

# Frege on Identity and Identity-Statements: A Reply to Caplan and Thau\*

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In ‘What’s Puzzling Gottlob Frege?’ Michael Thau and Ben Caplan argue that, contrary to the common wisdom, Frege never abandoned his early view that, as he puts it in *Begriffsschrift*, a statement of identity “expresses the circumstance that two names have the same content” (Bg, §8) and thus asserts the existence of a relation between names rather than a relation between (ordinary) objects. The arguments at the beginning of ‘On Sense and Reference’ do, they agree, raise a problem for that view, but, they insist, Frege does not, as the ‘standard’ interpretation has it, take these arguments to refute it. Rather, they claim, Frege is out to *defend* (a version of) his earlier view against these objections: Indeed, the defense he there offers is pretty much the same defense offered in *Begriffsschrift* against what are pretty much the same objections.

In defense of this novel interpretation, Thau and Caplan (henceforth, T&C) offer two sorts of evidence. First, they call our attention to six passages from Frege’s writings, of which the following will serve as an illustration:<sup>1</sup>

What is expressed in the equation ‘ $2 \times 2^3 = 18$ ’ is that the referent of the left-hand complex of signs is the same as that of the right-hand one. (FC, p. 138, op. 3)

T&C are impressed by the way Frege speaks here, and in the other passages, of an identity-statement’s *expressing* a relation between names—namely, that they are co-referential—and they urge us to take this language seriously. Frege’s so speaking, they suggest, shows that he

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<sup>1</sup> Three further passages come from *Grundgesetze* (Gg, v. II, §§64, 103, and 138). Another is from a letter to Peano: (PMC, letter xxxiv, p. 128). The last passage is from “Formal Theories of Arithmetic” (FTA, p. 118, op. 101). Note that this last paper was published in 1885 and so predates the development of the distinction between sense and reference.

still held the *Begriffsschrift* theory of identity even when he wrote this passage, which was at about the same time he wrote ‘On Sense and Reference’. T&C argue further that a close reading of ‘On Sense and Reference’ itself—in particular, of its opening paragraph—reveals that Frege is there defending, not attacking, his old theory of identity.

I find T&C’s arguments for their interpretation unconvincing. I shall look in section 3 at the specific textual evidence they offer. First, however, I shall argue, in section 1, that T&C misidentify the issue that is being raised at the beginning of ‘On Sense and Reference’: Specifically, they misunderstand the “challenging questions” about identity to which Frege refers in its opening sentence. Their interpretation can survive this error. But, as I shall argue in section 2, it cannot survive their misconstrual of the puzzle about cognitive value that Frege uses to motivate his distinction between sense and reference.

## 1 Frege on Identity

Frege begins ‘On Sense and Reference’ as follows:

Equality gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? (SM, p. 157, op. 25)

T&C take what is at issue here to concern what identity-statements *express*. The issue, they believe, is whether ‘Hesperus is Phosphorus’ expresses the thought that Hesperus is the same object as Phosphorus or instead the thought that ‘Hesperus’ and ‘Phosphorus’ have the same referent (Thau and Caplan, 2001, see p. 161). But that is a misreading. What is at issue here is *what identity is*. Russell puts the problem this way:

The question whether identity is or is not a relation, and even whether there is such a concept at all, is not easy to answer. For, it may be said, identity cannot be a relation, since, when it is truly asserted, we have only one term, whereas two terms are required for a relation. (Russell, 1903, p. 63)

Frege simply assumes an affirmative answer to the first question he poses: Identity is indeed a (binary) relation. Now, for Frege, a relation is a binary function, one whose value is always a truth-value. So the

remaining question, as Frege would have understood it *circa* 1892, is what the *arguments* of this function are: The question is whether they may be any objects one likes (people, planets, or what have you) or must always be names; the question is whether the relation of identity holds between Hesperus and Phosphorus or between ‘Hesperus’ and ‘Phosphorus’. Once the question has been put this way, it should be obvious what Frege’s mature answer was: The arguments of the relation of identity may be any objects at all; they need not, and typically will not, be names. But if one wants textual evidence, there is plenty to be had. Compare the official explanation of the identity-sign in *Grundgesetze*:

“ $\Gamma = \Delta$ ” shall denote the True if  $\Gamma$  is the same as  $\Delta$ ; in all other cases it shall denote the False. (Gg, v. I, §7)

with the corresponding explanation in *Begriffsschrift*:

... [L]et

$$\vdash A \equiv B$$

mean that *the sign A and the sign B have the same conceptual content, so that we can everywhere put B for A and conversely.* (Bg, §8, emphasis in original)

There is simply no hint of a meta-linguistic view of identity in the explanation given in *Grundgesetze*. On the contrary, Frege’s view in *Grundgesetze* is explicitly that identity is a binary relation like any other:

The functions of two arguments  $\xi = \eta$  and  $\xi > \eta$  always have a truth-value as value (at least if the signs “=” and “>” are appropriately defined). Such functions it will be appropriate to call *relations*. In the first relation, for example, 1 stands to 1, and in general every object to itself; in the second, for example, 2 stands to 1. (Gg, v. I, §4)

Identity is a two-place, first-level function, one whose arguments are ordinary objects—for example, numbers—and whose value is always a truth-value, just like the relation *greater than*. This fact is essential to Frege’s justification of Leibniz’s Law:

According to §7 the function of two arguments  $\xi = \zeta$  always has as value a truth-value, viz., the True if and only if the

$\zeta$ -argument coincides with the  $\xi$ -argument. If  $\Gamma = \Delta$  is the True, then

$$\underbrace{\quad}_{\mathfrak{F}} \begin{array}{l} F(\Gamma) \\ F(\Delta) \end{array}$$

is also the True; i.e., if  $\Gamma$  is the same as  $\Delta$ , then  $\Gamma$  falls under every concept under which  $\Delta$  falls; or, as we may also say: then every statement that holds for  $\Delta$  holds also for  $\Gamma$ . But also conversely. . . . Thus,  $\Gamma = \Delta$  is always the same truth-value as

$$\underbrace{\quad}_{\mathfrak{F}} \begin{array}{l} F(\Gamma) . \\ F(\Delta) \end{array}$$

(Gg, v. I, §20)

The *Begriffsschrift* theory of identity would render this argument incoherent. If the arguments to identity were names, rather than objects from the domain, the arguments of the concept indicated by the variable  $F$  would also have to be names. But that they simply are not: The arguments of the concept indicated by such a variable are objects in the domain, in this case, either truth-values or value-ranges.

Leibniz's Law of course appears in *Begriffsschrift*, too, as proposition (52). Frege there glosses it as follows:

The case in which the content of  $c$  is identical with the content of  $d$  and in which  $f(c)$  is affirmed and  $f(d)$  is denied does not take place.

Note how Frege must struggle with use and mention here. Frege is just not sensitive to such matters in *Begriffsschrift*. He says in §8 that "all signs. . . stand at times for their content, at times for themselves", but he seems unaware how problematic that statement is. He never makes any effort to explain what precisely it might mean or, crucially, how it might be understood when these signs are not names but variables. Indeed, if I had to guess, I would say that Frege's dissatisfaction with his old theory of what identity is emerged from reflection on just this sort of problem: As a theory of the semantics of identity, the *Begriffsschrift* view is completely inadequate. The identity-sign does not occur only in construction with names, but also in construction with variables. So how is a sentence such as ' $\forall x(x = 0 \rightarrow Fx)$ ' to be understood, on the *Begriffsschrift* view? What precisely is the variable supposed to range

over? Its two occurrences—once as argument of the identity-sign, once as argument of ‘ $F\xi$ ’—place incompatible demands on its range if identity is a relation between names.<sup>2</sup>

There is another piece of evidence that is worth mentioning, though it is somewhat less reliable than those we have just been discussing. In his lengthy 1910 exposition of Frege’s work, Philip Jourdain writes:<sup>3</sup>

We may sum up the advances made by Frege from 1879 [*Begriffsschrift*] to 1893 [*Grundgesetze*] as follows: . . . Fourthly, the sign ‘=’ ceased to be considered as the expression of a relation between *names*. (PMC, p. 204)

As is well known, Frege provided Jourdain with extensive comments on this piece, some of which were published with it as (sometimes very long) footnotes. It seems unlikely that Frege would not have corrected such a horrible misunderstanding of his position, if that were what it was.

## 2 Frege on Identity-Statements

So, clearly, Frege had abandoned the *Begriffsschrift* theory of identity—understood as a theory about what relation identity *is*—by the time he wrote *Grundgesetze*.

T&C might, however, simply concede this point: It amounts to abandoning what they call the ‘name view’ in favor of what they call the ‘hybrid view’ (Thau and Caplan, 2001, pp. 169–70, fn. 21). Their main

<sup>2</sup> For some relevant discussion, see my paper “*Grundgesetze der Arithmetik* I §10” (Heck, 1999).

<sup>3</sup> Another possible piece of evidence is found in the Introduction to *Grundgesetze*, where Frege is detailing the “internal changes” he has made to his formal language since *Begriffsschrift*. He writes:

The primitive signs used in *Begriffsschrift* occur here also, with one exception. Instead of the three parallel lines I have adopted the ordinary symbol of equality, since I have persuaded myself that it has in arithmetic precisely the meaning that I wish to symbolize. That is, I use the word ‘equal’ to mean the same as ‘coinciding with’ or ‘identical with’; and the sign of equality is actually used in arithmetic in this way. (Gg, v. I, p. ix)

One might take this passage to imply that Frege’s conception of what identity was, in *Begriffsschrift*, has since been jettisoned and, with it, his old sign for identity of content. But the question is complicated by the fact that the change in question may have more to do with Frege’s understanding of the notion of arithmetical equality than with his understanding of identity. Jourdain explicitly lists this change of mind as the second of Frege’s “advances”, separately from the fourth.

thesis is that “Frege never rejects the view that identity-statements *express meta-linguistic thoughts*” (Thau and Caplan, 2001, p. 163, my emphasis): Their view is thus not really one about what Frege takes identity to be, but about what he takes identity-statements to express. They might say, further, that dividing his earlier notion of ‘conceptual content’ into thought and truth-value allowed Frege to hold precisely the combination of views they would then be attributing to him: He could have rejected his old view of what identity *was* while retaining his old view about what identity-statements *express*; the two can now come apart, in a way they could not before. So let us consider this interpretation in its own right.

It is standardly supposed that the puzzle Frege uses to motivate the distinction between sense and reference has nothing special to do with identity. Frege does, of course, often mention identity-statements when introducing that distinction. The most famous example is perhaps this one:

- (1) The Morning Star is identical with the Evening Star.

But the problem can equally well be illustrated using these two sentences:

- (2) The Morning Star is a planet with a shorter period of revolution than the Earth.
- (3) The Evening Star is a planet with a shorter period of revolution than the Earth.

The problem, in this case, is not, of course, that one of these is informative and the other is not. The problem is that they are informative in different ways. As I would put it, they *differ* in cognitive value:<sup>4</sup> To know that the Morning Star is a planet with a shorter period of revolution than the Earth is one thing; to know that the Evening Star is a planet with a shorter period of revolution than the Earth is another. But that is the same problem that is illustrated by (1). What is puzzling, after all, is not simply that (1) should be informative—there is nothing obviously

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<sup>4</sup> Robert May has suggested to me that Frege himself speaks of cognitive value only in binary terms: Thoughts either have it or lack it. Most of his uses of this terminology probably do fit the binary reading, but I am not sure all do. See, in particular, the remark in the letter to Peano quoted below (note 7 and the accompanying text). In any event, we need some term to be used as I am using “cognitive value” here, as admitting of degrees. Since this use of the term has gained some currency, I shall continue so to use it.

puzzling about that, in itself—but rather that (1) should be informative, although

(4) The Morning Star is identical with the Morning Star.

is not. What is puzzling is, once again, that (1) and (4) *differ* in cognitive value. The problem is thus uniformly that cognitive value is not preserved by substitution of co-referential expressions, even in referentially transparent contexts. That, or so it is standardly said, is Frege's puzzle about cognitive value.

If so, however, T&C's interpretation looks to be in trouble. Even if we could solve the special case regarding identity-statements—the puzzle, as it arises with respect to (1) and (4)—by taking them to express meta-linguistic propositions about the names occurring in them, that would do nothing to solve the general case of the puzzle. In order to generalize the solution for identity-statements so as to apply to (2) and (3), one would have to regard them as expressing meta-linguistic propositions, too: A meta-linguistic view of what identity-statements express would then emerge as a special case of a meta-linguistic view of the thoughts expressed by *all* sentences containing proper names. But whatever the merits of that sort of view, there is no trace of such a view in Frege's writings. If that were Frege's view, he would not have said, in *Begriffsschrift*, that names “stand at times for their content, at times for themselves” (Bg, §8). On the contrary, they would always stand for themselves.

T&C are not unaware of objections of roughly this kind: Section VII of their paper constitutes an attempt to answer them. Their response, ultimately, is that “the problem about cognitive value. . . and the problem about identity. . . are not the same problem” (Thau and Caplan, 2001, p. 195). My initial reaction to this remark was to wonder what the problem about identity is supposed to be if not the problem about cognitive value. So far as I can tell, the only puzzle about identity that Frege discusses in the opening paragraph of ‘On Sense and Reference’ concerns a difference of cognitive value between sentences such as (1) and (4). If so, it is a special, and especially impressive, instance of the problem about cognitive value, not a different one.<sup>5</sup>

<sup>5</sup> Of course, in one sense, the problem about identity is a different problem: It is the problem we just finished discussing, about what identity is. But the point here is that what makes that problem pressing, in Frege's mind, is the problem about cognitive value.

After further reflection, however, I decided that T&C's understanding of the problem about cognitive value is different from mine. They say repeatedly that the problem about cognitive value concerns how two *sentences* with the same *truth-value* can have different cognitive values—tell us different things or express different thoughts. What they mean to deny is that the problem concerns (2) and (3) any more than it concerns a pair of sentences such as ' $2^2 = 4$ ' and ' $2 > 1$ ': The "problem arises for [these sentences] simply because they have the same truth-value" (Thau and Caplan, 2001, p. 193). So, according to T&C, the problem about cognitive value does not concern whether cognitive value is preserved under substitution of co-referential expressions, as Nathan Salmon and pretty much everyone else thinks it does (Thau and Caplan, 2001, p. 193, fn. 37).

I take it, then, that the view is this: The notion of sense is introduced to allow Frege to explain how, say, ' $2^2 = 4$ ' and ' $2 > 1$ ' can differ in cognitive value, though they have the same truth-value; the notion of sense is *not* to be used to explain why (1) and (4) differ in cognitive value. More precisely, Frege does not think he can explain why (1) and (4) express different thoughts simply in terms of the fact that the sense of 'the Morning Star' differs from that of 'the Evening Star'. For this latter purpose, Frege also appeals (and presumably thinks that he must also appeal) to the claim that identity-statements express meta-linguistic thoughts. Difference of sense may play some role, but it cannot be the whole story.

In support of their interpretation of the problem about cognitive value, T&C (2001, p. 192) quote a passage from *Function and Concept*:

[W]hat ' $2^2 = 4$ ' means is the True just as, say, ' $2^2$ ' means 4. Accordingly ' $2^2 = 4$ ', ' $2 > 1$ ', ' $2^4 = 4^{2^2}$ ', all mean the same thing, viz. the True, so that in ' $(2^2 = 4) = (2 > 1)$ ' we have a correct equation.

The objection here suggests itself that ' $2^2 = 4$ ' and ' $2 > 1$ ' nevertheless tell us quite different things, express quite different thoughts; but likewise ' $2^4 = 4^{2^2}$ ' and ' $4 \times 4 = 4^{2^2}$ ' express quite different thoughts; and yet we can replace ' $2^4$ ' by ' $4 \times 4$ ', since both signs have the same meaning. Consequently, ' $2^4 = 4^{2^2}$ ' and ' $4 \times 4 = 4^{2^2}$ ' likewise have the same meaning. We see from this that from sameness of meaning there does not follow sameness of thought expressed. (FC, pp. 144–5, opp. 13–4)

T&C take this passage to support their interpretation because they think

Frege is here illustrating the problem about cognitive value not just with identity-statements, though he does mention them, but also with the pair ' $2^2 = 4$ ' and ' $2 > 1$ '. So "[t]he problem... is not a problem about identity per se" (Thau and Caplan, 2001, p. 193). But this is a terrible misreading.

Just before this passage, Frege has been arguing that the meaning of a sentence is its truth-value. But now his interlocutor objects that, since ' $2^2 = 4$ ' and ' $2 > 1$ ' do not express the same thought, they cannot have the same meaning (that is, reference). Frege's response is that, if we think that pairs of sentences like ' $2^4 = 4^2$ ' and ' $4 \times 4 = 4^2$ ' express different thoughts, we are already committed—by a principle of compositionality to which Frege tacitly appeals but does not state—to allowing that two sentences can have the same meaning but express different thoughts. Frege is thus not, as T&C believe, using the two sentences ' $2^4 = 4^2$ ' and ' $4 \times 4 = 4^2$ ' to provide another example of a problem he has already illustrated using ' $2^2 = 4$ ' and ' $2 > 1$ '. Rather, he uses ' $2^4 = 4^2$ ' and ' $4 \times 4 = 4^2$ ' in an argument that is designed to answer an objection to his view of what sentences denote. The problem of cognitive value is thus not even raised in the passage T&C quote, let alone illustrated by ' $2^2 = 4$ ' and ' $2 > 1$ '.

There is thus no direct textual support for T&C's new interpretation of the problem of cognitive value. Nor, I shall now argue, does it withstand critical scrutiny. Frege certainly does think that ' $2^2 = 4$ ' and ' $2 > 1$ ' have the same meaning but express different thoughts. And he does need an explanation of how they can do so. But that fact poses no threat to the usual interpretation of the problem of cognitive value. Indeed, as the passage we have been discussing makes clear, Frege also thinks that ' $2^4 = 4^2$ ' and ' $4 \times 4 = 4^2$ ' express different thoughts, and he is acutely aware that he needs an explanation of why they do so. The two sentences immediately following the passage just quoted make it clear, moreover, that Frege is aware of the now familiar generalization of the problem about identity to one about substitution:

If we say 'the Evening Star is a planet with a shorter period of revolution than the Earth', the thought we express is other than in the sentence 'the Morning Star is a planet with a shorter period of revolution than the Earth'; for somebody who does not know that the Morning Star is the Evening Star might regard one as true and the other as false. And yet both sentences must mean the same thing: for it is just a matter

of interchange of the words ‘Evening Star’ and ‘Morning Star’, which mean the same thing, i.e. are proper names of the same heavenly body. (FC, p. 145, op. 14)

And in a letter to Peano, written a few years later, Frege explicitly characterizes the problem about cognitive value, even as it arises in the case of identity-statements, in terms of substitution.<sup>6</sup>

In the proposition, ‘The evening star is the same as the evening star’ we have only [a mere instance of the principle of identity]; but in the proposition ‘The evening star is the same as the morning star’ we have something [with a higher cognitive value]. How can the substitution of one proper name for another designating exactly the same heavenly body effect such changes? (PMC, p. 127, letter xxxiv/11)

Frege is thus clearly aware that the problem of cognitive value generalizes from the problem he had discussed in *Begriffsschrift*: He has come to realize that the problem has nothing particular to do with identity. Indeed, Frege’s use of the sentences ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ as his example in *Function and Concept* serves to remind us that the problem of cognitive value, even as it arises for identity-statements, is more general than the problem illustrated by (1) and (4). Of course, ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ are identity-statements, but neither of them is a triviality without cognitive value: What is significant, once again, is that they differ in cognitive value.

Frege thus wants an explanation of how two sentences can express different thoughts even if “it is just a matter of interchange of . . . proper names of the same” object (FC, p. 145, op. 14). And he has one. In the letter to Peano just cited, Frege briefly explains his distinction between sense and reference and writes:<sup>7</sup>

<sup>6</sup> The bracketed parts of the quote fill in references to an earlier part of the discussion. This particular letter is undated, but is presumably from about 1896.

<sup>7</sup> Similar remarks are contained in a letter to Russell (PMC, pp. 164–5, letter xxxvi/18).

In this passage and the preceding one, Frege is tacitly appealing to principles of compositionality he does not state. Since the meaning of the whole is determined by the meanings of the parts, and since ‘ $2^4$ ’ and ‘ $4 \times 4$ ’ have the same meaning, ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ must have the same meaning. Since the sense of a part is part of the sense of the whole (Gg, v. I, §32), and ‘ $2^4$ ’ and ‘ $4 \times 4$ ’ have different senses, ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ also have different senses.

... [T]he sense of the name ‘morning star’ is indeed different from that of the word ‘evening star’. *And so it happens* that the thought of [‘The evening star is the same as the evening star’] is different from that of [‘The evening star is the same as the morning star’]...; for the thought we express in a proposition is the sense of the proposition. (PMC, pp. 127–8, letter xxxiv/11, my emphasis)

And in *Function and Concept*, he writes, immediately following the passages quoted above:

We must distinguish between sense and meaning. ‘ $2^4$ ’ and ‘ $4 \times 4$ ’ certainly have the same meaning, i.e. are proper names of the same number; but they have not the same sense; *consequently*, ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ mean the same thing, but have not the same sense (i.e., in this case [viz., the case of a sentence]: they do not contain the same thought). (FC, p. 145, op. 14)

Frege could hardly be more explicit that an explanation is being offered here. In both passages, we are told, in effect, that the thoughts expressed by certain sentences differ *because* the senses of the names contained in them differ.

Now, as said above, T&C need not deny that the fact that ‘ $2^4$ ’ and ‘ $4 \times 4$ ’ have different senses plays *some* role in Frege’s explanation of why ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ express different thoughts. What they must deny is that the explanation appeals *only* to this fact and does not also appeal to the claim that identity-statements express meta-linguistic thoughts. The problem, however, is that Frege clearly thinks that the problem illustrated by ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ can equally well be illustrated by ‘The Morning Star is a planet with a shorter period of revolution than the Earth’ and ‘The Evening Star is a planet with a shorter period of revolution than the Earth’. Now, T&C nowhere say how they think Frege would have explained why these sentences—(2) and (3)—express different thoughts. Plainly, however, they must deny that he would have done so simply in terms of the fact that ‘the Morning Star’ and ‘the Evening Star’ have different senses. Since, as we have seen, Frege himself regards these two puzzles as one, it would just have been too obvious that such a solution would generalize to the case of identity-statements, thus making the meta-linguistic view of identity unnecessary. To put the point differently, the context in which the preceding passage

occurs makes it undeniable that Frege would have been equally happy to say the following:

‘The Morning Star’ and ‘the Evening Star’ certainly have the same meaning, i.e. are proper names of the same object; but they have not the same sense; *consequently*, ‘the Morning Star is a planet with a shorter period of revolution than the Earth’ and ‘the Evening Star is a planet with a shorter period of revolution than the Earth’ mean the same thing, but have not the same sense.

What we are being offered here is thus a uniform explanation of how the substitution of one name for another with the same reference can change the thought expressed. There is no suggestion that, in the case of identity-statements, the explanation must or should appeal to something beyond the fact that those names may have different senses.<sup>8</sup>

T&C have mistaken what’s puzzling Frege because they are concentrating on the easy case rather than the hard case. It is true enough that Frege thinks ‘ $2^2 = 4$ ’ and ‘ $2 > 1$ ’ express different thoughts. But to characterize the problem about cognitive value as a problem that concerns pairs like this one, rather than pairs like ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’, is to miss its point. If that were what the problem were about, it would be obscure why Frege even bothers to distinguish the senses of ‘ $2^4$ ’ and ‘ $4 \times 4$ ’. No such distinction is needed to explain why ‘ $2^2 = 4$ ’ and ‘ $2 > 1$ ’ express different thoughts. That is shown by the fact that this is not the problem that frustrates Russellians. On the other hand, the claim that ‘ $2^4$ ’ and ‘ $4 \times 4$ ’ have different senses *is* needed to explain why ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ express different thoughts. The case of ‘ $2^2 = 4$ ’ and ‘ $2 > 1$ ’, then, is the easy case; the case of ‘ $2^4 = 4^2$ ’ and ‘ $4 \times 4 = 4^2$ ’ is the hard case; and the fact that the problem also concerns such pairs as ‘ $2^4 > 3$ ’ and ‘ $4 \times 4 > 3$ ’ makes any view that appeals to peculiarities in the meaning of identity-statements insufficiently general. Solutions to the easy case thus do not generalize to the hard case. That is what makes the problem of cognitive value interesting.

So there is no saving T&C’s interpretation by distinguishing the problem about identity from the problem about cognitive value.

There is yet other textual evidence against T&C’s interpretation. Frege’s explanations of the symbols of his formal language, which are

<sup>8</sup> Of course, as said above, someone committed to the view that, for Frege, identity-statements express meta-linguistic thoughts could yet insist that (2) and (3) express meta-linguistic thoughts. But that would be sheer desperation.

given throughout Part I of *Grundgesetze*—including, of course, his explanation of the identity-sign, given in section 7 and quoted above—are supposed to specify not only the referents of the various expressions of that language, but also their *senses*. As Frege writes in *Grundgesetze*:

[N]ot only a denotation, but also a sense, appertains to all names correctly formed from our signs. Every such name of a truth-value *expresses* a sense, a *thought*. Namely, by our stipulations it is determined under what conditions the name denotes the True. The sense of this name—the *thought*—is the thought that these conditions are fulfilled. (Gg, v. I, §32, emphasis in original)

Consider, then, ‘ $0 = 1$ ’. Under what condition, as determined by the stipulations, will this sentence denote the True? It will—in light of the stipulation regarding identity and Frege’s definitions of the symbols ‘0’ and ‘1’—denote the True if, and only if, a certain value-range—the extension of the concept ‘ $\xi$  is a value-range whose members can be put in one-one correspondence with those of  $\epsilon(\epsilon \neq \epsilon)$ ’—is the same as a certain other value-range—the extension of the concept ‘ $\xi$  is a value-range whose members can be put in one-one correspondence with those of  $\epsilon(\epsilon = 0)$ ’. The thought it expresses is thus that zero is one, not that ‘0’ denotes the same object as ‘1’.

Frege goes on to say that “[i]t will be the reader’s task to make clear to himself” what thoughts are expressed by the various formal sentences that occur in his proofs. In fact, Frege often reformulates the thoughts expressed by these sentences in German, thereby making good on his promise to help his reader “as much as possible at the outset”. Unsurprisingly, several of these formulae contain the identity-sign and, to the best of my knowledge, nowhere in Frege’s reformulations do we find any indication that ‘ $0 = 1$ ’ is actually about names rather than numbers. What we find is, on the contrary, what the standard interpretation would lead one to expect:<sup>9</sup>

The number zero is different from the number one.

$$\vdash \neg(0 = 1)$$

<sup>9</sup> These are taken from the Table of Important Theorems, at the end of Volume I. The translations are my own.

If no object falls under a concept, then zero is the number of objects falling under this concept.

$$\begin{array}{l} \vdash \text{---} 0 = \mathcal{N}\hat{\epsilon}(F\epsilon) \\ \vdash \text{---} Fx \end{array}$$

Contrast his reformulations in *Begriffsschrift*:<sup>10</sup>

The case in which the content of  $c$  is identical with the content of  $d$  and in which  $f(c)$  is affirmed and  $f(d)$  is denied does not take place.

$$\begin{array}{l} \vdash \text{---} f(d) \\ \vdash \text{---} f(c) \\ \vdash \text{---} c \equiv d \end{array}$$

The content of  $c$  is identical with the content of  $c$ .

$$\vdash c \equiv c$$

No trace of such language is to be found in *Grundgesetze*.

The real problem with T&C's interpretation, however, is not that it fails to fit the handful of passages just mentioned. The real problem is that identity-statements play an absolutely fundamental role in Frege's philosophy of mathematics, a role of which T&C's interpretation can make no sense. The central sections of *Die Grundlagen der Arithmetik*, sections 62–69, in particular, constitute a sustained discussion of numerical identities and how they are to be explained. That is not just because identity-statements are an important sort of arithmetic statement. It is because Frege thinks, as he writes in the heading to these sections, that “[t]o obtain the concept of number, we must fix the sense of a numerical identity”. Since there is no more central task in the book than characterizing the concept of number, identity-statements could hardly be more important. There is no trace at all of Frege's old view of identity or identity-statements in these sections. On the contrary, in §66, where he is introducing the so-called Julius Caesar problem, we find Frege writing: “In the proposition ‘the direction of  $a$  is identical with the direction of  $b$ ’ the direction of  $a$  plays the part of an object. . .”. More importantly, on the *Begriffsschrift* view, abstraction principles, such as the crucial

<sup>10</sup> These two examples are from sections 20 and 21, respectively.

The number of *F*s is identical with the number of *G*s iff the *F*s and *G*s are in one-one correspondence,

contain an odd sort of use-mention confusion. It is utterly unobvious how explaining numerical identities in this way could serve to explain the concept of number, if the identity-statements themselves were not about numbers but about numerals: How could 'line *a* is parallel to line *b*' be "taken as an identity"—as the identity 'the direction of line *a* is identical with the direction of line *b*'—if identity-statements were not about lines at all but about their names? T&C owe an interpretation of *Die Grundlagen* that accords with their interpretation of Frege's views about identity. I very much doubt one is forthcoming.<sup>11</sup>

### 3 Caplan and Thau's Textual Evidence

As noted earlier, T&C cite six passages in which, according to them, Frege more or less explicitly states his continued allegiance to the view that identity-statements express meta-linguistic thoughts. In this section, I shall argue that these passages provide no support for that interpretation. I shall take up the question whether "the text of 'Sense and Reference' *itself* gives the lie to the standard interpretation" (Thau and Caplan, 2001, p. 163, their emphasis) afterwards.

Five of the six passages T&C cite come from Frege's criticisms of formalism.<sup>12</sup> The formalists, as Frege understood them, held that all arithmetical statements—including, of course, arithmetical statements of identity—are about numerals rather than about numbers. In the passages that T&C cite, Frege is precisely *not* agreeing with this aspect of the formalists' view: On the contrary, he is emphasizing that, on his view, identity is not a relation between signs but a relation between objects, namely, the objects denoted by the signs flanking the identity-sign. (For the formalists, the denotation of these signs, if they have any, is irrelevant for mathematics.) There is no reason to worry if Frege sometimes puts this point by saying that identity-statements *express* or *state* that these names denote the same object. Neither the

<sup>11</sup> For a discussion of how Frege's views about identity evolve with his conception of his logicism, see Robert May's "Frege on Identity Statements" (May, 2001).

<sup>12</sup> These are the passage from *Function and Concept*, the one from 'Formal Theories of Arithmetic', and the three from *Grundgesetze*. See note 1 for references. For a detailed, and I think largely accurate, discussion of the relation between Frege's attacks on formalism and the development of his views about identity, see May's paper "Frege on Identity Statements" (May, 2001).

word translated 'express' nor that translated 'state' has an established technical meaning for Frege.

Of course, the phrase 'express the thought' has a well-known technical meaning, but even the following remark would not worry me:

An identity-statement does not express the thought that the name on its left-hand side is identical with the name on its right-hand side. Nor does such a statement express the thought that the sense of the name on its left-hand side is the same as the sense of the name on its right-hand side. Rather, an identity-statement expresses the thought that the *denotation* of the name on its left-hand side is identical with the *denotation* of the name on its right-hand side.

No such passage occurs in Frege's writings: It is, rather, what I always tell my students when I lecture on 'On Sense and Reference'.<sup>13</sup> In so speaking, I am employing a means of expression that, while potentially misleading, I suppose, is understandable in the context, and I don't expect it in fact ever has misled any of my students. The point of the remark is obviously that identity-statements are about the *objects* that the names occurring in them denote. It is, moreover, difficult to know how one might express this point—generalizing, as it does, over all identity-statements—without resorting to the formal mode, that is, to semantic ascent.<sup>14</sup> It would be badly over-reading to suppose that the author of such a remark thereby committed himself to the view that identity-statements express meta-linguistic thoughts: By itself, the occurrence of the phrase 'express the thought' is completely insufficient to support such an attribution.

Moreover, in *Function and Concept*, one of the papers from which T&C draw their evidence, we find these two passages:

... [B]y writing ' $\vdash 2 + 3 = 5$ ' we assert that  $2 + 3$  equals 5. (FC, p. 149, op. 22)

Accordingly, e.g., ' $\neg(2^2 = 5)$ ' stands for the True, and we may add the assertion-sign: ' $\vdash \neg(2^2 = 5)$ '; and in this we assert

<sup>13</sup> There is, however, a similar passage in the letter to Peano that T&C claim "makes it almost out of the question that Frege is rejecting the name view in the opening paragraph of 'Sense and Reference'" (Thau and Caplan, 2001, p. 169). But the context makes it clear that Frege is attempting to forestall any misimpression that, according to him, an identity-statement expresses identity of *sense*. The fact that Frege uses the verb '*zu meinen*' is not, by itself, significant.

<sup>14</sup> Thanks to Kent Bach and Robert May for emphasizing this point to me.

that  $2^2 = 5$  is not the True, or that  $2^2$  is not 5. (FC, p. 150, op. 23)

Here, Frege talks of identity-statements as *asserting* propositions that are not meta-linguistic. He comes close to using both forms of expression in this passage:

... [I]f I wrote ' $(2 + 3 = 5) = (2 = 2)$ ' and presupposed that we knew  $2 = 2$  to be the True, I should not have asserted thereby that the sum of 2 and 3 is 5; rather I should only have designated the truth-value of ' $2 + 3 = 5$ 's *designating the same as* ' $2 = 2$ '. We therefore need another special sign to be able to assert something as true. For this purpose I let the sign '⊢' precede the name of the truth-value, so that for example in '⊢  $2^2 = 4$ ' it is asserted that the square of 2 is 4. (Gg, v. I, §5)

Now, I do not myself take these passages to support the attribution to Frege of any specific view about identity: I think it unwise to put much weight on Frege's choice of words here. But such passages ought to be worrying to T&C: These passages obviously provide as much support for the standard interpretation as the passages they cite provide for theirs. And there are about the same number of each—so long as we don't count Frege's verbal reformulations of theorems of *Grundgesetze*. If we do—and I don't see why we shouldn't—then there are overwhelmingly more passages supporting the standard interpretation.

T&C think, however, that a careful reading of the end of the first paragraph of 'On Sense and Reference' itself demonstrates that Frege must be defending, not attacking, the view that identity is a relation between names. But their objection to the standard reading is unconvincing. As most do, I take Frege to suppose that he has, at the end of what T&C call section III, reduced his old view to absurdity: If it were correct, identity-statements "would no longer refer to the subject matter, but only to its mode of designation"; no *astronomical* knowledge—that being the sort of thing Frege means by proper knowledge—would be expressed by 'Hesperus is Phosphorus', because no reference to celestial bodies would even be made in this sentence.<sup>15</sup> "But", Frege says—reiterating a point that he makes at the beginning—"in many cases this is just what we want to do." And with that, Frege ends his criticism, having contradicted an established consequence of his target.

<sup>15</sup> This wonderful way of putting the point is due to David Kaplan (1990, p. 118).

Now, what is T&C's problem with this reading? One of their worries<sup>16</sup> is that, if the standard interpretation is correct, then the next sentence—'If the sign "a" is distinguished. . .', which begins what they call section IV—"constitutes a fantastic non sequitur". Specifically, Frege would not "have made it clear that his discussion of the name view had come to an end and that the defects of the view that the discussion revealed force a reconsideration of object view", the view that identity is a relation between objects. But it is really a bit much to say that, if so, Frege would have been "a numbingly disorganized thinker" (Thau and Caplan, 2001, p. 185).<sup>17</sup> Maybe his signposting could have been better. But *most* of Frege's readers—namely, those of us who have been reading Frege as *rejecting* the *Begriffsschrift* view all these years—have been clear enough about this aspect of the paragraph's structure. If T&C are not, I'm not sure that's Frege's fault. (I think we can all agree that other aspects of 'On Sense and Reference' could have been clearer.)

Another concern that T&C express is that Frege is here charging his old view with the very same flaw against which he had defended it in *Begriffsschrift*, without indicating why he now takes it to be unable to meet the objection (Thau and Caplan, 2001, pp. 177, 183). But this question will only seem pressing if one assumes that Frege's *reason* for abandoning his old view is that he no longer takes it to be able to meet those objections. If one does not assume that, there's no cause for concern. And Frege's main reason for abandoning his old view, or so it seems to me, was that he came to realize that the puzzle that seemed to require it is really a more general problem about substitution, a problem that has nothing special to do with identity and which the meta-linguistic view of identity therefore cannot solve.<sup>18</sup> If so, I don't see what's wrong with supposing Frege's old response to these objections always seemed

<sup>16</sup> They are also worried about why the word 'essentially' occurs in the first sentence of what they call section IV. But maybe it's because the sentence reads better that way. Or perhaps Frege is simply wary of making the strong claim that the cognitive value would, in this case, be *exactly* the same, a claim he just doesn't need. In any event, T&C are surely wise not "to rest too much of [their] case on Frege's use of the word 'essentially'" (Thau and Caplan, 2001, p. 185).

<sup>17</sup> The way in which this charge is over-stated is unfortunately characteristic of the tone of T&C's paper: Many of their arguments evaporate when the rhetoric is replaced by a balanced statement of what they have actually shown. This particular charge is especially surprising, given that T&C are forced to the view that, when Frege writes at the end of 'On Sense and Reference', "Let us return to our starting point", he does not actually return us to our starting point (Thau and Caplan, 2001, pp. 194–5).

<sup>18</sup> Once again, for an extensive discussion of the evolution of Frege's views about identity-statements, see May's "Frege on Identity Statements" (May, 2001).

wrong to him, but, lacking anything else to say, he bit that particular bullet, and now that he's got an alternative, he's going to take it. But the question might nonetheless seem puzzling: Why *does* Frege take the view he had defended in *Begriffsschrift* to be unable to answer the objections he brings against it in 'On Sense and Reference', when he defends it against those very same objections in *Begriffsschrift*? The question, however, has a false presupposition, for Frege is *not* pressing the same objection both times.<sup>19</sup> The difference is subtle—it's almost just one of emphasis—but it is nonetheless real.

Frege's objection to the *Begriffsschrift* view in 'On Sense and Reference' is that it implies that identity-statements do not "refer to the subject matter, but only to its mode of designation" (SM, p. 157, op. 26): If that view were correct, identity-statements would never be about matters geometrical, but only about matters linguistic. Nothing in Frege's discussion in *Begriffsschrift* answers this particular charge. On the contrary, in *Begriffsschrift*, the objection is that the meta-linguistic view implies that identity-statements concern only "irrelevant question[s] of form" (Bg, §8), and Frege there responds by explaining why we need identity-statements in geometry, despite the fact that they concern signs rather than lines. To make the difference of emphasis clear: In *Begriffsschrift*, the charge is that, if the meta-linguistic view is correct, then identity-statements concern *irrelevant* questions of form; in 'On Sense and Reference', the charge is that identity-statements concern *questions of form*. Frege's response in *Begriffsschrift* just concedes that identity-statements concern questions of form, but he insists that these questions arise naturally within mathematics and so are not irrelevant. That does not even begin to answer the objection he raises in 'On Sense and Reference', which I myself would regard as conclusive.

## 4 Closing

My sense, for what it is worth, is that T&C's interpretation derives from their pressing natural questions that arise from two common misconceptions about 'On Sense and Reference'. What we learn from their

<sup>19</sup> Another option would be to hold, as May (2001) does, that Frege is not criticizing the *Begriffsschrift* view in 'On Sense and Reference' at all, but a different view. I'm not sure May is wrong, but I'm not sure he's right, either, so I offer the response in the text as an alternative or supplement, as the case may be.

My reading of the relevant parts of *Begriffsschrift* contradicts T&Cs, for which see section IV of their paper.

discussion is thus just how problematic these misconceptions can be. Let me close by making them explicit.

Consider this thesis:

- (5) ‘Hesperus is Phosphorus’ expresses the thought that the referent of ‘Hesperus’ is the same as the referent of ‘Phosphorus’.

I’ve not done a poll, but I think many suppose that ‘On Sense and Reference’ is, at least in part, directed against this sort of claim. But what T&C observe, quite correctly, is that there is no argument against (5) in ‘On Sense and Reference’. That realization might make one think Frege accepts (5). But that would simply be an understandable mistake, one deriving from a failure to remember that what is at issue in the first paragraph of ‘On Sense and Reference’ is what identity is. The options Frege considers are:

- (6) Identity is a relation between *objects*: Hesperus stands in it to Phosphorus if, and only if, Hesperus is the very same object as Phosphorus.
- (7) Identity is a relation between *names*: ‘Hesperus’ stands in it to ‘Phosphorus’ if, and only if, ‘Hesperus’ and ‘Phosphorus’ are intersubstitutable *salva veritate*.

Which of these conceptions of identity is present in (5)? The relation expressed by the words ‘is the same as’, as they are used in (5), clearly takes *objects* as arguments: Despite its use of meta-linguistic descriptions to refer to them, then, (5) treats identity as a relation between objects and so embodies the conception of identity in (6). A thesis in the spirit of (5) but embodying the conception of identity in (7) is:

- (8) ‘Hesperus is Phosphorus’ expresses the thought that ‘Hesperus’ and ‘Phosphorus’ are intersubstitutable *salva veritate*.

It is this sort of view that Frege is concerned to reject in ‘On Sense and Reference’. Again, however, that does not mean that he accepts (5). As we saw earlier, he does not: Unless it is generalized to the view that *all* sentences containing proper names express meta-linguistic thoughts, it fails to solve the generalized problem of cognitive value. And, as said above, Frege *never* held that view.

So the first common misconception from which T&C proceed is that (5) is one of Frege’s targets in ‘On Sense and Reference’. The other is that

Frege is arguing in ‘On Sense and Reference’ that names are associated with modes of presentation: In particular, in what T&C call section IV, Frege is arguing that, if an identity-statement is to contain “proper knowledge”, the names occurring in it must be associated with different modes of presentation (Thau and Caplan, 2001, p. 184). Now, if one reads ‘On Sense and Reference’ that way, then section 8 of *Begriffsschrift* can only come as a shock, perhaps enough of a shock to make one wonder what exactly Frege has changed his mind about. But Frege is *not* arguing that different modes of presentation must be associated with two names if the identity-statement conjoining them is not to be empty of cognitive value. Frege simply *asserts* that the modes of presentation must be different and then, using an example not very different from that used in *Begriffsschrift*, proceeds to illustrate what he means. What distinguishes Frege’s new view from his old one is thus not that he now takes names to be associated with modes of presentation: That he held already in 1879, and it is not terribly controversial. The difference lies in how Frege thinks names are *related* to modes of presentation. In *Begriffsschrift*, the mode of presentation associated with a name is explicitly *not* an aspect of its ‘conceptual content’: The whole point in *Begriffsschrift* is that names with the *same* content may be associated with *different* modes of presentation. What is new in 1892—and what is quite definitely controversial—is the view that a name’s association with a particular mode of presentation *is* an aspect of its content, one that by compositional operations becomes part of the content of any sentence containing it. The real challenge for Frege’s readers, it seems to me, is to understand what his argument for *that* conclusion might be.<sup>20</sup>

It’s comforting that orthodoxy should have things right, at least this once.<sup>21</sup>

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<sup>20</sup> For some ideas, see my papers “The Sense of Communication” (Heck, 1995) and “Do Demonstratives Have Senses?” (Heck, 2002).

<sup>21</sup> Thanks to Kent Bach, Robert May, Øystein Linnebo, Michael Rescorla, and Jason Stanley for discussion and for comments upon earlier drafts of this material. The comments of an anonymous referee also proved helpful.

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