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REMARKS ON *AL-FĀRĀBĪ*'S MISSING MODAL LOGIC AND ITS EFFECT ON *IBN SĪNĀ*

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ABSTRACT: We reconstruct as much as we can the part of al-Fārābī's treatment of modal logic that is missing from the surviving pages of his *Long Commentary on the Prior Analytics*. We use as a basis the quotations from this work in Ibn Sīnā, Ibn Rushd and Maimonides, together with relevant material from al-Fārābī's other writings. We present a case that al-Fārābī's treatment of the *dictum de omni* had a decisive effect on the development and presentation of Ibn Sīnā's modal logic. We give further evidence that the *Harmonisation of the Opinions of Plato and Aristotle* was not written by al-Fārābī.

KEYWORDS: Farabi, Ibn Sina, logic, modal, dictum de omni.

1. Introduction

The part of al-Fārābī's *Long Commentary on the Prior Analytics* (Al-Fārābī 1988) dealing with Book i, which includes everything in the *Prior Analytics* on modal logic, is famously missing.[i] Henceforth we refer to this work (Al-Fārābī 1988) as the *Long Commentary*. We know something about its missing contents from quotations and discussions in Ibn Sīnā, Ibn Rushd and Maimonides. Also, *Harmonisation* (Al-Fārābī 1999), which asks to be read as a work of al-Fārābī, contains some claims about the contents of the *Long Commentary*. So it's tempting to take these sources and try to reconstruct what we have lost in this missing work of al-Fārābī. This is a high-risk enterprise: a manuscript could come to light tomorrow and wreck all my calculations.

I will argue for the following points.

(A) Al-Fārābī didn't have his own modal logic and made no attempt to build one. (Section 2)

(B) Al-Fārābī did propose a closer study, within the framework of Aristotle's modal rules, of some sentence forms already suggested in the Peripatetic literature. (Section 3)

(C) In the course of (B), al-Fārābī started to develop a machinery for defining sentence meanings, which had a profound influence on the development of Ibn Sīnā's modal logic. (Sections 4, 5)

(D) Al-Fārābī didn't write *Harmonisation* (Al-Fārābī 1999). (Section 6. This is not a new claim, but I add further evidence.)

This paper takes up some questions that were studied in the course of writing Hodges and Druart (2019) but never made it into the finished article. Also, the material on *min ḥaythu* in Section 3 relates to some questions about Ibn Sīnā that Seyed N. Mousavian and I found we both had an interest in. I hope we will soon have something to report on that; and meanwhile I thank him for his kind invitation to submit this paper to the journal *Eshare*: *An Iranian Journal of Philosophy*.

2. A Farabian modal logic?

Aristotle himself proposed a modal logic that was wholly new. His students Theophrastus and Eudemus are reported to have asserted their independence by defending a modal logic different in some ways from Aristotle's. There were other classical Greek philosophers outside the Peripatetic tradition who had views relevant to the form that a modal logic should take; Diodorus Cronus is an obvious example.

By the second century AD we no longer hear of attempts to build a modal logic significantly different from that of Aristotle. The commentator Alexander of Aphrodisias, writing around AD 200, reports Aristotle's modal logic, tries to clarify it where it is obscure, and discusses various criticisms that have been made of it. But he never comes near proposing his own alternative system. Nor do Alexander's successors, at least up to the time of al-Fārābī. The first clear break with this tradition is in the work of Ibn Sīnā in the early 11th century; he develops in detail a system of temporal logic as opposed to Aristotle's logic of 'alethic' modalities (necessary, possible, contingent)[ii], and rejects Aristotle's alethic modal logic in favour of his own new version.

Ibn Sīnā followed the lead of al-Fārābī in many things. Did al-Fārābī perhaps point him towards the programme of designing a new modal logic? Possible indications of this are places where al-Fārābī gives his own opinion on what a modal logic should contain, as opposed to explaining or interpreting the views of Aristotle and other earlier logicians. Here are the three main candidates.

Candidate One: ampliation. Ibn Sīnā is one of a number of logicians who record that al-Fārābī proposed to read 'Every *B* is an *A*', at least in modal contexts, as quantifying not over things that are actual *B*s but over things that are either actual or

potential *B*s. Thus, we read that in a sentence of the form 'Every *C* is ...',

> (1) … if the subject is taken in the way that the outstanding[iii] later [scholar] preferred (*ikhtāra*), so that *C* is whatever could legitimately be a *C*, so that it includes whatever could legitimately be a *C*, even if it could have existed but didn't exist, or it wasn't in fact a *C* …
> (Ibn Sīnā 1964, 85.5–7)

This would have various consequences for the modal moods that al-Fārābī accepted as valid. (In a later terminology, this reading of the subject term is called 'ampliation to the possible'.)

Candidate Two: a modal syllogism. The work *Harmonisation* (Al-Fārābī 1999) has been attributed to al-Fārābī. It (Al-Fārābī 1999, 87.3–6) refers to the syllogism

> (2) Every *C* is a *B*;[iv]
> every *B* is an *A* necessarily.
> Therefore every *C* is an *A* necessarily.

(which Aristotle at *Prior Analytics* i.9, 30a15–23 says is clearly valid), and comments:

> So it is now clear that what Aristotle claimed about this syllogism is [correct] as he claimed it ... (Al-Fārābī 1999, 93.1–7)

Ibn Sīnā confirms that al-Fārābī took this view. At *Qiyās* (Ibn Sīnā 1964, 148.9) he says that al-Fārābī ('the outstanding one', see note iii) agreed with Aristotle about this same mood.

Candidate Three: a conversion. Both Ibn Sīnā and Ibn Rushd quote a place, presumably in the *Long Commentary*, where al-Fārābī argues in support of the claim that 'Every *B* is contingently an *A*' converts to 'Some *A* is contingently a *B*' (a claim that Aristotle made at *Prior Analytics* i.3, 25a40–b1). More precisely, Ibn Rushd in his *Maqālāt* (Ibn Rushd 1983) quotes al-Fārābī as follows (I add the labels (α) etc. for later convenience):

> (3) The sentence
> (α) 'Every animal is contingently (*mumkin*) sleeping'
> means that
> (β) every animal is contingently sleeping because of (*min jiha*) what sleeping is.
> And that being the case,
> (γ) something that is sleeping because of what sleeping is is contingently an animal, because its animality is not because of what sleeping is necessarily. (Ibn Rushd *Maqālāt* 1983, 102.13–16)

Ibn Sīnā quotes the same passage of al-Fārābī at his *Qiyās* (Ibn Sīnā 1964, 209.7–9). He begins his quotation at (β) rather than (α), but in all other relevant respects his wording is identical with that given by Ibn Rushd.

2.1. Candidate One

We know of only one discussion in al-Fārābī that is clearly relevant to Candidate One. Ibn Rushd quotes this discussion at length in his *Maqāla* (Ibn Rushd 1983, 1.3.5), translated in Passages A–D of the Appendix below. So we can read what al-Fārābī said and judge whether Ibn Sīnā has reported it correctly.[v]

Let me summarise what al-Fārābī says in these passages. He notes that Aristotle describes the following syllogism as 'perfect', i.e. as self-evidently valid:

(4) Every *C* is possibly a *B*;
every *B* is possibly an *A*.
Therefore every *C* is possibly an *A*.

(This is correct, *Prior Analytics* i.14, 32b34–40.) He also claims that Alexander takes 'every *B*' in Aristotle's text to mean 'everything that is an actual *B*'. Then he points out that something must be wrong. On this reading, some *C*s could be possible but not actual *B*s; and in that case the premise 'Every actual *B* is a possible *A*' gives no information about these *C*s. Al-Fārābī doesn't deduce that the syllogism (4) is invalid on Alexander's reading; it seems he allows that the conclusion might be reached by another route that is not self-evident. He does deduce that the syllogism is not perfect. He, then, concludes that Alexander's account of Aristotle's approach to this syllogism

(5) is not Aristotle's approach, nor does it fit the facts themselves. (Ibn Rushd 1983, 129.4)

He maintains that the syllogism can be made perfect by reading 'every *B*' as 'everything that is possibly or actually a *B*', and he suggests that this is what Aristotle intended.

It seems clear from this that al-Fārābī is not claiming to have a revised modal logic which ampliates to the possible. On the contrary, his claim is that ampliation to the possible would justify some of Aristotle's verdicts on syllogistic moods, and is probably what Aristotle intended in the first place.

Ibn Sīnā's statement about al-Fārābī and ampliation, (1) above, should be read in the context of Ibn Sīnā's own work on modal logic. Throughout the development of Ibn Sīnā's modal logic we can trace a drive towards eliminating ambiguities from the modal sentence forms; in Section 5 below we will see how this drive interlocks with Ibn Sīnā's reading of al-Fārābī. When Ibn Sīnā defines his new temporal sentence forms, he must eliminate the ambiguity in the quantifier, and he chooses to do this by reading 'every *B*' as 'every actual *B*'. Naturally he records that this reading is not the one that al-Fārābī 'preferred', but he doesn't go into the details of the context in which al-Fārābī preferred it and for what reason.

Later writers say more categorically that al-Fārābī read quantifiers in modal logic

as ampliating to the possible. Thus for example Rāzī in *Manṭiq al-Mulakhkhaṣ*:

> Al-Fārābī claimed that in the phrase 'Every *C*' one considers not things that are actually *C*s, but everything that it would be possible to describe as a *C*.
>
> (al-Rāzī 1961, 142.4f)

In his *Commentary on the Išārāt*, Naṣīr al-Dīn al-Ṭūsī (1995, 162.4–7), discussing the meaning of 'Every *C*' in affirmative statements, goes further and claims that according to al-Fārābī, what it means (*al-murādu bih*) is 'Everything that can legitimately be described as [a *C*], regardless of whether it is so described in fact or only in potentiality'. There is also a briefer statement in Ṭūsī's *Asās* (al-Ṭūsī, 2010, 93).

If we knew that Rāzī and Ṭūsī consulted al-Fārābī's text before writing these remarks, we could consider these authors as independent witnesses. But we don't know this, and in fact both Rāzī and Ṭūsī are most likely reporting what they thought Ibn Sīnā meant by (1).[vi] In sum, reports that al-Fārābī had a modal logic in which he ampliated to the possible are probably mild misunderstandings that grew with the telling.[vii]

2.2. Candidate Two

The work *Harmonisation* (Al-Fārābī 1999) may not be by al-Fārābī. We will come back to this issue in Section 6 below, where I will argue that the work is not by al-Fārābī but presents itself as having been written by him. So it may yet serve as evidence for the contents of the *Long Commentary*.

In any case Candidate Two is less likely than Candidate One to be evidence that al-Fārābī had his own version of modal logic. At least in Candidate One al-Fārābī was proposing something different from the mainstream view, even if he was claiming that it was Aristotle's real intention. But the validity of the syllogism (2) already *was* a mainstream view, explicitly endorsed by Aristotle himself. So why did Ibn Sīnā and the author of *Harmonisation* (Al-Fārābī 1999) even think it worth mentioning?

Passage C in the Appendix, quoted in Ibn Rushd (1983), includes an endorsement of the syllogism (2) by al-Fārābī. At 130.16–18 in the Passage, al-Fārābī calls attention to those syllogisms where the minor premise is either necessary or *de inesse*—this includes (2). He observes that all these syllogisms are valid because the minor premise states or implies that every *C* is an actual *B*; and that even if we ignore this fact, they are all made valid for Aristotle by assuming that Aristotle always reads 'Every *B*' as meaning 'Every actual or possible *B*'. There is nothing here to provide any reason for singling out the mood (2).

Ibn Sīnā's reason for mentioning the mood becomes clear if we look at the context of his remark. At *Qiyās* (Ibn Sīnā 1964, 144.15) Ibn Sīnā mentions the device used by al-Fārābī in Candidate Three (we will go further into details of this in Subsection 2.3 below). Al-Fārābī tries to justify an inference rule by first replacing a term *B* in it by '*B* insofar as it is a *B*', and then carrying out some manipulations which Ibn Sīnā will later describe (Ibn Sīnā 1964, 209.9) as 'pure sophistry'. For present

purposes let us refer to these moves by al-Fārābī as 'al-Fārābī's trick'.

After spending several pages discussing matters related to al-Fārābī's trick, Ibn Sīnā quotes the mood (2) above, notes that al-Fārābī, Aristotle and he himself all endorse it, and continues:

But why doesn't one of the two also say: 'This is not necessary, but one has to say

> "Every *B* is an *A* insofar as it is a *B*, with necessity" '?
> (Ibn Sīnā 1964, 148.13f)

The suggestion that Aristotle should have tried al-Fārābī's trick on a syllogism that Ibn Sīnā regarded as entirely straightforward can hardly be anything but irony. It's as if Ibn Sīnā said 'What a pity Aristotle didn't have al-Fārābī around to teach him this beautiful breakthrough'. Ibn Sīnā's only reason for mentioning syllogism (2) here is precisely what he says, namely that he, Aristotle and al-Fārābī all regard it as valid. Any of several other modal moods would have served the same purpose.

But then what reason did the author of *Harmonisation* have for mentioning that al-Fārābī endorsed this mood? At 87.3ff this author records that Themistius was one of many commentators who argued that the conclusion of (2) should be *de inesse*, not necessary. He holds that the disagreement between these commentators and Aristotle should be settled by invoking the *dictum de omni*; he claims that al-Fārābī (as author of the *Long Commentary*) did resolve the question in this way, coming down on Aristotle's side.

> (6) Alexander of Aphrodisias has summarised the meaning of the *dictum de omni*, while departing from [viii] what Aristotle claimed about it. We ourselves expounded what [Aristotle] had to say under this head in the book *Analytics*; [in particular] we showed the meaning of the *dictum de omni*, we gave a clear summary of the facts about it, and in this context (*fihi*) we made a distinction between the syllogistically necessary and the demonstratively necessary, in such a way that a person who studies it will be in a position to ignore all of the things that might cause him confusion under this head. So it is now clear that what Aristotle claimed about this syllogism is [correct] as he claimed it ...
> (Al-Fārābī 1999, 93.1–7)

Fortunately, we have Themistius's side of this story (Rosenberg & Manekin 1988, 100), a Hebrew paraphrase probably taken from Arabic, and with its help I think we can reconstruct what the author of *Harmonisation* is likely to be referring to.

Themistius argues from 'Every *C* is a *B*' and 'Every *B* is an *A* necessarily' as follows. Since every *B* is an *A* necessarily, '*A* will not be separated from *B* at any time'. I take this to mean 'For all times *t*, if a thing is a *B* at time *t* then it is an *A* at time *t*'. But since 'Every *C* is a *B*' is not said to be necessary, it allows that there is a time when some *C* is not a *B*. At this time the *C* will not have to be an *A* either.

Ibn Sīnā sets out his response to this argument at *Qiyās* (Ibn Sīnā 1964, 126.5–127.9). He translates the possibilities into times (in effect as Themistius did), and he

distinguishes between the two propositions

(7)
(*d*) Every sometimes-*B* is always (i.e. at all times of its existence) an *A*.
(*l*) Every sometimes-*B* is an *A* at all times when it is a *B*.

(In *Maš*ri*qīyūn* (Ibn Sīnā 1910, 65.5f), Ibn Sīnā proposes to describe sentences of these two forms as respectively *ḍarūrī* and *lāzim*; I abbreviate these to (*d*) and (*l*), as in Hodges, in preparation.) Read in terms of times, the non-modal minor premise says 'Every sometimes-*C* is sometimes a *B*', and together with (7)(*d*) this yields that every sometimes-*C* is always an *A*, i.e. a temporally necessary conclusion. Ibn Sīnā (1964, 126.6–10) remarks that some people failed to reach a necessary conclusion because they read the major premise as (*l*) rather than (*d*); they reckoned that (*l*) expresses a genuine necessity (*ḥasabūhu ḍarūrīyan ḥaqīqīyan*). This is fair comment on Themistius's argument.

Ibn Sīnā also says, though not at *Qiyās* 126ff, that sentences of the form (*l*) are counted as 'necessary' for purposes of demonstration (*burhān*) but not for purposes of syllogism. He spells this out in *Najāt*:

> The necessary in this case [i.e. in a discussion of demonstration, the topic of this part of *Najāt*] is not the same as the necessary which was in the book *Syllogism*. In the present case necessary means what is predicated permanently on what is posited as subject ... for as long as it is described by the subject term ...
> (Ibn Sīnā 1945, 133.14–134.2)

In other words, the modality in (*l*) is demonstratively necessary but not syllogistically necessary. He repeats the point at *Burhān* (Ibn Sīnā 1956, 123.14–17). But the distinction between (*d*) and (*l*) is due to Ibn Sīnā and has not been found in al-Fārābī;[ix] so can this really be the distinction that *Harmonisation* claims is in al-Fārābī's *Long Commentary*?

There is a clue at Ibn Sīnā's *Najāt* (1945, 133.6–9), where Ibn Sīnā makes one of his few references to the *dictum de omni*. This is the same paragraph in which we saw him distinguishing between syllogistically necessary and demonstratively necessary. There is no clear intrinsic connection between this distinction and the *dictum*, so one wonders why Ibn Sīnā mentions them together. A conjectural but entirely plausible reason is as follows. We know from Passage C in the Appendix that al-Fārābī endorsed Aristotle's acceptance of the syllogism (2), in a context where al-Fārābī also spoke of the *dictum*. Al-Fārābī was presumably aware of Themistius's argument against this mood, and it would have been natural for him to add a remark about why he rejected Themistius's argument. He could have said for example that he thought Themistius had misunderstood what Aristotle meant by 'necessary'. A recollection of such a passage in al-Fārābī could explain why Ibn Sīnā in *Najāt* associated the *dictum* with the distinction between (*d*) and (*l*). This reconstruction is highly conjectural, but I do submit that it makes everything fit. If correct, it gives a vivid insight into how Ibn

Sīnā developed his views through close study of the writings of his predecessors.

In sum, Candidate Two gives us some hints about Ibn Sīnā's thinking, and a suggestion of how al-Fārābī might have accounted for Themistius's rejection of the mood (2). But it leaves Passage C as our best evidence of al-Fārābī's views on this and related moods. Here al-Fārābī accepts Aristotle's verdicts on these moods and presents arguments to justify those verdicts, based on possible readings of the sentence forms involved.

2.3. Candidate Three

In Candidate Three al-Fārābī makes a novel argument in favour of a rule of conversion stated by Aristotle. Ibn Sīnā and Ibn Rushd both report the argument and then go on to discuss it. Ibn Rushd's discussion runs through *Maqālāt* (Ibn Rushd 1983, 102.13–105.10), assessing not only al-Fārābī's argument but also Ibn Sīnā's response to it. (In *Maqālāt* (Ibn Rushd 1983) this is part of *Maqāla* 1.3.2; the *Maqāla* was translated into Latin as *Quaesitum 3 De conversionibus*.) Ibn Sīnā's (1964) most direct discussion of al-Fārābī's argument is at *Qiyās* iv.4, 208.8–211.17, but it should be read alongside two other passages in *Qiyās* that are clearly discussing related matters. These are *Qiyās* ii.3, 98.14–103.14 and *Qiyās* iii.2, 145.11–149.13.

Before we assess what is novel in Candidate Three, we need to be clear about what argument al-Fārābī is putting forward in it. The following reconstruction seems the most straightforward. Al-Fārābī is arguing for the inference

> Every *B* is contingently an *A*. Therefore some *A* is contingently a *B*.

His *first step* is to take the first of these two formal sentences and replace it by a material instance:

> (α) Every animal is contingently sleeping.

Ibn Sīnā identifies this move at *Qiyās* 210.1f, and objects to it on the grounds that a conversion that works for this material instance might not work for other material instances of the same formal sentence.

Al-Fārābī's *second step* is to paraphrase (α) as

> (β) Every animal is contingently sleeping because of (*min jiha*) what sleeping is.

Here al-Fārābī has introduced a compound predicate 'sleeping because of what sleeping is'.[x] Ibn Sīnā objects (*Qiyās* 210.4f, cf. *Qiyās* 148.5–8) that this paraphrase is inadmissible because it alters the meaning. Ibn Rushd begs to differ:

> … this condition ['because of what sleeping is'] is understood regardless of whether it is uttered or not uttered. The two propositions are one and the same—I mean the one in which this condition is uttered and that in which it is not.
> (Ibn Rushd 1983, 104.6f)

Al-Fārābī's *third step* is to ignore the 'contingently' in the result of the second step,

and apply categorical *a*-conversion to the remainder, reaching

(γ−) Something that is sleeping because of what sleeping is is an animal.

This is unproblematic. The *fourth step* is to restore 'contingently':

(γ) Something that is sleeping because of what sleeping is is contingently an animal.

According to Ibn Rushd's quotation, this fourth step is justified 'because its animality is not because of what sleeping is necessarily'. This justification is problematic, because the phrase 'because of what sleeping is' is in the first part of the sentence (γ−) where it has no clear connection with 'an animal' at the end. Probably al-Fārābī has silently re-parsed (γ−) into

(γ!) Something that is sleeping is, because of what sleeping is, an animal.

As al-Fārābī correctly says in Ibn Rushd's quotation, nothing has to be an animal because of what sleeping is. So this something is only contingently an animal because of what sleeping is.

One argument to support this reading of the fourth step is that Ibn Sīnā at *Qiyās* 100.13–101.10 appears to be attacking precisely this move to take the 'because of' phrase as part of the predicate after it has been stated as part of the subject.[xi]

The *fifth step* is the reverse of the second. Slipping the 'because of ...' phrase quietly back into the subject term, we paraphrase 'sleeping because of what sleeping is' as simply 'sleeping', to reach our destination:

Something that is sleeping is contingently an animal.

It remains only to claim that this material instance is typical, so that we have justified the formal conversion rule.

Looking back over this argument, we can see that al-Fārābī is attempting something that may well be original, but it is hardly a step towards constructing a new variety of modal logic. Rather al-Fārābī has taken a conversion rule from Aristotle, and is suggesting that the rule is verified when it is taken as applying to a particular kind of sentence. The novelty consists in applying an old rule to a new kind of sentence. In the next section we will pursue this viewpoint.

3. Finding interpretations to verify laws

One of the chief ingredients of Peripatetic modal logic is a set of modal rules for finding conversions and contradictory negations of single formulas, and conclusions from pairs of formulas. At least up to the time of al-Fārābī, Aristotle's *Prior Analytics* i.8–22 served as a database and reference point for these rules.

If Aristotle's modal rules are simply accepted as a datum, then what remains for a commentator to do with them? One task is the formal task of checking the relationships and interderivabilities between the various rules. To judge from what

other writers reported, al-Fārābī himself contributed virtually nothing new along these lines, at least in modal logic.

Another task is to establish, for each rule, what its range of application is within ordinary discourse.[xii] There is no doubt that Peripatetic commentators, even up to Ibn Sīnā, were prepared to take Aristotle's modal system and interpret it piecemeal, examining one inference rule at a time and trying it out on various interpretations of the sentence forms involved. Aristotle himself had given the green light to this kind of commentary in passages such as

> (8) One must choose what belongs to all not with a limitation of time such as 'now' or 'at such-and-such a time', but without qualification.
> (*Prior Analytics* i.15, 34b7f; trans. Striker in Aristotle 2009, 23)

Al-Fārābī must surely have given his reaction to (8) in his *Long Commentary*, but nobody found his reaction interesting enough to tell us what it was.

Al-Fārābī was also aware of a standard Peripatetic classification of senses of 'necessary':

> The commentators ... arrive at three kinds of necessity: <a state of affairs can be> necessary as long as its subject exists, necessary as long as it exists itself, and necessary without qualification.
> (Al-Fārābī 1986b, 95.20–23; trans. Zimmermann 1981, 90)

He will also have known that Themistius discussed these three readings of 'necessary' in his *Prior Analytics* commentary (Rosenberg & Manekin 1988, 94). If the speculations in Subsection 2.2 above are sound, al-Fārābī in his *Long Commentary* may well have made some reference to the three readings of 'necessary' in an attack on Themistius's treatment of the mood (2). But beyond that, we have no explicit evidence that al-Fārābī applied these readings of 'necessary' to syllogistic moods.

However, there are three cases in which we have more substantial evidence that al-Fārābī proposed unusual interpretations of Aristotle's modal inference rules so as to justify those rules.

The first case is his reading of 'Every *B* is possibly an *A*' as 'Everything that is or could be a *B* is possibly an *A*', which he uses to rescue Aristotle's modal mood (4). We studied this reading already under Candidate One in Subsection 2.1. For the present I confine myself to the question where the reading came from.

At *Prior Analytics* i.13, 32b27–29 Aristotle draws a distinction between two readings of possibility statements. In this case we need to translate both the Greek and the Arabic, because they say different things. Here is the Greek version:

> (α) Since 'It is possible that this holds of this' can be taken in two ways, either 'of what this holds of' or 'of what this possibly holds of',
> (β) —for '*A* possibly holds of what *B* holds of' means either 'what *B* is said of' or 'what it is possibly said of', for there is no difference between '*A* possibly holds of what *B* is said of' and '*A* possibly holds of all *B*' ...
> (33a25–30)

And here is the Arabic:

> (α) For since the sentence 'it is possible for the thing to hold of the thing' can hold in two ways, either that it holds of it or that it is possible for it to hold of it,
>
> (β) —for the sentence '*C* can be said of *B*' signifies one of these two things, either that it is said of it or that it is possibly said of it ...
>
> (Jabre 1999, 227.2–6)

The difference, crucial for us, is that in the Greek version of (α) but not in the Arabic, Aristotle allows the subject term to be modalized with 'possible' in the same way as predicate terms are modalized in alethic possibility sentences. Reading the Arabic, al-Fārābī was probably unaware that Aristotle had made this suggestion (otherwise he would surely have quoted it in Passages A–D).[xiii]

Alexander in his commentary doesn't pick up the suggestion that the subject term might carry the modality 'possibly'. But there is a passage in his *On Conversion* (Alexander of Aphrodisias 1971, 78.18–20), where he seems to argue for Aristotle's suggestion by applying a conversion. He notes that

> Every human is contingently a laugher.

and takes the predicate to be 'contingently a laugher'. Then conversion gives

> Something that is contingently a laugher is human.

Here we have contingency modifying the subject instead of the predicate. As Alexander (1971, 78.25–79.1) says, in such cases 'the contingent ... ' forms a subject term. Al-Fārābī may well have known this passage, since it anticipates his second step under Candidate Three in Subsection 2.3 above.

The second case is illustrated by the following passage from al-Fārābī's *Long Commentary*, quoted by Maimonides in his *Medical Aphorisms* (Schacht & Meyerhof 1937):

> (9) This chapter [*Prior Analytics* i.15 on possible plus *de inesse*] is extremely helpful, more helpful than [the previous chapter i.14 on] the pure possible syllogisms. This is so because all the practical arts use this chapter, especially when they ask whether particular events will or will not occur in the future, in connection with medicine or agriculture or navigation or civic management or rhetoric or (other) well-known things, or in connection with anything that involves a need to know facts in advance.
>
> (Schacht & Meyerhof 1937, 81.7–10)

Al-Fārābī must have in mind some reading of 'possible' which yields knowledge about the future. This is puzzling because al-Fārābī is not on record as suggesting any sense of 'possible' that guarantees knowledge about the future. In fact, his view (for example at (Al-Fārābī 1986c, 160.14–19; Zimmermann 1981, 245.12–19) seems to be that a

contingent statement about the future can never be known to be true until it actually becomes true (and hence no longer contingent).

More likely al-Fārābī in (9) is using 'know' loosely, and he has in mind not possibility but *probability*. He will have read Aristotle's remark (translated from the Arabic Aristotle):

> (10) 'Possible' is used in two ways. The first way is what is true in most cases but doesn't establish necessity, like a human becoming grey-haired or growing taller or shorter, and in general what happens in the course of nature, because it is not permanently necessary, in view of the fact that humans don't last for ever ....
> (*Prior Analytics* i.13, 32b3–9; Jabre 1999, 225.16–226.1)

Aristotle suggests two readings of 'possible' here: 'most cases' and 'what happens in the course of nature'.

Al-Fārābī picks up the notion of 'most cases' in a passage of his *Burhan* which Ibn Rushd quotes—see Passage E in the Appendix. He remarks there that the sentence '*B*s are *A*s in most cases' could be read in any of three ways:

1. Most *B*s are always *A*s.
2. Every *B* is most of the time an *A*.
3. Most *B*s are most of the time *A*s.

This is a sound analysis, and as far as I know it is new with al-Fārābī (see note xiii). But less happy is his suggestion, in the quotation from Maimonides, that Aristotle's syllogisms with one premise possible and the other *de inesse* are valid when the possible premise is taken in the sense of 'most'. For example, in the mood *Barbara* Aristotle accepts

> Every *C* is a *B*.
> Every *B* is possibly an *A*.
> Therefore every *C* is possibly an *A*.
> (*Prior Analytics* i.15, 33b25-28).

But the following is not valid:

> Every ostrich is a bird.
> Most birds fly.
> Therefore most ostriches fly.

Perhaps if we find al-Fārābī's missing commentary we will see that he is aware of similar counter-examples. But the tone of the Maimonides quotation hardly suggests this.[xiv, xv]

The third case is al-Fārābī's attempt to use the notion 'insofar as' to justify Aristotle's modal inference rules.

The English 'insofar as' represents three different Arabic phrases: *bi-mā, min jiha* and *min ḥaythu*. Ibn Sīnā in his discussion of al-Fārābī's argument switches to *bi-mā* at

Qiyās (Ibn Sīnā 1964, 210.16), and at *Qiyās* 100.13 he takes *min jiha* and *min ḥaythu* together. Possibly al-Fārābī in his *Long Commentary* gave further examples using *bi-mā* or *min ḥaythu*.

As far as we know, al-Fārābī was the first commentator to use any of these 'insofar as' phrases for interpreting modal inference rules.[xvi] In Candidate Three we examined a case where he tries to use *min jiha* to justify a conversion rule. But beyond Candidate Three, our only evidence on how far he took the matter is the discussions in Ibn Sīnā's *Qiyās* (Ibn Sīnā 1964, 98.14–103.14, 145.11–149.13 and 208.8–211.17) (as mentioned at the beginning of Subsection 2.3 above). We can't be sure where Ibn Sīnā in these discussions is developing ideas in al-Fārābī and where he is making his own explorations.

It's worth mentioning that at *Qiyās* 100.7 Ibn Sīnā tries to read 'human insofar as he is a writer' in terms of whether 'human' is a part of the definition of 'writer'. This could just possibly be an indication that al-Fārābī explored 'insofar as' as a way of introducing definitions or natures into logical rules. That would be interesting both because we saw Aristotle in (10) remarking that 'possible' can refer to what happens 'by nature', and because al-Fārābī in his *Burhān* (Al-Fārābī 1986e) does suggest incorporating 'definition' and other essentialist notions *explicitly* into logical rules (cf. Strobino 2018).

That's about as much as we know about al-Fārābī's use of 'insofar as' notions in logic. But there is more to say about his sources. Although no earlier Arabic writer is on record as using these notions in logic, all of them were used to some extent in philosophy generally. However, their widespread use in philosophy, and particularly that of *min ḥaythu*, seems to be due to al-Fārābī himself.

bi-mā means literally 'by what' or 'through what'. It appears once in this literal sense in the *First Philosophy* (al-Kindī 1978, 100.7f) of al-Kindī (c. 801–c. 873). A more abstract usage as in 'number as number' appears in a lemma of Aristotle's *Metaphysics* used by Ibn Rushd (1967-73, 325.1); this translation of the *Metaphysics* was reportedly commissioned by al-Kindī (Martini Bonadeo 2003). From al-Fārābī onwards *bi-mā* is found fairly often in the sense 'insofar as' and similar senses.

jiha means literally the direction in which one faces. Metaphorically it means an aspect or a point of view. *min* means 'from', so that *min jiha* is 'from the point of view ...'. In the standard Arabic text of *Prior Analytics* i the phrase *min jiha* appears twelve times, and in eight of them it translates Greek *dià* 'because'. In a ninth case it is followed by a noun and translates *parà tò* 'because of'. (These nine cases can be quoted in support of the translation 'because' in (3) above.) In the three remaining cases it means '(viewed, known, imagined) as'; in two of these cases it translates Greek *hḗi*. This last meaning of *min jiha* brings it close to the other phrases *min ḥaythu* and *bi-mā*. The phrase also appears a dozen times in al-Kindī's *First Philosophy* (1978), for example on p. 99: 'The door and the bed are one in matter [since they are both made of wood], but they are also many in terms of (*min jiha*) matter since their matter is multiple'.

ḥaythu and its compound *bi-ḥaythu* both originally meant 'where', either literally or

metaphorically, and this meaning still survives in Ibn Sīnā. So *min ḥaythu* should mean 'from where', but at some point the *min* came to lose its literal meaning. There is evidence that this happened early and in legal discussions, for example in the interpretation of *min ḥaythu* in the *Qur'ān* verse ii.222.[xvii] I have not seen the phrase anywhere in the Arabic Aristotle. The earliest occurrences of the phrase that I know in philosophical texts are two in al-Kindī's *First Philosophy* (1978), for example at 105.12 'It is not multiple insofar as (*min ḥaythu*) it exists, though it is multiple and not one without qualification'. So it exists in philosophical writing before al-Fārābī but is rare. But with al-Fārābī *min ḥaythu* becomes almost a trademark of his language; we find it 103 times in his *Ḥurūf* (Al-Fārābī 1990), 37 times in his abridgment of *Maqūlāt* (Al-Fārābī 1986a) and 26 times in his *Jadal* (Al-Fārābī 1986f).

So it seems very likely that al-Fārābī was the main person responsible for making *min ḥaythu* part of the stock-in-trade of philosophical writing, and that his influence encouraged the use of *bi-mā* and *min jiha*.

4. Al-Fārābī's dictum de omni

In our discussion of Candidate Two above, we saw al-Fārābī invoking something called the *dictum de omni*. We will see that al-Fārābī's use of the *dictum* had a pervasive influence on Ibn Sīnā's modal logic, reaching far beyond the places where Ibn Sīnā explicitly mentions the *dictum*.

4.1. The *dictum* from Aristotle to Alexander

For reference below, we give a short summary of the contents of Aristotle's description of syllogisms in *Prior Analytics* i, as they would have appeared to the Arabic logicians:

> **i.1, 24a1–24b30** Definition of syllogism and its parts, finishing with explanation of universal quantification.
> **i.2, 25a1–26** *De inesse*, necessary and possible propositions introduced.
> **i.3, 25a27–25b25** Conversions.
> **i.4–7, 25b26–29b28** Categorical syllogisms, i.e. with both premises *de inesse*.
> **i.8–12, 29b29–32a16** Syllogisms with premises *de inesse* or necessary.
> **i.13–22, 32a16–40b16** Syllogisms with a possible premise.

In several places within these sections Aristotle uses a phrase meaning 'predicating of all'. Thus we have:

Prior Analytics	**Aristotle**	**Arabic Aristotle**
i.1, 24b28f	*tò katà pantòs katēgoreîsthai*	*maqūl ʿalā al-kulli*
i.4, 25b37–39	*tò katà pantòs katēgoreîsthai*	*maqūl ʿalā kulli*
i.4, 26b9	*toû pantòs katēgoreîtai*	*maqūl ʿalā kulli*
i.5, 26b34f	*tō*$_{\text{i}}$ *panti hupárkhē*$_{\text{i}}$	*maqūl ʿalā kulli*

Prior Analytics	**Aristotle**	**Arabic Aristotle**
i.8, 30a3	*tò katà pantòs*	*maqūl ʿalā al-kulli*

A Latin expression that covers all these phrases is *dictum de omni*. On the face of it, Aristotle uses these phrases simply as a way of referring to universally quantified statements. By contrast Alexander, al-Fārābī and Ibn Rushd all read him as referring to a particular device of modal logic which is used for determining logical properties of modalized premise-pairs in the mood *Barbara*.

Alexander, al-Fārābī and Ibn Sīnā (in his *Awsaṭ* and *Qiyās*) worked directly from the text of Aristotle, so that the placings of Aristotle's mentions of the *dictum* are likely to have affected how these later writers understood them. For example the first mention of the *dictum* comes in the middle of Aristotle's explanation of the types of sentence used in syllogisms; we will see in Section 5 below how this seems to have influenced Ibn Sīnā's use of the *dictum* when he began to introduce his own temporal sentences in his *Awsaṭ*.

Also the last mention is the only explicit reference to the *dictum* in the section of Aristotle's text that refers to modal syllogisms (*Prior Analytics* i.8–22). But it occurs right at the beginning of the section, in a position where readers could take it as applying to the whole section. Alexander took it this way, and applied the *dictum* on Aristotle's behalf in the following places:

Modes	**Aristotle**	**Alexander applies *dictum***
de inesse, necessary	i.9	125.34–126.8
possible, possible	i.14	167.14–20
de inesse, possible	i.15	174.3–28
possible, necessary	i.16	208.2–6, 209.35–210.8

Alexander's view (Alexander of Aphrodisias 1883, 174.20–28), for example, is that for a pair of modal premises, being able to derive a conclusion from them by the *dictum* is a necessary and sufficient condition for their being perfect, but not a necessary condition for them to have a conclusion.

What does Alexander count as using the *dictum de omni*? About this we can deduce two things from his text. First, Alexander of Aphrodisias (1883, 167.17) spells out that what is applied is the 'definition' (*horismós*) of the *dictum de omni*. Mueller in his translations (Alexander of Aphrodisias 1999 and 2013) reasonably takes this as a cue to expand some other mentions of the *dictum de omni* to 'the definition of the *dictum de omni*' (Alexander of Aphrodisias 1883, 126.1, 174.8 and 22, 208.5). On this reading the *dictum de omni* itself would simply be one or both of the universally quantified premises, and its 'definition' would be an explanation of its meaning. In effect, Alexander is saying that a premise-pair in mood *Barbara* is perfect if and only if we can pass to a conclusion immediately from an understanding of the meaning of one or both of the premises.

Second, we can infer from the detailed examples (Alexander of Aphrodisias 1883, 167.14–20) that Alexander is applying the *dictum* to give the meaning of the *major* premise. In particular suppose the major premise is 'Every *B* is an *A*' (maybe with some modality). Then the *dictum* is being used to determine exactly which things the major premise is telling us are *A*s (with the relevant modality).[xviii]

Turning to al-Fārābī, a phrase that he uses several times in Passages A–C (Ibn Rushd 1983, 128.1, 129.5, 129.9, 130.14, 130.18, 131.1) is 'the meaning of the *dictum de omni*'. Thus the *dictum* is used through its meaning—just as Alexander said it was used through its definition. On the other hand Ibn Rushd (1983, 154.9f; *Quaesitum* 4 in the Latin numbering) says that according to al-Fārābī, the sense (*mafhūm*) of the *dictum de omni* is that it is 'a meaning added to the sense of the universal premise'. So al-Fārābī says that the *dictum* is used through its meaning, and Ibn Rushd reports him as saying that the *dictum* itself is a meaning. I think there is no real discrepancy here: al-Fārābī's view is that the function of the *dictum* is to specify the meanings of sentences of the form 'Every *B* is an *A*', so 'the meaning of the *dictum*' is shorthand for 'the meaning specified by the *dictum*'.[xix]

We are left wondering whether the *dictum* is just a piece of obfuscation. Why not simply say that 'Every *B* is an *A*' means such-and-such? Why mention a *dictum*? In the case of al-Fārābī there is a good answer that we can give on his behalf.

As we saw in Section 3, al-Fārābī is inviting his readers to try out various new interpretations of the sentences of modal logic, with a view to applying the inference rules to them. So he needs to have a way of specifying the meanings of sentences. In fact it becomes clear quickly that he will need to have a way of specifying a *family* of meanings when he is considering a family of sentences. It would be prudent for him to have a consistent and deliberate procedure for doing this. The name *dictum* is a handle for referring to this procedure.

At this point we should look at some examples. The first three examples below are from Passages A–D, and the fourth is from a later *Maqāla*. All are phrased so as to explain the meaning of 'Every *B* is an *A*'.

(11) [The *dictum de omni* stipulates that:] *A* is made a predicate, either affirmatively or negatively and with whatever modality the predication happens to be made, of everything that is described as a *B* in such a way that *B* is affirmed as actually true of this thing.
(Ibn Rushd 1983, 128.5–7)

(12) [The meaning of the *dictum de omni* is that:] *A* is made a predicate, either affirmatively or negatively, and with whatever modality of predication it is—of things that are actual *B*s, and also of everything *X* such that every *X* is described as being an actual *B*.
(Ibn Rushd 1983, 129.6f)

(13) [The *dictum de omni* states that:] *A* is predicated affirmatively or negatively, with whatever modality it happens to be of the modalities of

> predication, of all of what is posited for *B*, and is described as a *B* by an affirmation only, with whatever modality it happens to be of the modalities of predication.
> (Ibn Rushd 1983, 133.2–4) 19

(14) [The *dictum de omni* is that:] The predicate extreme *A* is affirmed or denied, either as *de inesse* or as necessary or as possible, of everything that is an actual or possible or necessary *B*.
(Ibn Rushd 1983, 146.9f)

Ibn Rushd ascribes all four of these statements to al-Fārābī, though some are al-Fārābī reporting Alexander. The four statements have enough differences to make it likely that they are in fact four distinct pieces of al-Fārābī's text.

The four statements all have a similar form. In a moment we will extract this common form. But before we do that, we need to digress briefly on Peripatetic explanations of meanings. There are two main points to be made here.

First, when Peripatetic philosophers wanted to pin down the meaning of a phrase, they would sometimes take the phrase and add other words around it until the meaning was unambiguous. We can call this the 'Ajax-Ajax' method in the light of a standard example, which was that we disambiguate 'Ajax' by expanding it to 'Ajax the son of Oeleus of Locris' (Porphyry 1992, 44, 64.10–21). The method lends itself to explaining things in conversation, where the other person can say 'Got it, thanks' or 'Could you explain it a bit more, please?'. But in a book the author has to guess what is the right amount to add in order to make the meaning clear, and the result is that different explanations of the same meaning, even in the same book, tend to be fuller or thinner by chance.

The second point is that when explaining the meaning of a sentence S, Peripatetic philosophers tended to describe what is going on in the mind of a person who is (either aloud or purely mentally) intending S. So for example a person who intends 'This is a horse' mentally describes the relevant thing as a horse, and as a result the explanation of the meaning contains the phrase 'is described as a horse'. This style of explanation can come into conflict with the alternative style (which we also meet in Peripatetic philosophers) of describing the conditions under which the sentence S counts as true. For example the truth conditions for 'This is a horse' don't include anything about anybody making a description.

But the mentalistic style of explanation can have advantages. For example when describing the state of mind of a person intending S, we can add negative information about what this person is *not* intending, so as to say that some possibilities are left open by S. This can be trickier with the truth-conditional style, because a negative statement there is easily misunderstood as saying that the sentence S rules out certain possibilities.

Here is a diagram of the pattern common to all the four statements (10)–(13):

Everything that is said to	be not be	actually necessarily possibly	a *B*
is said to	be not be	*de inesse* necessarily possibly	an *A*

1. The phrases 'described as', 'affirmed as' and 'made a predicate' in the statements belong to the mentalistic style, and they show up in the diagram as 'is said to'.

2. The second 'be/not be' describes whether the predicate *A* is affirmed or denied. The first 'be/not be' makes a parallel distinction for the subject, namely whether the subject carries a negation or not. In fact the standard assumption in Aristotelian logic is that the subject term is not negated; so (13), which spells out that *B* is affirmative, is expressing the default position. Note that none of (11), (12) and (14) say that *B* is affirmative; they leave the default unstated. Differences of this kind between Ajax-Ajax definitions, about what to mention and what to leave assumed, are almost the norm.

However, we are right to allow the subject to be negated in al-Fārābī's *dictum*, given what he says about it elsewhere. Thus:

> Abū Naṣr [al-Fārābī] thought that the meaning of the *dictum de omni* makes some pairs of negative premises productive, and also makes a negative minor premise and an affirmative major premise productive. For example 'What is not an animal is not a human' combined with 'The statue is not an animal' yields that the statue is not a human. Or: What is not an animal is an inanimate object, and statues are not animals, so they are inanimate objects. (Ibn Rushd 1983, 154.17–24)

Al-Fārābī uses a negated subject term also in *Taḥlil* (Al-Fārābī 1986g, 114.11f), see Fallahi (2019).

3. In (13) al-Fārābī refers twice to 'the modalities of predication', once in connection with the predicate and once in connection with the subject. Here he is pointing out that his suggestion that Aristotle intended the modality 'possibly' on the subject allows us to think of the subject and predicate as equally open to being modalized. Strictly he uses 'actual' (*bil-fi'l*) on the subject to correspond to *de inesse* (*wujūdī*) on the predicate, but this is permissible.

By allowing both the subject term and the predicate term to carry either of two qualities and any of three modalities, al-Fārābī has given us a recipe for describing the meanings of as many as thirty-six (2×3×2×3) universally quantified sentence forms. This is many more than the six explicitly allowed by Aristotle's modal theory. It was also many more than al-Fārābī himself was equipped to handle. We have no record that he ever exploited the idea of putting 'necessary' on the subject term.

4. Note the phrase 'with whatever modality' (*bi-ayyi jihatin*) in (11), (12) and (13). The corresponding phrase in (14) is a simple disjunction over the three modalities. I read al-Fārābī as saying that every sentence captured by these forms of the *dictum* will have (perhaps only implicitly) just one of the three modalities at this part of its meaning. Here we see al-Fārābī using the *dictum* to describe a range of sentence meanings, not just a single one.

5. A puzzling phrase is 'posited for *B*' (*yūḍā'u li-b*) in (13), which Ibn Rushd (1983, 133.16 and 134.20) repeats. In Arabic the word for 'subject' means 'thing posited' (*mawḍū'*). This and the context seem to indicate that the phrase 'posited for *B*' is intended to mean 'described as a *B*, where *B* is the subject term'. If so then we lose nothing by omitting it; but I think we may be missing something here.

6. The last phrase in (13), 'everything *X* such that every *X* is described as being an actual *B*', is unexplained. Perhaps it is designed to allow what note xviii below calls the term reading.

5. Ibn Sīnā assembles his sentence forms

To place Ibn Sīnā's reaction to the *dictum de omni* in context, we need a short detour on the development of Ibn Sīnā's modal logic.

5.1. Ibn Sīnā's sextet

Ibn Sīnā gives accounts of aspects of modal logic in seven of his surviving works, in probable chronological order *'Uyūn al-Ḥikma* (Ibn Sīnā 1954), *Awsaṭ* (Ibn Sīnā 2018), *Najāt* (Ibn Sīnā 1945/2011), *Hidāya* (Ibn Sīnā 1974), *Qiyās* (Ibn Sīnā 1964), *Mašriqiyūn* (Ibn Sīnā 1910) and *Išārāt* (Ibn Sīnā 2000/1984). Two of these we will leave on one side: *'Uyūn* represents a very underdeveloped stage of Ibn Sīnā's modal logic where the only modalities are Aristotle's alethic modalities, and the modal material in *Hidāya* is too brief to be useful to us.

That leaves five works. In all five of these works Ibn Sīnā presents a list of ways of reading 'Every *B* is an *A*' using temporal notions so as to express some kind of necessity. The list is virtually the same in all five works, and normally consists of six items; accordingly I will call it the 'sextet'. A sixth version of the sextet appears in Ibn Sīnā's *Burhān* (Ibn Sīnā 1956). The six passages are:

Awsaṭ 105.6–107.3,
Najāt 35.2–36.13,
Qiyās 31.14—33.10,
Burhān 120.15–121.23,
Mašriqiyūn 68.6–20,
Išārāt 88.9–89.11,

The first of these works, *Awsaṭ*, must be significantly earlier than the logic part of *Najāt*, which is dated to 1013. The last work, *Išārāt*, comes from around 1030. It's striking that the sextet is a fixed feature through such a long span of Ibn Sīnā's career

and throughout his maturity.

Closer examination shows that in different works Ibn Sīnā puts the sextet in different positions relative to the contents of the *Prior Analytics*. In *Awsaṭ* he puts it alongside the end of *Prior Analytics* i.1, whereas in *Qiyās* he moves it forwards to the discussion of modal syllogisms. In *Najāt* he sends it back to the discussion of modalities in the *De Interpretatione*. All of this indicates that the sextet is driving Ibn Sīnā's move from Aristotle's view of modal logic to his own view. Certainly any account of the development of his modal logic will need to give it a central place.

Let me give such an account; it will have to be brief and short on references. (A fuller account is in preparation in Hodges (in preparation). Meanwhile two recent works Chatti (2019) and Nusseibeh (2018) cover important parts of the story.)

Thanks to the full treatment in *Qiyās* i, we know where the sextet came from. It was the result of Ibn Sīnā's efforts to remove ambiguities from an earlier list of three sentences which Ibn Sīnā calls the 'trio', *tathlīth*. The three sentences appear in Themistius's *Commentary on the Prior Analytics* (Rosenberg & Manekin 1988), and can be traced back to the commentary of Theophrastus on the *Prior Analytics*. As Themistius reports them the trio are:

1. Every *B* is an *A* at all times.
2. Every *B* is an *A* at all times when *B* exists.
3. Every *B* is an *A* at all times when *A* exists.

(Compare al-Fārābī's versions, namely Al-Fārābī (1986b, 95.20–23) and Al-Fārābī (1986c, 162.15–19), which we have already mentioned.) Ibn Sīnā took 1 as unambiguous, but he split 2 into two cases:

2.1 = (*d*). Every *B* is an *A* for as long as it exists.
2.2 = (*l*). Every *B* is an *A* for as long as it is a *B*.

Here (*d*) and (*l*) are as in Subsection 2.3 above, and the discussion in that subsection illustrates how Ibn Sīnā came to the conclusion that this disambiguation was needed. He split 3 into three cases 3.1, 3.2, 3.3 which need not concern us here. The six forms 1, 2.1, 2.2, 3.1, 3.2, 3.3 constitute the sextet.

Reading through the works of Ibn Sīnā listed above, we can see Ibn Sīnā's temporal logic gradually taking shape as it evolved out of the sextet. There were two main processes involved.

The first process was to eliminate sentence forms that were inappropriate for the logic. Ibn Sīnā eliminated all but (*d*) and (*l*), giving different reasons in different cases. He discarded 1 because it could be regarded as the special case of 2.1 where all *B*s are eternal. He discarded 3.1 because he regarded it as a useless tautology, and 3.2 and 3.3 probably because they mixed temporal and alethic modalities.

The second process was to close off the list under operations that would be needed in logic: passing from universal to existential and from affirmative to negative, and forming contradictory negations. Ibn Sīnā carried out these expansions mainly in *Qiyās* i and *Mašriqiyūn*. They were required because the sextet still maintained the

original viewpoint of Theophrastus, that the sentences being classified were versions of 'Every *B* is an *A*' (which is universal affirmative).

Side by side with these developments from the Theophrastan trio, Ibn Sīnā carried out similar refinements of proposals by Theophrastus and Alexander for reading 'Every *B* is an *A*' as a non-modal or *de inesse* proposition. These led to modalities which Ibn Sīnā named 'broad absolute' and 'narrow absolute'.

At the end of this process, Ibn Sīnā found himself working with a modal system that contained eight modalities: Aristotle's three alethic modalities 'necessary', 'possible' and 'contingent', and five temporal modalities ((*d*), (*l*) and three others[xx]). As he was putting this system into place, he needed to specify the meanings of the sentence forms that he was proposing. At this point he chose to follow al-Fārābī's version of the *dictum de omni*, as we will see below.

5.2. Ibn Sīnā's semantic expansions

Ibn Sīnā's works contain some two dozen places in which he uses a stylized explanation of the meaning of a modalized formal sentence of the form 'Every *B* ...'. Here are three typical examples:

(15) *Awsaṭ* 103.12–16: Those things that are described, in act in the intellect, as being a *B*, in any way and at any time, given that they are described as being a *B*, no such thing is described as being an *A*, but we don't know whether it is [not] an *A* at a particular time or permanently, regardless of whether it is together with this description of it at some specific time or at all times and that time together with its being a *B* permanently, or together with its being a *B* at some time of its being a *B*, or before that or after that.

(16) *Qiyās* 26.18–27.2: Each one of the things described and assumed to be actually (*bil-fi'l*) a *B*, permanently or not permanently, in fact is also described as being an *A* without referring to when, and to which of the three segments of time it is in.

(17) *Išārāt* 93.9–14: Each one of the things that are described as a *C*, regardless of whether it is described as a *C* in mental assumption or in the world, and regardless of whether it is so described permanently or non-permanently, but rather however it is (*kayfa kāna*), is a thing described as a *B* without any addition that it is so described at such-and-such a time or in such-and-such a case.

I will refer to passages of this type, using a similar format to explain the meaning of a modalized sentence form, as 'semantic expansions'. They are clearly Ajax-Ajax definitions; they explain the meaning of a sentence by adding pieces to the sentence. In fact they mostly look like al-Fārābī's *dictum de omni* with further components added. Many of these added components have the form 'regardless of whether ...' or 'without saying whether ...' or something equivalent.

Here is a table showing twenty-three such passages. For each passage, I indicate in the second column which modality of sentences it is supposed to be defining. The components recorded in the other columns will be explained below.

passage	**mode**	**actual**	**permanent**	**however**	**mind**	**exist**	**assume**
AWS 100.16–19	-	Y	N	N	Y	N	Y
AWS 100.19–101.1	-	Y	N	N	Y	N	Y
AWS 102.17–19	-	Y	N	N	Y	N	N
AWS 103.7–11	-	Y	Y	N	N	N	N
AWS 103.12–16	-	Y	Y	Y	Y	N	N
AWS 108.7–11	(*t*)	Y	N	N	N	N	N
AWS 108.21f	(*d*)	N	Y	Y	N	N	N
AWS 131.2–6	*nec*	N	Y	Y	N	N	Y
AWS 132.10–12	-	N	N	Y	N	N	Y
NAJ 42.10–12	(*d*)	N	Y	N	Y	N	N
NAJ 43.12–14	*pos*	N	N	Y	N	N	Y
NAJ 66.6–9	(*d*)	N	Y	N	N	N	Y
QIY 26.18–27.2	-	Y	Y	N	N	N	Y
QIY 31.15–32.1	(*d*)	Y	Y	N	N	N	N
QIY 33.11–14	*con*	N	Y	N	N	N	N
QIY 33.15–34.6	*pos3*	Y	Y	N	N	N	N
QIY 127.11f	*nec*	N	Y	N	N	N	N
MAS 64.3–6	-	Y	Y	N	N	Y	Y
MAS 64.14–16	-	Y	N	N	N	N	N
MAS 68.6–8	(*d*)	Y	N	N	N	N	N
MAS 69.12–14	(*d*)	N	N	Y	N	N	N
ISH 72.17–73.3	-	N	N	N	Y	Y	Y
ISH 93.9–14	(*t*)	N	Y	Y	Y	N	Y

Ibn Sīnā himself describes a few of these semantic expansions as *dictum de omni*—for example those at *Awsaṭ* 132.10–12 and *Qiyās* 127.11f. Perhaps more telling, the large cluster of semantic expansions in *Awsaṭ* 100–103 fall in a place corresponding to Aristotle's listing (at *Prior Analytics* i.1 end to i.2 beginning) of the sentence forms that occur in syllogisms; this is where Aristotle first mentions the *dictum*. *Awsaṭ* is the first work in which Ibn Sīnā introduces his new temporal forms, and he chooses the format of the *dictum* to explain what they are.

Ibn Sīnā uses the *dictum* format for explaining the meanings of various different modalities, including the temporal (*d*) and (*t*) and the alethic necessary, possible and contingent, as shown in the second column of the table. Where the mode is left blank in this column, generally Ibn Sīnā is using the *dictum* to introduce universal predicative sentences in general. The modality *pos3* will be explained below.

We turn to the six columns recording that various components are present in or

absent from the semantic expansion. Every one of these columns contains both Y and N, in fact the same mode can be listed once with Y and once with N in the same column. For example the explanations of (*t*) at *Awsaṭ* 108.7–11 and *Išārāt* 93.9–14 make almost opposite choices of Y and N. We have seen that this kind of variation is typical of Ajax-Ajax definitions.

One conspicuous feature of al-Fārābī's formulations of the *dictum* is that he splits the sentence into two parts, one associated with the subject and containing the quantifier, and the other associated with the predicate and containing the sentence modality. Ibn Sīnā takes over this feature and lets it organize his presentation. Thus each sentence has two 'sides' (*jānib*), one for the subject and one for the predicate. He normally takes the subject side first, then the predicate side. Thus at *Awsaṭ* 102.12 he records that he is moving to 'the side of the predicate'; at *Qiyās* 21.12 he says he has finished with 'the subject side'. At *Mašriqīyūn* 64 he discusses first the subject of the predicative proposition, then the predicate.

The predicate side of Ibn Sīnā's *dictum* does the same as al-Fārābī's: it records the modality of the sentence and may also distinguish affirmative from negative. For example both the explanations of the necessary sentence, at *Awsaṭ* 131.2– 6 and at *Qiyās* 127.11f, duly record that the predicate is said necessarily of the subject. Since this part of the *dictum* is very predictable I didn't put a column for it into the table above.

The situation changes completely for the subject side of the *dictum*. Here al-Fārābī's main concern was to record that the subject term can carry either of the modalities 'actual' and 'possible'; we will see that Ibn Sīnā disagrees with al-Fārābī over this. But also Ibn Sīnā has a number of other things to say about the subject side. The columns in the table record some of them, and I comment on these below.

The third column records whether or not the *dictum* says that in a universally quantified sentence 'Every *B* ...' the quantification is over those things that are actual *B*s. Recall that for al-Fārābī, some of Aristotle's modal moods indicate that Aristotle was quantifying over both actual and possible *B*s. A Y in this column means that Ibn Sīnā disagrees: he says explicitly that the things quantified over are actual *B*s. But an N need not mean that the sentence being defined quantifies also over things that are not actual *B*s. In fact for (*t*) sentences Ibn Sīnā has a Y at *Awsaṭ* 108.7–11 and an N at *Išārāt* 93.9–14; there is a similar variation for (*d*). On this evidence, the variation is simply that of Ajax-Ajax definitions: sometimes Ibn Sīnā mentions that the quantification is over actual *B*s, but sometimes he doesn't bother. His further remarks about quantification confirm this interpretation as far as temporal sentences are concerned, but I won't pursue this here.

The keen-eyed reader may notice that the table contains five definitions of alethic modalities, and that all but one of these have an N in the 'actual' column. The exception is the modality *pos3*, which is in fact a temporal modality: if a thing is an *A* with modality *pos3* then it will be an *A* at some time in the future (*Qiyās* 34.1). This leaves it open to argue that Ibn Sīnā allows some kind of ampliation of the subject for

sentences with pure alethic modalities. I think myself that this would be an oversimplification. Ibn Sīnā consistently says he doesn't ampliate the subject, without ever giving any hint of any exceptions. But the question needs more analysis than we have space for here.[xxi]

The fourth column records a Y if the semantic expansion says that the quantification is over individuals that are *B*s 'either permanently or non-permanently'. The variation of Ys and Ns, even for sentences with the same modality, is just as great as for 'actual', and this time there are no exceptions for the pure alethics. So the default assumption is that Ibn Sīnā intends that all of these sentences quantify over things that are *B*s regardless of whether those things are *B*s permanently or only temporarily. More precisely, 'Every *B* ...' quantifies over everything that is a *B* at least once, regardless of whether it is permanently a *B*.

Ibn Sīnā does in some places (for example *Qiyās* i.4, 29.6f) consider sentences of the form 'Everything that is a *B* at time *t* is an *A* at time *t'*'. These sentences are not covered by any of Ibn Sīnā's semantic expansions; but neither are they used in his logical system, for reasons that he sets out at *Qiyās* iii.1, 134.4–136.7.

The fifth column records a Y if the semantic expansion says that the things quantified over are the *B*s 'however they are' (*kayfa kāna*) or 'however they are described' (*kayfa wuṣifa(t)*). Ibn Sīnā leaves some clues about what he has in mind here. For example the phrase 'however it is described' at *Najāt* 66.10 must be a reference back to 'either necessarily or not necessarily, and either permanently or not permanently' at *Najāt* 66.8. So the purpose of the phrase here is to repeat the point that things that are sometimes *B*s are included in the quantification regardless of whether they are *B*s necessarily or permanently.

The sixth column records a Y if the semantic expansion says that the things quantified over are those that are described as *B*s 'in the mind' (or the intellect), or 'either in the mind or in the world'. There are seven such cases, but I think they are not homogeneous. In the five cases in *Awsaṭ* and *Najāt* there is no disjunction 'either in the mind or in the world'. In these cases Ibn Sīnā is probably pointing out that the proposition is a mental entity, so that anything included in it has to be a mental entity too. This view is problematic—even though every individual has its individual essence, it is hard to see how the individual essence of every flower in the world enters my mind if I intend a sentence about all flowers. That or some related worry might be why this particular locution disappears from the semantic expansions after *Najāt*.

The locution that appears in the two cases in *Išārāt* is different: 'either in the mind or in the world'. After writing *Najāt*, Ibn Sīnā has had time to reflect on the sense in which the individuals quantified over in logic are in the mind or in the world. So we find him in *Qiyās* 21.6 maintaining that some mental objects, for example mathematical constructions, should count as 'actual' just as much as any real-world objects: 'this actuality is not just the kind of actual existence that material things have'—and he goes on to cite icosahedra. This is likely to be what he has in mind at *Išārāt* 93.9–14.

But this explanation hardly fits *Išārāt* 72.17–73.3, where the thing in our mind is a human being, and Ibn Sīnā goes on to say that it could exist or not exist in the real world. In fact this is one of the two cases with a Y in the seventh column, meaning that the subject side of the semantic explanation includes 'whether or not it exists'. The other case is *Mašriqīyūn* 64.3–6, where the quantification is said to be over actual *B*s; it seems that some things can be actual *B*s but nonexistent. One possible explanation of this case is that Ibn Sīnā has confused definition by truth- conditions with mentalistic definition: for the sentence to be true, all actual *B*s must be *A*s, but I might believe that all *B*s are *A*s without having correct beliefs about what *B*s exist.

The eighth and last column of the table records with a Y those semantic expansions that require that the individuals quantified over are 'assumed' (*furiḍa*) or 'posited' (*wuḍi'a*) to be *B*s, where this seems to be a further condition over and above their being 'described as' *B*s. I can't explain what this column is for, but I suspect there is some connection with the occurrence of 'posited' in some of al-Fārābī's statements of the *dictum*, as noted near the end of Section 4 above.

We could have added other columns, for example to record cases where Ibn Sīnā includes a clause 'given that ...'. There is clearly a complex theory of the subject side behind these semantic expansions, ranging from logic to ontology.

6. *Harmonisation* again

The book *Harmonisation* (Al-Fārābī 1999) presents itself as having been written by al-Fārābī. At 93.2f the author claims to have written a commentary on the *Prior Analytics*, and his statement about the content of this commentary consists mostly of things that we know were in al-Fārābī's *Long Commentary*. Also the overall philosophy has much in common with al-Fārābī's.

But Lameer (1994, 30–39) and Marwan Rashed (2009) have argued that *Harmonisation* could not have been written by al-Fārābī. Rashed's evidence turns on points of general philosophy that hardly touch our concerns in this paper. Lameer's (1994, 36) evidence does mention a logical item, but unfortunately it is his weakest argument. He holds, reasonably, that al-Fārābī would not have made the confusion between major and minor premises that appears at *Harmonisation* (Al-Fārābī 1999) 89.1–3. But the author of *Harmonisation* doesn't make this confusion. It is part of a claim (*zaʿamū*) that he ascribes to some earlier commentators, namely that certain syllogisms occur in Plato. True, the author doesn't point out their error; but he may have decided not to bother with it because he has some more global condemnations to make of these people (for example that the syllogisms they ascribe to Plato are mostly second-figure with two affirmative premises (Al-Fārābī 1999, 91.10–12), a cardinal error of logic).

Nevertheless some other facts that have come to light in the present paper support a conclusion that *Harmonisation* was written at least a generation after al-Fārābī's death. (But not much later, since in the early 11th century Ibn Sīnā refers to the work as a work of al-Fārābī, cf. (Bīrūnī and Ibn Sīnā 1973, 40.12f). The first such

piece of evidence is the remark at *Harmonisation* (Al-Fārābī 1999, 79.7f) that

> (18) The scope of philosophy is measured by the expressions *min ḥaythu* and *min jiha*. As has been said, if one removed *min ḥaythu* and *min jiha* then the sciences and philosophy would be nullified.

In Section 3 above we saw evidence that before al-Fārābī, *min ḥaythu* was rarely used in philosophy. So the remark (18) can hardly have come from al-Fārābī himself, and is unlikely to have been made before al-Fārābī's work had become widely influential. This places it no earlier than the late 10th century AD.

A second consideration is that when the author of *Harmonisation* mentions the *Long Commentary*, the only items in it that he calls attention to are all in the section on modal logic. This would be natural if the author shared the perspective of Ibn Sīnā, who was a systematizing formal logician. But there is no evidence that al-Fārābī had this perspective. One indication that he didn't is the contents of his *Qiyas* (Al-Fārābī 1986d), an abridgment of the *Prior Analytics*. The work has slightly different contents in different manuscripts; the table below is based on the contents of the chief manuscript used by 'Ajam in his edition (Al-Fārābī 1986d) of *Qiyās*.

Al-Fārābī *Qiyās*	**Aristotle**
1–5. Propositions–Opposites	*De Interpretatione*
6–13. Conversion–Categorical syllogisms	*Prior Analytics* i.1–7
14. Hypothetical syllogisms	*Prior Analytics* i.25
15. Syllogism of absurdity	*Prior Analytics* i.24
16–20. Induction–Jurisprudence	*Prior Analytics* ii.23,24

Sections 1–13 cover material in *De Interpretatione* and in the first part of *Prior Analytics* i on categorical syllogisms, and they take up about 20 pages in 'Ajam's edition. Sections 14–20 take up about 34 pages, and they relate to items in *Prior Analytics* i and *Prior Analytics* ii, all of it later than the part of *Prior Analytics* i that covers modal syllogisms.

Thus al-Fārābī in his *Qiyās* (Al-Fārābī 1986d) includes no section devoted to modal logic. This is in spite of the fact that almost two-thirds of the book relates to material after the categorical syllogisms. To put it bluntly, al-Fārābī in his abridgment of the *Prior Analytics* ignores the part of the *Prior Analytics* that the author of *Harmonisation* thought was the most important part to mention.

In principle this could be an indication that al-Fārābī changed his mind about the significance of modal logic sometime late in his career. But no other surviving texts of his suggest such a change in his views. It remains unclear why the author of *Harmonisation* thought the modal syllogism (2) was worth mentioning.

Appendix: Al-Fārābī in Ibn Rushd *Maqāla* 1.3.5

The five passages A–E in this Appendix are all translated from al-ʿAlawī's edition (Ibn Rushd 1983) of the *Maqālāt* (Latin *Quaesita*) of Ibn Rushd. In al-ʿAlawī's numbering, passages A–D come from *Maqāla* 1.3.5, which is not known in Latin translation. Passage E comes from *Maqāla* 1.3.4 (*Quaesitum* 2 in the Latin numbering). It is a paraphrase of a paragraph from al-Fārābī's *Burhān* (Al-Fārābī 1986e), and it is included here alongside a translation of al-Fārābī's text in order to illustrate Ibn Rushd's style of paraphrasing. Passage A has a partial doublet at *Maqāla* 1.3.6 (*Quaesitum* 6; Ibn Rushd 1983, 146.4–11).

Throughout these passages, 'this book' is Aristotle's *Prior Analytics* and Abū Naṣr is al-Fārābī.

PASSAGE A

Alexander, as Abū Naṣr reports him, answered ... as follows.

> "Speaking about syllogistic moods where the premises are mixed, one being necessary and the other *de inesse*, the Philosopher just said that the modality of the conclusion follows /128/ [that of] the major premise, and that this is a consequence of the meaning of the *dictum de omni* which is used as a condition in this book. Also he said about [syllogisms that are] mixes of a possible premise and a necessary one, or a possible premise and a *de inesse* one, that when the minor premise is the possible premise, [the syllogisms] are imperfect because the condition of the *dictum de omni*, in the interpretation used in this book for mixed [syllogisms], fails [in these syllogisms]. More precisely he claimed that the meaning /128.5/ of the *dictum de omni*, as it is used as a principle in this book, is just that it stipulates the condition that [the major extreme] *A* (for example) is made a predicate, either affirmatively or negatively and with whatever modality the predication happens to be made, of everything that is described as a *B* in such a way that *B* is affirmed as actually true of this thing. When this condition is met in a syllogism that is a compound of a *de inesse* premise and a necessary one, the modality of the conclusion has to follow the modality of the major premise. And likewise when the meaning of the sentence is that
> Every *B* is an *A*, with necessity or *de inesse* on every *B*. /128.10/ and [every] *C* is an actual *B*, it follows necessarily that [every] *C* is an *A* with necessity or *de inesse* on *C*."

[Alexander] said:

> "The same holds for [syllogisms that are] mixes of a possible premise and a necessary one, or a possible premise and a *de inesse* one, when the major premise is the possible one and the minor premise is either *de inesse* or necessary. This is because [the syllogism] satisfies the condition of the *dictum de omni*, which is that the middle term *B* is actually true of [the individuals that

satisfy] the minor extreme *C*. But when /128.15/ the minor premise is a possible proposition, this condition fails. This is why the Philosopher said, about syllogisms of this kind, that they are imperfect and that [the modality of] their conclusion doesn't follow the modality of the major premise as it did in the previous case." (Ibn Rushd 1983, 127.17– 128.17)

PASSAGE B

/129.3/ But Abū Naṣr says: Alexander says that this is Aristotle's approach to the *dictum de omni* as used in this book. But it is not Aristotle's approach, nor does it fit the facts themselves. /129.5/ Namely, suppose the meaning of the *dictum de omni*, when it is used as the principle behind productive syllogisms, is that *A* is made a predicate—affirmatively or negatively, and with whatever modality of predication it is—of things that are actual *B*s, and also of everything *X* such that every *X* is described as being an actual *B*. Then the syllogisms that are composed of two possible premises in the first figure, which [Aristotle describes as] perfect, are not self-evidently (*ẓāhiratan*) productive. The problem is that the meaning of the *dictum de omni* in it fails to satisfy this condition. This is because /129.10/ the middle term is made a predicate of the minor extreme only possibly and not actually. This is incompatible with what Alexander claimed was the condition of the *dictum de omni* used in this book. Referring to the premise-pair consisting of two possible premises, he claimed that every *B* is an *A* with possibility, in the sense that everything that is an actual *B* is a possible *A*, giving the major premise 'Every *B* is an *A* with possibility'. Then when we attach to this the [minor premise] 'Every *C* is a *B* with possibility', then in terms of the *dictum de omni* it clearly doesn't follow /129.15/ that 'being a possible *A*' is made a predicate of every *C*, because *C* is not wrapped up (*tanṭawi*) below the middle term [*B*], I mean when the major extreme [*A*] is made a predicate of the middle term [*B*]. This is because the modality with which the middle term [*B*] is made a predicate of the minor [extreme *C*] is not the modality imposed on the middle term [*B*] by its having the major extreme [*A*] made a predicate of all of it. (Ibn Rushd 1983, 129.3–17)

PASSAGE C

/130.12/ [Aristotle says,] about [syllogisms that are] mixes of contingent and necessary premises, that [the modality of] the conclusion always follows [that of] the major premise. [But he adds] that when the minor premise is the contingent premise, then the syllogisms are not perfect, just meaning [to refer to those syllogisms that are] in matters where the meaning of the general *dictum de omni* which we spoke of is not true. When the meaning of the *dictum de omni* is true in them according to the condition that we mentioned, /130.15/ then the syllogisms in this kind of mix will be perfect too, just as in the other moods which he said were perfect. And those other moods—I mean those composed from *de inesse* and necessary premises, and from possible with either *de inesse* or necessary, where the major premise is the possible premise—have only perfect syllogisms occurring in them, either because the meaning

of the *dictum de omni* is true in the general version [that doesn't distinguish] the three modalities, or else because it is true [when the minor premise has its predicate] actually true [of its subject individuals]. In each of these cases /130.20/ the syllogisms will be perfect because the minor premise in them [has its predicate] actually true [of its subject individuals]. It could be argued that Aristotle just declared the syllogisms [with contingent minor premise] to be imperfect even when /131/ they could be taken as satisfying the meaning of the *dictum de omni*, because he intended the permanent meaning [of 'conclusion', i.e. one for all matters]. This is the sense in which he said that the mood consisting of two affirmative premises in the second figure doesn't yield a conclusion, even though it does yield one in some matters. So this is how Abū Naṣr interpreted the statement of the Philosopher ... (Ibn Rushd 1983, 130.11–131.3)

PASSAGE D

/133/ As for Abū Naṣr, his approach is, as we said, that the *dictum de omni*, as it is used as a condition in this book, imposes the same condition in all matters. Namely that *A* is predicated affirmatively or negatively, with whatever modality of predication it happens to be, on everything that is a subject for *B*, and is described as *B* by an affirmation only, with whatever modality of predication it happens to be. (Ibn Rushd 1983, 133.1–4)

PASSAGE E

/44.16/ What is non-necessarily true, either absolutely or in some thing, is of two kinds: the first of the two is what is true for most of the time or for most of the subject; or it is what combines these two things together. The second is what is true in a smaller or an equal number. This second kind is not studied in any science at all, but what is true for the majority is studied by many sciences. /44.20/ The premises which fit this description ... can be counted with the necessaries in most of the arts since they behave like them .... (Al-Fārābī 1986e *Burhān*, 44.16–23)

(Abū Naṣr was explicit /118.15/ that in his book on *Demonstration*, where he says:) There are two kinds of not necessary: either being true most of the time, or being true in most of the subject, or both together. As for what is true of fewer and of an equal number, these two subdivisions are the only ones which the sciences don't investigate; they do investigate the other two subdivisions. Included among the premises of this sort there are the necessary premises that are not [really] necessary, for example that every /115.20/ crow is black and all snow is white. (Ibn Rushd 1983, 118.14–20)

Endnotes

i The text that we do have, published by Dāneshpazhūh (Al-Fārābī 1988) and running to 290 pages of Arabic, covers *Prior Analytics* ii.11, 61b1 to ii.27, 70a21. The *Prior Analytics* as a whole runs from 24a10 to 70b38; so in terms of the Greek text we have roughly 10 pages out of 47. Hence we should expect the whole of al-Fārābī's commentary to take up roughly 290 × (47/10) = 1363 pages of Arabic, and 1363 − 290 = 1073 pages of this are missing. This missing part contains everything on modal logic, which is about 13 pages of Greek, making an estimated 377 pages of Arabic.

ii In the Arabic terminology that al-Fārābī used, Aristotle had four quantified non-modal sentence forms, namely 'Every *B* is an *A*', 'No *B* is an *A*', 'Some *B* is an *A*', 'Not every *B* is an *A*'. Al-Fārābī knew that Aristotle wrote them differently, with the *A* before the *B*. In some places al-Fārābī used the Aristotelian order, writing '*A* holds of every *B*', '*A* holds of no *B*', '*A* holds of some *B*', '*A* doesn't hold of all *B*'. In this paper I have used the *B*-*A* versions, which became standard in Arabic logic from al-Fārābī onwards. Al-Fārābī called these four sentence forms (in either order) *wujūdī*, which means 'expressing being'; the medieval Latins translated this as *de inesse*, and I have followed this translation. Al-Fārābī got modal forms by adding 'necessarily' or 'possibly' to the *de inesse* forms. Like Aristotle, he used the same word for 'possibly' and 'contingently', and in some cases I had to give up working out which al-Fārābī meant. The name 'alethic' for the modalities 'necessarily', 'possibly' and 'contingently' is modern, from von Wright (1951, 1f).

iii See Street (2001) for the evidence that 'the outstanding one'(*al-fāḍil*) in Ibn Sīnā means al-Fārābī.

iv Butterworth's translation (Butterworth 2001, 137) incorrectly puts a modality 'contingently' here, it seems misunderstanding *wujūdī*.

v This assumes we can trust Ibn Rushd to report a text accurately. In Passage E of the Appendix I quote side by side a paragraph of al-Fārābī's *Burhān* (1986e) and Ibn Rushd's quotation of it. Ibn Rushd is careless with some details but he has not altered the gist.

vi In Spain in the twelfth century Ibn Rushd clearly had a copy of al-Fārābī's *Long Commentary*, but there is some doubt how easy it was to find texts of al-Fārābī later and further east. Pourjavady and Schmidtke (2015, 261f) give some evidence on what works of al-Fārābī were available in Persia in the 12th and 13th centuries. They quote Bayhaqī (12th century) as reporting that in his time 'most of the works of Fārābī were available only in Syria but not in Khurāsān'.

vii There are two other passages in Ibn Sīnā reporting another reading of quantifiers, and this reading might be due to al-Fārābī.

> Some people say: An absolute proposition is one that asserts something about subject individuals that have become well-defined (*ḥuṣṣila*) existing things, so that when one says 'Every *C* is a *B*' it means that everything that has been described as a *C* in the past or the present has been found to be described as a *B*.
> (Ibn Sīnā 1964, 82.13–83.1)

Ibn Sīnā mentions the same view again in *Išārāt*:

> (19)
> According to some people's approach, the sentence 'Every *C* is a *B*', about things that

> exist and things that don't, means 'Everything that is a *C*, taking together those things that are in the present and those in the past (*ma'an fi al-ḥāli aw fi al-maḍi*), is described as being a *B* at those times when it exists (*waqta wujūdihi*)'. And in this case what we say to refer to past, present and future is 'Every *C* is a *B* with necessity'.
> (Ibn Sīnā 2000, 95.11–14)

The people referred to here seem to think that only well-defined individuals should be quantified over, and that the only individuals that are well-defined are those in the present or the past. This should be compared with al-Fārābī's view that the only determinate truths about the future are necessary truths; see *Long Commentary on De Interpretatione* (Al-Fārābī 1986b, 97).

Street (2016, 20f) cites the passage (19), but silently deletes the reference to the past and translates *fi al-ḥal* as 'at a given time'. I haven't seen this textual variant suggested elsewhere. But it may be relevant that Ṭūsī in his commentary on the passage (al-Ṭūsī 1995, 166.1–6) reads *waqta wujūdihi* as meaning a certain fixed time at which things are assessed as being or not being *C*s, so that the view being reported is not al-Fārābī's but the one that Aristotle rejects at *Prior Analytics* i.15, 34b7f. I think Ṭūsī's reading is not sustainable for several reasons, among them that the view described by Ibn Sīnā takes past and present 'together', and that Aristotle in the passage in question makes no distinction between past and future. Could Street have agreed with Tūsī's interpretation and adjusted Ibn Sīnā's text in the light of it?

viii I follow the reading *fāḍil* in one manuscript; it fits what al-Fārābī says at (5). The edition (Al-Fārābī 1999) follows other manuscripts in reading *nāḍil*, but its translation 'a défendu' is difficult on this reading.

ix The distinction doesn't appear in places where one would expect it if al-Fārābī was conscious of it, such as the abridgment of *De Interpretatione* (Al-Fārābī 1986c, 162.15–19). Ibn Sīnā (2000, i.5.2, 111.2–5) says he thinks his predecessors didn't grasp the distinction.

x *na'imun min jihati mā huwa nā'imun*. In fact *min jiha* has other meanings besides 'because'; I come back to this in Section 3. The reading 'because' here is justified by the fact that it makes sense of al-Fārābī's text, but we will see that the use of *min jiha* in the Arabic translation of the *Prior Analytics* points the same way.

xi Street (2004, 550) translates (γ) as 'Some of that which is sleeping is in so far as it is sleeping possibly an animal'. By moving 'in so far as it is sleeping' to the righthand side of the central 'is', Street moves part of the subject to the predicate in precisely the way that Ibn Sīnā attacks. (In Arabic the phrase 'is possibly' is a single word *yumkinu*, and Street's paraphrase becomes a re-parsing rather than a rephrasing.) If Street is pointing out how al-Fārābī could have got to (γ)—and not simply repeating al-Fārābī's mistake—then his account agrees with mine.

xii For most modern logicians this is an odd suggestion; surely the modal rules should be taken all together, and we should be asking what kinds of modality correspond to this system as a whole? Perhaps the nearest equivalent in modern logic to the old viewpoint is the discussion around the ranges of interpretation of subsets of Girard's Linear Logic; see for example 'Linear Logic' in the *Stanford Encyclopedia of Philosophy* (Di Cosmo & Miller 2016).

xiii But note the confusion in Passage E about whether 'most' should be taken as a kind of 'possible' or a kind of 'necessary'. Al-Fārābī says that these majority statements don't state a necessity but 'can be counted with the necessaries in most of the arts since they behave like them' (i.e. obey the same inference rules?). Ibn Rushd describes these statements as 'necessary

premises that are not [really] necessary' (!).

xiv This example is not as anachronistic as you might suppose. The first steps in permutations and combinations can be traced to al-Khalīl's *Kitāb al-ʿAyn* in the eighth century, and al-Kindī's work on cryptography (Al-Kindī 2003) in the ninth century is sometimes described as the first manual of statistics. But nothing in al-Fārābī's surviving text indicates that he saw the relevance of these mathematical advances to the logic of probability. If he had seen it, the result would have been a striking parallel to the *Port-Royal Logic* (Arnauld & Nicole 1970), whose authors included a final chapter on using probability to reason about the future, based on a recent breakthrough in the mathematics of lotteries by their friend Blaise Pascal.

xv Alexander of Aphrodisias (1883, 165.11–14) gives a syllogism to illustrate a demonstration from a 'most cases' premise. But his syllogism is hypothetical and not derived from a modal mood.

xvi Bäck (1996) traces the history of Peripatetic and Scholastic attempts to build a logic of these sentences. He finds relevant work already in Aristotle's *De Interpretatione* 11, assessing the inference 'Socrates is good, and Socrates is a cobbler; therefore Socrates is a good cobbler' (Bäck 1996, 59). But this is not a modal inference rule, and in any case al-Fārābī doesn't associate 'insofar as' with compound nouns like 'good cobbler'.

xvii See Maghen (2006, 182) on the *fiqh* interpretation of *min* in this verse with reference to sexual positions.

xviii There is still an ambiguity between two readings of the *dictum*; roughly speaking these are the individual reading (where the quantification is over individual *B*s) and the term reading (where the quantification is over terms *D* such that 'Every *D* is a *B*' holds). A recent paper arguing that Alexander intended the term reading is Gili (2015).

xix In Passages A–D al-Fārābī refers five times to 'the *dictum de omni* as it is used in this book' (Ibn Rushd 1983, 128.3f, 128.5, 129.3f, 129.11, 133.1). Here 'the book' is the *Prior Analytics*. By contrast there are other places where we find al-Fārābī talking about what locutions 'are used in the sciences', for example at *Burhān* (Al-Fārābī 1986e, 22.15, 44.23, 74.9) or *Ḥurūf* (Al-Fārābī 1990, 213.18). 'The sciences' are not mentioned in Passages A–D. This contrast confirms our general conclusion in Section 3 that al-Fārābī's discussions of modal logic are mainly with a view to interpreting or justifying Aristotle's modal rules.

xx (*d*) (Every *B* is) an *A* all the time it exists.
(*l*) ... an *A* whenever it is a *B*.
(*m*) ... an *A* sometime while it is a *B*.
(*t*) ... an *A* sometime while it exists.
(*w*) ... an *A* sometime but not always while it exists.

xxi Street has often referred to the semantic expansion (17), which is the final item in the big table above, for an understanding of Ibn Sīnā's view of quantification; see for example Street (2002, 134) and Street (2016, Appendix Text 1). He is clearly right to go to this semantic expansion. But I know of no place where he mentions that this is just one of many semantic expansions that Ibn Sīnā gives, and that they need to be interpreted as a group. Also I know of no place where he mentions the discussion of the subject side in *Qiyās* pages 19 to 21. If he had examined all this evidence I don't think he would have said about (17), to quote his (Street 2002, 134): 'I would therefore tend to take the [*C*s] "in mental supposition" ... as licensing a kind of ampliation to the possible'.

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