Chains of Inferences and the New Paradigm in the Psychology of Reasoning

Abstract: The new paradigm in the psychology of reasoning draws on Bayesian formal frameworks, and some advocates of the new paradigm think of these formal frameworks as providing a computational-level theory of rational human inference. I argue that Bayesian theories should not be seen as providing a computational-level theory of rational human inference, where by "Bayesian theories" I mean theories that claim that all rational credal states are probabilistically coherent and that rational adjustments of degrees of belief in the light of new evidence must be in accordance with some sort of conditionalization. The problems with the view I am criticizing can best be seen when we look at chains of inferences, rather than individual inferences. Chains of inferences have been neglected almost entirely within the new paradigm.

1 Introduction

The new paradigm in the psychology of reasoning, which developed over the past 20 years or so, "puts subjective degrees of belief center stage, represented as probabilities" (Elqayam & Over 2013, p. 249; see also Oaksford & Chater 2001; Oaksford & Chater 2007). On the new paradigm, the attitudes involved in reasoning are seen as partial beliefs.[1] Hence, it can hardly be surprising that the new paradigm in the psychology of reasoning is often seen as closely connected to Bayesian epistemology (e.g. Pfeifer & Douven 2014). Indeed, many advocates of the new paradigm in the psychology of reasoning want to use the Bayesian formal apparatus to

[1] I will sometimes also use the terms "subjective probabilities," "degrees of belief," or "credences" for these attitudes.

model human reasoning. As Elqayam and Over (2012, p. 29) put it: "Approaches in the new paradigm vary widely, but what they share is a commitment to psychological principles which fit within a broadly Bayesian paradigm." Advocates of the new paradigm assume that a broadly Bayesian formal apparatus can be used to model human reasoning. In this paper, I will argue that this assumption is mistaken.[2]

Some philosophers hold that Bayesianism is "a theory of consistent probabilistic reasoning [,… that] gives rise automatically to an account of valid probabilistic inference" (Howson & Urbach 2006, p. 301). Such a view might suggest that Bayesianism tells us how agents without cognitive limitations should reason. And indeed, some advocates of the new paradigm psychology of reasoning think that some kind of Bayesian theory should be used as a normative standard for assessing human reasoning (Oaksford & Chater 1998, pp. 307-308). Others hold that some version of Bayesianism is an adequate (or close enough) descriptive theory of the computational level of human reasoning (for references see Elqayam & Over 2012).

In this paper, I argue that Bayesian theories are neither helpful normative theories for understanding human reasoning nor helpful descriptive theories because they cannot adequately describe or evaluate chains of inferences, i.e. series of inferences such that the conclusion of the first is a premise of the second and so on. For my present purposes, a "Bayesian theory" is one that holds

[2] To get a first idea of what I have in mind, notice that the epistemological literature on subjective Bayesianism does not address the question what role, if any, partial beliefs play in human reasoning (see Staffel 2013, p. 3536). Consequently, it is not clear what, if anything, Bayesian epistemology can tell us about reasoning.
One might think that if that is right, then this is a problem for Bayesianism — and not only for the new paradigm psychology of reasoning. Suppose, for example, that we should "avoid talk about knowledge and acceptance of hypotheses, trying to make do with graded belief" (Jeffrey 1970, p. 183; see also Maher 1993, pp. 152-55) — as some Bayesian epistemologists claim we should. Then we would need an account of the rationality of reasoning with partial beliefs, if we want an account of rational reasoning at all. As John Broome has recently put it: "Bayesians owe us an account of the active reasoning processes by which you can bring yourself to satisfy Bayesian requirements" (Broome 2013, p. 208). However, I want to put the question whether it is a problem for Bayesianism that it has little to say about reasoning to one side. Whether or not it is a problem for Bayesianism, it surely is a problem for the new paradigm in the psychology of reasoning.

(a) that all rational credal states are probabilistically coherent and

(b) that rational update of a credal state in light of new evidence must be in accordance with some sort of conditionalization.

As I will argue, such theories cannot provide a computational-level account of (perfectly rational) chains of inferences. At best, they can give us necessary conditions for a chain of inference being rational, but it is unclear how anyone — even an agent with unlimited cognitive resources — could or would meet these conditions in forming a chain of inferences, i.e., it is unclear what computations are performed at the individual steps of such a chain.

Before I begin, I must forestall a potential misunderstanding. The new paradigm in the psychology of reasoning is often described as opposing a traditional paradigm that "anchored psychology of reasoning in classical, bivalent logic" (Elqayam & Over 2012, p. 28). The new paradigm is often seen as superior in handling reasoning under uncertainty, the paradoxes of material implication, and non-monotonic reasoning. In the present context, this contrast can be misleading. It is clear that we want accounts of what reasoning is, of what makes rational reasoning rational, and a descriptive psychological theory of how reasoning happens in humans. To think that classical logic can tell us what constitutes rational reasoning or gives us a computational-level theory of rational reasoning is wrong (see Harman 1986). However, we can acknowledge this and still think that the right descriptive and normative theories of non-monotonic reasoning or reasoning under uncertainty are not based on the concepts of partial belief, probabilistic coherence and conditionalization (see Stenning & van Lambalgen 2009). And these concepts are the cornerstones of Bayesianism. I am attacking the idea that reasoning from a new piece of evidence to a consequence of it should, at the computational level and in the fully rational case, be seen as a transition between probabilistically coherent sets of partial beliefs that crucially involves conditionalization. However, in attacking this idea, I am not advocating a return to a paradigm based on classical logic. We need new ways of thinking about reasoning — in all three respects: what it is, what makes it rational, and how it happens in humans.

Finally, it might be worth pointing out that I am not simply advocating a version of what Elqayam and Evans (2013) call "soft Bayesianism" as opposed to "strict Bayesianism." My point is not that our partial beliefs often do not conform to Bayesian assumptions. My point is that it is unclear how we could describe perfectly rational chains of inferences at the computational level of analysis within a Bayesian framework. Bayesianism, soft or strict, does not tell us anything about the computations that underlie (descriptively) or should underlie (normatively) chains of inferences.

2 The New Paradigm and Degrees of Belief

Let me begin by describing how degrees of belief are used within the new paradigm and what might have led advocates of the new paradigm astray. As I will argue below, the inadequacy of a Bayesian formal apparatus for modeling human reasoning comes out clearest when we look at chains of inferences, i.e. cases in which first a conclusion is drawn from some premises and then this conclusion is used as a premise (and hence as an input) in another inference and so on. Chains of inferences are, however, not discussed in the literature within the new paradigm. Both, theoretical discussions and empirical studies, focus exclusively on cases of single-step inferences.

On the side of theoretical discussion, recent overviews, reviews and critical discussions of the new paradigm in the psychology of reasoning do not mention chains of inferences (see Elqayam & Evans 2013; Elqayam & Over 2012; Evans 2012; Over 2009; Chater & Oaksford 2009; Oaksford & Chater 2001; for a general discussion of Bayesian models in cognitive science see Jones & Love 2011). Discussion focuses on topics like the nonmonotonicity of everyday inferences (Over 2009; Chater & Oaksford 2009), the status of Bayesianism as a normative or a descriptive theory of human reasoning (Elqayam & Evans 2013), or algorithmic level accounts within the new paradigm (Elqayam & Over 2012). The question how to model chains of reasoning simply does not come up.

On the side of empirical studies, research also seems to focus exclusively on single-step inferences. In a typical experiment, e.g., Oaksford, Chater and Larkin (2000) presented subjects with different scenarios that put constraints on the distribution of symbols on cards; they then asked subjects to either rate how likely it is that a certain prediction about the symbols on the cards is correct or to rate the acceptability of a conclusion about the cards given certain premises. These ratings are then taken to reflect the degree of belief the subject has in the conclusion (given the premises). The study was designed to compare four different kinds of single-step inference that subjects could make: *modus ponens*, *modus tollens*, denying the antecedent, and affirming the consequent. The same is true of other empirical investigations of such inferences (e.g. Singmann, Klauer, & Over 2014). Questions about chains of inferences do not arise in the context of such studies. The same holds for Chater and Oaksfords (1999) account of syllogistic reasoning. They gave subjects two premises and four possible conclusions, and subjects have to tick a box next to the conclusions they think follow from the two premises. Here again Chater and Oaksfords only discuss single-step inferences, and they do so in terms of the notion of *p*-validity that I shall discuss below.

Similarly, Oaksford and Chater (1994) can provide a probabilistic model for the well-known Wason Selection Task[3] only because they model the selection as a single-step computation of the maximal reduction of uncertainty. Problems regarding chains of inferences don't become visible when one is modeling this classic experimental paradigm in this way.

How might one empirically investigate chains of inferences within the new paradigm? First, we would need tasks that require subjects to form chains of inferences. Such tasks have been used in studies based on mental model theory (Cherubini & Johnson-Laird 2004; Van der Henst, Yang, & Johnson-Laird 2002). Second, we would have to measure the subjects' credences in intermediary and final conclusions after each step in the chain of inferences. In this way we could measure how adjustments of credences propagate through a set of credences via inference. However, if my arguments below are sound, Bayesian theories cannot model the computation underlying such a propagation in fully rational cases.

To sum up, there is no systematic treatment of chains of inferences, i.e. multi-step inferences, within the new paradigm. As we will see momentarily, this neglect of chains of inferences has important consequences. For it is not at all clear how chains of inferences can be modeled within a Bayesian framework. By contrast, single-step inferences can be modeled as the subject updating her credal state by conditionalizing (or Jeffrey-conditionalizing) on the newly acquired information represented by the premises. Thus, given the focus of theoretical and empirical research in the new paradigm, it is understandable how advocates of the new

[3] Here is an example of the task: Four cards are lying in front of you. Printed on them you see "A", "K", "2", and "7", respectively. Each card has a letter on one side and a number on the other. You are then given the statement "If there is a vowel on one side, then there is an even number on the other side"; you must then select those cards that you must turn over to determine whether the statement is true of false.

paradigm could have overlooked the problem I will be raising below. This does not, however, make the problem any less pressing.

It is worth pointing out that chains of inferences can be, and have been, investigated using mental model theory (e.g. Cherubini & Johnson-Laird 2004; Van der Henst, Yang, & Johnson-Laird 2002).[4] Mental model theory can, e.g., explain why reasoners find it more difficult to make chains of inferences (from quantified premises) than single-step inferences (Cherubini & Johnson-Laird 2004). And it is consistent with the finding that there are inter-individual differences in how people form chains of inferences from the same array of basic components, i.e. possible steps (Van der Henst, Yang, & Johnson-Laird 2002). Thus, more traditional approaches like mental model theory can give us some insight into how humans perform multi-step inferences, while the new paradigm psychology of reasoning cannot even model such inferences — or so I shall argue.

3 Chains of Inferences with Degrees of Belief

We often make inferences from premises that are themselves the conclusions of earlier inferences. In mathematics, e.g., we may first infer a lemma from some axioms and then infer a theorem from the lemma. We can also reason in this way about matters that involve uncertainty or in non-monotonic ways, e.g., about our moral obligations (an area where hardly any inference is monotonic). Thus, human reasoning often proceeds stepwise; we often reason by stringing together chains of inferences.[5] Given the ubiquity of chains of inferences, any theory of

[4] Thanks to an anonymous referee for alerting me to this work.

[5] I am not sure whether a process by which an entire credal state changes at once should be called "inference." However, even if it is not necessary, it is true that we can reason by forming chains of inferences. By doing philosophy or mathematics anyone can generate examples for her own case. It is easy to find examples that concerns empirical, non-necessary facts that can only be known *a posteriori*.

reasoning — be it normative or descriptive — should have the resources to give an account of such chains. So let us ask: How can Bayesian theories, as characterized above, describe or evaluate such chains of inferences?

Let us begin with three conditions that an account of chains of inferences in a Bayesian framework must meet.

(i) Reasoning is an act or process that brings one from some attitudes to others. For the new paradigm psychology of reasoning these attitudes are not outright beliefs but partial beliefs; so we are interested in chains of inferences where the attitudes involved are partial beliefs.[6]

(ii) A chain of inferences typically begins with one or more new pieces of evidence; in reasoning we are drawing out the consequences of the new evidence. According to Bayesian theories, some sort of conditionalization plays a crucial role in rational changes of credal states in light of new evidence. So, on any Bayesian theory, some sort of conditionalization must play a crucial role in describing or evaluating such chains of inferences.

(iii) Chains of inferences can be entirely rational and correct. In mathematics, e.g., we can form chains of inferences that are entirely rational and correct. Hence, the account shouldn't make chains of inferences *as such* irrational; there must be a correct way of forming such chains.

[6] Note that I am not interested in reasoning with outright beliefs about probabilities; I am only concerned with reasoning in which the involved attitudes are degrees of belief.

Given these constraints, an advocate of a Bayesian account of chains of inferences can take a couple of different general views on what such an account should look like. To see what these options are, let's say *S* is in a probabilistically coherent credal state and encounters new evidence *E*, *S* first infers *P* from *E* (i.e. *S* inferentially adjusts her credence in *P*), and then infers *Q* from *P* (i.e. *S* inferentially adjusts her credence in *Q*). This is a chain of inferences. Let us represent the three credal states of *S*, i.e. the one before the inference, the one after the first step, and the one after the second step, by Pr_{old}, Pr_{step1}, and Pr_{step2} respectively. What can a Bayesian theory tell us about it?

View 1: All that a Bayesian theory can and should tell us is that if *S*'s reasoning is rational, then the values of her new partial beliefs in *P* and *Q* equal the probabilities assigned to *P* and *Q* in a probability function that is the result of conditionalizing the probability function that characterizes *S*'s old credal state on *E* (or adjusting this function to a change in the partial belief in *E*, e.g., via Jeffrey Conditionalization).

View 2: If *S*'s reasoning is rational, each step in the chain of inferences happens by some computational process that can be thought of as a kind of "conditionalization" that operates on just a single partial belief — not a whole probability function at once. Let's call this computational process "stepwise-conditionalization." *S* first stepwise-conditionalizes *P* on *E*; *S* thereby arrives at a new partial belief in *P*. This is the first inference in the chain. Next, *S* stepwise-conditionalizes *Q* on *P* — using her new partial belief in *P*. *S* thereby arrives at a new partial belief in *Q*. This is the second inference in the chain.

View 3: If *S*'s reasoning is rational, *S* does two quite different things in forming the chain of inferences. First, *S* adjusts her partial belief in *P* in light of *E* via a computation like stepwise-conditionalization (see View 2). Second, a computation that is different from any kind of conditionalization brings *S* from her new partial belief in *P* to a new partial belief in *Q*. If *S* draws further conclusions from *Q*, this also happens via this second kind of process. Chains of inferences begin with an application of stepwise-conditionalization and then continue by iterations of a quite different computation, e.g., the application of rules of inference.

These seem to be the most plausible views on the matter, given a commitment to a "broadly Bayesian paradigm" in the psychology of reasoning. For what it is worth, I cannot see a fourth view that seems plausible (I will return to this issue in Section 6). In the remainder of this paper, I shall argue that on none of these three views, Bayesian theories provide a computational-level account for describing or evaluating chains of inferences. In the following three sections, I will address these three views in turn.

4 View 1

On View 1, all we get from Bayesian theories when it comes to chains of inferences is a necessary condition on the partial beliefs that are generated in such chains: their values must coincide with the values we would get by conditionalizing the whole credal state on *E*. Now, familiar kinds of conditionalization are operations on entire credal states "at once;" it is not clear what it would mean to perform a conditionalization operation in steps that could match the steps in a chain of inferences. Hence, the computation underlying the steps in a rational chain of inferences cannot be any familiar kind of conditionalization. Adopting a distinction due to Herbert Simon (1976), we might say that, on View 1, conditionalization provides merely a constraint on "substantive rationality" and does not tell us anything about "procedural rationality." This means that, on View 1, Bayesian theories don't tell us what computations are performed at each step in a chain of (rational) inferences — at least after the first step.

Let us look at a possible motivation for View 1 in a bit more detail, which will also foreshadow the discussion of View 2 below. What could Bayesianism tell us about the computations at issue? Perhaps the computation performed in the first step from E to P is the calculation of the value of the conditional probability of P given E, i.e. $Pr_{old}(P|E)$, given the old credal state of S, i.e. $Pr_{old}(P\&E)$ divided by $Pr_{old}(E)$ — or something the like, e.g., a computation of the new value of the partial belief in P according to Jeffery Conditionalization. However, after the first step, the function computed in the second step must yield $Pr_{old}(Q|E)$ when given the new partial belief in P, i.e. $Pr_{old}(P|E)$, as input (or the analogs of these for Jeffery Conditionalization). Bayesianism gives us no clue as to how such a function may be computed. In particular, it is utterly unclear what role the new partial belief in P could play in this computation. Of course, S might compute $Pr_{old}(Q|E)$ just as she computed $Pr_{old}(P|E)$, but S's updated partial belief in P would not play any role in this computation. Thus, it would be wrong to speak of a *chain* of reasoning here; S would perform a number of independent computations. There would be no sense in which the result of the first step is used as an input to the second step. The response of View 1 to this problem is to simply leave us in the dark about what happens or ought to happen when we are forming chains of inferences.

To see the full extent of the problem, notice that there is no function that, for any A and B, computes the value of $Pr_{old}(B|E)$ when given $Pr_{old}(A|E)$. After all, there can be two bodies of evidence E_1 and E_2 that have the same effect on S's credence in A, i.e. $Pr_{old}(A|E_1)=Pr_{old}(A|E_2)$, but different effects on S's credence in B, i.e. $Pr_{old}(B|E_1)\neq Pr_{old}(B|E_2)$. The function that computes $Pr_{old}(B|E)$ cannot solely depend on $Pr_{old}(A|E)$ but must also depend on the evidence the agent acquires. Thus, it seems that S must bring to bear her initial evidence E directly in the second step of her inference again.

In light of these problems, View 1 holds that Bayesian theories do not tell us anything about the computations underlying chains of inferences. These theories only tell us what the right results are. After all, even if the partial beliefs generated in a chain of inferences ought to match the results of applying some sort of conditionalization, this does not tell us anything about how chains of inferences are formed or ought to be formed. Thus, on View 1, Bayesian theories neither provide a computational-level theory of actual human chains of inferences nor of completely rational chains of inferences (i.e. a normative theory). Anyone who adopts View 1 must, therefore, say that the "wide consensus that Bayesianism, at least in the broader sense, best captures the computational level of analysis of the new paradigm" (Elqayam & Over 2012, p. 28) is misguided.

One might think that this is still the right view. Perhaps Bayesianism is merely a theory of substantive rationality. Perhaps it is silent on norms governing and computations underlying chains of inferences. In any event, we have seen that even if such a view is correct, it cannot support the ideas behind the new paradigm.

5 View 2

On View 2, there is some computational process that is at the same time a stepwise procedure and some sort or variant of conditionalization, adjusted so as to operate on individual credences and not on whole probability functions. Perhaps this is the kind of view Ralph Wedgwood has in mind when he writes:

> More precisely, I suggest, your […] credences are not only disposed to be probabilistically coherent; they are also disposed to change in response to experience, and the changes dictated by experience are propagated throughout the whole set of credences by means of some kind of conditionalization. These are the only kinds of changes in one's […] credences that are involved in these […] credences' essential functional role.
>
> (Wedgwood 2012, p. 320)

On this view, changes of partial beliefs can be "propagated throughout the whole" credal state "by means of some kind of conditionalization." This suggests that there is a stepwise computation that is "some kind of conditionalization" and whose steps correspond to the steps in a chain of inferences. Let's call this computation "stepwise-conditionalization," to distinguish it from the familiar kinds of conditionalization that operate globally on a whole credal state "at once." It is not clear that the idea of such an operation makes sense, but let us assume that it does. The idea is that *S* first stepwise-conditionalizes her partial belief in *P* on *E* and then uses her new partial belief in *P* in a second stepwise-conditionalization of *Q* on *P*. What should the transitions from Pr_{old} to Pr_{step1}, and from there to Pr_{step2} look like, on this view?

A natural suggestion is that stepwise-conditionalization is a computation that yields the conditional probability of, e.g., *P* given *E* in *S*'s old credal state and sets the value of *S*'s new partial belief in *P* to the value of this conditional probability. If we work with simple conditional probabilities, this would mean that $Pr_{step1}(P)=Pr_{old}(P|E)=Pr_{old}(P\&E)/Pr_{old}(E)$. It seems plausible that $Pr_{step2}(P)=Pr_{step1}(P)$. After all, *S* has already adjusted her credence in *P* in the first inference of the chain. Similarly, $Pr_{step1}(E)$ should stay unchanged, i.e. it should equal $Pr_{old}(E)$, which is the value after S has received the new piece of information. Given such a view, what should $Pr_{step2}(Q)$ be, i.e. S's credence in Q after S goes through the chain of inferences from E to P and from P to Q? If the second step in the chain of inferences works like the first one, we should have $Pr_{step2}(Q)=Pr_{step1}(Q|P)=Pr_{step1}(Q\&P)/Pr_{step1}(P)$. If we think that simple conditionalization gives the right answers to what our partial beliefs should be in the light of new evidence, S's new credence in Q ought to match the subjective probability in Q that results from conditionalizing S's initial credal state on E. I.e., we should have: $Pr_{step2}(Q)=Pr_{old}(Q|E)$. However, $Pr_{step2}(Q)=Pr_{old}(Q|E)$ does not always hold. Here is a counterexample. Let $Pr_{step1}(P) = Pr_{old}(P|E)=.4$, and let $Pr_{old}(E) =.5$, $Pr_{old}(P\&Q\&E) = Pr_{old}(Q\&P) =.1$ and $Pr_{old}(Q\&E) =.2$. If we now use the old credence in $Q\&P$ to compute $Pr_{step1}(Q|P)$, we get $Pr_{old}(Q|E)=.4 \neq Pr_{step1}(Q|P)=.25$. If we conditionalize the credence in $Q\&P$ and use this updated credence to calculate $Pr_{step1}(Q|P)$, we also get a wrong result: $Pr_{step1}(Q\&P) = Pr_{old}(Q\&P|E) = Pr_{old}(P\&Q\&E)/ Pr_{old}(E) = .2$ and, so, $Pr_{step1}(Q|P) = .2/.4 = .5 \neq Pr_{old}(Q|E)=.4$. There simply is no way to make this idea work. In order to get the right result for $Pr_{step2}(Q) = Pr_{step1}(Q|P)$, the value we used for $Pr(Q\&P)$ would have to be .16, which is neither the old nor the updated value of this credence. Parallel counterexamples can be constructed if we use formulae for calculating the results of Jeffrey Conditionalization for individual propositions — rather than conditional probabilities.

The underlying problem is that calculating conditional probabilities is not and cannot be transitive, which would be required in order to make the current proposal work. Indeed, on reflection, the whole idea seems absurd. If $Pr_{old}(Q|E)$ equals the result of stepwise-conditionalizing P on E and then Q on the new partial belief in P, it must also be equal to the result of first stepwise-conditionalizing an arbitrary *R* on *E* and then *Q* on the new partial belief in *R*. After all, $Pr_{old}(Q|E)$ is a definite value, given the probability function Pr_{old}. But surely what conclusions we can draw in the second step of a chain of inferences actually depends on the inference we made in the first step. In both, actual and completely rational human reasoning, it matters a lot what the first inference in a chain of inferences is. This complaint is independent of the specific way we calculate the particular values and of what we think the right version of conditionalization is.

The whole idea of calculating the results of conditionalization in a stepwise fashion that can form chains, i.e. where the result of the first computation is used as input to the second and so on, simply does not make sense. We cannot mimic conditionalization by a "chain-forming computational process." There is no operation of "stepwise-conditionalization" by which we could compute the credences that would result from "all at once" conditionalization in a way that maps onto the steps in a chain of inferences. Hence, we cannot make sense of View 2; there is no coherent version of this view.

6 View 3

On View 3, there are two quite different computations involved in forming chains of inferences. In our example, *S* first computes the value that conditionalizing her whole credal state on *E* would dictate for *P*; *S* then uses this new partial belief in *P* to arrive at a new partial belief in *Q*, but this second transition is accomplished by a quite different computation. If the chain continues with further inferences, they are treated like the second one.

How might such a view be fleshed out? For the first step, we can use familiar ways of calculating the results that various kinds of conditionalization would dictate. If we think, e.g., that simple conditionalization tells us what credences we ought to have in light of new evidence, we can calculate the credence in *P* that results in the first step as the conditional probability of *P* given *E* on the probability function that characterizes *S*'s initial credal state; i.e., we would again have $Pr_{step1}(P)=Pr_{old}(P|E)=Pr_{old}(P\&E)/Pr_{old}(E)$. If we think that Jeffrey Conditionalization yields the correct results, we can calculate $Pr_{step1}(P)$ as $Pr_{old}(P|E)\times P_{old}(E) + Pr_{old}(P|{\sim}E)\times P_{old}({\sim}E)$.[7]

We now need a way of computing $Pr_{step2}(Q)$ from $Pr_{step1}(P)$. If *Q* happens to be ~*P*, the answer seems obvious: $Pr_{step2}(Q)$ must be $1–Pr_{step1}(P)$. Can we generalize the idea behind this answer? The idea is that *S*'s degree of belief in *P* dictates a degree of belief in ~*P*, on pain of probabilistic incoherence. So what we need would be something like inference rules that capture what probabilistic coherence dictates given some degrees of belief that are already adjusted in light of evidence *E*.

Recall that if we presuppose the ability to reason arithmetically, four axioms suffice to characterize the classical probability calculus. Here is a possible axiom set:

A1: $\forall A\ (0 \leq Pr(A))$.

A2: $\forall A\ (\models_{\text{FOL}} A \rightarrow Pr(A) = 1)$.

A3: $\forall A \forall B\ (Pr(A\&B) = 0 \rightarrow Pr(A \vee B) = Pr(A) + Pr(B))$.

A4: $\forall A \forall B\ (Pr(A\&B) = Pr(A) \times Pr(B|A))$

[7] The two conditional probabilities here come from the credal state of *S* before the change of the credence in *E*. So Pr_{old} is not a probabilistically coherent credal state, but this is unsurprising because it is *S*'s credal state right after her credence in *E* has changed and we are wondering how this change might influence other credences, e.g. the credences in *P*. Note that *S*'s credal state before the change in the credence in *E* might have been probabilistically coherent.

If we assume that rational degrees of belief are "by nature," as it were, within the unit interval and that all theorems of first order logic have degree of belief 1 and all classical contradictions are believed to degree 0, it is straightforward to turn these axioms into rules of inference by considering all possible ways of exploiting A3 and A4. If we follow a suggestion by John Broome (2013, Chap. 10.4) and represent partial beliefs as the proposition followed by the degree to which it is believed — in the format "[Proposition *P*] (Cred: [degree of belief in *P*])" —, our axioms give rise to the following set of inference rules.

(1)
$$\begin{array}{ll} A & (\text{Cred: } n) \\ B & (\text{Cred: } m) \\ A\&B & (\text{Cred: } l) \\ \hline A\lor B & (\text{Cred: } n+m-l) \end{array}$$

(2)
$$\begin{array}{ll} A\lor B & (\text{Cred: } n) \\ B & (\text{Cred: } m) \\ A\&B & (\text{Cred: } l) \\ \hline A & (\text{Cred: } n+l-m) \end{array}$$

(3)
$$\begin{array}{ll} A & (\text{Cred: } n) \\ B & (\text{Cred: } m) \\ A\lor B & (\text{Cred: } l) \\ \hline A\&B & (\text{Cred: } n+m-l) \end{array}$$

(4)
$$\begin{array}{ll} A & (\text{Cred: } n) \\ B|A & (\text{Cred: } m) \\ \hline A\&B & (\text{Cred: } n\cdot m) \end{array}$$

(5)
$$\begin{array}{ll} A & (\text{Cred: } n) \\ A\&B & (\text{Cred: } m) \\ \hline B|A & (\text{Cred: } m/n) \end{array}$$

(6)
$$\begin{array}{ll} B|A & (\text{Cred: } n) \\ A\&B & (\text{Cred: } m) \\ \hline A & (\text{Cred: } m/n) \end{array}$$

The rule that allows one to infer "~*A* (Cred: 1–*n*)" from "*A* (Cred: *n*)" can be derived from rule (2), given that $A\lor{\sim}A$ is believed to degree 1 and $A\&{\sim}A$ is believed to degree 0.

We can now put the pieces together. On the view under consideration, *S* computes $Pr_{step1}(P)$ in accordance with a formula like $Pr_{old}(P\&E)/Pr_{old}(E)$, and in the second step of the chain *S* arrives at $Pr_{step2}(Q)$ by applying one of the rules (1)-(6). If all the partial beliefs *S* uses as premises when applying these rules are already adjusted in light of *E*, the resulting partial beliefs will equal the degrees of belief dictated by conditionalizing *S*'s whole initial credal state on *E*. Hence, these rules yield the right results, i.e. the results dictated by our preferred version of conditionalization.

Have we found a way of describing and evaluating chains of inferences in a "broadly Bayesian" framework? Unfortunately, there are at least two problems with this view — one minor and one major. The minor problem is that these rules are very "weak," i.e. you need a lot of already updated partial beliefs in order to derive interesting new partial beliefs. You need, e.g., three premises to conclude that $A \vee B$ (with some credence), while classical logic would, e.g., allow you to infer $A \vee B$ from *A* alone. This means that you need to start by applying the procedure of the first step (i.e. directly calculating individual degrees of belief dictated by your preferred version of conditionalization) to many partial beliefs; otherwise you do not have enough partial beliefs that you can use to trace out interesting consequences by using rules (1)-(6). However, rational chains of inferences do not seem to require that much input. Note that the issue is not that these rules require a lot of computational power.

Second, there is a major problem. The advocate of such an account must tell us what happens to all the partial beliefs that are neither updated in the first step nor (indirectly) via applications of rules (1)-(6). These partial beliefs cannot in general stay unchanged; for nothing would guarantee that the resulting credal state is probabilistically coherent — even if the reasoning is fully rational and correct and starts from a probabilistically coherent credal state.

After all, it seems implausible that fully rational and correct chains of inferences beginning with a probabilistically coherent credal state can lead to a probabilistically incoherent credal state — especially given that on Bayesian theories (as defined above) all rational credal states are probabilistically coherent. However, it also seems highly undesirable to simply get rid of these credences (thus creating gaps in the probability function that characterizes the resulting credal state). Getting rid of these credences would mean that every time we learn something new, we lose every part of our view of the world that — as far as we can tell — has nothing to do with what we just learned. It seems that there is no way of guaranteeing that we arrive at a probabilistically coherent credal state (even when starting with one) by the two-phase process that we are envisaging.

Maybe we could use the process from the first step once more to fill-in the gaps in the resulting probability function. However, if that is an option, why did we use rules (1)-(6) in the first place? Why should anyone reason in a stepwise fashion if she could simply apply conditionalization to her whole credal state, or at least arbitrarily large parts of it, all at once? Surely, we sometimes have to form chains of inferences — even fully rational and correct ones — because we cannot adjust arbitrary large parts of our credal state by some operation like conditionalization.[8]

[8] Note that accounts of reasoning with outright beliefs do not have these problems. Regarding the first, their rules typically require less information as input. Regarding the second problem, we can say — just to give a toy example of how such a theory can deal with the problem — that an ideal agent, who is not subject to computational limitations, keeps all her beliefs that are not changed by any possible chain of correct inferences starting (*inter alia*) with the new information. The resulting belief-state (if there is a stable one) is necessarily coherent if the agent can, e.g., use a rule like *reductio ad absurdum* and she eliminates a belief when she adopts a belief in the negation of the original belief. After all, if the resulting belief-state were incoherent, the agent could get rid of one of the beliefs by deriving the negation of the content of the belief by *reductio*. Of course, it is not a trivial matter to give rules for rational reasoning with outright beliefs. As is well known, we cannot simply take the rules of classical logic (see Harman 1986). And the problems I am pointing out in this paper apply with equal force to AGM-style theories (Alchourrón, Gärdenfors, & Makinson 1985) if one tries to use them as computational-level theories of (rational) inferences. At least, the same problems arise as long as updating and revising are conceived as global operations on belief-states. However, some promising work has been done in this area (see, e.g., Jago 2009). In any event, I am not trying to provide such a theory here.

An opponent might reply to the minor problem by providing a system of stronger inference rules. In fact, the considerations I mentioned motivated some authors to come up with stronger "probability logics," which they want to use to model human reasoning within the new paradigm (e.g. Pfeifer 2013; Pfeifer & Kleiter 2006). In order to allow for stronger inferences, such logics work with interval-valued credences, instead of point-valued credences. We can think of these intervals as representing sets of (coherent) probability distributions, such that the agent is "undecided" between the distributions in the set. In such systems we have, for example, rules like the following—corresponding to versions of modus ponens, conjunction-introduction, and cut respectively (see Gilio 2012; Pfeifer & Kleiter 2006):

(7) *B*|*A* (Cred: $[x_1,y_1]$)

<u>*A* (Cred: $[x_2,y_2]$)</u>

B (Cred: $[x_1x_2, 1–x_2+y_1x_2]$)

(8) *B*|*A* (Cred: $[x_1,y_1]$)

<u>*C*|*A* (Cred: $[x_2,y_2]$)</u>

B&C|*A* (Cred: $[Max(0,x_1+x_2–1), Min(y_1,y_2)]$)

(9) *C*|*A&B* (Cred: $[x_1,y_1]$)

<u>*B*|*A* (Cred: $[x_2,y_2]$)</u>

C|*A* (Cred: $[x_1x_2,y_1x_2+1–x_2]$)

Unfortunately, there are again at least two problems with this proposal. One immediate problem with such views is that, as Adam Elga (2010) has argued, agents with interval-valued credences are susceptible to variants of diachronic Dutch Books.[9] Hence, a fully rational thinker does not follow rules (7)-(9) because she does not have interval-valued credences. (Note that these rules give you interval-valued credences for the conclusion even if you have point-valued credences regarding the premises.)

A second problem is that these rules do not give us the interval-valued credences that we would get by (direct) conditionalization. The rules do not yield the right results. Here is an example using simple conditionalization: Suppose someone applies rule (7) with premise-attitudes that are already adjusted in light of the new evidence; yielding as conclusion-attitude a partial belief in *B* with the interval-value $[x_1x_2, 1–x_2+y_1x_2]$.[10] Suppose furthermore that the updated degree of belief in $A\vee B$, if the agent used conditionalization, would be [w, z]. Under these assumptions, the degree of belief in *B* after updating by conditionalization should be: $[x_1x_2+w–x_2, x_2y_1+z–x_2]$. This equals $[x_1x_2, 1–x_2+y_1x_2]$ only if z=1 and $w=x_2$ (which is the lowest value w can take, given x_2). Therefore, if z<1 or $w>x_2$, the result of applying rule (7) to arrive at a partial belief in *B* is not the same as the result of conditionalizing one's credence in *B* on one's evidence, i.e. the evidence one used to arrive at one's degree of belief in *A* and one's conditional degree of belief in *B* given *A* (i.e. the premise-attitudes of the inference under consideration). This is unacceptable. If we used this method and updated a partial belief via conditionalization and later on also reasoned to this same proposition from some premises, we could get different results. This way of reasoning is not a way to adjust one's credal state in light of new evidence. Rather, it is a way of figuring out what minimal conditions certain degrees of belief must meet if we pretend that all partial beliefs we are not considering take extreme values (within the

[9] For a response to Elga see (Chandler 2014). I think that Elga's claim can be defended against Chandler's critique, but that would lead us too far afield.
[10] I use the variables as in rule (7).

constraints given by the degrees of belief we are considering). If we apply rules like (7)-(9), we do not in general arrive at the credal state that would be dictated by conditionalization.

To sum up, if we want to have a formal system that is reasonably strong and we hold on to the ideas that rational credal states are probabilistically coherent and that some sort of conditionalization tells us what degrees of belief we ought to have in light of new evidence, we cannot accept View 3 as a computational-level account of fully rational chains of inferences. The view does not guarantee that the credal states that result from such chains are probabilistically coherent — even if they begin with a probabilistically coherent credal state and are fully rational. Inference rules for point-valued degrees of belief are implausibly weak. Moving to interval-valued degrees of belief does not help; it is in itself unattractive and does not yield the right results, i.e. degrees of belief underwritten by conditionalization. Therefore, View 3 is not a way of developing Bayesian theories into helpful normative or descriptive computational-level accounts of chains of inferences.

7 P-Validity: Another View?

Advocates of the new paradigm in the psychology of reasoning often appeal to the notion of probabilistic validity (p-validity) as the successor concept to classical validity (see Elqayam & Over 2012; Over 2009; Oaksford & Chater 2007). Following Adams (1998, pp. 131-132) an inference is defined to be p-valid just in case, for all uncertainty functions, the uncertainty of the conclusion is less or equal to the sum of the uncertainty of the premises, where an uncertainty function, u, is obtained from a probability function, Pr, by the principle that, for any proposition Φ, $u(\Phi)=1-Pr(\Phi)$. Can we construct a computational-level account of chains of inferences by exploiting the notion of p-validity?

Notice that p-validity does not tell us what the degree of belief in the conclusion should be; it merely puts a lower bound on it.[11] Hence, p-validity does not settle what computations underlie fully rational inferences; by itself, it does not provide a computational-level account of fully rational inferences, let alone chains of inferences. As long as a computation yields a degree of belief above the lower bound set by p-validity, it is not ruled out as a candidate computation underlying a particular fully rational inference (see Adams 1998).

How might one think about the computations underlying our ability to make p-valid inferences? There are at least two ways to think about them. First, one might hold that the partial belief in the conclusion that results from a rational inference is point-valued. In a fully rational inference, this point-value might either be settled by partial beliefs that do not occur in the premises or by some other mechanism. If the agent has partial beliefs that fix a unique degree of belief for the conclusion, on pain of probabilistic incoherence, it would be irrational for the agent to arrive at any other degree of belief in the conclusion. After all, that would make her credal state probabilistically incoherent. This would mean that these further partial beliefs must play a role in a rational inference to the conclusion. If all rational inferences are like that, the account reduces to the version of View 3 that uses rules (1)-(6) from the previous section. (With the only difference that some premises are not called "premises.") That view tried to squeeze an account of rational inference out of the notion of probabilistic coherence. If, on the other hand, the agent's partial beliefs do not settle a point-value for the degree of belief in the conclusion, the agent is either at liberty to adopt any degree of belief in the conclusion that does not make her credal state probabilistically incoherent or there is a general rule that settles which degree of belief within the allowed interval the agent should choose. On neither of these two options is the

[11] It is also worth noticing that p-validity defines a monotonic consequence relation. Adams was explicit about this, and Over (2009, p. 437) pointed it out again in a discussion of the new paradigm.

result of rationally revising one’s credal state in light of new evidence settled by some sort of conditionalization. On such views, the degrees of belief in the last conclusion of a chain of inferences can be very different from what conditionalization would dictate. Thus, such views don’t count as Bayesian theories, as I defined them in Section 1. Hence, if we understand the computations underlying rational p-valid inferences in this way, the view reduces to a version of View 3 or it is no longer a Bayesian theory in the sense defined above.

A second way of understanding the computations underlying the rational p-valid inferences is that they yield interval-valued degrees of belief. However, we have already seen in the discussion of View 3 that accounts based on interval-valued degrees of belief should be rejected. In fact, the version of View 3 that appeals to rules like (7)-(9) is a late descendant of accounts based on p-validity (see Pfeifer & Kleiter 2009).

To sum up, the notion of p-validity does not provide the resources to construct an account that goes beyond those we have already considered. If we flesh it out so as to contain an account of the computational level of rational chains of inference, such an account collapses into a version of View 3.

8 Conclusion

I have argued that Bayesian theories, i.e. theories according to which rational credal states are probabilistically coherent and some sort of conditionalization settles what degrees of belief we ought to have in the light of new evidence, cannot be used to give an account of rational chains of inferences at the computational level of analysis. I conclude that the “wide consensus that Bayesianism, at least in the broader sense, best captures the computational level of analysis of the new paradigm” (Elqayam & Over 2012, p. 28) must rest on a mistake. Or else it rests on an understanding of Bayesianism that is significantly broader than the notion of “Bayesian theories” I used in this paper. However, on such a broad or “soft” sense of “Bayesianism,” the role of the probability calculus, Dutch Book arguments, and rules of conditionalization is difficult to make out. It is not just that their role in describing actual chains of inferences by humans is unclear; their role in a computational account of rationally ideal chains of inferences is unclear. I don’t know what the meaning of “Bayesianism” on such a broad “understanding” of the term could be. In any event, such an account would have little to do with Bayesian epistemology.

Given all this, it is difficult to see what role Bayesian theories can play in the psychology of reasoning. Thus, when Elqayam and Over (2012, p. 29) say that “[a]pproaches in the new paradigm vary widely, but what they share is a commitment to psychological principles which fit within a broadly Bayesian paradigm,” it is unclear what these psychological principles might be.

Assuming that we should expect our epistemology to give an account of the rationality of reasoning — including chains of inferences —, one might conjecture that the inability of Bayesian theories to adequately describe or evaluate chains of inferences is not only a problem for the new paradigm in the psychology of reasoning but also points to general limitations of Bayesian epistemology. This is a promising line of further research, I think.

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References

Adams, E. W. (1998). *A primer of probability logic*. Stanford, Calif.: CSLI Publications.

Alchourrón, C. E., Gärdenfors, P. & Makinson, D. (1985). On the logic of theory change: partial meet contraction and revision functions. *Journal of Symbolic Logic* 50: 510-530.

Broome, J. (2013). *Rationality through reasoning*. Oxford: Wiley-Blackwell.

Chandler, J. (2014). Subjective probabilities need not be sharp. *Erkenntnis*: 1-14. doi: 10.1007/s10670-013-9597-2

Chater, N. & Oaksford, M. (1999). The probability heuristics model of syllogistic reasoning. *Cognitive Psychology* 38: 191-258. doi: 10.1006/cogp.1998.0696

—— (2009). Local and global inferential relations: Response to Over (2009). *Thinking & Reasoning* 15: 439-446. doi: 10.1080/13546780903361765

Cherubini, P. & Johnson-Laird, P. (2004). Does everyone love everyone? The psychology of iterative reasoning. *Thinking & reasoning* 10: 31-53.

Elga, A. (2010). Subjective probabilities should be sharp. *Philosophers' Imprint* 10.

Elqayam, S. & Evans, J. S. B. T. (2013). Rationality in the new paradigm: strict versus soft Bayesian approaches. *Thinking & Reasoning* 19: 453-470. doi: 10.1080/13546783.2013.834268

Elqayam, S. & Over, D. E. (2012). Probabilities, beliefs, and dual processing: the paradigm shift in the psychology of reasoning. *Mind & Society* 11: 27-40. doi: 10.1007/s11299-012-0102-4

—— (2013). New paradigm psychology of reasoning: An introduction to the special issue edited by Elqayam, Bonnefon, and Over. *Thinking & Reasoning* 19: 249-265. doi: 10.1080/13546783.2013.841591

Evans, J. S. B. T. (2012). Questions and challenges for the new psychology of reasoning. *Thinking & Reasoning* 18: 5-31. doi: 10.1080/13546783.2011.637674

Gilio, A. (2012). Generalizing inference rules in a coherence-based probabilistic default reasoning. *International Journal of Approximate Reasoning* 53: 413-434. doi: 10.1016/j.ijar.2011.08.004

Harman, G. H. (1986). *Change in view: principles of reasoning*. Cambridge, Mass.: MIT Press.

Howson, C. & Urbach, P. (2006). *Scientific reasoning: the Bayesian approach* (3rd ed.). Chicago: Open Court Publishing.

Jago, M. (2009). Epistemic logic for rule-based agents. *Journal of Logic, Language and Information* 18: 131-158.

Jeffrey, R. C. (1970). Dracula meets Wolfman: acceptance vs. partial belief. In M. Swain (ed.), *Induction, acceptance, and rational belief* (pp. 157-185). Dordrecht: D. Reidel.

Jones, M. & Love, B. C. (2011). Bayesian fundamentalism or enlightenment? On the explanatory status and theoretical contributions of Bayesian models of cognition. *Behavioral and Brain Sciences* 34: 169-188. doi: 10.1007/BF01454201

Maher, P. (1993). *Betting on theories*. Cambridge: Cambridge University Press.

Oaksford, M. & Chater, N. (1994). A rational analysis of the selection task as optimal data selection. *Psychological Review* 101: 608-631.

—— (1998). *Rationality in an uncertain world: essays on the cognitive science of human reasoning*. Hove: Psychology Press.
—— (2001). The probabilistic approach to human reasoning. *Trends in Cognitive Sciences* 5: 349-357. doi: 10.1016/S1364-6613(00)01699-5
—— (2007). *Bayesian rationality: the probabilistic approach to human reasoning*. Oxford: Oxford University Press.
Oaksford, M., Chater, N. & Larkin, J. (2000). Probabilities and polarity biases in conditional inference. *Journal of Experimental Psychology: Learning, Memory, and Cognition* 26: 883-899.
Over, D. E. (2009). New paradigm psychology of reasoning. *Thinking and Reasoning* 15: 431-438.
Pfeifer, N. (2013). The new psychology of reasoning: a mental probability logical perspective. *Thinking and Reasoning* 19: 329-345.
Pfeifer, N. & Douven, I. (2014). Formal epistemology and the new paradigm psychology of reasoning. *Review of Philosophy and Psychology* 5: 199-221. doi: 10.1007/s13164-013-0165-0
Pfeifer, N. & Kleiter, G. D. (2006). Inference in conditional probability logic. *Kybernetika* 42: 391-404.
—— (2009). Framing human inference by coherence based probability logic. *Journal of Applied Logic* 7: 206-217. doi: 10.1016/j.jal.2007.11.005
Simon, H. A. (1976). From substantive to procedural rationality. In S. J. Latsis (ed.), *Method and appraisal in economics* (pp. 129-148). Cambridge: Cambridge University Press.
Singmann, H., Klauer, K. C. & Over, D. E. (2014). New Normative Standards of Conditional Reasoning and the Dual-Source Model. *Frontiers in Psychology* 5. doi: 10.3389/fpsyg.2014.00316
Staffel, J. (2013). Can there be reasoning with degrees of belief? *Synthese* 190: 3535-3551. doi: 10.1007/s11229-012-0209-5
Stenning, K. & van Lambalgen, M. (2009). “Nonmonotonic” does not mean “probabilistic”. *Behavioral and Brain Sciences* 32: 102-103. doi: 10.1017/S0140525X0900048X
Van der Henst, J.-B., Yang, Y. & Johnson-Laird, P. N. (2002). Strategies in sentential reasoning. *Cognitive Science* 26: 425-468.
Wedgwood, R. (2012). Outright belief. *Dialectica* 66: 309–329.