Self, World, and Art: Metaphysical Topics in Kant and Hegel. Ed. Dina Emundts. Berlin: de Gruyter, 2013.

Gary Hatfield

Russell's Progress

Spatial Dimensions, the From-Which, and the At-Which

Kant's critical philosophy brought space (and time) to the forefront of philosophical attention. Regarding space as the form of outer sense, his critical doctrines purported to establish that synthetic a priori knowledge is possible, knowledge that describes and constrains any possible human experience of space. The implication was that the very same form of intuition governs all appearances of spatial structure, including all possible sensory experience and the pure intuition of a priori imagination.

During the nineteenth century, philosophers, sensory physiologists, and sensory psychologists responded to Kant's doctrine of space in various ways.[1] Some took his doctrine to be in fact a psychological proposal about the innateness of spatial perception and sought to support or refute it on those grounds. Others believed that the doctrine was intended to explain the possibility of geometrical knowledge. Some philosophers, including Bertrand Russell, held that Kant was seeking to establish a necessary relation between geometry and physical space, founded upon space as a form of intuition. Russell held that Kant's doctrine was refuted by the existence of non-Euclidean geometries, which made the question of the actual geometrical structure of physical space into an empirical matter.[2]

There is a variety of opinion on the relation between Kant's arguments that space is an a priori form of intuition and his theory of geometrical knowledge, which draws on that doctrine. Some interpreters contend that Kant's arguments for the apriority and the intuitional nature of space are separate from his theory of geometry,[3] while others find them to be more closely related.[4] In this literature, Strawson emphasized the "phenomenal interpretation" of Kant's theory of geometry: that Kant was addressing, in the first instance, the structure of space as experienced phenomenally and claiming that this space is of necessity described by Euclid's geometry. Citing Russell, he reports that the progress of pure mathe-

1 Gary Hatfield, *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz* (Cambridge: MIT Press, 1990).

2 Bertrand Russell, *The Principles of Mathematics*, 2nd ed. (New York: Norton, 1938), 372; the first edition was published in 1903.

3 Rolf-Peter Horstmann, "Space as Intuition and Geometry: Observations on Kant's Transcendental Aesthetic," in *Ratio* 18 (1976), 18–30.

4 P. F. Strawson, *The Bounds of Sense: An Essay on Kant's Critique of Pure Reason* (London: Methuen, 1966), 60, 66–7, 277.

matics and physics in the twentieth century rendered the phenomenal structure of space less important, because:

> on the one hand, rigour in mathematics turns on logic, not on looks – which is why Russell said that the development of mathematics showed Kant's *Anschauung* to be superfluous; and, on the other hand, the physical applications of geometry turn on physical tests and measurements of many kinds, and not on the mere contemplation of phenomenal appearances.[5]

Strawson accurately reports that Russell himself separated geometry as pure mathematics from the empirical determination of the geometry of physical space and that he believed that the rigorization of mathematics took it away from appeals to spatial experience. But Strawson underestimates the importance of phenomenal spatial appearances in Russell's theory of knowledge and hence in any knowledge of physical space.

I have rehearsed these themes concerning geometry and physical space in part so as to specify my topic more closely. I want to focus on a question in Russell that has to do not with the foundations of geometry or, to begin with, the specific geometry of physical space, but rather with the relation between the sensory experience of the spatial properties of objects and knowledge of a public spatial order. Thus, although Strawson's observation about the inadequacy of bare phenomenal looks may apply to specific problems addressed in Russell's philosophy of physics (viz., determining the precise structure of physical space), it does not hold with respect to his more general epistemology as applied to the spatial properties of things. Any claims about the spatial properties of objects in physical space, including measuring instruments, are for Russell based on perceptual experience.

My topic concerns some aspects of Russell's epistemological turn in the period after 1911. In particular, it focuses on two aspects of his philosophy in this period: his attempt to render material objects as constructions out of sense data, and his attitude toward sense data as "hard data." These aspects are epistemological and metaphysical. They involve commitments to the existence of certain sorts of entities, paired, not with the denial of other entities, but with a decision not to include other entities in the general scheme of being that Russell develops; and they involve a discussion of the sort of metaphysical commitments that would allow, as regards the objects around us, the positing of easily knowable entities as their basis.

5 Ibid., 286.

In considering this topic, I focus on Russell's "breakthrough" of early 1914, in which he concluded that, viewed from the standpoint of epistemology and analytic construction, space has six dimensions, not merely three. In this scheme, Russell posits a three-dimensional personal or "perspective" space that is inhabited by sense data. This space then forms the basis for constructing the three-dimensional space of physics (and of public things). I am concerned with the specifics of this construction: with the properties of the private spaces, the relations among those spaces, and their relation to physical space and to constructed "things," such as pennies or tables. I find that there are difficulties of interpretation with respect to these relations, which stem from the difficulty of finding a coherent interpretation of Russell's claim that objects such as tables and pennies look smaller at a greater distance (or look trapezoidal or elliptical from some points of view). I don't mean to challenge the phenomenal claim that objects do, in some sense, look small in the distance. Rather, I raise difficulties with Russell's analysis of this fact, in which he appeals to both phenomenal experience and the findings of sensory psychology. I hold that if he wishes to maintain his phenomenal claim about objects appearing smaller with greater distance, he must alter or redescribe aspects of his construction of ordinary things. However, if his construction of things and physical space is based on a problematic description of the private spaces, then his claim that private or perspective spaces are very well known and provide the hard data for knowledge of the physical world faces a challenge.

In the first section, I review the place of sense data in Russell's philosophical development after 1911. I then consider: his "breakthrough"; the findings of sensory psychology that he invoked in connection with it; some problems that arise, both for Russell and textbook psychology, in connection with the perception of size; and finally some problems for Russell's use of this psychology in describing the "hard" sense data at the basis of his constructions, along with some directions in which he might seek a solution. I end with a brief consideration of why experientially based knowledge of spatial items is crucial for Russell in a way that it is not for Kant.

1 Russell and Sense Data

The part of Russell's metaphysics that is our focus concerns his effort to render the material world, including the world of physical science, as a construction out of sense data. Russell worked at this project from sometime in 1912 through the *Analysis of Matter* in 1927. He did not sustain the same metaphysics throughout: indeed, within the range of years just indicated, he switched from endorsing a representative realism with inferred physical entities in *Problems of Philosophy*; to adopting a metaphysics of sense data and subjects who apprehend them later in 1912, according to which matter is constructed; to accepting the position that there exists only neutral stuff, that is, to his accepting late in 1918 the neutral monism of William James.[6]

Russell's effort to construct physical objects and theoretical entities from sense data begins with the sense data themselves. From sometime in the first decade of the century onward, Russell was a realist about sense data. In his autobiographical reconstruction of this period, he emphasizes the initial pluralistic realism that he shared with Moore: the real is not one, but many, and objects are known directly, as in naïve realism.[7] In his writings on sense data from 1912–14, he stresses a second position that he shared with Moore, the mind-independence of sense data: that they exist independently of our perception of them, even while we perceive them.[8] Accordingly, Russell distinguished sensation, as the mental act by which we are aware of sense data, from sense data as objects of which we are aware. Considering the question of whether the immediate objects of sense are mental entities or at least are in some sense mind-dependent, in *Our Knowledge of the External World*, the Lowell lectures in Boston from 1914, Russell held them to be physiologically conditioned but mind-independent:

> I think it must be admitted as probable that the immediate objects of sense depend for their existence upon physiological conditions in ourselves, and that, for example, the coloured surfaces which we see cease to exist when we shut our eyes. But it would be a mistake to infer that they are dependent on the mind, not real while we see them, or not the sole basis for our knowledge of the external world.[9]

6 On Russell's adoption of neutral monism, see Hatfield, "Sense-Data and the Philosophy of Mind: Russell, James, and Mach," in *Principia* 6 (2003), 203–30, and "Sense Data and the Mind–Body Problem," in *Perception and Reality: From Descartes to the Present*, ed. R. Schumacher (Paderborn: Mentis Verlag, 2004), 305–31.

7 Russell, *My Philosophical Development* (New York: Simon and Schuster, 1959), 61–2; G. E. Moore's well-known "Refutation of Idealism," in *Mind* n.s. 12 (1903), 433–53, in effect endorses a naïve realism, according to which we are directly aware of "the existence of a table in space" in the same way in which we are directly aware of the conscious element that accompanies every mental fact (453).

8 Moore, in "The Nature and Reality of Objects of Perception," in *Proceedings of the Aristotelian Society* 6 (1905–6), 68–127, distinguished "sense-contents" as non-mental from the mental acts by which we apprehend them; in "The Subject-Matter of Psychology," in *Proceedings of the Aristotelian Society* 10 (1909–10), 36–62, he made the same point, now using the term "sense data."

9 Russell, *Our Knowledge of the External World as a Field for Scientific Method in Philosophy* (Chicago: Open Court, 1914), 64; hereafter, *OKEW*.

In a paper from the same time, "The Relation of Sense-Data to Physics," he asserted that sense data are "physical," not mental, by which he meant that they are mind-independent and did not mean that they are composed of the elementary particles of physics.[10]

Why posit sense data as distinct from ordinary external objects? Russell accepted the arguments from perceptual variation. The table appears rectangular from one view, trapezoidal from another; brown from one view, glaring whitish grey from another. The coin appears now elliptical, now circular, now larger, now smaller, depending on viewing angle and distance. We believe that the color and shape of the real table and coin do not change from moment to moment; but the "data" that we see do change. Russell concluded that sense data are not parts of the objects themselves, and he posited them as entities distinct from really existing physical objects.[11] But then in 1912–14 he came to believe that he could do away with really existing physical objects as usually conceived and achieve a more economical ontology involving only two types of entity: subjects and data. Ordinary objects and microphysical processes had, in *Problems*, been things known by inference from sense data. Now they would be regarded not as inferred entities but as logical constructions or indeed as logical fictions.[12]

One of Russell's motivations for retaining sense data and jettisoning physical matter was epistemological: sense data are very well known. In the language of the Lowell lectures, he described them as the "hard data" from which he would construct the "things" of common sense and science. In calling the thing a "mere logical construction" or comparing it to a "fiction," he meant to say that it doesn't exist, or rather it needn't be held to exist.[13] Accordingly, only these particulars exist: sense data; the subjective acts of their apprehension; and, most likely, the domain of sensibilia – entities very like sense data that exist at places where there isn't anyone to see them at present. The sensibilia are not physiologically conditioned, which is why they are only "very similar" to sense data. The fact of physiological conditioning is an empirical regularity within our sense data: e.g., that staring at a bright light yields an afterimage that affects our

10 Russell, "Relation of Sense-Data to Physics" (hereafter, *RSDP*), in *Scientia* 16 (1914), 1–27, as reprinted in Russell, *Mysticism and Logic and Other Essays* (London: George Allen and Unwin, 1963), 108–31, on p. 112.

11 Russell, *The Problems of Philosophy* (New York: Oxford University Press, 1997), 12, 22–5; the first edition was published in 1912.

12 *OKEW*, vi; *RSDP*, 115.

13 Russell, *OKEW*, 70–2, 89; "Ultimate Constituents of Matter" (hereafter, UCM), in *Monist* 25 (1915), 399–417, as reprinted in *Mysticism and Logic*, 94–107, on p. 102; *RSDP*, 115.

visual experience for several minutes and about which we can construct explanations in the fictional language of physics and physiology.

If sense data are the "hard data" Russell takes them to be, the well-known items that are the basis for all other knowledge of the external world, then he should be clear about their phenomenal properties. For vision, these properties are colors, shapes, distances, and sizes. Russell worked on the structure of sense data and their spatial relations to one another in 1912–14. In January, 1914, he had what he considered a breakthrough, which caused him to emend the manuscript of the Lowell Lectures.[14] This breakthrough led him to the notion of a six-dimensional spatial construction, which related the private spaces of sense data to a constructed public space that could be known by all. I want to explore the spatial structure especially of the private spaces, as these are the (ostensibly) well-known data at the foundation of Russell's system.

2 Russell's "Breakthrough" and Six-Dimensional Space

In *Problems*, Russell offered a familiar description of sense data according to which, from most normal points of view, a table is seen as having a trapezoidal shape; or rather, if we carefully attend to our sense data of the table, we will notice that they are trapezoidal:

> We are all in the habit of judging as to the "real" shapes of things, and we do this so unreflectingly that we come to think we actually see the real shapes. But, in fact, as we all have to learn if we try to draw, a given thing looks different in shape from every different point of view. If our table is "really" rectangular, it will look, from almost all points of view, as if it had two acute angles and two obtuse angles. If the opposite sides are parallel, they will look as if they converged to a point away from the spectator; if they are of equal length, they will look as if the nearer side were longer. All these things are not commonly noticed in looking at a table, because experience has taught us to construct the "real" shape from the apparent shape, and the "real" shape is what interests us as practical men.[15]

There are two things to notice about this passage. First, Russell describes what he otherwise calls the sense data (here: appearances) of the table as if they were perspective projections; he describes the apparent shape as trapezoidal and compares it to the shape of the table as we would draw it. He does not indicate that the sense datum is experienced at a distance, but he does not say it is experienced

14 Kenneth Blackwell, "Our Knowledge of *Our Knowledge*," in *Russell: The Journal of Bertrand Russell Studies* 12 (1973), 11–13. *RSDP* records the breakthrough.

15 Russell, *Problems*, 10–11.

internally or in the eye, either, and I believe the presumption is that the trapezoidal shape is experienced roughly in the location of the table. Second, he tacitly appeals to psychological theories of shape perception, according to which, as a result of experience, we perceive (or think that we perceive) the table top as a rectangle in its proper location.[16]

Russell's position in *Problems* was subject to two sorts of objection. One type of objection was more generally abroad concerning sense data, and has become known as Stout's postulate, according to which, as described recently by Nasim, "one and the same thing cannot have more than one sensible quality at one and the same place" (at the same time, one presumes).[17] If four people view the same rectangular table from positions standing back from each of the four sides, they see four different trapezoidal sense data; even if the trapezoids seen from either end should by coincidence be congruent (the viewers have the same height and are in exactly similar positions), they show different sides of the table as being longer and shorter (for each, the near side is the longer). If one assumes that they are seeing these data as being at the position of the real table, then the data violate Stout's postulate. But T. P. Nunn replied that, if the various sense data are in the private spaces of distinct observers, then they are not literally together occupying the same location at the same time. In this regard, the public object must be seen as a construction based on the private elements. We shall see that Russell, in his "breakthrough," draws from Nunn's suggestion (and he in fact cites Nunn).[18]

The second objection was conveyed to Russell by Whitehead, as part of a fourteen-page critical response to a prepublication manuscript version of *Problems*. Whitehead wrote to his former pupil: "As to the 'shape of the table'. Why assume that our perception of space is two-dimensional? Perhaps you don't. I

16 G. E. Stout, *Analytical Psychology*, 2 vols. (London: Swan Sonnenschein, 1896), 2:21–3, offers a position similar to Russell's, asserting that we merely think of the real shape, without forming an image of it, which goes along with Russell's statement that "we come to think we actually see the real shapes," which might suggest that we don't actually see them (form an image of them). W. James, *Principles of Psychology*, 2 vols. (New York: Holt, 1890), 2:238–40, by contrast, suggests that we habitually replace the sensation of a projective shape with an image of the real shape seen from an ideally revealing point of view (so as to capture symmetry and equality of sides, if these exist). This could be Russell's position if we think that the "construction" of the real shape is a phenomenal construction. Russell read these works by Stout and James in 1894–96; see Russell, "What Shall I Read?" in *Cambridge Essays, 1888–99*, ed. K. Blackwell et al. (London: George Allen and Unwin, 1983), 345–70, on pp. 354–5, 357.

17 Omar W. Nasim, *Bertrand Russell and the Edwardian Philosophers* (Houndmills, England: Palgrave Macmillan, 2008), p. 6.

18 *RSDP*, 113, citing T. P. Nunn, "Are Secondary Qualities Independent of Perception?" in *Proceedings of the Aristotelian Society* 10 (1909–10), 191–218.

can't get a decisive instance just now. But the general impression on my mind is that you do. Surely such an assumption is false psychology."[19] Whitehead assumes from Russell's description that the sense data are two-dimensional, but he also correctly points out that there is no passage from which it can be seen that Russell decisively held this position (at least there is none to be found in the published version of *Problems*). All the same, in the six-dimensional construction of the "breakthrough," Russell explicitly renders his private spaces as three-dimensional. He also may have gleaned an idea from Whitehead's materials intended for the fourth volume of *Principia Mathematica* (but never published in the form Russell saw them, and later destroyed),[20] involving the distinction between the "from which" and the "at which."[21]

Russell's breakthrough from January, 1914, is recorded in the paper "The Relation of Sense-Data to Physics" (written in the first week of January). Russell responded to the above problems with his six-dimensional account of space.

Russell's six-dimensional space consists of two three-dimensional spaces: a private one and a public one that is constructed from it.[22] The private spaces are each experienced by an individual. These spaces have a three-dimensional structure that constitutes a "perspective" or point of view. As Russell explains, "two places of different sorts are associated with every 'sense datum', namely the place *at* which it is and the place *from* which it is perceived."[23] The from-which may be thought of as the phenomenal position of the subject or percipient, the place from which the subject perceives the world. The at-which is the location of the datum,

19 Victor Lowe, "Whitehead's 1911 Criticism of *The Problems of Philosophy*," in *Russell: Journal of the Bertrand Russell Archives* 13 (1974), 3–11, on p. 5.

20 Ibid., p. 3. Some of the ideas contained in these papers were published by Whitehead in 1919 and 1920, on which, see Elizabeth Ramsden Eames, *Bertrand Russell's Dialogue with His Contemporaries* (Carbondale: Southern Illinois University Press, 1989), 121. Famously, in the preface to *OKEW* (p. vi), Russell credited Whitehead with his changed position from *Problems*, saying: "I owe to him the definition of points, the suggestion for the treatment of instants and 'things,' and the whole conception of the world of physics as a *construction* rather than an *inference*." As is well known (see, e.g., Eames, *Russell's Dialogue*, 103, 118–21), Whitehead was not pleased with the use to which Russell put these ideas, and he refused to send Russell any more of his "notes" and asked him not to discuss his ideas (in a letter from 1917).

21 A distinction similar to Russell's between the from-which and the at-which appears in A. N. Whitehead, *An Enquiry Concerning the Principles of Natural Knowledge* (Cambridge: Cambridge University Press, 1919), 85, 189, and *The Concept of Nature* (Cambridge: Cambridge University Press, 1920), ch. 5. The distinction is not worked out in such a way that it adds to Russell's version, and the relation between these later publications and Whitehead's earlier "notes" is not clear.

22 *RSDP*, 119: "The world which we have so far constructed is a world of six dimensions, since it is a three-dimensional series of perspectives, each of which is itself three-dimensional."

23 *RSDP*, 117.

perceived to be at a certain distance (the space is three-dimensional) and having a shape and size as experienced from the from-which. As Russell elaborates:

> each person, so far as his sense-data are concerned, lives in a private world. This private world contains its own space [...] . The place *at* which a sense datum is, is a place in private space. This place therefore is different from any place in the private space of another percipient.[24]

These private spaces would seem to speak to both of the problems from *Problems*. They obviate, in Russell's words, the problem "of combining what we call different appearances of the same thing in the same place,"[25] since the place of each at-which occurs only in the private space of the individual. Further, Russell here makes clear that the private spaces are three dimensional; the relation between the from-which and the at-which must be supposed to include a distance at which the datum is seen. Although not crucial for our purposes, Russell augments the actual private spaces of individuals with additional locations, from-whiches, that correspond to the points of view from which a sense datum would be observed if anyone were there; in these cases, the postulated entity located in the at-which is called a "sensibile" and amounts to an unsensed sense datum (occurring minus the effects of any physiological conditioning, as can accrue with actual perceivers).[26]

The second three-dimensional space is the public or physical space of "perspectives" or points of view. It is essentially an ordering of the from-which locations, that is, the places from which each perceiver experiences a datum. He introduces this space as follows:

> In addition to the private spaces belonging to the private worlds of different percipients, there is, however, another space, in which one whole private world counts as a point, or at least as a spatial unit. This might be described as the space of points of view, since each private world may be regarded as the appearance which the universe presents from a certain point of view. I prefer, however, to speak of it as the space of *perspectives*, in order to obviate the suggestion that a private world is only real when someone views it.[27]

The various from-whiches, or perspective units, can, according to Russell, be ordered so as to produce "the one all-embracing space of physics."[28] To each po-

---

24 *RSDP*, 117. Sajahan Miah, *Russell's Theory of Perception, 1905–1919* (London: Continuum, 2006), 158, implausibly makes the "at-which" a location in public space. Rather, locations in public space are constructed from the from-whiches.
25 *RSDP*, 118.
26 *RSDP*, 110–11.
27 *RSDP*, 118.
28 *RSDP*, 119

sition relative to a table, or, more conveniently, to a penny, an entire domain of private worlds can be imagined, each addressing the penny from a location so as to yield an appearance of a specific size and shape. These sizes and shapes are similar enough to be compared among perceivers through language,[29] and they therefore can serve as a basis for ordering the positions from-which into a set of locations that define the penny considered as a "public" object. Once these locations are determined, the notion of the penny as a real material thing is no longer of use, and it is reduced to the class of its appearances.

For our purposes, the crucial aspect of the construction of public space and material things is that it is accomplished, in Russell's view, by an ordering of private spaces according to their similarities and relations to one another. Russell explains the process:

> The arrangement of perspectives in a space is effected by means of the differences between the appearances of a given thing in the various perspectives. Suppose, say, that a certain penny appears in a number of different perspectives; in some it looks larger and in some smaller, in some it looks circular, in others it presents the appearance of an ellipse of varying eccentricity. We may collect together all those perspectives in which the appearance of the penny is circular. These we will place on one straight line, ordering them in a series by the variations in the apparent size of the penny. [...] By such means, all those perspectives in which the penny presents a visual appearance can be arranged in a three-dimensional spatial order.[30]

It must be remembered that there is no material penny, in the usual sense. The penny is the class of its appearances. In considering a material penny as a fictive construction, we may hold that "the matter of a thing is the limit of its appearances as their distance from the thing diminishes."[31] Talk of the penny "appearing" in the perspectives is a way of characterizing the class of sense data that are all "as of" a single material thing. This classification must be able to be achieved simply by comparing sense data empirically. Whether this project is workable in general is not my present concern.[32] Assuming for the sake of argument that the appearances can be ordered as he says, I want to look into the specific structure of the private spaces. In doing so, I will raise some questions about the coherence of this description, questions that were available within the conceptual framework in the psychology of perception of his time, and which apply more generally to the no-

29 *RSDP*, 117.
30 *RSDP*, 118–19.
31 *RSDP*, 121.
32 Miah, *Russell's Theory of Perception*, ch. 7, collects many objections to Russell's constructivist project.

tion of the penny looking smaller at a greater distance and looking elliptical from an angle.

3 Russell among the Psychologists

In ordering the sense data of the penny, Russell appeals to their "apparent sizes." The smaller the apparent size (as of the same penny), the farther away the from-which from the constructed location of the penny. Russell might also have appealed to their apparent shapes, to the effect that, the greater the eccentricity of the ellipse, the farther the from-which is (as regards viewing angle) from a line perpendicular to and through the center of the upward facing disc of the penny.

Russell also offers a different description of the series of phenomenal sizes. Instead of equating the series with apparent sizes, he relates them to the amount of visual field taken up:

> When a number of people are said to see the same object, those who would be said to be near to the object see a particular occupying a larger part of their field of vision than is occupied by the corresponding particular seen by people who would be said to be farther from the thing. By means of such considerations it is possible, in ways which need not now be further specified, to arrange all the different spaces in a three-dimensional series.[33]

While this may seem equivalent to talk of apparent size, it is not. Occupying a larger part of the visual field is a matter of the visual angle an object subtends. Visual angle does vary as Russell says: for an object of constant size viewed head on (as with the penny), the object will take up more of the visual field the closer the vantage point (or from-which) is to the object. But, as discussed in the next section, the perceived size of an object is a function of both visual angle and the perceived distance to the object.

Both of Russell's descriptions connect with the language used by psychologists, and indeed in his discussion of "hard data" in *Our Knowledge*, he makes clear his dependence on the findings of psychology. He observes that:

> Psychologists […] have made us aware that what is actually given is much less than most people would naturally suppose, and that much of what at first sight seems to be given is really inferred. This applies especially in regard to our space-perceptions. For instance, we instinctively infer the "real" size and shape of a visible object from its apparent size and shape, according to its distance and our point of view.[34]

33 UCM, 103–4.
34 *OKEW*, 68.

Here again we have talk of apparent size and shape, and now distance is brought in as something that is added to what Russell may well have considered to be a two-dimensional sense datum. Russell notes that the "real" size "at first sight seems to be given," suggesting that the real size is in some sense phenomenally present; but it remains unclear whether he still holds the position from *Problems*, according to which we merely "come to think we actually see the real shapes" and presumably sizes, a phrasing which allows that we merely mistakenly believe that we see the real size. The above passage occurs in the portion of the Lowell lectures thought to have been composed before the breakthrough in January, 1914.[35] The part inserted after the breakthrough goes through the six-dimensional space, using much the same language as "Relation of Sense-Data to Physics."

Russell's description of the appearances of the penny accords with some descriptions of apparent size and shape in a psychological account with which he was familiar. G. E. Stout, whose psychology textbook Russell read and who was one of his teachers at Cambridge, offers a position that is similar to Russell's pre-breakthrough discussion in *Our Knowledge*. He reports that in visual perception we actually experience a visual magnitude that accords with visual angle and a visual shape that accords with the projective shape of the object as seen from a particular point of view. We then infer the real size and forget the apparent size. But, interestingly, he asserts that this real size is not present to us phenomenally. Here is what Stout says:

> If we attend to the visual appearance rather than to the object, we find that it varies in size according to our distance from the thing seen. At great distances this variation is forced upon our notice. Looking down from a high tower on men walking in the streets of a town, we remark with a kind of surprise how small they look. But for comparatively short distances the variation passes unnoticed unless we expressly measure the visual angle. The reason is that the visual magnitude, as such, is habitually disregarded, serving only as a sign of the real magnitude. But this real magnitude, in its distinction from apparent magnitude, is neither actually seen nor mentally visualized. Nor is it represented by an ideally revived tactual or motor complex. There is no trace of such a complex in consciousness.[36]

There are three points to note. First, Stout affirms a phenomenology of things looking smaller at a distance, according to visual angle. Second, he remarks that, such appearances are overlooked in favor of the real magnitude (by a process he has characterized as an inference). Third, he reports that this real magnitude is not present in consciousness as a visual content, that is, as a phenomenal spatial extent that corresponds to the real magnitude. These positions accord at least

35 Blackwell, "Our Knowledge", 13.
36 Stout, *Analytical Psychology*, 2:22–3.

partially with Russell's pre-breakthrough report of what "the psychologists" have revealed. A potential discrepancy is that Stout suggests that the men are both seen at a distance and look small, while Russell in the passage above writes that, when we take distance into account, we get to real size.

As it happens, the relations among visual angle, apparent size, distance, and real size are a problem for both Stout and Russell, and could have been seen to be so on the basis of other psychological literature produced in England, some at Cambridge, during the latter decades of the nineteenth century and the beginning of the twentieth century.

4 Problems with Perceived Size

By Russell's day, the relations among visual angle, apparent size, distance, and real size had long been studied in the optical tradition and were under intense investigation in sensory psychology. From the time of Ibn al-Haytham and Descartes, it was known that, for adult perceivers, perceived size depends on both visual angle and perceived distance.[37] Visual angle varies inversely with distance. The perception of size involves both visual angle and perceived distance. Consider an example in which a smaller object at a closer distance takes up the same visual angle as a larger object at a farther distance (Fig. 1a). Assuming that visual angle is accurately recorded by the visual system[38] and that distance is accurately perceived, both objects are perceived with their true sizes (a perceptual result later known as "size constancy"). If two objects of different size are at the same distance from the eye, then the smaller one subtends a smaller visual angle (Fig. 1b); if the distance is accurately perceived, then the sizes of the two objects are accurately perceived.

A further factor arose with Kepler's discovery of the retinal image. Visual angle can be defined by relations among directions in the field of view. Once Kepler accurately characterized the optics of the retinal image, it became known that the visual angle subtended by an object corresponds to the size of the image that the

37 See Gary Hatfield and William Epstein, "The Sensory Core and the Medieval Foundations of Early Modern Perceptual Theory," in *Isis* 70 (1979), 363–84.

38 In what follows, I assume that the visual system is basically accurate in recording visual angles, or visual directions. This is a separate matter from whether perceivers are good at judging visual angles. It simply assumes that we see things in the direction that they actually are. When both eyes are open, especially for near distances visual angle is constructed in relation to the "cyclopean eye." For a discussion, see Maurice Hershenson, *Visual Space Perception: A Primer* (Cambridge: MIT Press, 1999), ch. 2.

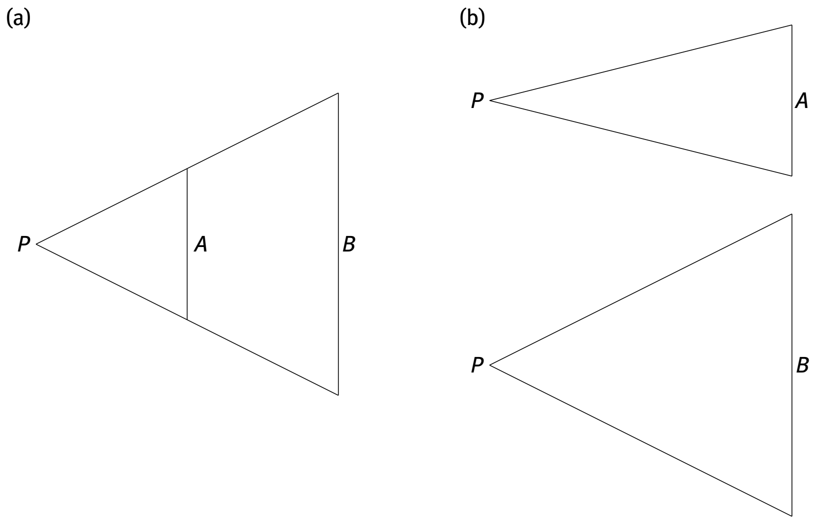

Fig. 1: Part (a) shows how two objects of different sizes (*A* and *B*) can subtend the same visual angle when placed at different distances from the perceiver *P*. Assuming that visual angle (or visual direction) is accurately registered by the visual system, then, if distance is perceived veridically, *A* and *B* are perceived with their true sizes. Part (b) shows how the same two objects relate to the perceiver when they are the same distance away. The smaller object (*A*) subtends a smaller visual angle. Again, if distance is perceived veridically, then both *A* and *B* are perceived with their true sizes.

object projects onto the retina. The larger the retinal projection, the larger the visual angle. This relation includes the previous one: a larger and a smaller object at the same distance have larger and smaller retinal projections, but if distance is accurately perceived their differing sizes are accurately perceived; further, they can be placed in such a way that they project the same size onto the retina and, again, if distance is accurately perceived, the objects are perceived with their true sizes.

Kepler's finding led to the question of whether perceivers are aware of retinal image size and also of the shapes that objects project onto the retina. In the case of the penny, the retinal image size varies inversely with distance, so that, when farther away, the penny projects a smaller size. Similarly, when viewed straight on, the penny projects a circle, but when viewed from an angle, as this angle diverges from straight on, the penny projects ellipses of increasing eccentricity (until the penny projects an elongated rectangle, when viewed from the side).[39]

These facts thus far only specify the sizes and shapes of images projected on the retina. Except when the retinal values are combined with a perception of distance, these facts don't specify a visual experience of an object with a shape and size at a distance. Theorists varied in their interpretation of what sorts of experiences perceivers have of projective values. Some theorists spoke of perceiving or having sensations or visual appearances that correspond to the retinal projections: for a given object, it is smaller in sensation at greater distance, and its shape in sensation varies with viewing angle. But the question of where these sensations appear to be became problematic. Berkeley placed them "in the mind," Reid had them at "no distance from the eye." Stout didn't specify.[40]

Others, such as James Sully, in his widely known textbooks, contended that the notion of a two-dimensional sensation that is experienced in adult life is a fiction. He held that adults always localize and experience sensations as being at some distance.[41] This distance may be perceived more or less accurately. Writers on vision at Russell's Cambridge, including W. H. R. Rivers and C. S. Myers, observed that perceived size (or magnitude) is determined by retinal image size together with perceived distance. Here is how Rivers explained the relation and an instance of its empirical manifestation:

> The two chief factors on which depends the perception of the size of an object are the size of the retinal image and the estimated distance of the object. The importance of the latter factor is shown by an after-image experiment. If the after-image of an object is projected on a moving screen, it will be seen to change in size, becoming smaller as the screen approaches the eyes, larger as it recedes. The retinal image upon which the after-image depends remains constant in size, and the changes in apparent magnitude depend on the projection. Emmert measured the after-image at different distances, and found the linear size of the image equal to the linear size of the object, multiplied by the distance at which the image was seen.[42]

---

39 Although one might carry through this discussion using the examples of both diminishing size with distance and altered shape with viewing angle, henceforth I focus on the former. It is emphasized by Russell in *RSDP*, and the psychological literature of his time is better developed on this topic.

40 George Berkeley, *An Essay Towards a New Theory of Vision* (Dublin: Rhames and Papyat, 1709), § 41; Thomas Reid, *An Inquiry into the Human Mind*, 4th ed. (Edinburgh: Bell and Creech, 1785), ch. 6, sec. 23, p. 416; Stout, *Analytic Psychology*, 2:22–3.

41 James Sully, *Outlines of Psychology* (New York: Appleton, 1886), 149, 181, and *The Human Mind: A Text-Book of Psychology*, 2 vols. (New York: Appleton, 1892), 1:208, 245.

42 W. H. R. Rivers, "Vision," in *Text-Book of Physiology*, ed. E. A. Schäfer, 2 vols. (Edinburgh: Pentland, 1898–1900), 2:1026–1148, on pp. 1139–40. An abbreviated account is given by C. S. Myers, *A Text-Book of Experimental Psychology, Part I*, 2nd ed. (Cambridge: Cambridge University

In speaking of "projection," Rivers means that a subjective phenomenon, such as an afterimage, is perceived as localized at a specific location – in this case, on the surface of a screen. He notes that in Emmert's experiment, the size of the afterimage varies directly with the distance at which the screen is placed. This relation, which specifies that perceived size is a function of visual angle and perceived distance, is known as "Emmert's law."[43] Rivers generalizes the function to apply to the relations among visual angle, perceived distance, and perceived size in normal perception; the function later became known as the size-distance invariance relation.[44] According to this relation (Fig. 2a), if distance is veridically perceived, size is veridical (size constancy is attained); if distance is underperceived, size is underperceived, etc.

For present purposes, the most interesting feature of Rivers's description is that, as the afterimage is perceived as being at a greater distance, its size *increases*. This is exactly the opposite of Russell's account of the penny, in which apparent size decreases with distance. What is going on?

Russell, in describing the penny, is describing the perceived size of an object of fixed physical size as it recedes (Fig. 2b). It is unclear from his description whether he means to be describing the diminishing retinal-image size of the penny, its diminishing visual angle, or merely to be reporting a fact about how phenomenal size changes with increasing distance. Rivers is describing a situation in which a constant visual angle or retinal size (the physiological process underlying the afterimage, which remains constant on the retina) is paired with differing distances. He finds that, for a constant visual angle, perceived size increases with perceived distance, in accordance with Emmert's law.

Thus far, there is no direct contradiction between Rivers and Russell. But Rivers's description entails that, in Russell's situation, if the distance to the penny

---

Press, 1911), ch. 22. Russell was at Cambridge when Rivers and Myers published these works. Although he may have met Myers before 1914, that matters not for my point, which is to indicate what knowledge was available whether Russell availed himself of it or not.

43 On Emil Emmert's law, see Edwin G. Boring, *Sensation and Perception in the History of Experimental Psychology* (New York: Appleton-Century-Crofts, 1942), 292–3, 310.

44 The connection between the size–distance invariance relation and Emmert's law has frequently been discussed. I assume here that Emmert's law relates visual angle (as perceived) with distance (as perceived). For a review of the earlier literature that raises interpretive questions about Emmert's own intentions, see William Epstein, John Park, and Albert Casey, "The Current Status of the Size-Distance Hypotheses," in *Psychological Bulletin* 58 (1961), 491–514. In any event, a recent review of the constancy literature finds that the "vast majority" of studies validate the size–distance invariance relation: Mark Wagner, *The Geometries of Visual Space* (Mahwah, N. J.: Erlbaum, 2006), 226. Many psychologists use the term "size–distance invariance hypothesis" rather than "relation."

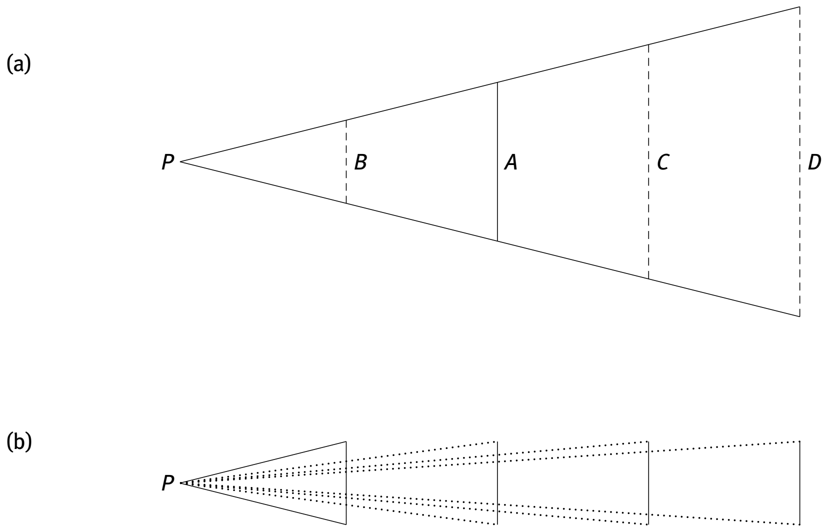

Fig. 2: Part (a) illustrates the size–distance invariance relation and Emmert's law. Assume that the afterimage is formed by segment *A*, which is perceived to be at the distance shown from the perceiver *P*. Then, if the afterimage is perceived at distances *B*, *C*, or *D*, it is perceived as being correspondingly smaller or larger. These relations work whether the perceiver veridically perceives the distances, or underrepresents them. Part (b) shows the geometrical relation in which the visual angle subtended by a penny decreases with increasing distance from the perceiver.

is perceived accurately, then the penny should appear of constant size as it recedes.[45] That is, its size should be perceived veridically, so long as its distance is perceived veridically. Again there is no direct contradiction, because Russell has not made explicit whether or not the distance is perceived veridically. But trouble is brewing, since Russell indicates (by calling the private spaces "three-dimensional") that the penny is seen at a distance. Further, if Russell accepts Stout's problem as stated above, that the various sense data are supposed to be "in the same place," then he must assume that the distance is perceived to increase as the viewer's actual distance from the penny increases; otherwise, the differing sense data of the penny would not be in danger of being perceived as being at the same

45 For convenience's sake, I speak of Russell's penny as if it were a material object at a location in relation to a perceiver. Of course, for Russell, this notion of the penny is constructed from private spaces in which a penny datum is at a distance from the from-which, that is, from the location from which the datum is perceived.

place at the same time. In that case, a contradiction with Rivers ensues; for, if the members of the series of sense data of the penny at the at-which are perceived as all being in the same place (that is, at correspondingly appropriate distances in private space so that they would be constructed as being in the same place), then as the from-which recedes from the at-which, the perceived distance between them must increase appropriately.[46] But if the increasing distance between at-which and from-which is perceived veridically, then the sense data will all be perceived as having the object's true size (the actual size of a penny), not as being smaller with increasing distance, as Russell reports them to be.

5 Problems and Prospects for Russell's Ordering

The more sophisticated psychological accounts of Sully and Rivers raise problems for understanding the structure of the private spaces in the six-dimensional space of Russell's "breakthrough." The problems are, to begin with, not technical but interpretive. They are not technical, for in this case a technical problem can arise only once we have settled on an interpretation of what Russell means to be asserting in the various passages scouted above, when he speaks of "apparent size" and describes the size of the penny as diminishing with increasing distance between its location and the place from which it is perceived.

Reflecting on the relations among apparent size, visual angle, and perceived distance as reviewed above, there are several options for interpreting what Russell means. Although it is not clear that any of the more plausible interpretations of what Russell intended to say are in the end coherent, we may be able to repair things on his behalf. Here are the positions that are initially plausible as interpretations of Russell's intent:

1. The penny data are located in the same place at-which, and they appear in that place to be increasingly smaller as the distance to the from-which increases.

2. The penny data that diminish in size are perceived as being in the eye or at the eye, and these data are then, as result of experience, overlooked in favor of a perception of the penny in its true location with its true size.

46 I am assuming for now that the distances in question in the private spaces can be ascertained by reference to or comparison with that distance as constructed in the public space. This assumption is reasonable, as the public space locations are built upon relations among the spatial structures of the private spaces. Of course, Russell could decide that this was not the case, because assuming it to be so creates a problem for something else he believes. I take up this possibility in the next section.

3. The penny is perceived as being at its true distance and with its true size, but the visual angle diminishes as the distance between the at-which and the from-which increases and the perceiver notices this and says that the penny "looks smaller" with increasing distance.

The first position (1) would be Russell's intent if he accepted Stout's problem as formulated, according to which the data at the at-which are in danger of all being in the same place, and so in private space they are at appropriate distances from the from-which so as to be related to the same location in public space. The main problem with (1) is that it violates the size-distance invariance relation (and Emmert's law). If the penny data appear to be at increasing (veridical) distances as the from-which moves back and yet also appear to be smaller, then their visual angles must decrease even more than is called for by geometry. For if the visual angle decreased as geometry requires and if distance were perceived veridically, the penny data would appear with their veridical sizes, not with diminishing sizes. So if Russell sticks with (1), he violates the basic geometry of size and distance perception.

Position (2) would have Russell relying on the historical positions of Berkeley and Reid or the notion of overlooked "sensations" employed by Stout. It conforms to texts that seem to hint of particulars or data radiating from the object;[47] in the case of the penny, the smaller "at-which" sizes would occur at the from-which locations and would diminish regularly with distance. There are two problems with this interpretation of Russell's intent. First, he doesn't explicitly say that the ever smaller penny data appear to be in the eye or at the eye. Indeed, in describing his private spaces or perspectives as three-dimensional, he suggests that the at-which is perceived to be some distance from the from-which. Second, and perhaps more decisively, after the breakthrough he doesn't describe the decreasing apparent sizes of the penny as something that must be uncovered. His report that penny data appear smaller with increasing distance is offered as a description of perceptual experience and he doesn't describe the "real size" as something that is constructed by taking distance into account in relation to penny data (as he did before the breakthrough). Rather, he speaks of the "real size" of a thing as the size that it appears to have as the distance to it approaches zero.[48] (For an extent near the surface of the eye, linear size is close to retinal size.) In any case, (2) doesn't

47 e. g., UCM, 102–3: "At every place between us and the sun, we said, there is to be a particular which is to be a member of the sun as it was a few minutes ago." However, such passages might also be interpreted as indicating that the various from-whiches of the perspectives are at "every place" between us and the sun.
48 *RSDP*, 121–2.

fit his post-breakthrough position very well.

Position (3) has the advantage that it accords with Emmert's law and offers Russell a way to fit his description to the textbook perceptual science of his time. It has him suggesting that size constancy is achieved with the penny, but nonetheless visual angle decreases with increasing distance. Accordingly, Russell's description of the penny data as looking smaller with increasing distance would be a remark, not on the perceived sizes of the penny data, but on the decreasing visual angle. As an interpretation of Russell's intent, (3) is not very plausible. Although he briefly mentions increasing size in the "field of vision" with decreasing distance in "Ultimate Constituents," he does not integrate this into a discussion that includes a description of size constancy. In any case, in the more extensive discussion in "Relation of Sense-Data," he regularly speaks of the penny as appearing smaller with greater distance.

None of positions (1) to (3) is satisfactory as an interpretation of Russell's intent, but for different reasons: (1) is unsatisfactory on technical grounds, while (2) and (3) do not harmonize well with the relevant texts.

We can, however, formulate a fourth position that is technically acceptable and that accords with his basic phenomenal observation that the penny both (a) looks to be at a distance and (b) looks smaller with greater distance.

4. The relation among perceived size, visual angle, and perceived distance in Emmert's law is preserved, but as the from-which recedes from the location of the penny, the distance to the "real" location of the penny is underperceived in such a way that the penny data are perceived as being closer and smaller than they would be if size constancy obtained.

Position (4) is technically sound with respect to Emmert's law. It has the advantage of preserving Russell's phenomenal description, in which the penny looks smaller (the penny data appear smaller) with increasing viewing distance. So far, it is better off than the previous positions. However, it introduces phenomenal facts that Russell doesn't mention, and it makes his constructive project harder to carry out.

The unmentioned phenomenal facts concern the underrepresentation of distance, an underrepresentation that increases with viewing distance. The fact that Russell doesn't mention these facts is consistent with his scattered remarks according to which distance perception isn't particularly good, especially at farther distances.[49] It is not so consistent with the implication that the data in the private spaces are very well known, for it entails that Russell overlooked or didn't

49 *OKEW*, 73: "It seems probable that distances, provided they are not too great, are actually feel the need to describe a basic fact about his hard data. Nonetheless, it may be consistent with his use of the private spaces as the basis for the construction of the common physical space.

We have seen that, in this construction, Russell must rely on comparisons among private spaces in establishing a serial order of the from-whiches. This serial order then establishes points or spatial units in public space. Leaving aside problems of ambiguity that might arise (e.g., in distinguishing on the basis of phenomenal size a penny at a shorter distance from a quarter at a longer distance), he would be able to establish a series. For according to (4), the penny data do diminish regularly in size with increasing distance. Unlike (1), these data aren't all referred to the same location in relation to physical space (or to counterpart distance relations in private space). Nor do they track veridical distance in the private spaces, as in (3). As the from-which recedes from the "real" location of the penny, the phenomenal distance to the penny data at the at-which increases but not at the same rate as the increasing distance to the "real" location. This means that the private spaces include a regular separation, with increased viewing distance, between the at-which and the "real" location. That fact puts an extra wrinkle in Russell's constructive problem. But it may only make the construction more complicated, without (on the grounds we are discussing) making it impossible.

Position (4) may not be the best interpretation of Russell's intent, as it is unclear that he was responsive to the phenomenal relations that it implies. However, (4) might well be an improvement on his actual position. It takes into account some experimental findings from Russell's day that suggest just such a contraction in visual space (an underrepresentation of distance that increases with distance). These findings were made by the German-speaking sensory psychologists Franz Hillebrand and Walter Blumenfeld.[50] Their work has received only sporadic

---

given more or less roughly in sight"; he backs off slightly in *Philosophy* (New York: Norton, 1927), 138: "It is still an open question whether the space of sight has depth, or is merely a surface, as Berkeley contended."

50 Franz Hillebrand, "Theorie der scheinbaren Grösse beim binokularen Sehen," in *Denkschrift der Kaiserlichen Akademie der Wissenschaften Wien, Mathematisch-Naturwissenschaftliche Classe* 72 (1902), 255–307; Walter Blumenfeld, "Untersuchungen über die scheinbare Grösse im Sehraume," in *Zeitschrift für Psychologie und Physiologie der Sinnesorgane* 65 (1913), 241–404. For discussions of their work and its application to the problem of constancy and the observation that things look smaller in the distance, see: Boring, *Sensation and Perception*, 294–6, and "Visual Perception as Invariance," in *Psychological Review* 59 (1952), 141–8; and Hatfield, "Representation and Constraints: The Inverse Problem and the Structure of Visual Space," in *Acta Psychologica* 114 (2003), 355–78, "On Perceptual Constancy," in *Perception and Cognition: Essays in the Philosophy of Psychology* (Oxford: Clarendon Press, 2009), 178–211, and "Phenomenal and Cognitive Factors in Spatial Perception," in *Visual Experience: Sensation, Cognition, and Constancy*, recognition in the century since it was published.[51] But the results remain interesting today. In the context of Russell's project, they would seem to offer him a better description of his private spaces, one that accords with the phenomenal diminution of the penny with distance, without violating the basic laws of size and distance perception. If distance is increasingly underrepresented, then the penny data will appear at a distance, which is consistent with the three-dimensionality of the private spaces; but they will also appear increasingly smaller with distance. In this way, these results would allow Russell to put psychology in the service of epistemology, a stated aim of *Our Knowledge*.[52] The psychology would be more sophisticated than the textbook commonplaces that Russell incorporates into his own discussion.

6 Russell's Progress

If this reconstruction of Russell's position after the breakthrough is correct, it suggests that his new insight was not as path-breaking as he might have imagined. The more textually plausible readings of his position after the breakthrough either have dire technical problems or turn out not to fit the texts very well. Position (4), which is technically more acceptable, does not fully fit the texts, either, although it allows Russell to retain the decrease in phenomenal size with distance. With respect to Russell's ordering in terms of phenomenal size, the technically better position would permit him to pursue his ordering of perspectives into series that allow for construction of the all-embracing physical space. From our viewpoint, it permits us to see how Russell could have elaborated his insight, had he delved more deeply into the psychological literature of his time.

As it happens, Russell's subsequent fascination with the project took him first to an encounter with behaviorism (though by no means a wholesale adoption of its tenets) in the *Analysis of Mind*, and then, in the *Analysis of Matter*, on to a deeper analysis of the theory of matter as constructed from sense data now rendered (un-

---

ed. G. Hatfield and S. Allred (Oxford: Oxford University Press, 2012), 35–62.

51 The contraction found by Hillebrand and Blumenfeld has been used in arguing that visual space is hyperbolic non-Euclidean: Rudolf K. Luneburg, *Mathematical Analysis of Binocular Vision* (Princeton: Princeton University Press, 1947); also, Patrick A. Heelan, *Space-Perception and the Philosophy of Science* (Berkeley: University of California Press, 1983). For discussion, see Hatfield, "Phenomenal and Cognitive Factors."

52 On Russell's conception of how psychology can aid epistemology, see Hatfield, "Psychology, Epistemology, and the Problem of the External World: Russell and Before," in *The Historic Turn in Analytic Philosophy*, ed. E. Reck (London: Palgrave Macmillan, in press).

der the influence of James's neutral monism) as "momentary particulars."[53] In these further developments, and especially in the *Analysis of Matter*, the distinction between the from-which and the at-which that arose with the breakthrough remain important in ways that are crucial to understanding Russell's ongoing development. But to address the importance of this distinction in that work would take us beyond the breakthrough itself, a moment that yielded some progress in Russell's analysis of phenomenal space, but not as much as was available to him had he probed more deeply into the psychological science of his time.

Finally, we can conclude that, both at the time of the breakthrough and in *Matter*, empirical knowledge of phenomenal spatial looks is absolutely important to Russell. One cannot arrive at a theory of the spatial structure of physical space without it. In this way, Russell differs from Kant. In Kant's critical philosophy, the structure of physical space is determined by the geometrical structures that must unfold in experience in accordance with space as an a priori form of intuition.[54] Thus, physical bodies and physical space must conform a priori to geometry, that is, in Kant's thinking, to the geometry of Euclid.[55] This remains so even if (some of) Kant's arguments for the a priori intuitional nature of space are separate from this theory of geometry.

In contrast, for Russell in the period we have been considering, all knowledge of physical space, which for him means the constructed physical space of everyday experience and of science, arises from sense data. For the purposes of science, these data may be as of measuring instruments and the results of physical tests. Even so, or decidedly so, the scientist must rely on sense data for knowledge of physical space. It is not a matter of "contemplating" phenomenal appearances (as Strawson had it),[56] but of constructing even ordinary objects from similarities that are perceptually noted in the spatial structures at-which in relation to the place from-which. Exactly how to carry out the scientific construction of physical space on a large scale was becoming an ever more intricate problem during the time in

---

53 Russell, *The Analysis of Mind* (London: Allen and Unwin, 1921); *The Analysis of Matter* (London: Kegan Paul, Trench, Trubner, 1927). On the importance of the from-which for interpreting Russell's much maligned statement in *Matter* about the perceptual world being "in his head," see Hatfield, "Perception and Sense Data," in *Oxford Handbook of the History of Analytical Philosophy*, ed. M. Beaney (Oxford: Oxford Univ. Press, in press), footnote 7.

54 Kant, *Critique of Pure Reason*, trans. P. Guyer and A. Wood (Cambridge: Cambridge University Press, 1998), B 147; A 165/B 206; A 224/B 271; A 239–40/B 298–9.

55 Kant on the mathematics (geometry) of "physical space": *Prolegomena to Any Future Metaphysics*, trans. G. Hatfield (Cambridge: Cambridge University Press, 2004), First Part, Note 1 (Akademie edition, 4:287–8).

56 Strawson, *Bounds of Sense*, 286.

which Russell wrote. But that for Russell this process of construction must begin with comparisons among phenomenal spatial looks is beyond doubt. This is one of the ways in which, in relation to Kant, Russell was an empiricist.[57]

57 Thanks to Bernhard Thöle for helpful criticism of an earlier version given in Berlin in June, 2011, and to Rolf-Peter Horstmann for ongoing discussions on this and related topics, including comments on the penultimate draft.