophore.
5.2.1 Configuration phytochrome chromophore

The trans geometry of the chromophore in \( P_{fr} \) arises after red-light absorption of \( P_r \). We have \( P_r \xrightarrow{660 \, nm} P_{fr} \). The tetrapyrrole system is a relatively flat system of molecular \( \pi \)-orbitals. Hence, the drawings are a fair approximation of the planar geometry of the chromophore.

Subsequently we may note that in this molecular \( \pi \)-orbital system, the electrons in the \( \pi \)-orbital 'freely' may propagate over the plane of the molecule. This is mirrored by the consecutive double bonds in Fig. 1. Now, to follow the faith of the electrons in the configuration let us take a look at (2). For brevity, the upper (in the figure) ring is named the D pyrrole ring while the lower is denoted by C.

In (2) the two \( \pi \) electrons are 'loosely' related and (3) and (4) show the propagation of this electron in the D ring. In (4) the electron is positioned on O and hence there is the possibility that the H from the N moves to O, see (5). Subsequently, the double bond closest to the O can be 'opened' and a double bonded N can arise in the D ring such as in (6). This electron propagates through the D ring back to the C that makes the connection with the C ring such as pictured in (7).

5.2.2 Gravity photon influence

In the first place it is reasonable to suppose that the propagation of electrons in \( \pi \)-orbitals resembles the propagation through a wire. In that case it is a well known fact that electric and magnetic field vector are orthogonal. Noting the fairness of the approximative representation in (8), it is noted that e.g. the electric vector in the C ring associated to the double bonded N i.e. the

\[
\begin{align*}
&\text{C} \xrightarrow{N} \text{E} \xrightarrow{B} \text{H} \\
&\text{in the C ring of (8) is orthogonal to the electric field vector in the double bond of N in the D ring i.e.}
\end{align*}
\]

\[
\begin{align*}
&\text{O} \xrightarrow{H} \text{E} \xrightarrow{B} \text{N} \\
&\text{in the D ring of (8). The lines next to the bonds ending in either an E or a B represent the electro and magnetic field vectors along the } \pi \text{ bond.}
\end{align*}
\]

Hence, we may see an allignment of the \( \vec{E}_D \) and \( \vec{B}_C \) along a pair \( \vec{E}_{\delta g} \) and \( \vec{B}_{\delta g} \), showing \( \cos \angle (\vec{E}_D, \vec{B}_C) = \pm 1 \) similar to \( \cos \angle (\vec{E}_{\delta g}, \vec{B}_{\delta g}) = \pm 1 \). Hence,
the electric and magnetic form of the gravity photon can be considered to enhance the spontaneous $P_{fr} \rightarrow P_r$ decay. If the gravity stimulus finds a physiological response in the catalytic activity of $P_{fr}$ [9] then the gravity stimulus is also among the possible causes of 'deactivation' of the catalyst. The already mentioned hypocotyls of dark grown Arabidopsis seedlings that exhibit strong gravitropism which is reduced in in red-light, does not disprove the previous argument of gravireception on $P_{fr}$ but could also suggest that for Arabidopsis other processes related to active $P_{fr}$ prevail over geotropism after growing in the dark.

To the previous alignment hypothesis we could also add the following application of $\cos \angle (\vec{E}_{\delta g}, \vec{B}_{\delta g}) = \pm 1$ to a cytokinins Zeatin[6-(4-hydroxy-3-methyl-2-buteryl]aminopurine]. It is known that higher plants contain hormones called cytokinins that induce cell division in association with the auxins. The structure formula of Zeatin is

![Zeatin structure formula](image)

Fig. 2. E-B folded Zeatin alligned along $\vec{E}_{\delta g}$ and $\vec{B}_{\delta g}$.

In Fig. 2. the E-B folded form is presented. Of course, the side chain

![Zeatin side chain](image)

does not necessarily need to stand in the folded geometry of Fig. 2. to the purine base. However, the $\cos \angle (\vec{E}_{\delta g}, \vec{B}_{\delta g}) = \pm 1$ field enhances the purine $\text{C=\text{N}}$ bond orthogonal in a plane with the $\text{C=C}$ bond of the side chain.

It can be imagined that the permanent presence of the $\cos \angle (\vec{E}_{\delta g}, \vec{B}_{\delta g}) = \pm 1$ field, enhances the 'folding' of the Zeatin side chain with the effect that e.g. it has a higher probability to pass a barrier in a membrane to stimulate cell division or to fit a cavity in an enzyme as a cofactor for enzymatic activity. Concerning a possible cooperation between cytokinins and phytochrome, there are indications that e.g. the altering of the circadian-clock in plants which can
be influenced by cytokinins is also dependent on phytochrome (B) [31]. When a circadian-clock depends on cytokinins and phytochrome then the two are likely to be linked in their physiological activity which in turn might also find expression in geotropism.

6 Conclusion and discussion

In the paper an electromagnetic field was obtained from a weak metric disturbance, $\delta g$. The unifying principle that follows from the mathematics has been subsequently applied to a long standing problem in plant biochemistry, namely geotropism. In geotropism charge redistribution occurs. If, initially, no net charge and zero charge transport is assumed, then the gravity photic field with the use of the $U(1)$ gauge in the associated Dirac function, induces a non-zero net charge distribution. If, subsequently, a wave function transformation $\Psi \rightarrow e^{iR} \Psi = \Psi'$ can be associated to a cytochemical process, then we see that with a gravity induced electromagnetic field such as in e.g. (31) and (32) a redistribution of net charge, i.e. a $q \rightarrow q''$ transformation following a $\Psi \rightarrow e^{iR} \Psi = \Psi'$ transformation plus rewriting of Maxwell’s e.m. equations, can occur. Secondly, it was argued that the gravity photon form $\cos \left( \vec{E}_{\delta g}, \vec{B}_{\delta g} \right) = \pm 1$ enhances the decay of the active $P_{fr}$ to the inactive $P_r$.

The electrical charge re-distribution is in a number of experiments associated to the geotropical response. Note the mathematical transformation of a gravity disturbance, $\delta g$ into an electromagnetic field like in (31) and (32). From this reformulation of weak gravity into electromagnetic field vectors ‘inner’ $U(1)$ transformation shows a non-zero redistribution of electric charge, i.e. a gauge from $q = 0$ to $q'' \neq 0$. Hence, the fundamental gravity - to - electric field transformation in geotropism is in theory reduplicated through ‘inner’ gauge transformation. Note also that in $U(1)$ the $\cos \left( \vec{E}_{\delta g}, \vec{B}_{\delta g} \right) = \pm 1$ is ‘broken’.

Because initially the gravity field transformation resembles a photonic field and phytochrome is involved in signalling photonic changes like photoperiodism, the process of $U(1)$ transformation appears to be situated at or in phytochrome. Recall that it is experimentally demonstrated that physiologically red-light reception, meaning a phytochrome-light reaction, is involved in enhanced geotropic behavior. If a gravity disturbance of the Minkovski metric is disguised as a electromagnetic field such as in (31) and (32), the connection to phytochrome is likely. The fact that the gravity photon also is able to enhance ‘deactivation’ of the catalytic form of phytochrome, $P_{fr}$, nicely fits a physiological picture where exhaustion of resources is prevented by the stimulus itself.
As a consequence of the previous arguments, we may note that the cytoskeleton of the cell is most likely not a photo-receptive organelle. The transformations from gravity to electromagnetic field, when not an intrinsic physical property, together with the gauge transformations of the Dirac equation leading to a re-distribution of charge, are therefore most probably not situated there.

Hence, the theory proposed here at its least tries to pin point the molecular structure fit for aspects of gravisensing in phytochrome carrying plants. Of course the transformation is merely mathematical and theoretical. However, the formalism enables to draw a closer circle around the cellular mechanism of geotropism and gravisensing. The fact that the mathematics shows that gravity can be treated as a kind of electromagnetic radiation points to phytochrome because red-light radiation and physiological gravisensing find a common ground there. Note the findings of E. Liscum and R. P. Hangarter [27]. The application of \( \cos \left( \vec{E}_{\delta g}, \vec{B}_{\delta g} \right) = \pm 1 \) to Zeatin represents a new, modified, form of the Cholodny - Went hypothesis.

Concerning the gibberellins, many biological active gibberellins look like A1, i.e. contain only one double bond. Hence, it is unlikely that the proposed allignment of electro and magnetic field vector with \( \vec{E}_{\delta g} \) and \( \vec{B}_{\delta g} \) would play a role in the influence of gravity on intra-molecular configuration. Hence, from this perspective it is unlikely that a reformulation of the Cholodny - Went hypothesis such as for Zeatin is possible. If the modified Cholodny -Went hypothesis carries weight, then one may conclude that gibberellins only play a secondary role in geotropism. With the same admittedly face value reasoning from \( \cos \left( \vec{E}_{\delta g}, \vec{B}_{\delta g} \right) = \pm 1 \), a similar point can be made in case of e.g. IAA.

The discussion ends by noting that the provided mathematics also points at the ‘graviton problem’ when the unification of the fields is intrinsic and does not need a special biological substrate for the transformation of gravity into the electric and magnetic field vectors, \( \vec{E}_{\delta g} \) and \( \vec{B}_{\delta g} \). In any case, it makes sense to employ the presented mathematical insight into an elusive problem as the molecular mechanism of gravity sensing in organisms.
References


