The Eliminability of Higher Order Vagueness

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Abstract. It is generally supposed that borderline cases account for the tolerance of vague terms, yet cannot themselves be sharply bounded, leading to infinite levels of higher order vagueness. This higher order vagueness subverts any formal effort to make language precise. However, it is possible to show that tolerance must diminish at higher orders. The attempt to derive it from indiscriminability founders on a simple empirical test, and we learn instead that there is no limit to how small higher order tolerance may become. That means there is no limit to how precisely we may draw the boundaries of borderline cases, thus forestalling any requirement for higher order vagueness.

1. The Same Reason Argument

A conspicuous obstacle in the way of a satisfactory analysis of soritical vagueness is the problem of higher order vagueness (HOV). Ordinary vagueness concerns the applicability of a predicate; higher order vagueness arises with respect to the *definite* applicability of a predicate. The latter transforms a humble logical puzzle into a far-reaching metalinguistic quagmire: the impossibility of saying (precisely) what it is possible to say. There is general agreement as to why vagueness spills over into higher orders: it is *for the same reason* that soritical vagueness arises to begin with.

> Someone might seek to obtain precision in the use of words by saying that no word is to be applied in the penumbra, but unfortunately the penumbra itself is not accurately definable, and all the vaguenesses which apply to the primary use of words apply also when we try to fix a limit to their indubitable applicability.
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> – Bertrand Russell (1923, pp. 63-64)

> For if *IFa* can be true, then so surely can *IIFa*, *IIIFa*, and so on. Or again, if *a* can denote a borderline case of the predicate *F*, then surely the sentence *Fa* can be a borderline case of the predicate 'is neither true nor false'.
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> – Kit Fine (1975, p. 141)

> However, with most or even all vague predicates, it soon appears that the idea that there is a sharp division between the positive cases and the borderline ones, and between the borderline cases and the negative ones, can no more be sustained than can the idea that there is a sharp division between positive and negative cases.
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> – R. M. Sainsbury (1991, 168)

> The difficulties presented by the question 'When did Rembrandt become old?' are also presented by the question 'When did Rembrandt become clearly old?'; the point reiterates *ad infinitum*.
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> – Timothy Williamson (1994, p. 2)

These citations provide a family portrait of what can be called the *same reason argument* (SRA) for HOV. There is no obvious agreement as to what that reason is, but one conspicuous candidate is the *tolerance* of vague predicates: the fact that their applicability is unaffected by small differences in relevant parameters.[1] For example, the term 'bald' is tolerant because whether or not a man is bald is not decidable by the difference of a single hair.

I argue that the SRA fails for tolerance. The fact that a difference is insufficient to affect the applicability of a vague predicate does not entail it is insufficient to affect the definite applicability of the predicate (the definiteness of its applicability). This raises the issue of whether there is *any* difference insufficient to affect the definite applicability of a vague predicate – i.e. whether there is any limit to how precise 'definitely' can be made – i.e. whether HOV is eliminable. One influential approach derives tolerance from indiscriminability. Does the latter guarantee the existence of differences too small to affect the applicability of (at least some) predicates, as well as their definite applicability? I will argue that we are led to the opposite conclusion: there is no amount of difference too small to affect the definiteness of vague predicates. So far as tolerance and indiscriminability are concerned, the term 'definitely' may be made as precise as one pleases. There are other arguments for HOV than the SRA, and reasons other than tolerance and indiscriminability for thinking vagueness uncontainable.[2] But the discovery that tolerance and indiscriminability pose no obstacle to the elimination of HOV should at least subject alternative considerations to increased scrutiny.

2. Soritical Vagueness

A predicate *F* is *soritically vague* (or sorites susceptible) when three conditions are met: (A) there is an a_0 instance to which *F* applies, (B) there is an instance a_m to which *F* does not apply (to which ~*F* applies), and (C) there is a series of instances a_0, a_1, ... a_{m-1}, a_m such that differences between successive members are insufficient to affect the applicability of *F*, i.e. change it from true to false, or vice versa. We see that soritical vagueness inherently involves tolerance; we can call condition (C) the *tolerance principle*. Using standard (classical) logic, it is plausible to represent these conditions respectively as follows:

(1) Fa_0

(2) $\sim Fa_m$

(3) $(n)(Fa_n \rightarrow Fa_{n+1})$

(Equivalent versions of the argument switch *F* and ~*F* or count down instead of up.) Customary examples of predicates fulfilling these conditions are 'bald', 'heap', 'child', 'tall', 'far' and 'red'. For example, a man with no hairs on his head is clearly bald; there is some number of hairs that will guarantee he is not bald (e.g. 10,000); yet it seems undeniable that gaining a single hair is never sufficient to switch someone from bald to nonbald. The crux of the argument, of course, is (3). It plus (1) and ostensibly unproblematic inference rules of standard logic lead to the negation of (2), thus generating an explicit contradiction.

We have identified the "philosophically interesting kind" of vagueness[3] in terms of sorites susceptibility and tolerance. This contrasts with the more usual approach which defines it in terms of borderline cases.[4] From the perspective of tolerance, borderline cases provide a provisional solution of the paradox: (3) becomes inaccurate and misleading because there are cases which are neither true nor false – cases to which neither *F* nor ~*F* apply – cases which are *indefinite* (indeterminate, unclear, etc.). These are the borderline cases: *they account for tolerance*. In contrast with indefinite instances, we can call *definite* those which *are* either true or false. Given monotonicity, there will be just one range of indefinites, in between the respective ranges for the positive and negative instances. This span of instances, between the last for which *F* is true and the first for which *F* is false, constitutes the gap that small differences are insufficient to bridge. The degree of uncertainty with respect to small differences is a direct consequence of the inability to assess applicability.

This theory of vagueness, to the extent that it is one, is semantic: predicates are indefinite insofar as instances exist for which they insufficiently defined, instances for which one cannot say either the term applies or does not. Not all will accept its proposed features, even provisionally. In particular, the supposition that an indefinite instance *x* of a vague predicate *F* is such that neither *Fx* nor ~*Fx* is true will be rejected by those who trace vagueness to epistemic concerns or who do not wish to postulate any alternative to truth and falsity.[5] They will say, rather, that indefiniteness consists in *Fx* not being *knowably* or *definitely* true or false, allowing that it may be true or false nonetheless. No attempt will be made to resolve this particular dispute. The approach here takes seriously the possibility that indefiniteness represents a status distinct from truth or falsity.

3. The Significance of Indefiniteness

How does the proposed solution work? If we stick with standard logic, the claim is that the symbolization in (3) involves a faulty presumption. In the case of baldness, it is the presumption that every haircount is represented in the domain under consideration. However, haircounts for which *F* is indefinite are classically meaningless (being neither true nor false) and therefore must be excluded. Assume the instances in the classical domain remain sequentially enumerated: this means subscripts no longer map onto haircounts. For example, suppose haircounts i through j ($0 < i <= j < m$) represent a range of instances which are indefinite with respect to F, and therefore do not belong to the classical domain. Cases a_0 to a_{i-1} will correspond to haircounts 0 to $i-1$, but (because of the exclusions) a_i will jump ahead to haircount $j+1 > i$. Hence, the difference between a_{i-1} and a_i will exceed the tolerance limit of F, identified as the extent of the range of indeterminates i to j. On the assumption of monotonicity this jump will happen just once. But that's enough to provide a case – Fa_{i-1} & $\sim Fa_i$ – that contradicts (3) and yet remains compatible with tolerance requirements.

However, the winds of philosophical fashion have shifted to more inclusive approaches. The usual tactic is to alter or extend standard logic via the addition of *definitely* (D) and *indefinitely* (I) operators on the analogy of modal systems, treating indeterminacy as the formal equivalent of contingency:[6]

(4) $IFx \leftrightarrow \sim DFx \ \& \sim D\sim Fx$

Reformulating conditions (A)-(C) using these operators yields

(1') DFa_0

(2') $D\sim Fa_m$

(3') $(n)(DFa_n \rightarrow \sim D\sim Fa_{n+1})$

and again we have left paradox behind.

Unlike the previous approach, indefinite instances are included in the domain under discussion, and can be explicitly distinguished by use of the *I* operator. I.e., the assumption is that $(\exists x)IFx$ is sometimes true. Without that assumption, $\sim D\sim Fx$ implies DFx, and consequently (3') implies

(3'') $(n)(DFa_n \rightarrow DFa_{n+1})$

(a substitution instance of (3)), and paradox arises anew.

The use of operators may contribute to the illusion that we can eat our cake and have it too – that we can represent non-bivalence from a classically bivalent platform. But closer examination reveals the extent to which we have left standard logic behind. Consider the statement

(5) $Fx \rightarrow DFx$

It contains two occurrences of 'Fx' which presumably are to be given the same interpretation. If standard logic is in force, the occurrence on the left must be bivalent; yet the occurrence on the right is said to be definite, allowing the possibility of being indefinite, i.e., non-bivalent. To avoid ascribing 'Fx' inconsistent semantic characteristics, we must either abandon bivalence or the assumption that $(\exists x)IFx$ is sometimes true. Consequently, the gainful employment of the 'definitely' operator cannot be separated from non-bivalence and therefore – to borrow a distinction from Susan Haack (1996, p. 4ff) – represents a *deviation from*, as opposed to an *extension of*, standard logic. This is reflected in the non-classical interpretation that must be given the conditional '$\rightarrow$' in (5). Since 'definitely' applies to all true instances on the current approach, 'Fx' and 'DFx' will indeed have the same *positive* extension – if F is true of x, then DF will be also. However, in virtue of $(\exists x)IFx$, we cannot infer that if DF is false of x, then F will be also: the *negative* extensions of 'Fx' and 'DFx' will differ. Hence, the conditional does not support transposition.[7] This non-classical assymmetry between F and DF is crucial to the argument that follows.

It is no surprise that the inclusion of statements that are neither true nor false requires some deviation from bivalent classical logic. But the problem of HOV arises whether one includes them or not. And the operator approach has this undoubted benefit: by placing issues of definiteness vs. indefiniteness into the object language, it provides a convenient means for expressing and assessing claims concerning HOV.[8]

4. Higher Order Vagueness

The idea that tolerance can be explained by indefiniteness leads straight to the SRA for HOV. Just as it is not possible to find an exact haircount that separates baldness from nonbaldness, it seems impossible to find an exact haircount that separates definite from indefinite baldness. In the description of the vagueness of F, we assumed we could identify the boundary instances of the indefinite domain: that we could identify the greatest positive instance and the least negative instance and say precisely where the indefinite instances begin and where they end. But, just as the vagueness of F expresses itself as a zone of indefiniteness, the boundary of that zone will be indeterminate in the same way and *for the same reason*. If there can be borderline cases of baldness, surely there can be borderline cases of borderline cases of baldness. Indeed, once the possibility of borderline cases has been admitted, infinite orders of indefiniteness beckon forth.

Here is how that process works. Let us use 'D^w' to represent a sequence of w occurrences of the definiteness operator D. We also assume that each instance in the series in question differs from its successor by an amount within the bounds of tolerance. We begin with $w = 0$.

Stage One. In virtue of tolerance, deny the existence of any instance such that D^wF can be applied to it, and $\sim D^wF$ applied to its successor:

(6) $\sim(\exists n)(D^wFa_n \,\&\, \sim D^wFa_{n+1})$

However, (6) is classically equivalent to a premise that leads to soritical paradox:

(7) $(n)(D^wFa_n \rightarrow D^wFa_{n+1})$

Stage Two. To avoid paradox, postulate instances which are indefinite with respect to D^wF, so that there are enough indefinites between the last instance for which D^wF is true and the first instance for which $\sim D^{w}F$ is true to exceed tolerance.

Stage Three. Explicitly represent this possibility through the use of the definiteness operator, denying the existence of any instance such that $DD^{w}F$ can be applied to it and $D{\sim}D^{w}F$ applied to its successor:

(8) $\sim(\exists n)(DD^{w}Fa_{n}\ \&\ D{\sim}D^{w}Fa_{n+1})$

Stage Four. Given the SRA, it follows that $DD^{w}F$ is tolerant in the same way as $D^{w}F$.; therefore, increment the value of w to $w+1$ and return to Stage One.[9]

It will be apparent that there will be a different order of vagueness for every iteration of the definiteness operator, and that the process never ends. But why shouldn't it end? – why couldn't the SRA peter out after awhile, as it were, so that at some finite order $v > 0$ tolerance ceases to propagate upwards? Were there only *finite* orders of HOV, however great, we would have sharp boundaries for some order x. However, providing sharp boundaries for any order ($D^{x}F$) will provide them for the next lower order ($D^{x-1}F$) as well. This can be seen most clearly at the bottom: specifying where the indefinite instances for F begin is also to specify where its positive instances end, and specifying where its indefinite instances end also specifies where the negative instances begin. Hence, sharp boundaries at any order entail sharp boundaries throughout. Consequently, if the SRA is *ever* valid – if definiteness ever requires tolerance – there is no stopping short of infinity.[10]

5. The Discomforts of HOV

The position here should be distinguished from the claim that HOV is itself inherently paradoxical.[11] On the present approach, HOV is the frying pan in which we evade that fire. We opt for it with the avidity with which a utilitarian opts for killing five innocent babies instead of ten. Why should HOV be undesirable in itself, at best a lesser evil? While the sorites might be dismissed as just another prickly conundrum that will dissolve as soon as we sufficiently refine the concepts and theories involved, HOV subverts the very enterprise of conceptual refinement. Timothy Williamson, for example, speaks of "the hopeless demand for a precise meta-language" (1994, p. 162). HOV is a dagger in the very heart of formalism, rendering impotent any desire to explicate precisely the essential nature of vagueness.

But the unpleasant consequences of HOV extend beyond the invalidation of any effort to specify a vagueness-free metalanguage, and include the awkwardness of supposing the existence of *actual infinities*. To be sure, the success of the mathematical calculus has greatly desensitized traditional philosophical concerns in this regard. However there are considerable differences between HOV and the suppositions underlying the continuum, none of which redound to the advantage of the former.

Suppose in the whim of a moment I coin a vague expression, say by prefixing a nominally precise term with 'about' and/or postfixing it with 'ish'. What I have done, according to the supposition of HOV, is bring into existence a term whose meaning encompasses infinite orders of indefiniteness. What has enabled me to exercise such magic? On the analogy of Zeno's paradoxes, we may suppose I have done no more than Achilles when he overtakes the tortoise. The fallacy underlying those paradoxes is the presumption that the possibility of analyzing some act into an infinite series of components entails that the performance of that act requires completion of an infinite series of (intentionally distinct) subsidiary acts.[12] It overlooks what calculus has taught us, that sometimes there are shortcuts that allow us e.g. to determine the limit of a series without the need to calculate separately every constituent element of it. So if Achilles can in effect sum an infinite series simply by overtaking the tortoise, why can I not construct infinite orders of HOV with my 'about -ish'?

But there are critical differences between these two cases. I have experience with dividing lines into constituent segments, and therefore can *see* how the limit and the sum of the infinite series amount to the same thing. However, who has experience with dividing vague terms into constituent orders of indeterminacy? and who can purport to *see* how infinite orders of HOV accumulate to determine the meaning of a vague predicate? Indeed, even the philosophically sophisticated will experience difficulties in explaining just how this decomposition works. Suppose, for example, they represent the vagueness of the term 'child' as the culmination of the following infinite series of orders of indefiniteness:

Order 0: (provisionally) indefinite for ± 4 years of age 16

Order 1: (provisionally) indefinite for ± 2 years of order 0 boundaries

Order 2: (provisionally) indefinite for ± 1 year of order 1 boundaries

...

It seems necessary to construe the boundaries of each particular order as precise – at least provisionally, in isolation from the consequences of higher orders. Otherwise, one would presume the irrelevance of higher orders in accounting for vagueness. That is, we are supposing it is in virtue of higher order indefinites that the borders of any particular order become imprecise. However, we can always calculate or approximate the limits of such series (like the example above), and thereby – contrary to the supposition of the SRA – establish precise boundaries for those instances which are "unqualifiedly and unimpugnably" true or false.[13] So not only do we have no experience of compounding vagueness out of indefinitenesses of different orders, we seem to have no idea how that could be done. And while the accomplishments of mathematical analysis are abundant and far-reaching, all the defender of HOV can point to is the avoidance of soritical paradox. Hence, the awkwardness of supposing infinite orders of HOV is not palliated through comparison with the methodology and success of the calculus. This should establish the desirability of any account of vagueness that avoided reliance upon HOV. But how might that be achieved?

6. Tolerance and Higher Orders

Let us take conditions (A)-(C), plus the supposition of indefinite cases and the monotonicity of predicate *F*, as exhausting what we know about some soritical situation. We know there is at least one instance to which *F* applies, a_0, and at least one to which ~*F* applies, a_m, and at least one instance, call it a_i, to which neither *F* nor ~*F* apply. In virtue of monotonicity we know there is but one contiguous series of indefinite instances located between the *F* and the *~F* cases; i.e., $0 < i < m$. However, to avoid begging the question against HOV, we cannot assume the ability to identify the greatest *F* case (the largest *x* such that Fa_x is the case) nor the least *~F* case, nor indeed the least or greatest indefinite case.

Given that the positive extensions of a predicate and its definitization are the same, we know that the amount of difference between *F* and *~F* is the same as that between *DF* and *D~F*. However, because the indefinite cases are *between* the (definitely) true and (definitely) false, we also know that the amount of difference between *DF* and *IF* (between *DF* and *~DF*) must be less than that between *DF* and *D~F* (between *F* and *~F*). In other words, as we traverse upward along the series of cases, after leaving the (definitely) true, we will encounter the indefinite before reaching the (definitely) false. And this will hold no matter how many orders of HOV are presumed to muddle the situation. Therefore, an amount of difference insufficient to affect the applicability of a vague predicate *F* – insufficient to bridge the gap from cases to which the term applies to cases to which its negation applies – need not be insufficient to affect the applicability of *DF*.

Thus, *DF* is *less tolerant* than *F*: the possibility of a difference that is insufficient to affect *F*, yet sufficient to affect *DF*, entails that the tolerance of *F* must be greater than the tolerance of *DF*. This does not imply *DF* is intolerant, i.e. completely precise; it is possible that, while its tolerance is smaller than *F*, *DF* may nonetheless remain significantly tolerant. In that case the supposition of HOV could be sustained, though requiring careful qualification. Nonetheless, the fact that *DF* is less tolerant than *F* clearly undercuts the plausibility of the SRA. There is a literal sense in which 'same reason' is false; the *amount* of tolerance is not the same. Is the tolerance of 'definitely' simply less than that of the vague predicates to which it is applied, or completely nonexistent (zero)? Can HOV be eliminated?

7. The Indiscriminability Theory of Vagueness

In a pair of influential articles published side by side in *Synthese* in 1975, Michael Dummett and Crispin Wright proposed what can be called the *indiscriminability theory of vagueness* (ITV).[14] They claim that vagueness is a natural consequence of limitations on human powers of perception, at least for a fundamental category of predicates. The meanings that we are able to ascribe to *observational* or phenomenal predicates – those we can apply by e.g. "just looking" – are constrained by the kinds of features that we can discriminate, the circumstances in which we typically learn them, what we can and cannot be expected to remember about them, and so forth. The proposal is that, for such predicates, the tolerance principle is a consequence of an *indiscriminability principle*: if two things are indiscriminable (indiscernible, indistinguishable, etc.), then an observational predicate applicable to the one is applicable to the other. But the defining characteristic of indiscriminability, in contrast with identity, is its non-transitivity: case a_0 may be indiscriminable from case a_1, and case a_1 may be indiscriminable from case a_2, yet case a_0 may be discriminable from case a_2. This suggests that the conditions involved in indiscriminability have the same logical form as those involved in soritical vagueness: (a) there is an instance a_0 to which observational predicate F applies, (b) there is an instance a_m to which observational predicate F does not apply, and (c) there is a series of instances a_0, a_1, ... a_{m-1}, a_m each of which is indiscriminable from its successor in such a way that any observational predicate applicable to the one is applicable to the other. The symbolic formulae (1)-(3) are as reflective of conditions (a)-(c) as they are of those involved in soritical arguments, (A)-(C).[15]

This identity of logical form provides the basis for an appealing reduction. At the very least, observational predicates include those that simply describe the appearances of things – e.g. 'looks red' – and at the outside, they include any ostensively definable term. By deriving tolerance from observationality, Dummett and Wright can account for the prevalence, utility, and necessity of vagueness in ordinary language. Terms involving the appearance of things will be fundamental to any functional language, terms accommodating our perceptual limitations obviously will be more convenient than terms which do not, and terms reflecting such limitations can hardly be expunged from practical discourse. And it would provide an answer to our question: the ITV would show why both vague predicates and their definitizations are tolerant. The indiscriminability of successive cases guarantees their differences are too small to affect the applicability of observational predicates, or to affect their definite applicability.

8. Problems with Indiscriminability

Is the ITV true? At first sight, the basic principle seems unquestionable: if two things are observationally indiscriminable, then it seems hardly plausible that an observational predicate could apply to one but not the other. But a number of authors – including Linda Burns (1986), Mark Sainsbury and Timothy Williams (1997), and Wright himself (1987) – now argue that this superficial plausibility evaporates when confronted by hard cases.[16] Construed as an empirical claim, the indiscriminability principle turns out to be false. Assemble a group of competent speakers of English, and provide them a series of colored patches each of which is indiscriminable from its successor but which gradually shade from red at one end to orange at the other. About 32 patches should be enough, but feel free to increase that number to make the difference from one to the next as small as you please. Ensure the circumstances are such as to rule out any source of confusion, mistake or error. Now ask these speakers one by one to say, of each patch in succession starting at the red end, whether or not it looks red. You will find that each, well before getting to the orange end, reaches a last patch to which they are willing to apply the predicate 'looks red'. Indeed, if they did not you would have grounds for concluding that either they were not competent in English or you had not eliminated all grounds for error. For, after all, orange patches *don't look red*. The different speakers can hardly be expected to agree on *which* patch is the last which looks red; nor indeed the same speaker on different occasions. But nothing prevents us (and them) from agreeing that the *overall last* red-looking patch is the last for which there is agreement that 'looks red' applies (sort of a bastard supervaluationism):

> We have therefore to acknowledge ... that a sorites series of indistinguishable colour patches *can* contain a last patch which is definitely red: it will be a patch about whose redness there is a consensus ... and its immediate successor will be a patch about which the consensus breaks down ....[17]

This kind of empirical test leaves defenders of the ITV without attractive options.[18] They can claim that the last (definitely) red-looking patch is not really *indiscriminable* from the one that follows, since indeed it has been discriminated. However, this renders the principle vacuous because we are left without any reliable means of determining when instances really are indiscriminable: for looking the same clearly is not enough. Or, they can claim that 'looks red' is not in fact *observational* in the appropriate sense, since it does not apply to both of two indiscriminable items. But, in that case, there no longer would appear to be *any* observational predicates in ordinary language – if 'looks red' doesn't count as observational, what does? – and there goes any plausible account of the prevalence, utility and necessity of vagueness.[19]

Consequently, the ITV has failed to provide grounds for believing that the term 'definitely' cannot be made perfectly precise in applications to vague predicates. Quite the contrary: its failure has provided grounds for thinking such precision attainable. For, as we have seen, no matter how minimal the difference between succeeding patches in the series from red to orange, we expect competent speakers of English sooner or later to stop applying the predicate 'looks red'. This provides a *de facto* basis for drawing a precise borderline between *DF* and *~DF* – one that can be drawn regardless of how small the difference between successive cases. In other words, when the applicability of a vague predicate is cast as an empirical matter, HOV simply does not arise.[20] We have thus shown the SRA to be baseless; the infinite series of higher orders never gets off the ground.

9. A General Solution

However, the empirical test is not regarded as providing a general solution to the sorites or the problem of HOV. It is seen rather as a repudiation of the ITV: "observationality has no impact on the sorites paradoxes", conclude Sainsbury and Williamson (1997, 470). The test reveals that the indiscriminability principle is either false or vacuous: there *cannot* be a series of instances each of which is indiscriminable from its successor in such a way that any observational predicate applicable to the one is applicable to the other, not in any substantive sense.[21] But the tolerance principle – that there can be a series of instances such that differences between successive members are insufficient to affect the applicability of a vague predicate – does not depend on indiscriminability for its paradoxicality. For example, we *can* discriminate between a man with *n* hairs and the same man with one more hair (*n+1*); not by direct observation, of course, but by a process of counting. This ability to discriminate (albeit indirectly) between successive haircounts does not weaken the claim that that difference is too small to affect the applicability of the predicate 'bald'. It is not indiscriminability but indefiniteness that precludes any decision process taking minor differences of haircount into account.

But, granting that the tolerance principle does not depend on the indiscriminability principle, is there any reason to doubt that the empirical test applies just as well to the former? For example, select an equivalently competent group of speakers of English, and test them (in conditions precluding confusion, mistake or error) with the predicate 'bald', using a series of pictures that show a man going from bald to nonbald in tiny-but-just-discernible steps. Sure enough, sooner or later the consensus that the man is 'bald' will break, providing a sharp boundary for 'definitely bald' just like the earlier example provided a sharp boundary for 'definitely looks red'. Similar tests can be imagined for any term, however non-observational. It can be argued that different predicates present different circumstances and different challenges. But the term 'definitely' will always remain distinct from any predicate to which it is applied. The term is about *semantics* – the application of predicates to instances. Expressions like 'red', 'bald', 'child', 'heap', 'few', etc. are about very different things: 'bald' is about relative hairlessness, 'red' is about color, 'child' is about a stage of maturity, and so forth. There is no reason why terms about different things should not be differently vague – there is no reason in principle why 'bald', 'heap', 'red', child', etc. cannot be tolerant, and 'definitely' intolerant.

10. 'Definitely' vs. 'Bald', 'Heap', etc.

How may ordinary terms and their definitizations differ with respect to vagueness? The vagueness of familiar terms like 'bald', 'red', 'child', 'heap', etc. is largely empirical. It is a fact about their conventional usage, including their susceptibility to soritical paradox. Why not simply avoid problems by replacing vague terms with more precise equivalents? Because, it is argued, such an approach fails to appreciate the genius of ordinary language: it overlooks the utility and, some would say, the necessity of vagueness in the words we use. And even when it is possible to precisify language, any particular stipulation will be vulnerable to charges of arbitrariety. Why define 'heap' as 5000 grains, for example: why not 5001, 4999, etc.? But it should be clear that none of these considerations apply to the philosophical employment of terms like 'definitely', 'determinately', 'clearly', and so forth. Though those terms have established ordinary uses, that does not constrain their technical employment; this latter is linguistically prescriptive, not descriptive. Often enough, the reason for introducing a technical term is to be able to say things with more clarity than one could without it. This is not to deny that one might sometimes want a deliberately vague term for the purposes of some argument, or indeed for the purposes of leaving some argument deliberately vague. On the other hand, we sometimes want our claims to be true and our arguments to be valid and sound; we may not be satisfied with saying something merely indeterminate. Charges of arbitrariety are generally irrelevant when it comes to technical stipulations. The fact that there are many equally legitimate axiomatizations of propositional logic does not make any particular one a bad choice; rather, all are equally good.

Of course, the critical contrast is not between e.g. 'bald' and 'definitely', but between 'bald' and 'definitely bald'. According to the usual approach, the latter is something of a hybrid: the combination of an operator and a predicate. However, we can get around that with a quick definition:

> (5) The predicate *definitely F* (*DF*) applies to an instance x $=_{\text{df}}$ the operator *definitely* (*D*) applies to the application of predicate F to instance x.

However, 'definitely' is clearly dominant and 'bald' recessive, at least so far as the earlier considerations are concerned.[22] It is evident that 'definitely bald' is an expression bound not by ordinary use but by philosophical intent, whose utility lies – insofar as it is possible – in precision and clarity, to which ends arbitrariety would be an acceptable means. Confusion may be engendered by the fact that *DF* and *F* apply to the same instances (again, that identity of positive extension). But *DF* applies directly to *applications of F*, and only *indirectly* to the instances to which *F* applies. Hence, the precision of 'definitely bald' remains independent of (orthogonal to) the precision of 'bald'. While the latter is an ordinary language term which is vague as a matter of linguistic fact, the former is a technical term of art whose prescription is not hindered by such descriptive niceties. And we have discovered that, so far as indiscriminability and/or tolerance are concerned, we can make it as precise as we want.

11. Vagueness and Semantic Rules

To tidy things up, it would be nice to have an account of the semantic rules for soritically vague terms and the term 'definitely' that renders the eliminability of HOV intelligible. To a considerable degree the debate over vagueness has turned on conceptions of such rules. These are deep waters, to be sure. Still, one tentative lesson that might be drawn is the importance of distinguishing between descriptive characterizations of semantic rules and their (prescriptive) content, viz. how they say the term ought to be used. That a term ought to be used in a certain way is itself a descriptive truth of a sort, a truth about the accepted rules for some empirical language; but there are descriptive truths about semantic rules which do not form part of their content. For example, brevity truly characterizes the definition 'unmarried male', but is not an explicit component of it (unlike e.g. 'briefly, unmarried male'). Analogously, tolerance should be regarded as *true of* the semantic rules for 'bald' but not *part of* the rules for that term. It is not necessary to learn the meaning of 'tolerance' in order to know the meaning of 'bald'.

Wright argues against the existence of semantic rules for vague terms on the assumption that, were there such rules, they would be incoherent.[23] We now have grounds for believing this incoherence does not arise; regardless, he seems insufficiently cautious in inferences from facts about the vagueness of terms to the contents of their rules. At the very least, the failure of the ITV has casts doubt on the thesis that vague terms are *essentially* vague. There is a difference between semantic rules that are indefinite and rules that explicitly require their own indefiniteness. I submit the former is sufficient to account for tolerance: *pace* Wright, it does indeed represent a "hiatus .... the reflection of an omission" (1975, 330). 'Bald' does not need a rule that *says* that plus or minus a hair makes no difference with regard to baldness. All it needs is a rule so indefinite, when it comes to such minor differences, that you sometimes cannot tell whether it applies or not. For example, one person's semantic rule for 'looks bald' might be to apply it to those men who resemble his Uncle Vinnie. Vagueness will result from the inability, in some instances, of making a ruling either way.

Sainsbury correctly points out that a definition of 'child' that applied the term to humans up through 15 years and denied it to those 18 or older, leaving the status of 16- and 17-year-olders indefinite, would not be *vague* in any ordinary sense (1991, 173). Nonetheless, *a la* Wright, he supposes that *were* there truths about a term's definiteness, they *would* be part of its rules:

> There is no least red thing of which 'red' is unqualifiedly and unimpugnably true. Were there such an object, its identification would be an essential prerequisite for the full and perfect command of the language, yet evidently nothing plays this role. (1991, 169).

The argument is vitiated by the supposition that descriptive truths about the use of 'red' need be incorporated into the content of its semantic rules. Why require something through special rules that is already true as a matter of fact? The (in)definiteness of a predicate is not something spelt out in the semantic rules that prescribe its use, but rather (simply) an empirically detectable characteristic of linguistic behavior according to those rules.

In contrast with ordinary language terms like 'bald' and 'red', the semantic rules for 'definitely' can be quite definite indeed. The indefiniteness of predicate *F* will consist of the range of cases left unresolved by the predicate's semantic rules, those for which the predicate is neither true nor false. That range is something that is discoverable in actual practice, for example through tests that reveal which instances *F* is applied to, which instances *F* is denied to, and which instances cannot be decided one way or the other. The precision with which the (in)definiteness of a predicate can be measured is limited only by the extent to which one can minimize the difference between instances. The SRA fails: so far as tolerance and indiscriminability are concerned, HOV is eliminable.

Endnotes

1. The idea of tolerance is due to Wright (1975, 333): "a notion of a degree of change too small to make any difference". However, the present use differs from Wright's in (i) not supposing it inherently incoherent, and (ii) distinguishing it from indiscernibility. As will become evident, the latter distinction is crucial for the present argument. In (1988, p. 217), Sorensen uses an equivalent notion of "limited sensitivity".

2. For instance, there are arguments for HOV based on the (same) lack of sharp boundaries

(Williamson 1994), on the vagueness of 'vague' (Hyde 1994), on the vagueness of the analysis of 'vague' (Burgess 1990), and on the use of vague predicates (Sainsbury 1991).

3. Burns (1995) uses this manner of speaking though never directly as quoted.

4. This approach is widespread in the literature; the expression dates back at least as far as Black (1937, p. 71): "The vagueness of a term is shown by producing 'borderline cases', i.e., individuals to which it seems impossible either to apply or not to apply the term." Definition in terms of borderline cases is crucial to an argument for HOV in Hyde (1994); in this regard, see Sorensen (1985) and Reas (1989).

5. This group will include not just epistemicists like Williamson (1995) but also Wright in one of his more recent discussions of the subject (1995). This latter is peculiar, however, in that the author (unlike Williamson) also claims "there is no species of indefinite truth" (p. 143). How all that is supposed to fit together will not be explored here (see Wright 2001).

6. It is usual to find (4) stated as a definition rather than a biconditional. However, that would misrepresent the present approach; that indefiniteness should be defined in terms of definiteness is an illusion fostered by a need to observe the proprieties of standard logic.

7. The logical status of (5) has been the subject of spirited debate; for example, see Wright (1995), Sorensen (1995) and Williamson (1995).

8. Cf. Wright (1992, 130): "when dealing with vague expressions, it is essential to have the expressive resources afforded by an operator expressing *definiteness* or *determinacy*."

9. Wright (1992) describes an equivalent process.

10. See note 20.

11. Wright (1992) makes the argument that HOV is itself paradoxical; for criticism, see Edgington (1993) and Heck (1993).

12. Cf. Vlastos (1967).

13. The expression is Sainsbury's; see below, section 11.

14. Dummett (1975) and Wright (1975); Wright (1976) is largely an abbreviated version of the latter. Another notable (and contiguous) essay from *Synthese* **30** is Fine (1975).

15. "This is a paradox of exactly the [same] form", Dummett (1975, p. 111).

16. More peripherally, Schwartz (1989) and Burgess (1990) can be added to this list.

17. Wright (1987, p. 222).

18. The discussion here closely parallels Sainsbury and Williamson (1997); the empirical test is more or less an amalgam of all the mentioned sources.

19. There is a disingenuous element to the argument here, because the interpretation just given to the expression 'last red-looking patch' appears to invoke a non-observational sense of 'looks red', i.e. one that requires taking group surveys, and not "just looking". But that sense presupposes there is also a last patch in the phenomenal (non-group) sense of 'looks red'.

20. Burgess (1990) would contest this assessment: for Burgess there is HOV, but finitely many orders. His conclusion is just as fatal to the problematic nature of HOV, for reasons discussed earlier. Still, I would contend that the higher orders Burgess explores characterize the description of a certain way of measuring indefiniteness (schema (A*), essentially our empirical test), but are not part of the meaning of 'indefinitely'.

21. This is essentially Burns' conclusion (1986, 510): "*There are no series of the kind required for the paradoxical argument to work*." However, by failing to distinguish indiscriminability from tolerance, Burns inflates the significance of this result. Schwartz (1989) is a corrective, though it does not emphasize that distinction either.

22. Sorensen (1995, p. 164) claims that, while 'definite' is not itself vague, it "passively transmit[s] the vagueness of what it modifies", but he elucidates no mechanism for this.

23. A major thesis of Wright (1975) and Wright (1976), qualified but not abandoned in Wright

(1987).

Acknowledgements

I would like to thank Stephen P. Schwartz for his encouragement and helpful comments at various stages in the development of these arguments. He is not responsible, of course, for any errors that remain.

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