Individual-Actualism and Three-Valued Modal Logics, Part 1: Model-Theoretic Semantics

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INDIVIDUAL-ACTUALISM AND THREE-VALUED MODAL LOGICS, PART 1: MODEL-THEORETIC SEMANTICS

A logician's model is a set-theoretic object; it is interesting in so far as it models the sorts of facts and counterfactuals about reference, thus about the relations between linguistic objects and the world, which can underlie the assignment of truth-values to sentences in an interpreted language. Fixing such a language, an associated model-theoretic semantics gives rise to the following question: How is the satisfaction-in-a-model relation (between models, variable assignments and formulae in an *uninterpreted* language) related to satisfaction relation (between variable assignments and formulae in an *interpreted* language)? In what sense does the unproblematic former relation model the more problematic latter relation? Through an answer to this question, model-theory meets metaphysics. A philosopher's evaluation of a model-theoretic semantics should be based on the sort of answers available to the above question, and the commitments presupposed by such answers.

Consider for a moment the simplest case: classical model theory, in which the uninterpreted languages are first-order and extensional, those studied under the rubric "First-Order Predicate Logic". Even in this setting our question should be asked; but here the obviousness of the answer renders the question scarcely visible: a model has the form $\mathfrak{A} = (U, \mathscr{E}, \mathscr{N})$; members of U represent objects or individuals; $\mathscr{E}$, the extension-function, represents the application relation between interpreted predicates and n-tuples of individuals; $\mathscr{N}$, the naming-function, which represents the designation relation between names and individuals. When we direct our question towards the familiar two-valued model-theoretic semantics for modal languages, matters are less straightforward. The simplest response follows the pattern set in the non-modal case: where $(\mathfrak{A}, w)$ is a total modal model, with $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N})$ as described in §1, we'd say: members of W represent possible worlds; members of U represent possible individuals;

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for each $u \in W$ members of $\bar{U}(u)$ represent the individuals which exist at whatever world u represents. This response carries a commitment to the reality of possible worlds and possible individuals. In one respect, that of answering our question, it makes life easy. But the extravagent ontology presupposed by this answer may make one want an alternative response.

The above offer may be declined in three ways: the world-actualist refuses to posit the possible worlds; the individual-actualist refuses to posit the possible individuals; the actualist (sans phrase) refuses to posit either sort of entity. (Of course if the offer of modal realism comes from David Lewis, who takes a possible world to be a possible individual – the sum of the possible individuals existing in it – then individual-actualism collapses to actualism, and world-actualism without individual-actualism becomes very peculiar.) In this paper I speak as an individual-actualist. I'll follow the common policy of availing myself of "possible-world talk" without addressing the issue of whether or not to buy world-actualism.

The "cash value" of individual-actualism is, in part, a constraint on any definition of satisfaction for sentences of interpreted modal languages: such a definition must neither use variable-assignments whose values are non-actual individuals nor claim that any singular designators refer to such individuals. This constraint carries over to a constraint on any model-theoretic semantics that purports to model the semantic under-pinnings of an interpreted language. Where $\mathfrak{M} = (\mathfrak{A}, w)$ is a modal model, w is "the actual world" of $\mathfrak{M}$; the individual-actualist will maintain that relative to $\mathfrak{M}$, members of $\bar{U}(w)$, and only these members, represent individuals; so the individual-actualist should require that satisfaction (in-a-model) meet what I'll call "the actualist's constraint": if a formula ϕ is satisfied in model $\mathfrak{M}$ by a variable-assignment α then:

(i) for any variable v, $\alpha(v) \in \bar{U}(w)$;

(ii) for any individual constant τ occurring in ϕ, $\mathcal{N}(\tau) \in \bar{U}(w)$.

Can we simply impose the model-theoretic version of the actualist's constraint on the familiar two-valued semantics for modal languages? Unpacking satisfaction in $(\mathfrak{A}, w)$ for a formula containing '$\Box$' involves looking at questions of satisfaction in other models of the form $(\mathfrak{A}, u)$ for appropriate $u \in W$. The actualist's constraint is imposed on satisfaction-in-a-model in general, not on satisfaction in just one model based on a given structure $\mathfrak{A}$. In the conditions for satisfaction in $(\mathfrak{A}, w)$, members of $U - \bar{U}(w)$ do work, even though it's not the work of representing individuals; they help determine the conditions for satisfaction in $(\mathfrak{A}, w)$. They do this in a way which makes the following hold: relative to $(\mathfrak{A}, w)$, if the world represented by u had been actual then members of $\bar{U}(u)$ would have represented all individuals. Thus satisfaction in any model $(\mathfrak{A}, u)$ must also meet the actualist's constraint. With the constraint so understood, the only two-valued **S5** models the individual-actualist could regard as legitimate relata for satisfaction-in-a-model relation (if it is to model satisfaction) would be models based on structures meeting the constant-domain condition: for all $u \in W$, $\bar{U}(u) = U$.[1] But this restriction would be intolerable; it would make uncontroversial falsehoods into logical truths, e.g. "Everything necessarily exists" and "There couldn't be something which didn't actually exist".

The individual-actualist can't give a satisfactory answer to our initial question when that question is directed to the familiar modal model-theoretic semantics. The actualist constraint requires that the individual-actualist adopt a model-theoretic semantics in which $\mathcal{N}$ and α can be undefined on certain names and variables. The best way to permit this is to permit sentences to be neither true nor false; in other words, our semantics must open a truth-value (and thus a "satisfaction-value") gap. For non-modal languages, such semantics are discussed in detail in [2]; some acquaintance with that paper should make the reader more comfortable with what will follow. A semantics involving such a gap requires definition of a frustration relation, in addition to a satisfaction relation; so the actualist's constraint must be supplemented by requiring that if ϕ is frustrated in $\mathfrak{M}$ by α then (i) and (ii) hold.

This paper presents three model-theoretic semantics for disinterpreted modal languages; one will be actualistic in that it honors the model-theoretic version of the actualist's constraint; the other two flout this constraint, but differ in their treatment of identity and existence. We'll investigate the relationships between these semantics.

There will be translations from the actualistic semantics into both possibilistic semantics; this shows that the individual-actualist may pretend to posit non-actual possible individuals, and so "speak with" the individual-possibilist. The instrumental value of this pretence will be evident when we consider formalization of the logics based on these various semantics; see remarks at the end of §7.

Some logicians are reluctant to think about three-valued model-theoretic semantics, and so prefer to handle formulae containing non-denoting terms by adopting a conventional truth-value; the Falsehood Convention has been most popular: an atomic formula is not satisfied if it contains a non-denoting term; all other clauses in the definition of satisfaction run as usual. The relationship between three-valued model-theoretic semantics and two-valued semantics for non-modal languages is examined in [2] §3. Those results easily extend to both the possibilistic and the actualistic semantics for modal languages; in particular, results about these semantics easily collapse to results about their corresponding two-valued semantics. Formulation of these results is left to those readers who prefer the latter sort of semantics to the three-valued sort.

The following notational conventions guide our use of the expressions in the left-hand column:

$\ulcorner \ldots \urcorner\!\downarrow$: $\ulcorner \ldots \urcorner$ is defined, i.e. stands for something,
$\ulcorner \ldots \urcorner\!\uparrow$: $\ulcorner \ldots \urcorner$ is undefined;

$\ulcorner \ldots = \text{\_ \_ \_} \urcorner$: $\ulcorner \ldots \urcorner$ and $\ulcorner \text{\_ \_ \_} \urcorner$ are both defined and stand for the same thing;

$\ulcorner \ldots \simeq \text{\_ \_ \_} \urcorner$: either
$\ulcorner \ldots = \text{\_ \_ \_} \urcorner$ holds or else both $\ulcorner \ldots \urcorner$ and $\ulcorner \text{\_ \_ \_} \urcorner$ are undefined;

$\ulcorner \ldots \in \text{\_ \_ \_} \urcorner$: $\ulcorner \ldots \urcorner$ and $\ulcorner \text{\_ \_ \_} \urcorner$ are both defined and the former designates an element of what the latter designates.

1. THREE MODEL-THEORETIC SEMANTICS FOR MODAL LANGUAGES

Fix a countable set **Var** of variables, and these logical lexicons:

$$\mathrm{lex}_0 = \{`\bot', `\supseteq', `\underline{\exists}', `\underline{\Box}', `\approx'\};$$

$\text{lex}_1 = \{`\bot', `\supset', `\exists', `\Box', `\approx'\}$;

for $i \in 2$, $\text{lex}_{i,u} = \text{lex}_i \cup \{`\mathbf{u}'\}$;

for $i \in 2$, $\text{lex}_{i,T} = \text{lex}_i \cup \{`T'\}$;

$\text{lex}_{0,T,\mathbf{u}} = \{`\mathbf{u}', `\supseteq', `T', `\underline{\exists}', `\underline{\Box}', `\approx'\}$;

$\text{lex}_2 = \{`\mathbf{u}', `\supset', `T', `\exists', `\Box', `\approx'\}$.

Where lex_y is as above with 'T' $\notin \text{lex}_y$, form $\text{lex}_{y,s}$ by replacing '$\approx$' by '$\approx_s$' in lex_y. Fix a set **Pred** of predicate constants and a set **C** of individual constants. Where lex_y has been defined above, we introduce a language $L_y = L_y(\mathbf{Pred}, \mathbf{C})$ as follows. A term of L_y is a member of $\mathbf{Var} \cup \mathbf{C}$; the formulae of L_y are defined by selecting from the following formation-rules those which involve symbols in the lexicon of L_y:

> if $\theta \in \mathbf{Pred}$ is n-place and $\tau_0, \ldots, \tau_{n-1}$ are terms then $\theta(\tau_0, \ldots, \tau_{n-1})$ is a formula;
>
> if τ_0, τ_1 are terms then $(\tau_0 \approx \tau_1)$ and $(\tau_0 \approx_s \tau_1)$ are formulae;
>
> '$\bot$' and '$\mathbf{u}$' are formulae;
>
> if ϕ and ψ are formulae then $(\phi \supseteq \psi)$, $(\phi \supset \psi)$, $T\phi$, $\underline{\Box}\phi$, and $\Box\phi$ are formulae;
>
> if ϕ is a formula and $v \in \mathbf{Var}$ then $(\underline{\exists}v)\phi$ and $(\exists v)\phi$ are formulae.

Let $fml(L_y)$ be the set of formulae of L_y; let $\text{Sent}(L_y)$ be the set of sentences of L_y. A parameter-occurrence in $\phi \in fml(L_y)$ is an occurrence of a member of **C** or a free occurrence of a variable in ϕ; τ is a parameter of ϕ iff there is a parameter occurrence of τ in ϕ.

In what follows, the left entry will serve as an abbreviation of the right entry in all languages in which the left entry has not already been defined; most of these abbreviations are taken from [2]:

> $\bot : T\mathbf{u}$; $\neg\phi : (\phi \supseteq \bot)$; $\neg\phi : (\phi \supset \bot)$;
>
> $(\phi \,\underline{\vee}\, \psi) : (\neg\phi) \supseteq \psi$; $(\phi \vee \psi) : (\neg\phi) \supset \psi$;
>
> $(\phi \,\underline{\&}\, \psi) : \neg(\phi \supseteq (\neg\psi))$; $(\phi \,\&\, \psi) : \neg(\phi \supset (\neg\psi))$;
>
> $(\phi \supseteq \psi) : (\phi \supset \psi) \,\&\, (\phi \supset \psi) \,\&\, (\psi \supset \psi)$;

$F\phi : T(\neg\phi)$; $U\phi : \neg T(\phi \supset \phi)$; $U\phi : \neg T(\phi \supseteq \phi)$;

$(\phi \supset_w \psi) : (T\phi) \supset \psi$; $(\phi \supset_s \psi) : (F\phi) \vee \psi$;

$(\phi \equiv \psi) : (\phi \supseteq \psi) \mathbin{\underline{\&}} (\psi \supseteq \phi)$;

$(\phi \equiv \psi) : (\phi \supset \psi) \mathbin{\&} (\psi \supset \phi)$;

$(\underline{\forall} v)\phi : \neg(\underline{\exists} v)\neg\phi$; $(\forall v)\phi : \neg(\exists v)\neg\phi$;

$(\underline{\exists} v)\phi : (\exists v)\phi \mathbin{\&} (\forall v)(\phi \supset \phi)$;

$(\tau_0 \not\approx \tau_1) : \neg(\tau_0 \approx \tau_1)$; $(\tau_0 \not\approx \tau_1) : \neg(\tau_0 \approx \tau_1)$;

$E(\tau) : (\underline{\exists} v)(v \approx \tau)$, where v is not τ;

$E(\tau) : (\exists v)(v \approx \tau)$, where v is not τ;

$E_s(\tau) : (\underline{\exists} v)(v \approx_s \tau)$, where v is not τ;

$E_s(\tau) : (\exists v)(v \approx_s \tau)$, where v is not τ;

$E_s(\tau) : TE(\tau)$;

$(\tau_0 \approx_s \tau_1) : (\tau_0 \approx_s \tau_1) \mathbin{\&} (TE(\tau_0) \equiv TE(\tau_1))$;

$(\tau_0 \approx \tau_1) : (\tau_0 \approx_s \tau_1) \mathbin{\underline{\&}} (\tau_0 \approx_s \tau_0) \mathbin{\underline{\&}} (\tau_1 \approx_s \tau_1)$;

$\underline{\Box}\phi : \Box\phi \mathbin{\&} \Box(\phi \supset \phi)$;

$\Diamond\phi : \neg\Box\neg\phi$; $\underline{\Diamond}\phi : \neg\underline{\Box}\neg\phi$.

Where $L = L_y(\mathbf{Pred}, \mathbf{C})$ is one of the languages just introduced, we adopt the following definitions. A partial structure for L, or alternatively for **Pred**, **C**, has the form $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N})$, where:

W and U are non-empty sets; $R \subseteq W^2$;

$\bar{U}$ is a function from W into Power(U);

$\mathscr{N}$ is a function into U with dom($\mathscr{N}$) $\subseteq$ **C**;

$\mathscr{E}$ is a function on **Pred** such that for any n-place $\mathbf{P} \in \mathbf{Pred}$:

$\mathscr{E}(\mathbf{P})$ is a function into $2 = \{0, 1\}$ with $\mathrm{dom}(\mathscr{E}(\mathbf{P})) \subseteq W \times U^n$.

(Note: here $W \times U^o = W$.)

A partial model for L, or alternatively for **Pred**, **C**, has the form $\mathfrak{M} = (\mathfrak{A}, w)$ where $\mathfrak{A}$ is as above and $w \in W$. Hereafter partial structures and partial models shall simply be called 'structures' and 'models' respectively. Where $\mathfrak{A}$ and $\mathfrak{M}$ are as above:

Frame($\mathfrak{A}$) = Frame($\mathfrak{M}$) = (W, R);

$\mathfrak{A}$ is the structure for $\mathfrak{M}$;

w is the actual-world for $\mathfrak{M}$;

α is an $\mathfrak{A}$-assignment iff α is a function into U with dom(α) $\subseteq$ **Var**;

α is an $\mathfrak{M}$-assignment iff α is an $\mathfrak{A}$-assignment with rng(α) $\subseteq \bar{U}(w)$;

$\mathscr{N}$ is total iff dom($\mathscr{N}$) = **C**;

α is total iff dom(α) = **Var**;

$\mathscr{E}$ is total iff for each n-place **P** $\in$ **Pred**, dom($\mathscr{E}$(**P**)) = $W \times U^n$;

$\mathfrak{A}$ is total iff $\mathscr{E}$ and $\mathscr{N}$ are total;

$\mathscr{E}$ is actualistic iff for each n-place **P** $\in$ **Pred** and $n \geqslant 1$: for any $(w, a_0, \ldots, a_{n-1}) \in W \times U^n$, if $\mathscr{E}(\mathbf{P})(w, a_0, \ldots, a_{n-1})\downarrow$ then $a_0, \ldots, a_{n-1} \in \bar{U}(w)$;

$\mathscr{E}$ is actualistically total iff $\mathscr{E}$ is actualistic and for each n-place **P** $\in$ **Pred** with $n \geqslant 1$:
if $w \in W$ and $a_0, \ldots, a_{n-1} \in \bar{U}(w)$ then $\mathscr{E}(\mathbf{P})(w, a_0, \ldots, a_{n-1})\downarrow$.
$\mathfrak{A}$ and $\mathfrak{M}$ are extensionwise actualistic, hereafter *ea*, if $\mathscr{E}$ is actualistic;

$\mathfrak{A}$ and $\mathfrak{M}$ extensionwise actualistically total, hereafter *eat*, iff $\mathscr{E}$ is actualistically total;

$\mathfrak{M}$ is denotationwise actualistic, hereafter *da*, iff rng($\mathscr{N}$) $\subseteq \bar{U}(w)$;

$\mathfrak{M}$ is actualistic, or an *a*-model, iff $\mathfrak{M}$ is *ea* and *da*;

$\mathfrak{M}$ is actualistically total, or an *at*-model, iff $\mathfrak{M}$ is *eat* and *da*;

$\mathfrak{A}$ and $\mathfrak{M}$ are non-null, hereafter *nn*, iff for each $w \in W$, $\bar{U}(w)$ is non-empty.

For the philosophical points of this paper, we could restrict our attention to structures and models that are *at*, or even *at* and *nn*. I have introduced a broader class of structures because such greater generality is virtually free, is natural, and hopefully is not distracting.

Let a class **L** be logic-defining iff all members of **L** are binary relational systems (b.r.s.s.), i.e. of the form (W, R) where W is a non-empty set and $R \subseteq W^2$.

The following logic-defining classes, with their peculiar but traditional labels (except for '$\mathbf{B}^-$'), figure prominently in the modal logic literature:

K = the class of all b.r.s.s.;

T = the class of reflexive b.r.s.s.;

K4 = the class of transitive b.r.s.s.;

B^- = the class of symmetric b.r.s.s.;

S4 = **T** $\cap$ **K4**;

B = **T** $\cap$ $\mathbf{B}^-$;

S5 = $\{(W, W^2)$: W is non-empty$\}$.

Where **L** is logic defining and $\mathfrak{A}$ is a structure for **Pred**, **C**:

$\mathfrak{A}$ is an **L**-structure iff Frame($\mathfrak{A}$) $\in$ **L**;

$\mathfrak{A}$ is an $\mathbf{L}_x$-structure iff $\mathfrak{A}$ is an **L**-structure and is x, where 'x' is replaced by '*ea*', '*eat*', '*nn*', '*eat & nn*', etc.

Where $\mathfrak{M}$ is a model for **Pred**, **C**:

$\mathfrak{M}$ is an **L**-model iff Frame($\mathfrak{M}$) $\in$ **L**;

$\mathfrak{M}$ is an $\mathbf{L}_x$-model iff $\mathfrak{M}$ is an **L**-model and is x, where 'x' is replaceable by '*ea*', '*eat*', '*da*', '*a*', '*nn*', etc.

A class of models $\mathbf{L}_x$ (where we permit 'x' to be replaced by the empty symbol as well as those indicated above) together with a model-theoretic definition of satisfaction and frustration, hereafter called simply "a semantics", determine a modal logic. In this paper, we'll consider logics of the following forms, where the superscript indicates whether the semantics is possibilistic, semi-possibilistic or actualistic:

$$\mathbf{L}^{p}, \mathbf{L}^{p}_{ea}, \mathbf{L}^{p}_{eat}, \mathbf{L}^{p}_{ea\&nn}, \mathbf{L}^{p}_{eat\&nn}, L^{sp}, L^{sp}_{eu}, \mathbf{L}^{sp}_{eat}, \mathbf{L}^{sp}_{ea\&nn}, \mathbf{L}^{sp}_{eat\&nn}, L^{a}_{a}, L^{a}_{at}, \mathbf{L}^{a}_{nn}, \mathbf{L}^{a}_{at\&nn}.$$

Let $\mathfrak{A}$ be a structure for L, α be an $\mathfrak{A}$-assignment, and τ be a term of L. We define the denotation of τ relative to $\mathfrak{A}$ and α as follows:

$$\text{den}(\mathfrak{A}, \alpha, \tau) \simeq \begin{cases} \alpha(\tau) & \text{if } \tau \in \mathbf{Var}; \\ \mathscr{N}(\tau) & \text{if } \tau \in \mathbf{C}. \end{cases}$$

We'll now define the relations $\models_p$ and ${}_p\!\dashv$ (i.e. possibilistic satisfaction and frustration). Where $\mathfrak{M} = (\mathfrak{A}, w)$ for $w \in W$, let:

$\mathfrak{M}\ {}_p\!\dashv \bot[\alpha]$;

$\mathfrak{M} \not\models_p \mathbf{u}[\alpha]$ and $\mathfrak{M}\ {}_p\!\not\dashv \mathbf{u}[\alpha]$;

where $\mathbf{P} \in \mathbf{Pred}$ is 0-place,

$\mathfrak{M} \models_p \mathbf{P}[\alpha]$ iff $\mathscr{E}(\mathbf{P}) = 1$;

$\mathfrak{M}\ {}_p\!\dashv \mathbf{P}[\alpha]$ iff $\mathscr{E}(\mathbf{P}) = 0$;

where $\mathbf{P} \in \mathbf{Pred}$ is n-place for $n \geqslant 1$,

$\mathfrak{M} \models_p \mathbf{P}(\tau_0, \ldots\ \tau_{n-1})[\alpha]$ iff for each $i < n$ there is an a_i such that $\text{den}(\mathfrak{A}, \alpha, \tau_i) = a_i$ and $\mathscr{E}(\mathbf{P})(w, a_0, \ldots, a_{n-1}) = 1$;

$\mathfrak{M}\ {}_p\!\dashv \mathbf{P}(\tau_0, \ldots, \tau_{n-1})[\alpha]$ iff for each $i < n$ there is an a_i such that $\text{den}(\mathfrak{A}, \alpha, \tau_i) = a_i$ and $\mathscr{E}(\mathbf{P})(w, a_0, \ldots, a_{n-1}) = 0$;

$\mathfrak{M} \models_p (\tau_0 \approx \tau_1)[\alpha]$ iff there is an a so that $\text{den}(\mathfrak{A}, \alpha, \tau_i) = a$ for $i = 0, 1$;

$\mathfrak{M}\ {}_p\!\dashv (\tau_0 \approx \tau_1)[\alpha]$ iff there are distinct a_0, a_1, so that $\text{den}(\mathfrak{A}, \alpha, \tau_i) = a_i$ for $i = 0, 1$;

$\mathfrak{M} \models_p (\tau_0 \approx_s \tau_1)[\alpha]$ iff $\mathfrak{M} \models_p (\tau_0 \approx \tau_1)[\alpha]$;

$\mathfrak{M}\ {}_p\!\dashv (\tau_0 \approx_s \tau_1)[\alpha]$ iff either $\mathfrak{M}\ {}_p\!\dashv (\tau_0 \approx \tau_1)[\alpha]$ or for some $i < 2$ den$(\mathfrak{A}, \alpha, \tau_i)\downarrow$ and den$(\mathfrak{A}, \alpha, \tau_{1-i})\uparrow$;

$\mathfrak{M} \models_p (\phi \supseteq \psi)[\alpha]$ iff either $\mathfrak{M}\ {}_p\!\dashv \phi[\alpha]$ and either $\mathfrak{M} \models_p \psi[\alpha]$ or $\mathfrak{M}\ {}_p\!\dashv \psi[\alpha]$, or $\mathfrak{M} \models_p \phi[\alpha]$ and $\mathfrak{M} \models_p \psi[\alpha]$;

$\mathfrak{M}\ {}_p\!\dashv (\phi \supseteq \psi)[\alpha]$ iff $\mathfrak{M} \models_p \phi[\alpha]$ and $\mathfrak{M}\ {}_p\!\dashv \psi[\alpha]$;

$\mathfrak{M} \models_p (\phi \supset \psi)[\alpha]$ iff either $\mathfrak{M}\ {}_p\!\dashv \phi[\alpha]$ or $\mathfrak{M} \models_p \psi[\alpha]$;

$\mathfrak{M}\ {}_p\!\dashv (\phi \supset \psi)[\alpha]$ iff $\mathfrak{M} \models_p \phi[\alpha]$ and $\mathfrak{M}\ {}_p\!\dashv \psi[\alpha]$;

$\mathfrak{M} \models_p T\phi[\alpha]$ iff $\mathfrak{M} \models_p \phi[\alpha]$;

$\mathfrak{M}\ {}_p\!\dashv T\phi[\alpha]$ iff $\mathfrak{M} \not\models_p \phi[\alpha]$;

$\mathfrak{M} \models_p (\underline{\exists} v)\phi[\alpha]$ iff for some $a \in \bar{U}(w)$, $\mathfrak{M} \models_p \phi[\alpha^v_a]$ and for all $b \in \bar{U}(w)$ either $\mathfrak{M} \models_p \phi[\alpha^v_b]$ or $\mathfrak{M}\ {}_p\!\dashv \phi[\alpha^v_b]$;

$\mathfrak{M}\ {}_p\!\dashv (\underline{\exists} v)\phi[\alpha]$ iff for all $a \in \bar{U}(w)$, $\mathfrak{M}\ {}_p\!\dashv \phi[\alpha]$;

$\mathfrak{M} \models_p (\exists v)\phi[\alpha]$ iff for some $a \in \bar{U}(w)$, $\mathfrak{M} \models_p \phi[\alpha^v_a]$;

$\mathfrak{M}\ {}_p\!\dashv (\exists v)\phi[\alpha]$ iff for all $a \in \bar{U}(w)$, $\mathfrak{M}\ {}_p\!\dashv \phi[\alpha^v_a]$.

$\mathfrak{M} \models_p \underline{\Box}\phi[\alpha]$ iff for all $u \in W$, if wRu then $(\mathfrak{A}, u) \models_p \phi[\alpha]$;

$\mathfrak{M}\ {}_p\!\dashv \underline{\Box}\phi[\alpha]$ iff for some $u \in W$, wRu and $(\mathfrak{A}, u)\ {}_p\!\dashv \phi[\alpha]$, and for all u such that wRu, either $(\mathfrak{A}, u) \models_p \phi[\alpha]$ or $(\mathfrak{A}, u)\ {}_p\!\dashv \phi[\alpha]$.

$\mathfrak{M} \models_p \Box\phi[\alpha]$ iff for all $u \in W$, if wRu then $(\mathfrak{A}, u) \models_p \phi[\alpha]$;

$\mathfrak{M}\ {}_p\!\dashv \Box\phi[\alpha]$ iff for some $u \in W$, wRu and $(\mathfrak{A}, u)\ {}_p\!\dashv \phi[\alpha]$.

Clearly where $\mathfrak{A}$ and α are total $\models_p$ coincides with the usual two-valued notion of satisfaction, which we'll represent by '$\models_2$'.

The following further definitions shall be useful:

$\mathfrak{M} |_p \phi[\alpha]$ iff $\mathfrak{M} \not\models_p \phi[\alpha]$ and $\mathfrak{M}\ {}_p\!\not\dashv \phi[\alpha]$;

$\mathfrak{M} \models^w_p \phi[\alpha]$ iff $\mathfrak{M}\ {}_p\!\not\dashv \phi[\alpha]$;

$\mathfrak{M} \models_p \phi$ iff for all $\mathfrak{A}$-assignments α, $\mathfrak{M} \models_p \phi[\alpha]$, and similarly for $\mathfrak{M}\ {}_p\!\dashv \phi$, $\mathfrak{M} \models^w_p \phi$ and $\mathfrak{M} |_p \phi$;

where $\Gamma \subseteq fml(L)$, $\mathfrak{M} \models_p \Gamma[\alpha]$ iff for all $\phi \in \Gamma$, $\mathfrak{M} \models_p \phi[\alpha]$;

similarly for $\mathfrak{M} \models^w_p \Gamma[\alpha]$; similarly for $\mathfrak{M} \models_p \Gamma$ and $\mathfrak{M} \models^w_p \Gamma$.

As usual, possibilistic truth and falsity are possibilistic satisfiability and frustratability for sentences (i.e. formulae with no free variables). Note: '$\models^w$' represents "weak satisfaction"; the superscripted 'w' is not a metavariable or a variable, but an abbreviation of 'weak'.

The following should be noticed: $\models_p$ and ${}_p\dashv$ are possibilistic in that they flout the actualist's constraint; we might have $\mathfrak{M} \models_p \phi[\alpha]$ or $\mathfrak{M}\ {}_p\dashv \phi[\alpha]$ even though den$(\mathfrak{A}, \alpha, \tau) \notin \bar{U}(w)$, for τ a parameter in ϕ. But the clauses governing '$\underline{\exists}$' and '$\exists$' are actualistic in the sense that the relevant values for the quantified variables are confined to $\bar{U}(w)$ where w is the actual world for $\mathfrak{M}$.[2]

A sequent of L is an ordered triple (Γ, Δ, ϕ), where $\Gamma \subseteq \Delta \subseteq fml(L)$ and $\phi \in fml(L)$. Where 'x' may be replaced by the empty symbol, '*eat*', '*ea*' or '*nn*' ['*a*', '*at*', '*a* & *nn*', '*at* & *nn*'] let (Γ, Δ, ϕ) be $\mathbf{L}^p_x$-valid iff for every $\mathbf{L}_x$-model $\mathfrak{M}$ with structure $\mathfrak{A}$ and every $\mathfrak{A}$-assignment [$\mathfrak{M}$-assignment] α:

$$\text{if } \mathfrak{M} \models_p \Gamma[\alpha] \text{ and } \mathfrak{M} \models^w_p \Delta[\alpha] \text{ then } \mathfrak{M} \models_p \phi[\alpha];$$

let (Γ, Δ, ϕ) be weakly $\mathbf{L}^p_x$-valid iff for every $\mathfrak{M}$ and α as above:

$$\text{if } \mathfrak{M} \models_p \Gamma[\alpha] \text{ and } \mathfrak{M} \models^w_p \Delta[\alpha] \text{ then } \mathfrak{M} \models^w_p \phi[\alpha].$$

Let ϕ be $\mathbf{L}^p_x$-positively equivalent to ψ iff for all $\mathfrak{M}$ and α as above:

$$\mathfrak{M} \models_p \phi[\alpha] \text{ iff } \mathfrak{M} \models_p \psi[\alpha];$$

let ϕ be $\mathbf{L}^p_x$-equivalent to ψ iff ϕ is $\mathbf{L}^p_x$-positively equivalent to ψ and $\neg\phi$ is $\mathbf{L}^p_x$-positively equivalent to $\neg\psi$.

We'll now define the relations $\models_{sp}$ and ${}_{sp}\dashv$ (i.e. semi-possibilistic satisfaction and frustration). Where $\mathfrak{M} = (\mathfrak{A}, w)$ is any model for L and α is any $\mathfrak{A}$-assignment, we take over all clauses in the definition of $\models_p$ and ${}_p\dashv$, except those governing '$\approx$' and '$\approx_s$'; these are our novel clauses:

$$\mathfrak{M} \models_{sp} (\tau_0 \approx \tau_1)[\alpha] \text{ iff den}(\mathfrak{A}, \alpha, \tau_0) = \text{den}(\mathfrak{A}, \alpha, \tau_1) \in \bar{U}(w);$$

$$\mathfrak{M}\ {}_{sp}\dashv (\tau_0 \approx \tau_1)[\alpha] \text{ iff for both } i < 2, \text{den}(\mathfrak{A}, \alpha, \tau_i) \in \bar{U}(w) \text{ and den}(\mathfrak{A}, \alpha, \tau_0) \neq \text{den}(\mathfrak{A}, \alpha, \tau_1);$$

$$\mathfrak{M} \models_{sp} (\tau_0 \approx_s \tau_1)[\alpha] \text{ iff } \mathfrak{M} \models_{sp} (\tau_0 \approx \tau_1)[\alpha];$$

$\mathfrak{M}\ {}_{sp}\!\dashv (\tau_0 \approx_s \tau_1)[\alpha]$ iff either $\mathfrak{M}\ {}_{s}\!\dashv (\tau_0 \approx \tau_1)[\alpha]$ or for some $i < 2$, $\text{den}(\mathfrak{A}, \alpha, \tau_i) \in \bar{U}(w)$ and either $\text{den}(\mathfrak{A}, \alpha, \tau_{1-i})\uparrow$ or $\text{den}(\mathfrak{A}, \alpha, \tau_{1-i}) \notin \bar{U}(w)$.

We define $\models^{w}_{sp}$, $|_{sp}$, etc., in the obvious way. Where **L** is a logic defining class and 'x' may be replaced as usual, we define $\mathbf{L}^{sp}_x$-validity, weak $\mathbf{L}^{sp}_x$-validity, etc., in the obvious way. The distinctive feature of the semi-possibilistic semantics is that '$\approx$' is handled like a predicate in an *ea*-model, and '$\approx_s$' is handled as much like such a predicate as possible: if $\text{den}(\mathfrak{A}, \alpha, \tau_i) \in U - \bar{U}(w)$ then for $(\tau_0 \approx \tau_1)$ or $(\tau_0 \approx_s \tau_1)$ in $\mathfrak{M}$, τ_i is treated as if $\text{den}(\mathfrak{A}, \alpha, \tau_i)\uparrow$.

The following facts are obvious and shall be used in what follows:

$\mathfrak{M} \models_p (\tau \approx \tau)[\alpha]$ iff $\mathfrak{M} \models_p (\tau \approx_s \tau)[\alpha]$ iff $\text{den}(\mathfrak{A}, \alpha, \tau)\downarrow$;

$\mathfrak{M} \models_{sp} (\tau \approx \tau)[\alpha]$ iff $\mathfrak{M} \models_{sp} (\tau \approx_s \tau)[\alpha]$ iff $\mathfrak{M} \models_{sp} E(\tau)[\alpha]$ iff $\mathfrak{M} \models_{sp} E_s(\tau)[\alpha]$ iff $\text{den}(\mathfrak{A}, \alpha, \tau) \in \bar{U}(w)$;

$\mathfrak{M}\ {}_{p}\!\dashv E(\tau)[\alpha]$ iff either $\bar{U}(w) = \{\ \}$ or $\text{den}(\mathfrak{A}, \alpha, \tau) \in U - \bar{U}(w)$;

$\mathfrak{M}\ {}_{sp}\!\dashv E(\tau)[\alpha]$ iff $\bar{U}(w) = \{\ \}$;

where $\bar{U}(w) \neq \{\ \}$, we also have:

$\mathfrak{M}\ |_p\ E(\tau)[\alpha]$ iff $\text{den}(\mathfrak{A}, \alpha, \tau)\uparrow$;

$\mathfrak{M}\ |_{sp}\ E(\tau)[\alpha]$ iff $\mathfrak{M} \not\models_{sp} E(\tau)[\alpha]$ iff either $\text{den}(\mathfrak{A}, \alpha, \tau)\uparrow$ or $\text{den}(\mathfrak{A}, \alpha, \tau) \in U - \bar{U}(w)$.

It should be noticed that the following is weakly $\mathbf{L}^{sp}_x$-valid, but not weakly $\mathbf{L}^{p}_x$-valid:

$$(\{(\exists v)E(v)\}, \{(\exists v)E(v)\}, E(\tau));$$

similarly with '$\underline{\exists}$' replacing '$\exists$'. In this respect the semi-possibilistic modal logics are more like the non-modal three-valued logics discussed in [2] than are the possibilistic modal logics; this is one reason for preferring them. On the other hand, where $\mathfrak{M}$ and α are total, we may still have $\mathfrak{M}\ |_{sp}\ \phi[\alpha]$ for appropriate ϕ; thus confining our attention to total models and assignments, $\models_{sp}$ does not collapse to the familiar two-valued satisfaction relation $\models_2$.

Under either the possibilistic or the semi-possibilistic semantics, the clauses for '$\Box$' and '$\underline{\Box}$' lead us to examine $(\mathfrak{A}, u)$ and α for all u such that wRu; even if $\mathfrak{M}$ and α satisfy the actualist's constraint, that is even if $\mathfrak{M}$ is *da* and α is an $\mathfrak{M}$-assignment, $(\mathfrak{A}, u)$ and α might fail to meet that constraint. This consideration leads us to our actualistic semantics. For $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N})$ and any $u \in W$, let:

$$\mathscr{N}^u(\mathbf{c}) = a \text{ iff } \mathscr{N}(\mathbf{c}) = a \text{ and } a \in \bar{U}(w), \text{ for any } \mathbf{c} \in \mathbf{C};$$

$$\alpha^u(v) = a \text{ iff } \alpha(v) = a \text{ and } a \in \bar{U}(w), \text{ for any } v \in \mathbf{Var};$$

$$\mathfrak{A}^u = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N}^u);$$

$$\mathfrak{M}^u = (\mathfrak{A}^u, u), \text{ where } \mathfrak{M} = (\mathfrak{A}, w).$$

Obviously $\mathfrak{M}^u$ is *da*; so if $\mathfrak{A}$ is *ea* then $\mathfrak{M}^u$ is actualistic. Clearly α^u is an $\mathfrak{M}^u$-assignment. These facts will insure that $\vDash$ and $\Dashv$ are well-defined and honor the model-theoretic version of the actualist's constraint. Our definition of $\mathfrak{M} \vDash \phi[\alpha]$ and $\mathfrak{M} \Dashv \phi[\alpha]$ follows that of $\vDash_p$ and ${}_p\Dashv$, with the restriction that $\mathfrak{M}$ be an *a*-model and α be an $\mathfrak{M}$-assignment, and with this change:

$$\mathfrak{M} \vDash \Box\phi[\alpha] \text{ iff for all } u \in W, \text{ if } wRu \text{ then } \mathfrak{M}^u \vDash \phi[\alpha^u];$$

$$\mathfrak{M} \Dashv \Box\phi[\alpha] \text{ iff for some } u \in W, wRu \text{ and } \mathfrak{M}^u \Dashv \phi[\alpha^u].$$

$$\mathfrak{M} \vDash \underline{\Box}\phi[\alpha] \text{ iff for all } u \in W, \text{ if } wRu \text{ then } \mathfrak{M}^u \vDash \phi[\alpha^u];$$

$$\mathfrak{M} \Dashv \underline{\Box}\phi[\alpha] \text{ iff for some } u \in W, wRu \text{ and } \mathfrak{M}^u \Dashv \phi[\alpha^u], \text{ and for all } u \in W, \text{ if } wRu \text{ then either } \mathfrak{M}^u \vDash \phi[\alpha^u] \text{ or } \mathfrak{M}^u \Dashv \phi[\alpha^u].$$

Notice that evaluating the satisfaction or frustration of $\Box\phi$ or $\underline{\Box}\phi$ in $\mathfrak{M} = (\mathfrak{A}, w)$ involves consideration of models whose actual worlds may not be w; where u is such a world, members of $\bar{U}(u) - \bar{U}(w)$ "drop away" from the naming function and the variable assignment under consideration, since relative to a model with actual world u, they do not represent individuals. It is this "dropping away" phenomenon that forces the individual actualist to contend with non-denoting terms; so indirectly it is this that leads to a semantics incorporating a truth-value gap.

As above, let:

$$\mathfrak{M} \mid \phi[\alpha] \text{ iff } \mathfrak{M} \nvDash \phi[\alpha] \text{ and } \mathfrak{M} \not\Dashv \phi[\alpha];$$

$$\mathfrak{M} \vDash^w \phi[\alpha] \text{ iff } \mathfrak{M} \not\Dashv \phi[\alpha].$$

Define $\mathfrak{M} \vDash \Gamma[\alpha]$, $\mathfrak{M} \vDash^{w} \Gamma[\alpha]$, $\mathfrak{M} \vDash \phi$, etc., as usual.

Where (Γ, Δ, ϕ) is a sequent of L, let (Γ, Δ, ϕ) be $\mathbf{L}_x^a$-valid iff for all $\mathbf{L}_x$-models $\mathfrak{M}$ for L and all $\mathfrak{M}$-assignment α:

$$\text{if } \mathfrak{M} \vDash \Gamma[\alpha] \text{ and } \mathfrak{M} \vDash^{w} \Delta[\alpha] \text{ then } \mathfrak{M} \vDash \phi[\alpha];$$

let (Γ, Δ, ϕ) be weakly $\mathbf{L}_x^a$-valid iff for all $\mathfrak{M}$ and α as above:

$$\text{if } \mathfrak{M} \vDash \Gamma[\alpha] \text{ and } \mathfrak{M} \vDash^{w} \Delta[\alpha] \text{ then } \mathfrak{M} \vDash^{w} \phi[\alpha].$$

Let ϕ be $\mathbf{L}_x^a$-positively equivalent to ψ iff for all $\mathfrak{M}$ and α as above:

$$\mathfrak{M} \vDash \phi[\alpha] \text{ iff } \mathfrak{M} \vDash \psi[\alpha];$$

let ϕ and ψ be $\mathbf{L}_x^a$-equivalent iff ϕ is $\mathbf{L}_x^a$-positively equivalent to ψ and $\neg\phi$ is $\mathbf{L}_x^a$-positively equivalent to $\neg\psi$.

It should be noticed that the treatment of '$\approx$' and '$\approx_s$' under the actualistic semantics coincide with their treatment under the semi-possibilistic semantics; that is, where $\mathfrak{M} = (\mathfrak{A}, w)$ is *ea* and α is an $\mathfrak{A}$-assignment:

$$\mathfrak{M} \;{}^{w}\!\vDash (\tau_0 \approx \tau_1)[\alpha^{w}] \text{ iff } \mathfrak{M} \vDash_{sp} (\tau_0 \approx \tau_1)[\alpha];$$

$$\mathfrak{M} \;{}^{w}\!\dashv (\tau_0 \approx \tau_1)[\alpha^{w}] \text{ iff } \mathfrak{M} \;{}_{sp}\!\dashv (\tau_0 \approx \tau_1)[\alpha];$$

similarly with '$\approx$' replacing '$\approx_s$'. Thus the semi-possibilistic semantics makes a slight concession to actualism while remaining possibilistic in spirit (since it flouts the actualist's constraint). Clearly if $\bar{U}(w) \neq \{\ \}$, $\mathfrak{M}$ is actualistic and α is an $\mathfrak{M}$-assignment then:

$$\mathfrak{M} \nvDash E(\tau)[\alpha] \text{ iff } \mathfrak{M} \mid E(\tau)[\alpha] \text{ iff } \mathrm{den}(\mathfrak{A}, \alpha, \tau)\uparrow.$$

These biconditionals carry over from the non-modal semantics presented in [2]. Thus, as with a semi-possibilistic logic, $(\{(\exists v)E(v)\}, \{(\exists v)E(v)\}, E(\tau))$ is weakly $\mathbf{L}_x^a$-valid. We also have:

$$\mathfrak{M} \vDash E(\tau)[\alpha] \text{ iff } \mathfrak{M} \vDash E_s(\tau)[\alpha] \text{ iff } \mathrm{den}(\mathfrak{A}, \alpha, \tau)\downarrow.$$

Where $\mathfrak{M}$ can be any model for L and α any $\mathfrak{A}$-assignment, the corresponding biconditionals fail under the semi-possibilistic semantics, since $\mathrm{den}(\mathfrak{A}, \alpha, \tau)\downarrow$ does not entail that $\mathfrak{M} \vDash_{sp} E(\tau)[\alpha]$. In this respect the actualist semantics is more like the non-modal semantics presented in [2] than is the semi-possibilistic semantics.

As in [2], we have the following deduction theorems, where 'x' is replaced as usual, 'z' is replaced by 'p', 'sp', or 'a', and $\Gamma \subseteq \Delta \subseteq fml(L)$:

$(\Gamma \cup \{\phi\}, \Delta \cup \{\phi\}, \psi)$ is $\mathbf{L}_x^z$-valid iff $(\Gamma, \Delta, (\phi \supset_w \psi))$ is $\mathbf{L}_x^z$-valid;

$(\Gamma, \Delta \cup \{\phi\}, \psi)$ is weakly $\mathbf{L}_x^z$-valid iff $(\Gamma, \Delta, (\phi \supset_s \psi))$ is weakly $\mathbf{L}_x^z$-valid;

The reasons given in [2] for regarding '$\approx$' and 'E' as inadequate expressions of identity and existence respectively carry over to both our actualistic and our possibilistic model-theoretic semantics for modal languages. If they are accepted, '$\Box E(\tau)$' and '$\Diamond E(\tau)$' are inadequate representations of 'τ necessarily exists' and 'τ could exist' respectively; and '$\Box(E(\tau) \supset \mathbf{P}(\tau))$' is an inadequate representation of 'τ is essentially a $\mathbf{P}$'. But as in [2], if '$\approx_s$' and 'E_s' replace '$\approx$' and 'E', these inadequacies are avoided. Again, this applies to both the actualistic and the possibilistic semantics.

The difference between the actualistic and the possibilistic semantics gives rise to the following. For any term:

$(\{\ \}, \{\neg E(\tau)\}, \Box\neg E(\tau))$ is weakly $\mathbf{L}_x^a$-valid, but neither weakly $\mathbf{L}_x^p$-valid nor weakly $\mathbf{L}_x^{sp}$-valid;

$(\{\neg E_s(\tau)\}, \{\neg E_s(\tau)\}, \Box\neg E_s(\tau))$ is $\mathbf{L}_x^a$-valid, but neither $\mathbf{L}_x^p$-valid nor L_x^{sp}-valid.

The first weak $\mathbf{L}_x^a$-validity and the second $\mathbf{L}_x^a$-validity are model-theoretic counterparts of one of Kripke's central insights: if a singular term of an interpreted language lacks a referent, then relative to any possible world it lacks a referent; so if Vulcan doesn't exist then necessarily Vulcan does not exist. Of course 'Vulcan' might have referred in other worlds: in another world, 'Vulcan' refers (and perhaps even the referential intentions governing 'Vulcan' are roughly those which governed its brief currency in actual astronomical discourse). But this is a point about 'Vulcan', not about Vulcan; it must be expressed metalinguistically; 'Vulcan could have existed' or 'possibly Vulcan exists' (construed non-epistemically) does not express this metalinguistic truth. Granted the correctness of Kripke's point on this matter, the fact that these sequents are $\mathbf{L}_x^a$-valid but not $\mathbf{L}_x^p$-valid is a facet of the philosophical superiority of the actualistic over the possibilistic model-theoretic semantics.

The following notation shall be useful later. Where $V \subseteq W$, let:

$$\mathcal{N}^V(\mathbf{c}) = a \text{ iff } \mathcal{N}(\mathbf{c}) = a \text{ and for all } w \in V,\ a \in \bar{U}(w);$$

$$\alpha^V(v) = a \text{ iff } \alpha(v) = a \text{ and for all } w \in V,\ a \in \bar{U}(w).$$

Where $(w_0, \ldots, w_{n-1}) \in W^n$ let $\mathcal{N}^{(w_0,\ldots,w_{n-1})} = \mathcal{N}^{\{w_0,\ldots,w_{n-1}\}}$, $\alpha^{(w_0,\ldots,w_{n-1})} = \alpha^{\{w_0,\ldots,w_{n-1}\}}$. Finally, where $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathcal{N})$ and $\vec{w} \in W^n$, let $\mathfrak{A}^{\vec{w}} = (W, R, U, \bar{U}, \mathscr{E}, \mathcal{N}^{\vec{w}})$; where $\mathfrak{M} = (\mathfrak{A}, w)$ let $\mathfrak{M}^{\vec{w}} = (\mathfrak{A}^{\vec{w}}, w_{n-1})$; thus $\mathfrak{M}^{(w_0,\ldots,w_n)} = (\mathfrak{M}^{(w_0,\ldots,w_{n-1})})^{w_n}$ for $n \geq 1$.

The following fact will be used frequently in what follows. Suppose $\phi, \phi' \in fml(L)$, $\tau_0, \ldots, \tau_{n-1}, \tau'_0, \ldots, \tau'_{n-1}$ are terms of L. Suppose that ϕ' differs from ϕ only in containing parameter occurrences of τ'_i when ϕ contains parameter occurrences of τ_i. Suppose $\mathfrak{M} = (\mathfrak{A}, w)$ is an actualistic model for L, and $V, V' \subseteq W$. If for all $i < n$ $\text{den}(\mathfrak{A}^V, \alpha^V, \tau_i) \simeq \text{den}(\mathfrak{A}^{V'}, \alpha^{V'}, \tau'_i)$ then

$$(\mathfrak{A}^V, w) \models \phi[\alpha^V] \text{ iff } (\mathfrak{A}^{V'}, w) \models \phi'[\alpha^{V'}];$$

$$(\mathfrak{A}^V, w) \dashv \phi[\alpha^V] \text{ iff } (\mathfrak{A}^{V'}, w) \dashv \phi'[\alpha^{V'}].$$

Our chosen language L may be easily enriched by adding 'ι' to its logical lexicon and defining both the terms of L and the formulae of L by a simultaneous induction, with this new formation rule:

if ϕ is a formula and $v \in \mathbf{Var}$ then $(\iota v)\phi$ is a term.

With this change, denotation must be defined relative to a model rather than a structure; otherwise the definition of den is as before, with this additional clause:

$\text{den}(\mathfrak{M}, \alpha, (\iota v)\phi) \simeq$ the unique $a \in \bar{U}(w)$ such that $\mathfrak{M} \models \phi[\alpha^v_a]$, where w is the actual world of $\mathfrak{M}$.

Therefore if $\text{den}(\mathfrak{M}, \alpha, (\iota\tau)\phi)\downarrow$ then $\text{den}(\mathfrak{M}, \alpha, (\iota v)\phi) \in \bar{U}(w)$, as an individual-actualist would expect.

We have required that $\mathbf{C}$ consist of individual-constants; if we wish to allow n-place function-constants for $1 < n < \omega$, we must be careful. The simplest approach would be the following: where $\mathbf{f}$ is an n-place function constant let $\mathcal{N}(\mathbf{f})$ be a function into U with $\text{dom}(\mathcal{N}(\mathbf{f})) \subseteq U^n$; let

$\text{den}(\mathfrak{A}, \alpha, f(\tau_0, \ldots, \tau_{n-1}) \simeq \mathcal{N}(\mathbf{f})(\text{den}(\mathfrak{A}, \alpha, \tau_0), \ldots, \text{den}(\mathfrak{A}, \alpha, \tau_{n-1}))$.

Then our definition of a denotation-wise actualist model would require of $\mathfrak{M} = (\mathfrak{A}, w)$ that $\mathcal{N}(\mathbf{f})$ be into $\bar{U}(w)$. But even with this change, the actualistic semantics will not work correctly; for $u \in W$, if $\mathfrak{M}^u$ is defined as before, it needn't be denotationwise actualistic. Therefore the definition of $\mathcal{N}^u$ needs a revision; for **f** as above, let

$\mathcal{N}^u(\mathbf{f})(a_0, \ldots, a_{n-1}) = a$ iff $\mathcal{N}(\mathbf{f})(a_0, \ldots, a_{n-1}) = a$ and $a \in \bar{U}(u)$;

$\mathcal{N}^u(\mathbf{f})(a_0, \ldots, a_{n-1})\uparrow$ otherwise.

Letting $\mathfrak{M}^u = (\mathfrak{A}^u, u)$, the actualistic semantics will now run smoothly. Notice that in the definition of $\mathcal{N}^u(\mathbf{f})(a_0, \ldots, a_{n-1})$ there was no need to impose the condition that $a_0, \ldots, a_{n-1} \in \bar{U}(u)$; similarly in the definition of denotationwise actualistic model $\mathfrak{M} = (\mathfrak{A}, w)$, there was no need to require that $\text{dom}(\mathcal{N}(\mathbf{f})) \subseteq \bar{U}(w)^n$. Neither additional condition would effect the definition of $\models$ and $\dashv$. To keep things simple, this paper will not further consider languages with function-constants.

I'm inclined to regard lexicons of the form $\text{lex}_{0,\ldots}$ as of peripheral technical interest. I've discussed them in this paper because others may not share my attitude, and because they do raise curious technical difficulties which are worth noticing, if only to justify my attitude. The philosophical considerations which might lead one to prefer one non-modal lexicon to another are discussed in [2] §12; the considerations raised there seem to carry over to the modal lexica presented above. (If one thinks otherwise, one would have to regard modal operators are importantly disanalogous to quantifiers; this could make one want to consider "mixed" lexicons, e.g. one formed by replacing '$\underline{\Box}$' in $\text{lex}_{0,\ldots}$ by '$\Box$'. Space does not permit discussion of the logics appropriate to languages based on such lexicons.)

2. RELATIONSHIPS BETWEEN THESE MODEL-THEORETIC SEMANTICS

Given $L = L_y(\mathbf{Pred}, \mathbf{C})$, let $L' = L_{y'}(\mathbf{Pred}, \mathbf{C})$, where:

if y is either 0 or 0,**u** or 1 or 1,**u** then y' is 1,**u**;

if y is either 0,s or 0,**u**,s or 1,s or 1,**u**,s then y' is 1,**u**,s;

if y is either 0,T or 0,T,**u** or 1,T or 2 then y' is 2.

We'll define $s\colon fml(L) \to fml(L')$ which translates the semi-possibilistic into the possibilistic semantics, in this sense: for any $\phi \in fml(L)$, any model $\mathfrak{M}$ with structure $\mathfrak{A}$, and any $\mathfrak{A}$-assignment α:

$$\mathfrak{M} \vDash_{sp} \phi[\alpha] \text{ iff } \mathfrak{M} \vDash_p s(\phi)[\alpha]; \mathfrak{M}\ {}_{sp}\!\!\dashv \phi[\alpha] \text{ iff } \mathfrak{M}\ {}_{p}\!\!\dashv s(\phi)[\alpha].$$

Let:

$$E'(\tau) = (E(\tau) \vee \mathbf{u}); E_s'(\tau) = (E_s(\tau) \vee \mathbf{u});$$

$$s(\tau_0 \approx \tau_1) = (\tau_0 \approx \tau_1) \,\&\, E'(\tau_0) \,\&\, E'(\tau_1);$$

$$s(\tau_0 \approx_s \tau_1) = (\tau_0 \approx_s \tau_1) \,\&\, (E_s'(\tau_0) \vee E_s'(\tau_1)).$$

Given $\phi \in fml(L)$, form $s(\phi)$ by replacing each occurrence of $(\tau_0 \approx \tau_1)$ or $(\tau_0 \approx_s \tau_1)$ in ϕ by $s(\tau_0 \approx \tau_1)$ or $s(\tau_0 \approx_s \tau_1)$ respectively. It's easy to see that s is as claimed.

We now show why we needed to have $y' = 1, \ldots$ with '$\mathbf{u}$' $\in \text{lex}_{y'}^{m}$ if we were to get a translation $s\colon fml(L) \to fml(L')$, or even a formula $s(\tau_0 \approx \tau_1)$, meeting the above conditions. Consider **Pred** $=$ **C** $= \{\ \}$, $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N})$ a structure for **Pred**, **C**, α a total $\mathfrak{A}$-assignment, $w \in W$ and $\mathfrak{M} = (\mathfrak{A}, w)$; thus for any $\psi \in fml(L_{1,T})$ or $\psi \in fml(L_{0,T})$, either $\mathfrak{M} \vDash_p \psi[\alpha]$ or $\mathfrak{M}\ {}_{p}\!\!\dashv \psi[\alpha]$. For some $v \in \mathbf{Var}$, suppose $\alpha(v) \in U - \bar{U}(w)$; so $\mathfrak{M} \,|_{sp}\, (v \approx v)[\alpha]$; then no formula of $L_{1,T}$ or $L_{0,T}$ could serve as $s(v \approx v)$; so we need '$\mathbf{u}$' $\in \text{lex}_{y'}$ if we are to "translate" $(v \approx v)$. It's easy to see that any formula of $L_{0,T,\mathbf{u}}$ is $\mathbf{K}_x^{\rho}$-equivalent to either '$\mathbf{u}$' or to a formula of $L_{0,T}$; see [2] Lemma 1; thus no formula of $L_{0,T,\mathbf{u}}$ could serve as $s(v \approx v)$; so we needed $y' = 1, \ldots$.

We now consider translation from the actualistic into the possibilistic semantics; we'll construct $t\colon fml(L) \to fml(L')$ so that for every $\phi \in fml(L)$, every a-model $\mathfrak{M}$ for L and every $\mathfrak{M}$ assignment α:

$$\mathfrak{M} \vDash \phi[\alpha] \text{ iff } \mathfrak{M} \vDash_p t(\phi)[\alpha]; \mathfrak{M} \dashv \phi[\alpha] \text{ iff } \mathfrak{M}\ {}_{p}\!\!\dashv t(\phi)[\alpha].$$

Suppose $\mathscr{F}$ is a set of terms of L and $\phi \in fml(L)$. If '$\approx$' $\in \text{lex}_y$, form $\phi_{\mathscr{F}}$ by replacing each occurrence of an atomic formula in ϕ containing a parameter-occurrence of a member of $\mathscr{F}$ by '$\mathbf{u}$'. Where θ is an occurrence of a '$\approx_s$'-equation, let the 0-occurrence [1-occurrence] in θ be the leftmost [rightmost] occurrence of a term in θ. If '$\approx_s$' $\in$ lex_y, form $\phi_{\mathscr{F}}$ by replacing each occurrence in ϕ of an atomic formula containing a member of $\mathscr{F}$ but not containing '$\approx_s$' by $\mathbf{u}$, and replacing each occurrence θ in ϕ of the form $(\tau_0 \approx_s \tau_1)$ as follows:

if $\tau_0, \tau_1 \in \mathscr{F}$ and both the 0-occurrence and the 1-occurrence in θ are parameter-occurrences in ϕ then replace θ by '**u**';
if for some $i < 2$, $\tau_i \in \mathscr{F}$, the i-occurrence in θ is a parameter-occurrence in ϕ, and the $1-i$-occurrence in θ is not a parameter-occurrence in ϕ (i.e. $\tau_{1\text{-}i} \in$ **Var** and the $1-i$-occurrence in θ is bound in ϕ) then replace θ by '$\perp$'; if for some $i < 2$, $\tau_i \in \mathscr{F}$, $\tau_{1\text{-}i} \notin \mathscr{F}$, and both the 0-occurrence and the 1-occurrence in θ are parameter-occurrences in ϕ then replace θ by '$\perp$'.

Let $\text{Param}(\phi) = \{\tau\colon \tau \text{ is a parameter in } \phi\}$.

OBSERVATION 1. Where '$\approx$' $\in$ lex$_y$['$\approx_s$' $\in$ lex$_y$], $\mathfrak{M}$ is an a-model for L, α is an $\mathfrak{M}$-assignment, and for every $\tau \in \text{Param}(\phi)$, den$(\mathfrak{A}, \alpha, \tau)\uparrow$ if [iff] $\tau \in \mathscr{F}$, then:

$$\mathfrak{M} \models \phi[\alpha] \text{ iff } \mathfrak{M} \models \phi_{\mathscr{F}}[\alpha];\ \mathfrak{M} \mathrel{=\!\!|} \phi[\alpha] \text{ iff } \mathfrak{M} \mathrel{=\!\!|} \phi_{\mathscr{F}}[\alpha].$$

(In fact, where $\mathfrak{M}$ is any model for L with structure $\mathfrak{A}$ and α is any $\mathfrak{A}$-assignment, we may replace '$\models$' and '$\mathrel{=\!\!|}$' by '$\models_p$' and '${}_p\!\mathrel{=\!\!|}$' respectively in the preceding biconditionals.)

Where $\mathscr{F}$ and $\mathscr{G}$ are sets of terms of L, we adopt these abbreviations:

$$\Phi(\mathscr{F}, \mathscr{G})\colon \wedge \{E(\tau)\colon \tau \in \mathscr{F}\} \ \&\ \wedge \{\neg E(\tau)\colon \tau \in \mathscr{G}\};$$

$$\Phi_s(\mathscr{F}, \mathscr{G})\colon \wedge \{E_s(\tau)\colon \tau \in \mathscr{F}\} \ \&\ \wedge \{\neg E_s(\tau)\colon \tau \in \mathscr{G}\}.$$

For '$\approx$' $\in$ lex$_y$['$\approx_s$' $\in$ lex$_y$] we now define t inductively. If ϕ is atomic, $t(\phi)$ is ϕ; t commutes with '$\supset$', '$\underline{\supset}$', 'T', '$\exists$' and '$\underline{\exists}$'. Where $\mathscr{F} \subseteq \text{Param}(\psi)$, let $\Theta(\mathscr{F}, \psi)[\Theta_s(\mathscr{F}, \psi)]$ be:

$$(\Phi(\text{Param}(\psi) - \mathscr{F}, \mathscr{F}) \supset t(\psi_{\mathscr{F}}))$$
$$[(\Phi_s(\text{Param}(\psi) - \mathscr{F}, \mathscr{F}) \supset t(\psi_{\mathscr{F}}))];$$

where ϕ is $\Box\psi$, let $t(\phi)$ be:

$$\Box(\wedge \{\Theta(\mathscr{F}, \psi)\colon \mathscr{F} \subseteq \text{Param}(\phi)\})$$

$$[\Box(\wedge \{\Theta_s(\mathscr{F}, \psi)\colon \mathscr{F} \subseteq \text{Param}(\phi)\})];$$

where ϕ is $\underline{\Box}\psi$, replace '$\Box$' by '$\underline{\Box}$' in the previous clause. Since depth$(\psi_{\mathscr{F}})$ = depth(ψ), $t(\phi)$ is well-defined by induction on depth(ϕ).

We'll prove that t is as desired by induction on the construction of ϕ. The only case worth discussing is where ϕ is $\Box\psi$ or $\underline{\Box}\psi$. Suppose that $\mathfrak{M} = (\mathfrak{A}, w)$ is an a-model for L, α is an $\mathfrak{M}$-assignment, frame$(\mathfrak{A}) = (W, R)$, wRu, and that t is as desired for formulae of depth $<$ depth(ϕ).

CASE 1. '$\approx$' $\in$ lex$_y$. It suffices to show:

$$\mathfrak{M}^u \models \psi[\alpha^u] \text{ iff } (\mathfrak{A}, u) \models_p \wedge \{\Theta(\mathscr{F}, \psi)\colon \mathscr{F} \subseteq \text{Param}(\phi)\}[\alpha]$$

$$\mathfrak{M}^u \dashv \psi[\alpha^u] \text{ iff } (\mathfrak{A}, u)\ {}_p\!\dashv \wedge \{\Theta(\mathscr{F}, \psi)\colon \mathscr{F} \subseteq \text{Param}(\phi)\}[\alpha].$$

Fix $\mathscr{F} \subseteq$ Param(ϕ). If for some $\tau \in \mathscr{F}$, $\mathfrak{M}^u \models E(\tau)[\alpha^u]$, then $(\mathfrak{A}, u)\ {}_p\!\dashv$ $\Phi(\text{Param}(\phi) - \mathscr{F}, \mathscr{F})[\alpha]$; so $(\mathfrak{A}, u) \models_p \Theta(\mathscr{F}, \psi)[\alpha]$; suppose that for all $\tau \in \mathscr{F}$, $\mathfrak{M}^u \not\models E(\tau)[\alpha^u]$. If for some $\tau \in$ Param$(\phi) - \mathscr{F}$, den$(\mathfrak{A}, \alpha, \tau)\downarrow$ but $\mathfrak{M}^u \not\models E(\tau)[\alpha^u]$, then $(\mathfrak{A}, u)\ {}_p\!\dashv E(\phi)[\alpha]$; so as above $(\mathfrak{A}, u) \models_p \Theta(\mathscr{F}, \psi)[\alpha]$. Suppose that for all $\tau \in$ Param$(\phi) - \mathscr{F}$, if den$(\mathfrak{A}, \alpha, \tau)\downarrow$ then $\mathfrak{M}^u \models E(\tau)[\alpha^u]$. Clearly some $\mathscr{F}$ meets this condition. By the previous supposition and Observation 1:

(*) $\quad \mathfrak{M}^u \models \psi[\alpha^u]$ iff $\mathfrak{M}^u \models \psi_{\mathscr{F}}[\alpha^u]$; $\mathfrak{M}^u \dashv \psi[\alpha^u]$ iff $\mathfrak{M}^u \dashv \psi_{\mathscr{F}}[\alpha^u]$.

By our induction hypothesis, since depth$(\psi_{\mathscr{F}}) =$ depth(ψ):

(**) $\quad \mathfrak{M}^u \models \psi_{\mathscr{F}}[\alpha^u]$ iff $\mathfrak{M}^u \models_p t(\psi_{\mathscr{F}})[\alpha^u]$;

$\quad \mathfrak{M}^u \dashv \psi_{\mathscr{F}}[\alpha^u]$ iff $\mathfrak{M}^u\ {}_p\!\dashv t(\psi_{\mathscr{F}})[\alpha^u]$.

For any $\tau \in$ Param$(t(\psi_{\mathscr{F}}))$, $\tau \in$ Param$(\phi) - \mathscr{F}$; so if $\mathfrak{M}^u \not\models E(\tau)[\alpha^u]$ our last supposition yields den$(\mathfrak{A}, \alpha, \tau)\uparrow$; thus:

(***) $\quad \mathfrak{M}^u \models_p t(\psi_{\mathscr{F}})[\alpha^u]$ iff $(\mathfrak{A}, u) \models_p t(\psi_{\mathscr{F}})[\alpha]$;

$\quad \mathfrak{M}^u\ {}_p\!\dashv t(\psi_{\mathscr{F}})[\alpha^u]$ iff $(\mathfrak{A}, u)\ {}_p\!\dashv t(\psi_{\mathscr{F}})[\alpha]$.

Putting together these biconditionals:

$$\mathfrak{M}^u \models \psi[\alpha^u] \text{ iff } (\mathfrak{A}, u) \models_p \Theta(\mathscr{F}, \psi)[\alpha];$$

$$\mathfrak{M}^u \dashv \psi[\alpha^u] \text{ iff } (\mathfrak{A}, u)\ {}_p\!\dashv \Theta(\mathscr{F}, \psi)[\alpha].$$

From this and the semantics for '&' the desired biconditionals follow.

CASE 2. '$\approx_s$' $\in$ lex$_y$. It suffices to show:

$$\mathfrak{M}^u \models \psi[\alpha^u] \text{ iff } (\mathfrak{A}, u) \models_p \wedge \{\Theta_s(\mathscr{F}, \psi)\colon \mathscr{F} \subseteq \text{Param}(\phi)\}[\alpha];$$

$\mathfrak{M}^u \dashv \psi[\alpha^u]$ iff $(\mathfrak{A}, u)\ {}_p\!\dashv \wedge \{\Theta_s(\mathscr{F}, \psi): \mathscr{F} \subseteq \text{Param}(\phi)\}[\alpha]$.

Fix $\mathscr{F} \subseteq \text{Param}(\phi)$. If for some $\tau \in \mathscr{F}$, $\mathfrak{M}^u \vDash E_s(\tau)[\alpha]$, then as before $(\mathfrak{A}, u) \vDash_p \Theta_s(\mathscr{F}, \psi)[\alpha]$; so suppose that for all $\tau \in \mathscr{F}$, $\mathfrak{M}^u \nvDash E_s(\tau)[\alpha]$. If for some $\tau \in \text{Param}(\phi) - \mathscr{F}$, we have $\mathfrak{M}^u \nvDash E_s(\tau)[\alpha^u]$, then $(\mathfrak{A}, u)\ {}_p\!\dashv E_s(\tau)[\alpha^u]$, and so $(\mathfrak{A}, u)\ {}_p\!\dashv \Phi_s(\text{Param}(\phi) - \mathscr{F}, \mathscr{F})[\alpha]$; so $(\mathfrak{A}, u) \vDash_p \Theta_s(\mathscr{F}, \psi)[\alpha]$, so this time we may suppose that for all $\tau \in \text{Param}(\phi) - \mathscr{F}$, $\mathfrak{M}^u \vDash E_s(\tau)[\alpha^u]$. Clearly some $\mathscr{F}$ satisfies these suppositions. Using both suppositions, Observation 1 applies, yielding (*). The induction hypothesis yields (**). Our last supposition yields (***). So we have:

$$\mathfrak{M}^u \vDash \psi[\alpha^u] \text{ iff } (\mathfrak{A}, u) \vDash_p \Theta_s(\mathscr{F}, \psi)[\alpha];$$

$$\mathfrak{M}^u \dashv \psi[\alpha^u] \text{ iff } (\mathfrak{A}, u)\ {}_p\!\dashv \Theta_s(\mathscr{F}, \psi)[\alpha].$$

The desired biconditionals follow immediately. QED

We now show why we needed $y' = 1, \ldots$ and '**u**' $\in \text{lex}_{y'}$ if we were to have L' be our "target" language for t. Consider **Pred** $= \{\ \}$, $\mathbf{C} = \{\mathbf{c}\}$; there is a total a-model for **Pred**, **C**, so that $\mathfrak{M} \mid \Box E(\mathbf{c})$. (Where $\mathfrak{M} = (\mathfrak{A}, w)$, we make sure that $\mathscr{N}(\mathbf{c}) \in \bar{U}(w)$ and that for some u with wRu, $\mathscr{N}(\mathbf{c}) \notin \bar{U}(w)$.) For any total $\mathfrak{M}$-assignment α and any formula θ of $L_{1,T}$ or $L_{0,T}$, either $\mathfrak{M} \vDash_p \theta[\alpha]$ or $\mathfrak{M}\ {}_p\!\dashv \theta[\alpha]$. Thus no formula of $L_{1,T}$ or $L_{0,T}$ could serve as $t(\Box E(\mathbf{c}))$; so we needed '**u**' $\in \text{lex}^m_{y'}$. Since every formula of $L_{0,T,\mathbf{u}}$ is $\mathbf{K}^p_x$-equivalent either to '**u**' or to a formula of $\mathbf{L}_{0,T}$, this also shows that no formula of $L_{0,T,\mathbf{u}}$ could serve as $t(\Box E(\mathbf{c}))$; so we needed $y' = 1, \ldots$. Where was this used in our argument? Notice that from $(\mathfrak{A}, u)\ {}_p\!\dashv \Phi(\text{Param}(\phi) - \mathscr{F}, \mathscr{F})[\alpha]$ we can't conclude that $(\mathfrak{A}, u) \vDash_p (\Phi(\text{Param}(\phi) - \mathscr{F}, \mathscr{F}) \supset t(\psi_{\mathscr{F}}))[\alpha]$; so we needed '$\supset$' in forming $\Theta(\mathscr{F}, \psi)$; similarly for $\Theta_s(\mathscr{F}, \psi)$.

For $i < 2$, suppose $\mathfrak{A}_i = (W_i, R_i, U_i, \bar{U}_i, \mathscr{E}_i, \mathscr{N}_i)$ is a structure for L, $w_i \in W_i$ and $\mathfrak{M}_i = (\mathfrak{A}_i, w_i)$ are actualistic; let π be a narrow isomorphism from $\mathfrak{M}_0$ to $\mathfrak{M}_1$ iff π maps $\bar{U}_0(w_0)$ one-one onto $\bar{U}_1(w_1)$, for all n-place $\theta \in$ **Pred** and $\vec{a} \in \bar{U}_0(w_0)^n$: $\mathscr{E}_0(\theta)(w_0, \vec{a}) \simeq \mathscr{E}_1(\theta)(w_1, \pi\vec{a})$, and for all $\tau \in \mathbf{C}$, $\mathscr{N}_1(\tau) \simeq \pi\mathscr{N}_0(\tau)$.

Where π is a narrow isomorphism from $\mathfrak{M}_0$ to $\mathfrak{M}_1$, let π be a y-actualistic isomorphism iff for any $\mathfrak{M}_0$-assignment α and $\phi \in fml(L_y)$;

$$\mathfrak{M}_0 \models \phi[\alpha] \text{ iff } \mathfrak{M}_1 \models \phi[\pi \circ \alpha];$$

$$\mathfrak{M}_0 \dashv \phi[\alpha] \text{ iff } \mathfrak{M}_1 \dashv \phi[\pi \circ \alpha].$$

$\mathfrak{M}_0$ is y-actualistically isomorphic to $\mathfrak{M}_1$ iff there is a y-actualistic isomorphism from $\mathfrak{M}_0$ to $\mathfrak{M}_1$, in symbols $\mathfrak{M}_0 \cong^a_y \mathfrak{M}_1$. If π is a y-actualistic isomorphism from $\mathfrak{M}_0$ to $\mathfrak{M}_1$ then for the individual actualist $\mathfrak{M}_0$ and $\mathfrak{M}_1$ are not interestingly different: in an obvious sense the w_0-part of $\mathfrak{A}_0$ is literally isomorphic to the w_1-part of $\mathfrak{A}_1$ by the narrowness of π; since the sole role of any $u \in W_i - \{w_i\}$ and any $a \in \bar{U}_i(u) - \bar{U}_i(w_i)$ is to determine satisfaction and frustration conditions in $\mathfrak{M}_i$, the biconditions in the definition of a y-actualistic isomorphism suffice to make $\mathfrak{M}_0$ and $\mathfrak{M}_1$ not interestingly different; there is no need to require some closer sort of correspondence between non-actual worlds or individuals of $\mathfrak{M}_0$ and $\mathfrak{M}_1$.

Where $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N})$ is a structure for L, let $\mathfrak{A}$ be settled iff for every R-chain, $w_0, \ldots, w_n$ such that $w_i \neq w_{i+1}$ for all $i < n$: if $a \in \bar{U}(w_0) - \bar{U}(w_1)$ then $a \notin \bar{U}(w_n)$; a model $\mathfrak{M} = (\mathfrak{A}, w)$ is settled iff $\mathfrak{A}$ is settled.

OBSERVATION 2. If $\mathfrak{M}$ is settled and actualistic then relative to $\mathfrak{M}$ the actualistic and possibilistic semantics coincide; in other words for any $\mathfrak{M}$-assignment α and any $\phi \in fml(L)$:

$$\mathfrak{M} \models \phi[\alpha] \text{ iff } \mathfrak{M} \models_p \phi[\alpha];$$

$$\mathfrak{M} \dashv \phi[\alpha] \text{ iff } \mathfrak{M} \;{}_p\!\dashv \phi[\alpha].$$

This observation will follow from a more general observation. Suppose we're given $\phi \in fml(L)$, an R-chain $w = w_0, \ldots, w_n$, $(n_1, \ldots, n_n) \in \omega^n$, and arrays $\langle v^i_j \rangle_{i \leqslant n, 1 \leqslant j \leqslant n_i}$ of distinct variables, and $\langle a^i_j \rangle_{i \leqslant n, 1 \leqslant j \leqslant n_i}$ with $a^i_j \in \bar{U}(w_i)$ for all $i \leqslant n$ and $1 \leqslant j \leqslant n_i$. Let:

$$\alpha_0 = \beta_0 = \alpha^{v^0_1, \ldots, v^0_q}_{a^0_1, \ldots, a^0_q} \text{ for } q = n_0;$$

$$\alpha_{i+1} = (\alpha^{w_{i+1}}_i)^{v^{i+1}_1, \ldots, v^{i+1}_q}_{a^{i+1}_1, \ldots, a^{i+1}_q} \text{ and } \beta_{i+1} = (\beta_i)^{v^{i+1}_1, \ldots, v^{i+1}_q}_{a^{i+1}_1, \ldots, a^{i+1}_q}$$

for $q = n_{i+1}$ and $i < n$.
Then:

$$\mathfrak{M}^{(w_1, \ldots, w_n)} \models \phi[\alpha_n] \text{ iff } (\mathfrak{A}, w_n) \models_p \phi[\beta_n];$$

$$\mathfrak{M}^{(w_1, \ldots, w_n)} \dashv \phi[\alpha_n] \text{ iff } (\mathfrak{A}, w_n) \;{}_p\!\dashv \phi[\beta_n].$$

Observation 2 is the special case where $n = 0$ and $n_0 = 0$. Claim:

(i) if $\text{den}(\mathfrak{A}^{\{w_1,\ldots,w_n\}}, \alpha_n, \tau) = a$ then $\text{den}(\mathfrak{A}, \beta_n, \tau) = a$;

(ii) if $\text{den}(\mathfrak{A}, \beta_n, \tau) = a \in \bar{U}(w_n)$ then $\text{den}(\mathfrak{A}^{\{w_1,\ldots,w_n\}}, \alpha_n, \tau) = a$;

(i) is obvious. Assume the antecedent of (ii). If $\tau \in \mathbf{C}$ then $a = \mathcal{N}(\tau) \in \bar{U}(w_0) = \bar{U}(w)$, since $\mathfrak{M}$ is actualistic; since $\mathfrak{M}$ is settled, $a \in \bar{U}(w_i)$ for all $i \leqslant n$; so $\mathcal{N}^{\{w_1,\ldots,w_n\}}(\tau) = a$, as required. Suppose $\tau \in \mathbf{Var}$; fix $i_0 \leqslant n$ to be least so that for all j with $i_0 \leqslant j \leqslant n$, $\beta_j(\tau) = a$. Then $\alpha_{i_0}(\tau) = a \in \bar{U}(w_{i_0})$; since $\mathfrak{A}$ is settled, $a \in \bar{U}(w_j)$ for $i_0 \leqslant j \leqslant n$; so $\alpha_n(\tau) = a$, as required.

Suppose ϕ is atomic. By (i):

$$\text{if } (\mathfrak{A}^{\{w_1,\ldots,w_n\}}, w_n) \vDash \phi[\beta_n] \text{ then } (\mathfrak{A}, w_n) \vDash_p \phi[\beta_n].$$

If $(\mathfrak{A}, w_n) \vDash_p \phi[\beta_n]$ and τ is a parameter of ϕ, then $\text{den}(\mathfrak{A}, \beta_n, \tau) \in \bar{U}(w_n)$, since $\mathfrak{A}$ is *ea*; so by (ii), $(\mathfrak{A}^{\{w_1,\ldots,w_n\}}, w_n) \vDash \phi[\alpha_n]$. These arguments also apply when '$\dashv$' replaces '$\vDash$'. The inductive steps for '$\supseteq$', '$\supset$', 'T', '$\underline{\exists}$' and '$\exists$' are straightforward. If ϕ is $\Box\psi$, notice that if $w_n R w_{n+1}$:

$$(\mathfrak{A}^{\{w_1,\ldots,w_{n+1}\}}, w_{n+1}) \vDash \psi[\alpha_n^{w_{n+1}}] \text{ iff } (\mathfrak{A}, w_{n+1}) \vDash_p \psi[\beta_n];$$

$$(\mathfrak{A}^{\{w_1,\ldots,w_{n+1}\}}, w_{n+1}) \dashv \psi[\alpha_n^{w_{n+1}}] \text{ iff } (\mathfrak{A}, w_{n+1})\, {}_p\!\dashv \psi[\beta_n],$$

using the induction hypothesis, where $n_{n+1} = 0$. Similarly if ϕ is $\underline{\Box}\psi$.

Settled models are of technical rather than philosophical interest. (A referee has pointed out to me that the tense-logic analogue of settledness reflects the doctrine that an object can't go out of existence and then come back into existence.) We'll show that any actualistic model may be transformed into a settled actualistic model preserving frustration and satisfaction under the actualistic semantics; in combination with Observation 2, we have then a model-theoretic way to convert certain questions about that semantics into questions about the possibilistic semantics.

Where (W, R) is a b.r.s., and $w_0 \in W$, let (W, R) be a tree with root w_0 iff:

(i) for every $w \in W$ there is a unique R-chain from w_0 to w;

(ii) there is no R-cycles (i.e. for every $w \in W$ there is no R-chain from w to w).

Let $\mathfrak{M} = (\mathfrak{A}, w_0)$ be tree-based iff Frame($\mathfrak{A}$) is a tree with root w_0.

OBSERVATION 3. For any actualistic model $\mathfrak{M}_0 = (\mathfrak{A}_0, w_0)$ for L there is a settled tree-based actualistic model $\mathfrak{M}_1$ so that $\mathfrak{M}_0 \cong_2^a \mathfrak{M}_1$.

Suppose $\mathfrak{A}_0 = (W_0, R_0, U_0, \bar{U}_0, \mathscr{E}_0, \mathscr{N}_0)$. Let:

$$W_1 = \{\langle w_0, \ldots, w_n\rangle : w_0, \ldots, w_n \text{ is an } R\text{-chain for } n \geqslant 1\};$$

$$R_1 = \{(v, v')\text{: for some } n, v = \langle w_0, \ldots, w_n\rangle \text{ and } v_1 = \langle w_0, \ldots, w_n, w_{n+1}\rangle\}.$$

(Notation: with $\mathfrak{A}^{\langle w_0, \ldots, w_n\rangle}$, etc., we're regarding $\langle w_0, \ldots, w_n\rangle$ as a single world in W_1; $\mathfrak{A}^{(w_0, \ldots, w_n)}$, is to be understood as $(\ldots (\mathfrak{A}^{w_0}) \ldots)^{w_n}$ as in the definition at the end of §1.)

For $a \in \bar{U}_0(w_0)$ let $g(a, \langle w_0\rangle) = (a, \langle w_0\rangle)$; for $a \in \bar{U}(w_{n+1})$ and $\langle w_0, \ldots, w_{n+1}\rangle \in W_1$, let:

$$g(a, \langle w_0, \ldots, w_{n+1}\rangle) = \begin{cases} g(a, \langle w_0, \ldots, w_n\rangle) & \text{if } a \in \bar{U}_0(w_n); \\ (a, \langle w_0, \ldots, w_{n+1}\rangle) & \text{otherwise.} \end{cases}$$

For $v = \langle w_0, \ldots, w_n\rangle \in W_1$, let:

$$\bar{U}_1(v) = \{g(a, v)\colon a \in \bar{U}_0(w_n)\}.$$

Let $U_1 = \cup\{\bar{U}_1(v)\colon v \in W_1\}$; for $\tau \in \mathbf{C}$ let $\mathscr{N}_1(\tau) \simeq (\mathscr{N}_0(\tau), \langle w_0\rangle)$. For $b = (a, v) \in U_1$ let $fb = a$. For each m-place $\theta \in$ **Pred** and $b_0, \ldots, b_{m-1} \in U_1$ let:

$$\mathscr{E}_1(\theta)(\langle w_0, \ldots, w_n\rangle, b_0, \ldots, b_{m-1}) \simeq \mathscr{E}_0(\theta)(w_n, fb_0, \ldots, fb_{m-1}).$$

Let $\mathfrak{A}_1 = (W_1, R_1, U_1, \bar{U}_1, \mathscr{E}_1, \mathscr{N}_1)$, $\mathfrak{M}_1 = (\mathfrak{A}_1, \langle w_0\rangle)$, $\pi a = (a, \langle w_0\rangle)$ for all $a \in \bar{U}_0(w_0)$. Clearly π is a narrow isomorphism from $\mathfrak{M}_0$ to $\mathfrak{M}_1$ and $\mathfrak{M}_1$ is settled, tree-based and actualistic. Where $v = \langle w_0, \ldots, w_n\rangle \in W_1$ and $i \leqslant n$ let $v_i = \langle w_0, \ldots, w_i\rangle$. We will show the following: where β is an $\mathfrak{A}_1$-assignment, $\alpha = f \circ \beta$ and $\phi \in fml(L_2)$:

$$\mathfrak{M}_0^{(w_0, \ldots, w_n)} \models \phi[\alpha^{w_n}] \text{ iff } \mathfrak{M}_1^{(v_0, \ldots, v_n)} = \phi[\beta^{v_n}];$$

$$\mathfrak{M}_0^{(w_0, \ldots, w_n)} \dashv \phi[\alpha^{w_n}] \text{ iff } \mathfrak{M}_1^{(v_0, \ldots, v_n)} \dashv \phi[\beta^{v_n}].$$

Where $n = 0$, $\mathfrak{M}_0^{w_0} = \mathfrak{M}_0$ and $\mathfrak{M}_1^{v_1} = \mathfrak{M}_1$; so π will be a 2-actualistic isomorphism. We procede by induction on ϕ. For any term τ of L:

$$\mathrm{den}(\mathfrak{A}^{(w_0,\ldots,w_n)}, \alpha^{w_n}, \tau) \simeq f\,\mathrm{den}(\mathfrak{A}^{(v_0,\ldots,v_n)}, \beta^{v_n}, \tau)$$

so the required biconditionals hold if ϕ is atomic. Suppose ϕ is $(\exists v)\psi$. If $\mathfrak{M}_0^{(w_0,\ldots,w_n)} \models \phi[\alpha^{w_n}]$, fix $a \in U_0(w_n)$ so that $\mathfrak{M}_0^{(w_0,\ldots,w_n)} \models \psi[(\alpha^{w_n})_a^v]$; for $b = g(a, v_n)$, $\alpha_a^v = f \circ (\beta_b^v)$, $(\alpha^{w_n})_a^v = (\alpha_a^v)^{w_n}$ and $(\beta^{v_n})_b^v = (\beta_b)^{v_n}$; so by induction hypothesis $\mathfrak{M}_1^{(v_0,\ldots,v_n)} \models \psi[(\beta^{w_n})_b^v]$; so $\mathfrak{M}_1^{(v_0,\ldots,v_n)} \models \phi[\beta^{w_n}]$. The other steps for $(\exists v)\psi$ are analogous. The other inductive steps are easy.

Of course, where $\mathfrak{M}_1$ is constructed from an **L**-model $\mathfrak{M}_0$ as above, $\mathfrak{M}_1$ need not be an **L**-model. When **L** is **T**, $\mathbf{B}^-$ or **B**, versions of Observation 2 apply to **L**-models.

Let $(\mathfrak{A}, w_0)$ be an r-tree-based [s-tree-based, rs-tree-based] model iff Frame($\mathfrak{A}$) is the reflexive [symmetric, reflexive and symmetric] closure of a tree with root w_0. Where $\mathfrak{M}_0$ is a **T**-model, it's easy to see how to revise the construction of $\mathfrak{M}_1$ so as to ensure that $\mathfrak{M}_1$ is r-tree-based, and so is a **T**-model.

Let $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N})$ be s-settled iff:

> for every R-chain $w_0, \ldots, w_n$ where $w_{n-1} \neq w_n$ and for all $i < n - 1$, $w_i \neq w_{i+1}$ and $w_i \neq w_{i+2}$, if $a \in \bar{U}(w_0) - \bar{U}(w_1)$ then $a \notin \bar{U}(w_n)$.

The following versions of Observation 3 hold:

> if $\mathfrak{M}_0$ is a $\mathbf{B}^-$-model [**B**-model] then there is an s-settled s-tree-based [sr-tree-based] model $\mathfrak{M}_1$ so that $\mathfrak{M}_0 \cong_2^a \mathfrak{M}_1$.

The only difference in this: let:

> $W_1 = \{\langle w_0, \ldots, w_n\rangle : w_0, \ldots, w_n$ is an R-chain, $w_{n-1} \neq w_n$, and for all $i < n - 1$, $w_i \neq w_{i+1}$, $w_i \neq w_{i+2}\}$; R_1 is the symmetric-closure [reflexive, symmetric-closure] of $\{(v, v'): v' = \langle w_0, \ldots, w_{n+1}\rangle \in W_1$ and $v = \langle w_0, \ldots, w_n\rangle\}$.

The rest of the construction runs as above.

Because there is no reasonable version of settledness for **K4**-models, there is no plausible analog of Observation 3 restricted to **K4**-, **S4**- or **S5**-models. This fact seems deeply connected with the difficulty of constructing sound and complete formalizations of logics of the form $\mathbf{K4}^{a}_{x}$, $\mathbf{S4}^{a}_{x}$ and $\mathbf{S5}^{a}_{x}$.

3. ON POSSIBILISTIC QUANTIFICATION AND INDIVIDUAL ESSENCES

In this section we'll consider two enrichments of a given language L. Form $L^{\dot\exists}$ from L by adding '$\dot\exists$' to the logical lexicon of L. The corresponding addition to our possibilistic semantics is:

$$\mathfrak{M} \models_p (\dot\exists v)\phi[\alpha] \text{ iff for some } a \in U,\ \mathfrak{M} \models_p \phi[\alpha^v_a];$$

$$\mathfrak{M}\ {}_p\!\!\dashv (\dot\exists v)\phi[\alpha] \text{ iff for some } a \in U,\ \mathfrak{M}\ {}_p\!\!\dashv \phi[\alpha^v_a].$$

There is no sense in applying our actualistic semantics to $L^{\dot\exists}$, since $\mathfrak{M} \models \phi[\alpha^v_a]$ and $\mathfrak{M} \dashv \phi[\alpha^v_a]$ require that $a \in \bar{U}(w)$, where $\mathfrak{M} = (\mathfrak{A}, w)$.

Form L^1 from L by introducing a countable set $\mathbf{Var(1)}$ of type-1 variables, governed by these formation rules:

if $\Upsilon \in \mathbf{Var(1)}$ and τ is a term then $\Upsilon\tau$ is a formula;

if ϕ is a formula and $\Upsilon \in \mathbf{Var(1)}$ then $(\exists\Upsilon)\phi$ is a formula.

Where $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N})$, an $\mathfrak{A}$-essence is a subset of U; so an $\mathfrak{A}$-individual essence is of the form $\{a\}$ for $a \in U$; see [1] for a discussion of these notions under the two-valued possibilistic semantics. Since we're here concerned only with individual essences, we may replace $\{a\}$ by a, letting β be an $\mathfrak{A}$-individual-essence (hereafter $\mathfrak{A}$-i.e.) assignment iff β is a function into U with $\text{dom}(\beta) \subseteq \mathbf{Var(1)}$. Where $\mathfrak{M} = (\mathfrak{A}, w)$ is actualistic and α is an $\mathfrak{M}$-assignment, we define $\models$ and $\dashv$ relative to α and β, with these novel clauses:

$$\mathfrak{M} \models \Upsilon\tau[\alpha, \beta] \text{ iff } \text{den}(\mathfrak{A}, \alpha, \tau) = \beta(\Upsilon);$$

$$\mathfrak{M} \dashv \Upsilon\tau[\alpha, \beta] \text{ iff } \text{den}(\mathfrak{A}, \alpha, \tau) \neq \beta(\Upsilon);$$

$$\mathfrak{M} \models (\exists\Upsilon)\phi[\alpha, \beta] \text{ iff for some } a \in U\ \mathfrak{M} \models \phi[\alpha, \beta^{\Upsilon}_a];$$

$$\mathfrak{M} \dashv (\exists\Upsilon)\phi[\alpha, \beta] \text{ iff for all } a \in U\ \mathfrak{M} \dashv \phi[\alpha, \beta^{\Upsilon}_a].$$

Where $\mathfrak{M}$ is any model and α is any $\mathfrak{A}$-assignment, the definition of $\models_p$ and ${}_p\!\!\dashv$ for formula of L^1 uses the above novel clauses with '$\models$' and '$\dashv$' replaced by '$\models_p$' and '${}_p\!\!\dashv$'. Clearly quantification of type-1 variables is almost possibilistic quantification over all of U; but under the possibilistic semantics, it isn't quite that. The argument used in [1] p. 443 can be easily made to show that "some possible non-actual object (i.e. a member of $U - \bar{U}(w)$) is **P**" is **not** expressible in L^1 under the possibilistic semantics. ($\mathbf{P}\Upsilon$ is not a formula; as for $(\exists x)(\mathbf{P}(x) \,\&\, \Upsilon x)$, in $(\mathfrak{A}, w)$ '$(\exists x)$' is restricted to $\bar{U}(w)$.)

OBSERVATION 4. There is a translation $t^*: fml(L^{\dot\exists}) \rightarrow fml(L^1)$ such that for any actualistic model $\mathfrak{M}$ for L, any $\mathfrak{M}$-assignment α and any $\phi \in fml(L)$:

$$\mathfrak{M} \models_p \phi[\alpha] \text{ iff } \mathfrak{M} \models t^*(\phi)[\alpha];$$

$$\mathfrak{M} \;{}_p\!\dashv \phi[\alpha] \text{ iff } \mathfrak{M} \dashv t^*(\phi)[\alpha].$$

Associate with each $v \in \mathbf{Var}$ a corresponding $\Upsilon_v \in \mathbf{Var(1)}$. Suppose that $\phi \in fml(L^{\dot\exists}_y)$ where '$\supset$' $\in \text{lex}_y$. Where ϕ is $\mathbf{P}(\tau_0, \ldots, \tau_{n-1})$, suppose $v_0, \ldots, v_{m-1}$ are the variables among the τ_i's; let $\overline{\phi}$ be

$$(\exists v_0) \ldots (\exists v_{m-1}) \left(\bigwedge_{i<m} \Upsilon_{v_i}(v_i) \,\&\, \phi \right);$$

define $\overline{\phi}$ similarly when ϕ is $(\tau_0 \approx \tau_1)$ or $(\tau_0 \approx_s \tau_1)$. Let $\overline{(\phi \supset \psi)}$ be $(\overline{\phi} \supset \overline{\psi})$, $\overline{T\phi}$ be $T\overline{\phi}$, $\overline{(\dot\exists v)\phi}$ be $(\exists \Upsilon_v)\overline{\phi}$, $\overline{(\exists v)\phi}$ be $(\exists \Upsilon_v)((\exists v)\Upsilon_v v \,\&\, \overline{\phi})$, and let $\overline{\Box\phi}$ be $\Box\overline{\phi}$. Finally, if $v_0, \ldots, v_{m-1}$ are the variables free in ϕ, let $t^*(\phi)$ be:

$$(\exists \Upsilon_{v_0}) \ldots (\exists \Upsilon_{v_{m-1}}) \left(\bigwedge_{i<m} \Upsilon_{v_i}(v_i) \,\&\, \overline{\phi} \right);$$

it's easy to see that $t^*(\phi)$ is as required. Where '$\supseteq$' $\in \text{lex}^m_y$, a similar construction works.

It is even easier to construct the translation $\hat{t}: fml(L^1) \rightarrow fml(L^{\dot\exists})$ so that for any actualistic model $\mathfrak{M}$ for L, any $\mathfrak{M}$-assignment α and $\phi \in fml(L^1)$:

$$\mathfrak{M} \models \phi[\alpha] \text{ iff } \mathfrak{M} \models_p \hat{t}(\phi)[\alpha];$$

$$\mathfrak{M} \dashv \phi[\alpha] \text{ iff } \mathfrak{M} \;{}_p\!\dashv \hat{t}(\phi)[\alpha];$$

simply replace type-**1** variables by new type-0 variables, and where v replaces Υ, replace $\Upsilon\tau$ by $(v \approx \tau)$.

The discussion of t in §2 suggests that for an individual-actualist who accepts the logic $\mathbf{L}^a_x$, ϕ under the possibilistic semantics represents a meaningful formula if ϕ is $\mathbf{L}^a_v$-equivalent to $t(\psi)$ for some ψ. Observation 4 suggests that if an individual-actualist accepts the actualistic model-theoretic semantics for L^1 given above, then she may regard all formulae of L^3 under the possibilistic semantics as representing meaningful formulae. But should an individual actualist accept the actualistic semantics assigned to L^1?

It's important to keep in mind that this model-theoretic semantics presupposes that individual essences are not individuals; they cannot be said to exist or fail to exist at worlds. Rather they are predicative entities, the referents of predicates which uniquely apply to an individual at every world in which that individual exists.[3]

Of course the introduction of such a predicate may rely on the existence of that unique individual; for example if Richard Nixon didn't exist, 'is Richard Nixon' would not refer to the individual essence of Richard Nixon (hereafter called 'being Richard Nixon'). But other such predicates do not rely on the existence of that individual; thus suppose Richard Nixon developed from Spermy and Eggy (where 'Spermy' and 'Eggy' rigidly denote the sperm and egg from which Nixon developed); if we accept the plausible thesis that Richard Nixon essentially developed from Spermy and Eggy, and we ignore the possibility that a single zygote develops into twins, then 'is the person that developed from Spermy and Eggy' also refers to being Richard Nixon; so from any world in which Spermy and Eggy exist, being Richard Nixon is referentially accessible, even if Richard Nixon does not exist in that world, e.g. because Spermy never fertilizes Eggy in that world.

The following second actualist constraint is quite plausible: a definition of satisfaction and frustration must allow assignment of type-1 variables ranging over individual essences only to referentially accessible individual essences. Granting this constraint, the individual actualist should accept our actualistic model-theoretic semantics for L^1 only under the idealizing assumption that all possible individual-essences are referentially accessible from every possible world.

This assumption is surely false; even granting highly idealized biological assumptions, surely there are worlds from which being Richard Nixon is not referentially accessible; perhaps any world in which either Spermy or Eggy does not exist would be such a world.

To represent the additional actualist constraint within a model-theoretic semantics, we'd have to expand our structures to the form $\mathfrak{A} = (W, R, U, \bar{U}, \hat{U}, \mathscr{E}, \mathscr{N})$, where $\hat{U}$ maps W into Power(U); $\hat{U}(w)$ represents the class of individual-essences accessible from w. If one is willing to commit oneself to individual essences at all, it seems that one should believe that any actualized individual essence must be referentially accessible; in this case we should impose this condition on all models for L^1: for all $u \in W$, $\bar{U}(u) \subseteq \hat{U}(u)$.

Where $\mathfrak{M} = (\mathfrak{A}, w)$, let β be an $\mathfrak{M}$-i.e. assignment iff dom(β) $\subseteq$ Var(**1**) and β is into $\hat{U}(w)$; for any $u \in W$, let $\beta^u(\Upsilon) = \beta(\Upsilon)$ if $\beta(\Upsilon) \in \hat{U}(u)$; $\beta^u(\Upsilon)\uparrow$ otherwise. Where $\mathfrak{M}$ is actualistic, α is an $\mathfrak{M}$-assignment and β is an $\mathfrak{M}$-i.e. assignment, we define $\mathfrak{M} \models \phi[\alpha, \beta]$ and $\mathfrak{M} \dashv \phi[\alpha, \beta]$ as above, with these revisions:

$$\mathfrak{M} \models \Box\phi[\alpha, \beta] \text{ iff for all } u, \text{ if } wRu \text{ then } \mathfrak{M}^u \models \phi[\alpha^u, \beta^u];$$

$$\mathfrak{M} \dashv \Box\phi[\alpha, \beta] \text{ iff for some } u, wRu \text{ and } \mathfrak{M}^u \dashv \phi[\alpha^u, \beta^u];$$

a similar revision applies to $\underline{\Box}\phi$. Clearly if for all $u \in W$; $\hat{U}(u) = U$, this definition coincides with the previous one.

Although this is not the place for an extended discussion of the actualistic semantics for L^1 just presented, three points deserve mention. Observation 3 does not hold for our final semantics; but an analog does hold. Let $\mathfrak{A}$ be special iff for every R-chain $w_0, \ldots, w_n$, $\bar{U}(w_0) \subseteq \hat{U}(w_n)$; let $\mathfrak{M} = (\mathfrak{A}, w)$ be special iff $\mathfrak{A}$ is special. There is a translation $\bar{\imath}^*$: $fml(L) \to fml(L^1)$ so that for every actualistic special mode $\mathfrak{M}$ and every $\mathfrak{M}$-assignment α:

$$\mathfrak{M} \models_p \phi[\alpha] \text{ iff } \mathfrak{M} \models \bar{\imath}^*(\phi)[\alpha];$$

$$\mathfrak{M} \;{}_p\!\dashv \phi[\alpha] \text{ iff } \mathfrak{M} \dashv \bar{\imath}^*(\phi)[\alpha].$$

However, restricting our attention to actualistic special models is the formal correlate of another idealizing assumption: that every individual essence of an actual individual is referentially accessible from any world ancestrally possible from the actual world. Surely this is as false as our previous idealizing assumption; for example there are accessible worlds from which being Richard Nixon is not referentially accessible.

Our conclusion: commitment to individual essences does not systematically enlarge the class of formulae of L which, under the possibilistic semantics, an individual-actualist may regard as representing meaningful formulae.

4. ON $\mathbf{L}^{a}_{at}$-BIVALENCE AND DE DICTO SENTENCES

Suppose that **L** is a logic-defining class. Where 'x' is replaceable as usual and $\phi \in fml(L)$, let ϕ be $\mathbf{L}^{a}_{x}$-bivalent iff for all $\mathbf{L}^{a}_{x}$-models $\mathfrak{M}$ for L and all $\mathfrak{M}$-assignments α: either $\mathfrak{M} \models \phi[\alpha]$ or $\mathfrak{M} \dashv \phi[\alpha]$. Let ϕ be de dicto iff ϕ contains no subformula $\Box\psi$ which contains an occurrence of a variable free in ψ but bound in ϕ.

OBSERVATION 5. Where 'x' is replaceable by 'at' or 'at & nn', $\phi \in fml(L_{i,\mathbf{u}})$, and $i < 2$:

> ϕ is $\mathbf{L}^{a}_{x}$-bivalent iff ϕ is $\mathbf{L}^{a}_{x}$-equivalent to a de dicto sentence of $L_i(\mathbf{Pred}, \{\ \})$.

Since the argument if $i = 0$ is just like that for $i = 1$, assume $i = 1$.

($\Leftarrow$) Let ϕ be $\mathbf{L}^{a}_{x}$-bivalent* iff for every $\mathbf{L}^{a}_{x}$-model $\mathfrak{M}$ and every $\mathfrak{M}$-assignment α, if for all variables v free in ϕ $\alpha(v)\downarrow$ then: either $\mathfrak{M} \models \phi[\alpha]$ or $\mathfrak{M} \dashv \phi[\alpha]$. If ϕ is $\mathbf{L}^{a}_{x}$-equivalent to ψ, where ψ is an $\mathbf{L}^{a}_{x}$-bivalent* sentence of $L_1(\mathbf{Pred}, \{\ \})$, then ψ is $\mathbf{L}^{a}_{x}$-bivalent, and thus so is ϕ. So it suffices to show that any de dicto sentence ψ of $L_1(\mathbf{Pred}, \{\ \})$ is $\mathbf{L}^{a}_{x}$-bivalent*. Clearly all atomic formulae of $L_1(\mathbf{Pred}, \{\ \})$ are $\mathbf{L}^{a}_{x}$-bivalent*; if θ and θ' are $\mathbf{L}^{a}_{x}$-bivalent* formulae then so are $(\theta \supset \theta')$ and $(\exists v)\theta$. If θ is an $\mathbf{L}^{a}_{x}$-bivalent* sentence of $L_1(\mathbf{Pred}, \{\ \})$ then $\Box\theta$ is $\mathbf{L}^{a}_{x}$-bivalent*. But if $\Box\theta$ is a subformula of ψ then θ is a sentence, since ψ is a de dicto sentence. So by induction on the construction of ψ, ψ is $\mathbf{L}^{a}_{x}$-bivalent*.

($\Rightarrow$) Firstly, we mention an obvious point. Where $\mathfrak{A}_i = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N}_i)$ for $i < 2$, let $\mathfrak{A}_0 \subseteq \mathfrak{A}_1$ iff $\mathscr{N}_0 \subseteq \mathscr{N}_1$; where $\mathfrak{M}_i = (\mathfrak{A}_i, w)$, let $\mathfrak{M}_0 \subseteq \mathfrak{M}_1$ iff $\mathfrak{A}_0 \subseteq \mathfrak{A}_1$. Suppose α and α' are $\mathfrak{M}_i$-assignments, and $\phi \in fml(L_{1,\mathbf{u}})$; if $\alpha \subseteq \alpha'$ and $\mathfrak{M}_0 \subseteq \mathfrak{M}_1$ then:

$$\text{if } \mathfrak{M}_0 \models \phi[\alpha] \text{ then } \mathfrak{M}_1 \models \phi[\alpha'];$$

$$\text{if } \mathfrak{M}_0 \dashv \phi[\alpha] \text{ then } \mathfrak{M}_1 \dashv \phi[\alpha'].$$

Proof is by an easy induction on ϕ.

Suppose $\mathfrak{A} = (W, R, U, \bar{U}, \mathscr{E}, \mathscr{N})$ is a structure for L; fix $w_0 \in W$. Let: $U^* = U \times W$; $\bar{U}^*(w) = \{\langle a, w\rangle: a \in \bar{U}(w)\}$; $\pi(\langle a, w\rangle) = a$; $\mathscr{N}^*(\mathbf{c}) = \langle \mathscr{N}(\mathbf{c}), w_0\rangle$; for $\mathbf{P} \in \mathbf{Pred}$ and n-place, let:

$$\mathscr{E}^*(\mathbf{P}) \simeq \mathscr{E}(P) \text{ if } n = 0;$$

$$\mathscr{E}^*(\mathbf{P})(w, b_0, \ldots, b_{n-1}) \simeq \mathscr{E}(\mathbf{P})(w, \pi b_0, \ldots, \pi b_{n-1}) \text{ if } n \geqslant 1, \text{ for } b_0, \ldots, b_{n-1} \in U^*;$$

let $\mathfrak{A}^* = (W, R, U^*, \bar{U}^*, \mathscr{E}^*, \mathscr{N}^*)$; where $\mathfrak{M} = (\mathfrak{A}, w_0)$ is actualistic, so is $\mathfrak{M}^* = (\mathfrak{A}^*, w_0)$.

LEMMA 1. *Suppose $w \in V \subseteq W$, α is an $\mathfrak{A}^*$-assignment, τ is a term and $\phi \in fml(L_{I,\mathbf{u}})$;*

$$\text{if } \mathrm{den}(\mathfrak{A}^{*V}, \alpha^{w}, \tau) = b \text{ then } \mathrm{den}(\mathfrak{A}^V, (\pi \circ \alpha)^{w}, \tau) = \pi b;$$

$$\text{if } (\mathfrak{A}^{*V}, w) \vDash \phi[\alpha^{w}] \text{ then } (\mathfrak{A}^V, w) \vDash \phi[(\pi \circ \alpha)^{w}];$$

$$\text{if } (\mathfrak{A}^{*V}, w) \mathrel{=\!\!|} \phi[\alpha^{w}] \text{ then } (\mathfrak{A}^V, w) \mathrel{=\!\!|} \phi[(\pi \circ \alpha)^{w}].$$

Since $(\pi \circ \alpha)^{w} = \pi \circ (\alpha^{w})$, the first conditional holds. The induction on ϕ is straightforward; we'll consider the case in which ϕ is $\Box\psi$. Suppose wRu and $(\mathfrak{A}^{*V}, w) \vDash \phi[\alpha^{w}]$. If $w \neq u$ then $\alpha^{(w,u)} = \{\ \}$; so $(\mathfrak{A}^{*V \cup \{u\}}, u) \vDash \psi[\{\ \}]$; by induction hypothesis, $(\mathfrak{A}^{V \cup \{u\}}, u) \vDash \psi[\{\ \}]$; by the obvious point, $(\mathfrak{A}^{V \cup \{u\}}, u) \vDash \psi[(\pi \circ \alpha)^{(w,u)}]$. If $w = u$, $(\mathfrak{A}^{*V}, u) \vDash \psi[\alpha^{u}]$; by induction hypothesis $(\mathfrak{A}^V, u) \vDash \psi[(\pi \circ \alpha)^{u}]$. Thus $(\mathfrak{A}^V, w) \vDash \phi[\alpha^{w}]$. By a similar argument the third biconditional also holds for $\Box\psi$. QED

Where $\phi \in fml(L_{I,\mathbf{u}})$, suppose without loss of generality that no variable free in ϕ also occurs bound in ϕ. Consider an occurrence of the variable v in a subformula θ of ϕ; we'll call that occurrence bad for θ iff θ is a subformula of some ψ, $\Box\psi$ is a subformula of ϕ, and that occurrence of v is free in ψ but bound in ϕ. Let an occurrence of an atomic formula θ in ϕ be bad iff either it contains an occurrence of a variable free in ϕ, of a member of $\mathbf{C}$, or of a variable bad for θ. Form ϕ_0 from ϕ by replacing each bad occurrence in ϕ of an atomic formula by '**u**'. Clearly ϕ_0 is a de dicto sentence of $L_{I,\mathbf{u}}(\mathbf{Pred}, \{\ \})$. Where ψ is an occurrence of a subformula of ϕ, let ψ_0 be the corresponding occurrence of a subformula of ϕ_0. Let $\mathfrak{A}_0 = (W, R, U, \bar{U}, \mathscr{E}, \{\ \})$, $\mathfrak{A}_0^* = (W, R, U^*, \bar{U}^*, \mathscr{E}^*, \{\ \})$.

LEMMA 2. *Let R be irreflexive. Let ψ be an occurrence of a formula in ϕ; suppose $w \in V \subseteq W$ and α is an $(\mathfrak{A}_0^{*w}, w)$-assignment such that for all variables v, if v is free in ϕ or an occurrence of v in ψ is bad for ψ then $\alpha(v)\uparrow$. Then*:

$$\text{if } (\mathfrak{A}_0^{*V}, w) \vDash \psi[\alpha] \text{ then } (\mathfrak{A}_0^{*V}, w) \vDash \psi_0[\alpha];$$

$$\text{if } (\mathfrak{A}_0^{*V}, w) \dashv \psi[\alpha] \text{ then } (\mathfrak{A}_0^{*V}, w) \dashv \psi_0[\alpha].$$

Where ψ is atomic and bad then $(\mathfrak{A}_0^{*V}, w) \mid \psi[\alpha]$; if ψ is atomic and not bad then ψ is ψ_0; so for atomic ψ the lemma holds. If ψ is $(\psi_0 \supset \psi_1)$ the inductive step is straightforward. If ψ is $(\exists v)\theta$ then no occurrence of v in ψ is free in ϕ or bad for ψ; using this fact, the inductive step is straightforward, since for any $b \in \bar{U}^*(w)$, α_b^v still meets the condition in the lemma. Suppose ψ is $\Box\theta$ and wRu. Since $w \neq u$, by the construction of $\mathfrak{M}_0^*$, $\alpha^u = \{\ \}$. If $(\mathfrak{A}_0^{*V}, w) \vDash \psi[\alpha]$ then $(\mathfrak{A}_0^{*V \cup \{u\}}, u) \vDash \theta[\{\ \}]$; since $\{\ \}$ meets the condition on assignments in this lemma, we may apply the induction hypothesis to get $(\mathfrak{A}_0^{*V \cup \{u\}}, u) \vDash \theta_0[\{\ \}]$; so $(\mathfrak{A}_0^{*V}, w) \vDash \psi_0[\alpha]$. A similar argument yields the second conditional. QED.

Thus supposing that R is irreflexive, $\mathfrak{M}$ actualistic and α an $\mathfrak{M}$-assignment, we have:

$$\text{if } \mathfrak{M}_0^* \vDash \phi[\{\ \}] \text{ then } \mathfrak{M} \vDash \phi[\alpha] \text{ and } \mathfrak{M}_0 \vDash \phi_0[\alpha];$$

$$\text{if } \mathfrak{M}_0^* \dashv \phi[\{\ \}] \text{ then } \mathfrak{M} \dashv \phi[\alpha] \text{ and } \mathfrak{M}_0 \dashv \phi_0[\alpha].$$

Suppose that $\mathfrak{M}_0^* \vDash \phi[\{\ \}]$; by Lemma 1, $\mathfrak{M}_0 \vDash \phi[\{\ \}]$; so by the obvious point $\mathfrak{M} \vDash \phi[\alpha]$; but by Lemma 2 our supposition yields $\mathfrak{M}_0^* \vDash \phi_0[\{\ \}]$; by Lemma 1, $\mathfrak{M}_0 \vDash \phi_0[\{\ \}]$; so by the obvious point $\mathfrak{M}_0 \vDash \phi[\alpha]$. A similar argument yields the second conditional.

CLAIM. If ϕ is $\mathbf{L}_x^a$-bivalent then ϕ is $\mathbf{L}_x^a$-equivalent to ϕ_0. It suffices to show that for any actualistic model $\mathfrak{M}$ with Frame($\mathfrak{M}$) $= (W, R)$ and R irreflexive, and any $\mathfrak{M}$-assignment α:

$$\mathfrak{M} \vDash \phi[\alpha] \text{ iff } \mathfrak{M} \vDash \phi_0[\alpha];\ \mathfrak{M} \dashv \phi[\alpha] \text{ iff } \mathfrak{M} \dashv \phi_0[\alpha].$$

Assuming that ϕ is $\mathbf{L}_x^a$-bivalent, either $\mathfrak{M}_0^* \vDash \phi[\{\ \}]$ or $\mathfrak{M}_0^* \dashv \phi[\{\ \}]$; by the previously established conditionals, these biconditionals hold.

To prove Observation 5, form ϕ_1 from ϕ_0 by replacing each occurrence of '**u**' in ϕ_0 by '$\bot$' or '$\neg\bot$'. By a trivial variation of the obvious point:

$$\text{if } \mathfrak{M} \vDash \phi_0 \text{ then } \mathfrak{M} \vDash \phi_1$$

$$\text{if } \mathfrak{M} \dashv \phi_0 \text{ then } \mathfrak{M} \dashv \phi_1.$$

Clearly ϕ_1 is a de dicto sentence of $L_1(\mathbf{Pred}, \{\ \})$ and is $\mathbf{L}_x^a$-equivalent to ϕ.

NOTES

[1] More generally, the only two-valued modal models $(\mathfrak{A}, w)$ the individual-actualist could accept would meet this expanding-domain condition: for any $u, v \in W$ so that an R-chain runs from w to u and uRv, $\bar{U}(u) \subseteq \bar{U}(v)$. For suppose there is an R-chain of length n from w to u. Unpacking the conditions for $(\mathfrak{A}, w) \vDash_2 \Box^n(\exists v)\Box(v \approx v)$, for each $a \in \bar{U}(u)$ we must ask whether $(\mathfrak{A}, u) \vDash_2 \Box(v \approx v)[a]$; thus we must ask whether $(\mathfrak{A}, v) \vDash_2 (v \approx v)[a]$; for that condition to make sense, the actualist's constraint requires that $a \in \bar{U}(v)$.

[2] To reiterate: the distinction between the possibilistic and semi-possibilistic model-theories on the one hand and the actualistic model-theory on the other hand concerns the treatment of free-variables (or more generally, of parameters). Unlike the former treatment, the actualistic treatment *insures* that the quantifiers are actualistic in the sense just explained. (See the first paragraph of §3.) But the treatment of quantifiers is not the crux of the matter.

[3] I hope that the replacement of $\{a\}$ by a in the definition of an $\mathfrak{A}$ – i.e. assignment does not obscure this point. One should be guided by the syntax of L^1 on this matter. Many contributors to the philosophical literature do speak of individual essences as existing or not existing at or in possible worlds. Either they have a conception of individual essences different from that motivating the model-theoretic semantics presented above (one according to which an individual essence is itself an individual) or else by 'exist' in such contexts they mean what I'm calling 'referential accessibility'.

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