The Cut-Free Approach and the Admissibility-Curry*

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Abstract: The perhaps most important criticism of the nontransitive approach to semantic paradoxes is that it cannot truthfully express exactly which metarules preserve validity. I argue that this criticism overlooks that the admissibility of metarules cannot be expressed in any logic that allows us to formulate v-Curry sentences and that is formulated in a classical metalanguage. Hence, the criticism applies to all approaches that do their metatheory in classical logic. If we do the metatheory of nontransitive logics in a nontransitive logic, however, there is no reason to think that the argument behind the criticism goes through. In general, asking a logic to express its own admissible metarules may not be a good idea.

Until recently, philosophical common sense had it that, on pain of triviality, you cannot accept all of classical logic, allow for self-reference and let

*This is the pre-peer- reviewed version of the following article *The Cut-Free Approach and the Admissibility-Curry* (doi:10.1002/tht3.267), which will be published in final form in Thought: A Journal of Philosophy.

your language contain its own truth predicate (satisfying all Tarski biconditionals). The nontransitive approach to semantic paradoxes has proved common sense wrong. It offers a supra-classical logic, called STT, with a transparent truth predicate (i.e. a truth predicate, T, such that A and $T\langle A\rangle$ are everywhere intersubstitutable) and self-reference (Cobreros et al., 2013, 2012; Ripley, 2013, 2012). Transitivity, i.e. Cut, fails in STT. But it only fails for paradoxical sentences, like the liar sentence or Curry sentences. That seems like a small price to pay for the full strength of classical logic together with a transparent truth predicate. With that much to recommend itself, I think we should try to hold on to the nontransitive approach as long as we can. My goal in this paper is to show that, contrary to recent criticism (Rosenblatt, 2017; Barrio et al., 2017), we shouldn't reject the nontransitive approach because it cannot express the admissibility of its own metarules.

Along the way, we can observe some general facts about the possibility of expressing, in the object language, which metarules are admissible. In particular, I point out that no one who does her metatheory in classical logic and allows for self-reference can allow her logic to truthfully express which metarules are admissible, according to her own logic.

In Section 1, I provide some background and explain the recent criticism of the nontransitive approach. I respond to the criticism in Section 2.

1 The Cut-Free Approach and Naive Validity

The criticism of the nontransitive approach on which I will focus says that the nontransitive approach cannot give us a validity predicate that does what it should do, namely express validity. In particular, critics charge that we cannot express validity in the nontransitive approach because that would require that we can express the admissibility of metarules, i.e., express that particular metarule applications preserve validity. But that cannot be done in the nontransitive approach. In this section, I will explain this criticism and bring out the central underlying assumption, which I call the faithfulness requirement.

It is helpful to start with the v-Curry (for validity-Curry) paradox (Beall and Murzi, 2013). Suppose you want to have a validity predicate, in your object language. Let's say, for any set Γ of object language sentences, our object language contains a canonical name $\langle \Gamma \rangle$, and it also contains a two place validity predicate Val, where $Val(\langle \Gamma \rangle, \langle \Delta \rangle)$ is meant to express that the argument from premises Γ to conclusions Δ is valid (in whatever sense is codified by our logic, including the logic of validity).[1] As Beall and

[1] As Ketland (2012) and Cook (2014) have shown, the v-Curry paradox can be avoided if we codify just the validities of first-order logic by the *Val* predicate. In that case, the VP rule below must be rejected because the fact that A follows from Γ in the "logic of validity" (i.e. classical logic enriched with a validity predicate) doesn't imply that the argument from Γ to A is valid in first-order logic. After all, the fact that $\Gamma \vdash A$ may depend on the rules governing the validity predicate. Unfortunately, this response to the v-Curry isn't one that typical advocates of the nontransitive approach, like David Ripley, can embrace. Ripley (2017) is a semantic inferentialist (of the bilateralist variety), and the notion of validity he is interested in is therefore the notion of validity that can be used in his inferentialist account of meaning. But logical validity can at best explain the

Murzi (2013) point out, it seems natural to think that *Val* must obey the following metarules:

$$\dfrac{}{\Gamma, Val(\langle\Gamma\rangle, \langle\Delta\rangle) \vdash \Delta}\,\text{VD} \qquad\qquad \dfrac{\Gamma \vdash \Delta}{\vdash Val(\langle\Gamma\rangle, \langle\Delta\rangle)}\,\text{VP}$$

Let us accept this for now (as it won't really matter). Unfortunately, given Contraction,[2] Cut, and our language's resources for self-reference,[3] these rules yield triviality. To see this, let κ be (or be intersubstitutable with) the sentence $Val(\langle\kappa\rangle\ , \langle\bot\rangle)$,[4] and call the move of substituting one for the other "κ-Def" (if they are the same object language sentences, this is a purely meta-linguistic move). We can now argue as follows:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\kappa, Val(\langle\kappa\rangle, \langle\bot\rangle) \vdash \bot}\,\text{VD}}{\kappa \vdash \bot}\,\kappa\text{-Def}}{\vdash Val(\langle\kappa\rangle, \langle\bot\rangle)}\,\text{VP}}{\vdash \kappa}\,\kappa\text{-Def} \qquad \dfrac{\dfrac{}{\kappa, Val(\langle\kappa\rangle, \langle\bot\rangle) \vdash \bot}\,\text{VD}}{\kappa \vdash \bot}\,\kappa\text{-Def}}{\vdash \bot}\,\text{Cut}$$

meaning of logical vocabulary. The upshot is that if Ripley restricts his validity predicate to the validity of first-order logic, then his theory doesn't offer a treatment of the word "valid" that he takes to express the basic semantic notion, namely validity in a wider sense (which he explains in terms of the normative status of collections of assertions and denials). This would put Ripley into a position analogous to someone who takes truth to be the basic semantic notion but whose semantic theory doesn't offer a treatment of the word "true" that expresses this notion. I shall assume that it is part of the motivation for adding a validity predicate to STT to avoid being in such a position.

[2] I am working with sets on both sides of the turnstile and thus build in structural contraction and permutation.

[3] I assume self-reference by fiat. Thus, I am avoiding questions about how to add arithmetic to STT and how we should think about the Diagonal Lemma in STT.

[4] I am omitting set-brackets for singletons and I assume that we have $\bot$ in the language.

That is the v-Curry paradox. Advocates of the nontransitive approach claim that they can solve this paradox (Ripley, 2013, p. 154). Of course, the nontransitive solution is to reject the last step, i.e. the application of Cut.

This solution to the v-Curry is interesting only if, in STT plus *Val* (henceforth STV),[5] *Val* really does what it is supposed to do, namely express validity. The validity it is supposed to express is the validity that we express in the metalanguage by "$\vdash$" (which is not purely logical validity, but includes the logic of validity and, perhaps, truth, etc.). The core of the criticism is that the nontransitive approach cannot have a predicate that expresses validity; i.e., *Val* in the nontransitive approach cannot do what it is meant to do (Barrio et al., 2017; Rosenblatt, 2017).

Let me explain. Critics of the nontransitive approach point out that if *Val* expresses validity, then it allows us not only to express that particular arguments are valid, but also that if certain arguments are valid, then another argument is valid. That is, the validity predicate should allow us to express that an instance of a metarule is admissible.[6]

Let me be clear what I mean. We can think of a metarule as a set of ordered pairs of a collection of premise-sequents and a conclusion-sequent; i.e., metarules are sets of pairs like this one: $\langle \{\Theta_1 \vdash \Xi_1, ..., \Theta_n \vdash \Xi_n\}, \Gamma \vdash \Delta \rangle$.

[5]It won't matter whether STV contains a truth predicate. If it does, the name STTV may be more appropriate.

[6]Sometimes Barrio, Rosenblatt and Tajer focus on derivable or primitive metarules. I argue elsewhere that their claims don't hold if we restrict the faithfulness requirement below to derivable metarules . Hence, I interpret their claims here as applying to admissible metarules.

A metarule is admissible in a logic, L, just in case, for all its elements, if all the premise-sequents, $\Theta_1 \vdash \Xi_1$,..., $\Theta_n \vdash \Xi_n$, hold in L, then so does the conclusion-sequent, $\Gamma \vdash \Delta$.

The idea of the critics of the nontransitive approach is this: If, e.g., $\Gamma \vdash \Delta$ holds in a logic if $\Theta \vdash \Xi$ holds (where our logic $\vdash$ includes the logic of validity), then the logic should also prove some object language sentence that expresses this fact, and vice versa. The object language sentence is plausibly this: $Val(\langle\Theta\rangle, \langle\Xi\rangle) \rightarrow Val(\langle\Gamma\rangle, \langle\Delta\rangle)$. So our logic should prove $Val(\langle\Theta\rangle, \langle\Xi\rangle) \rightarrow Val(\langle\Gamma\rangle, \langle\Delta\rangle)$ just in case $\langle\{\Theta \vdash \Xi\}, \Gamma \vdash \Delta\rangle$ is an element of an admissible metarule. After all, on the face of it, $Val(\langle\Theta\rangle, \langle\Xi\rangle) \rightarrow Val(\langle\Gamma\rangle, \langle\Delta\rangle)$ says that if the argument from Θ to Ξ is valid, then the argument from Γ to Δ is valid. But if that holds, then $\{\langle\{\Theta \vdash \Xi\}, \Gamma \vdash \Delta\rangle\}$ is an admissible metarule. And if there is an admissible metarule that contains $\langle\{\Theta \vdash \Xi\}, \Gamma \vdash \Delta\rangle$ and the argument from Θ to Ξ is valid, then the argument from Γ to Δ is also valid.

We can formulate this idea as a requirement. Critics assume that the validity predicate in STV does what it is supposed to do (namely express validity) only if it satisfies the following requirement:

Faithfulness Requirement:

$\vdash_{STV} \; (Val(\langle\Theta_1\rangle, \langle\Xi_1\rangle) \& ... \& Val(\langle\Theta_n\rangle, \langle\Xi_n\rangle)) \rightarrow Val(\langle\Gamma\rangle, \langle\Delta\rangle)$ holds just in case if $\Theta_1 \vdash_{STV} \Xi_1$,..., $\Theta_n \vdash_{STV} \Xi_n$, then $\Gamma \vdash_{STV} \Delta$.

Critics of the nontransitive approach argue that STV doesn't satisfy the faithfulness requirement. Since faithfulness fails, STV cannot truthfully capture which metarule applications preserve validity, according to STV. Barrio, Rosenblatt and Tajer summarize this criticism as follows:

> We will suggest three things. Firstly, [...] we will show that ST plus a validity predicate satisfying (generalized versions of) VD and VP does not provide a correct characterization of its own notion of validity. The difficulty [... is that] there are certain metarules that hold in this theory but that cannot be proved to hold, even though we can express them in the language of the theory. Secondly, [...] VD and VP can be strengthened in a very natural way so that those facts about metarules can in fact be represented. Thirdly, [...] the resulting system faces [... the problem] that the most obvious way to strengthen VD and VP will allow us to prove an internalized version of Cut. (Barrio et al., 2017, Sec. 1)

Let's grant their first and second points and focus on the third. By "an internalized version of Cut" they mean that, in STV, we can prove of applications of Cut that are not admissible in STV that they preserve validity. As an example, let Π be the singelton set containing a v-Curry sentence, namely the sentence that says that the argument from Π to $\emptyset$ is valid. Notice that in STV, $\emptyset \vdash_{STV} \Pi$ and $\Pi \vdash_{STV} \emptyset$ both hold but $\emptyset \vdash_{STV} \emptyset$ doesn't. This is how rejecting Cut blocks the v-Curry's threat of triviality. However, STV proves:

Int-Cut $\quad \vdash_{STV} (Val(\langle\emptyset\rangle, \langle\Pi\rangle) \& Val(\langle\Pi\rangle, \langle\emptyset\rangle)) \rightarrow Val(\langle\emptyset\rangle, \langle\emptyset\rangle)$

To see why Int-Cut holds in STV, notice that, given VD and Contraction, we get $Val(\langle\Pi\rangle, \langle\emptyset\rangle) \vdash_{STV} \emptyset$. So $Val(\langle\emptyset\rangle, \langle\Pi\rangle), Val(\langle\Pi\rangle, \langle\emptyset\rangle) \vdash_{STV} Val(\langle\emptyset\rangle, \langle\emptyset\rangle)$, by Weakening. And metarules for conjunction (on the left) and the conditional (on the right) will then give us Int-Cut.

Int-Cut seems to say that it is a "logical" truth (where logic includes the logic of validity) that if the arguments from $\emptyset$ to Π and from Π to $\emptyset$ are both valid, then the argument from $\emptyset$ to $\emptyset$ is also valid. As we know, however, this claim and, hence, Int-Cut are false about STV. So STV seems to say something false about its own metarules.

Barrio et al. argue that if we weaken the rules for validity, then STV won't prove of all instances of admissible metarules that they preserve validity.[7] So, whether we adopt strong or weak rules for validity, STV cannot truthfully express which applications of metarules preserve validity, according to STV.

Let's take stock. Critics of the nontransitive approach, in effect, suggest that a logic with a validity predicate (that is meant to solve paradoxes like the v-Curry) must meet the faithfulness requirement. Moreover, they argue that there is no plausible way to add a validity predicate to the logic of the nontransitive approach such that the resulting logic meets the faithfulness requirement. Hence, we should reject the nontransitive approach.

[7] That is their first point in the summary quoted above.

2 A Problem with Expressing Admissibility

The above criticism of the nontransitive approach has real bite only if the faithfulness requirement is plausible. In this section, I will first argue that the faithfulness requirement is plausible only if we are willing to go non-classical in our metatheory. The criticism rehearsed in the previous section actually applies to everyone who conducts her metatheory in classical logic (and who allows for sufficient self-reference to be worried about the paradoxes). Next, I will argue that if we conduct our metatheory in a nontransitive logic, there is no reason to think that the argument for Int-Cut goes through. The upshot is that there is no special problem for the nontransitive approach.

Recall that the faithfulness requirement says that a logic should be able to express that something is an application of one of its admissible metarules. To see whether that is really a good requirement, let's try to add such an expression to an object language. For any sequent $\Gamma \vdash \Delta$ that can be formulated in our language, let $\langle \Gamma \vdash \Delta \rangle$ be its canonical object language name. For our purposes, it will suffice to look at metarule applications with just one premise sequent. So a two-place predicate, *Adm*, will suffice. We want $Adm(\langle \Gamma \vdash \Delta \rangle, \langle \Theta \vdash \Xi \rangle)$ to express that if $\Gamma \vdash \Delta$ hold in our logic, then so does $\Theta \vdash \Xi$. The following is a variation on the faithfulness requirement using *Adm*.

ADM $\quad \vdash Adm(\langle \Gamma \vdash \Delta \rangle, \langle \Theta \vdash \Xi \rangle)$ just in case

if $\Gamma \vdash \Delta$, then $\Theta \vdash \Xi$.

Suppose we have a predicate that obeys ADM. And suppose that our object language allows for self-reference, such that, for your favorite absurd object language sentence A, the object language contains a Curry-like sentence κ such that:

ACu $\quad \kappa = Adm(\langle \vdash \kappa \rangle, \langle \vdash A \rangle)$.[8]

This gives rise to a version of Curry's paradox that I call the a-Curry (for admissibility-Curry), which is a variant of what Wansing and Priest (2015) call an "external Curry."

Proposition. *If ADM and ACu hold, then* $\vdash A$.

Proof. Suppose that $\vdash \kappa$. By ACu, this implies that $\vdash Adm(\langle \vdash \kappa \rangle, \langle \vdash A \rangle)$.[9] By ADM, it follows that if $\vdash \kappa$, then $\vdash A$. By modus ponens (in the metalanguage), $\vdash A$. Discharging our assumption by conditional proof (in the metalanguage) we get: if $\vdash \kappa$, then $\vdash A$. By ADM, it follows that $\vdash Adm(\langle \vdash \kappa \rangle, \langle \vdash A \rangle)$. By ACu, this means that $\vdash \kappa$. By modus ponens (in the metalanguage), we conclude that $\vdash A$. ■

The proof works like an ordinary Curry paradox in most respects. We use a conditional, "If $\vdash \kappa$, then $\vdash A$," that is true just in case its antecedent

[8]That is, our expression "κ" picks out the same sentence in the object language as our expression "$Adm(\langle \vdash \kappa \rangle, \langle \vdash A \rangle)$." If the object language were visible, we would see just one sentence here.

[9]Here I am assuming that "$\vdash$" is extensional. Alternatively, one could define κ in such a way that $\vdash \kappa$ holds iff $\vdash Adm(\langle \vdash \kappa \rangle, \langle \vdash A \rangle)$ holds.

is true. The difference to the traditional Curry paradox lies in the division of labor between object language and metalanguage. Our self-referential starting point, κ, and the absurdity, A, are in the object language. However, ADM bounces us out to the metatheory, and we then use the classicality of our metatheory to reason in the usual Curry-fashion to the claim that your favorite absurdity can be derived in the object language. Consequently, which rules hold in the object language doesn't matter. We just need ADM and ACu.

As Wansing and Priest (2015) point out, this kind of external Curry can be avoided by rejecting Contraction in the metatheory. So why should we care about the a-Curry? Well, precisely because it forces us to go non-classical in the metatheory if we want to express admissibility in our object language.

If we replace "$Adm(\langle \Gamma \vdash \Delta \rangle, \langle \Theta \vdash \Xi \rangle)$" in ADM with "$Val(\langle \Gamma \rangle, \langle \Delta \rangle) \rightarrow Val(\langle \Theta \rangle, \langle \Xi \rangle)$," it is easy to see that a proof exactly parallel to the one above goes through for every logic that meets the faithfulness requirement (and the corresponding variant of ACu).[10] So, upon reflection, the faithfulness requirement should be tempting only for people who are willing to do their metatheory in a non-classical setting.

Before moving on, notice that we can avoid the a-Curry while staying classical in our metatheory by taking issue with ADM. In parallel to

[10] I am assuming that every logic that is meant to solve the semantic paradoxes allows for stipulations like ACu. Hence, I will assume that rejecting ACu and its variants is not an option.

what Ketland (2012) and Cook (2014) say about the v-Curry, the a-Curry doesn't arise for admissibility predicates that codify admissibility in (just) logic, where logic doesn't include a theory of admissibility or validity but just classical first-order logic (FOL). To see this, suppose that $\vdash_{ADM}$ is the consequence relation of a calculus that includes first-order logic and also non-logical rules or axioms that govern the *Adm* predicate, and let $\vdash_{FOL}$ be the consequence relation of FOL, over the same language. Now, let's adjust ADM in the following way:

ADM* $\vdash_{ADM} Adm(\langle \Gamma \vdash \Delta \rangle, \langle \Theta \vdash \Xi \rangle)$ just in case
if $\Gamma \vdash_{FOL} \Delta$, then $\Theta \vdash_{FOL} \Xi$

As before, by ACu, assuming $\vdash_{FOL} \kappa$ gives us $\vdash_{ADM} Adm(\langle \vdash \kappa \rangle, \langle \vdash A \rangle)$, which by ADM* jointly imply $\vdash_{FOL} A$. By conditional proof and ADM*, it follows that $\vdash_{ADM} Adm(\langle \vdash \kappa \rangle, \langle \vdash A \rangle)$. This time, however, that doesn't imply $\vdash_{FOL} \kappa$, and so the proof above no longer goes through. In fact, it is easy to see that $\nvdash_{FOL} \kappa$. We just have to notice that κ has the form $F(x, y)$ and that no formula of this form is a truth of FOL because FOL is closed under substitution (see Cook, 2014). Given substitution, if κ were a truth of FOL, every metarule could be proved to be admissible.[11] So if critics of the nontransitive approach restrict the faithfulness requirement along the lines of ADM*, they avoid the a-Curry. Perhaps they could insist on

[11] If you believe that logic is essentially formal, that closure under substitution is the hallmark of logic's formality and that any validity and admissibility worth their names are logical relations, then you may think that what I say here means that the a-Curry isn't a genuine problem for anyone. Discussing such large topics in the philosophy of logic is beyond the scope of this paper.

their adjusted faithfulness requirement in a classical setting in this way.[12] In fact, however, Barrio et al. (2017) seem unwilling to take that route. They briefly consider a right-rule for the validity predicate, which they call VP^K, that captures all logical metarules in the object language, but they reject this rule because the rule leads to the admissibility of metarules that it cannot capture in the object language. They do this, I take it, because they think that the crucial philosophical question is not whether an extension of STT can capture the validities of first-order logic but whether it can capture its own consequence relation. For semantic inferentialists, like Ripley, this is an important question because the calculus in which they are ultimately interested is supposed to serve as a theory of meaning. Hence, the question whether this calculus can adequately capture its own consequence relation is, for them, the question whether their theory of meaning applies to itself. Understood in this way, the criticism of Barrio et al. amounts to the claim that the nontransitive semantic inferentialist cannot give an account of what she means when she is presenting her theory of meaning. To make that point, they need ADM and not ADM*. So I will assume that it is ADM and not ADM* that is at issue.

As we have seen, if we want to hold on to ADM and ACu, we must go non-classical in the metatheory. I will now argue that even if we go

[12]Since Peano Arithmetic can be added to STT (Cobreros et al., 2013), it is an interesting question whether a faithful validity predicate that captures first-order logical validities can be defined in Peano Arithmetic. Since this question is independent of the viability of the nontransitive approach, I will not pursue it here.

non-classical in our metatheory, there is no special problem for the nontransitive approach. Before I do so, I want to point out that if there are convincing reasons why we shouldn't do our metatheory in classical logic (at least for solutions to the paradoxes), we don't need Barrio et al.'s criticism of the nontransitive approach in order to reject it in its current form. After all, advocates of the nontransitive approach always work in a classical metatheory. This is standard in most other approaches as well. So the criticism would apply not just to the nontransitive approach. There are, of course, exceptions; some people do their metatheory in a nonclassical logic (Weber et al., 2016; Zardini, 2014, 2013; Bacon, 2013). Let's assume that this is the way to go.

Barrio et al. (2017) use Zardini (2014) as an example of a non-contractive theorist who avoids the problems they raise for the nontransitive approach. Zardini uses his preferred non-contractive logic $\mathrm{IKT}_{\Rightarrow \mathsf{tf}}$ as the logic in which he does the metatheory of $\mathrm{IKT}_{\Rightarrow \mathsf{tf}}$. In order to level the playing field, we would have to look at ST from a nontransitive perspective.

I do not have a nontransitive metatheory that parallels Zardini's non-contractive metatheory. Fortunately, that isn't necessary. Barrio et al.'s argument—rehearsed above—for the claim that STV proves a statement saying that a problematic instance of Cut preserves validity depends on the claim that, in STV, we have $Val(\langle \Pi \rangle, \langle \emptyset \rangle) \vdash \emptyset$. Their argument uses this claim as a crucial intermediary conclusion. They derive it via VD, and they use it as a premise in an application of Weakening. In other words, they cut on "$Val(\langle\Pi\rangle, \langle\varnothing\rangle) \vdash \varnothing$" in the metalanguage. Notice that "$Val(\langle\Pi\rangle, \langle\varnothing\rangle) \vdash \varnothing$" translates to $Val(\langle Val(\langle\Pi\rangle, \langle\varnothing\rangle)\rangle, \langle\varnothing\rangle)$ in the object language and, hence, to $Val(\langle\Pi\rangle, \langle\varnothing\rangle)$. This is a v-Curry sentence in the object language. Hence, it is the kind of paradoxical sentence for which Cut should fail, according to the nontransitive approach. That suggests that Cut should also fail for "$Val(\langle\Pi\rangle, \langle\varnothing\rangle) \vdash \varnothing$" in the metalanguage. Thus, if we do our metatheory in a nontransitive setting, we shouldn't expect to be able to cut on "$Val(\langle\Pi\rangle, \langle\varnothing\rangle) \vdash \varnothing$." So, we shouldn't expect that Barrio et al.'s argument that STV proves a statement saying that problematic applications of Cut preserve validity goes through.

Of course, standard formulations of STV assume that we can use claims like $Val(\langle\Pi\rangle, \langle\varnothing\rangle) \vdash \varnothing$ as lemmas in sequent proof-trees, i.e., that we can cut on them. But that is because standard formulations of STV do their metatheory in classical logic.

To sum up, if we think that we should do the metatheory of the nontransitive approach in classical logic, then we should dismiss Barrio et al.'s criticism because we should reject the faithfulness requirement. If, however, we should do the metatheory of the nontransitive approach in a nontransitive logic, then there is no reason to think the argument behind Barrio et al.'s criticism goes through.

It may be a fair request to ask advocates of the nontransitive approach to do their metatheory in their preferred nontransitive logic. And it is not obvious that advocates of the nontransitive approach can do for STT what Zardini did for $\text{IKT}_{\Rightarrow\text{tf}}$. That may be a problem for the nontransitive approach, but it is not the problem that Barrio, Rosenblatt and Tajer raised. They point out, in a classical setting, that ST cannot meet the faithfulness requirement. Regarding that problem, we are all in the same situation.

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