Perceiving Abstract Objects

Inheriting Ohmori Shōzō's Philosophy of Perception

Takashi Iida[1]

[1] Department of Philosophy, College of Humanities and Sciences, Nihon University

1. Introduction

This paper is about geometrical shapes and our knowledge of them. My interest in this subject comes from my work on the ontology and epistemology of linguistic types and tokens (Iida 2009). We see the linguistic entities like alphabets and Chinese characters in two different ways. Usually we see them as types, but sometimes we see them as tokens. I think it a very significant fact that linguistic entities sometimes appear as abstract entities in the form of types, and they sometimes appear as concrete objects or events in the form of tokens. For, our recognition and handling of the linguistic entities show that we have a capacity to recognize and handle abstract entities in concrete situations.

In trying to account how we recognize an abstract type in a concrete token, I have found Ohmori Shōzō's work on the philosophy of sense perception very helpful. His account of the perception of a material object has helped me to get a clearer picture of the relation between a type and its tokens.

This experience encouraged me to extend Ohmori's account further. It is not difficult to realize that linguistic characters are after all geometrical shapes. Hence, it seems natural to see whether my Ohmori-like account of linguistic entities can be extended to geometrical shapes in general.

2. Ohmori Shōzō 's philosophy of perception

Ohmori Shōzō (1921--1997) was a leading figure in the postwar Japanese analytic philosophy. In the 150 year history of modern Japanese philosophy, he was one of the very few who thought their own original thoughts independent of the ever-changing philosophical fashions, which almost always come from abroad. I was very fortunate to have had him as my first teacher in philosophy.

Throughout his life, Ohmori was much concerned with the philosophical problems of sense perception. In an important paper "*Mono to Chikaku* (Things and Perception)" (reprinted in (Ohmori 1971)), he set his problem as that of explaining the relation between two languages; one is a language which describes the world as consisting of things, and the other is a language which describes the world as being perceived. A little later, he gave up speaking of two languages, and tried to integrate two different ways of speaking about the world into one language. It was made possible by his insight that the perception of an object is always accompanied by some very closely related thoughts. Such a picture of sense perception is presented in his book *Mono to Kokoro* (Matter and Mind) published in 1974 (Ohmori 1974). My description of Ohmori's theory is based on it.

His starting point is the following observation. When we perceive a material object, we see it as a three-dimensional object that has a back, sides, and an inside, even though, strictly speaking, we see only a "surface" that the object presents to us. We may add that a material object is also perceived to be an object that endures in time, even though we seldom watch it for the entire period of its existence and sometimes we see it for only an instant.

Ohomori thinks that, in general, there are two modes in which an object is presented to us. In some cases, an object is presented to us by being perceived as in sense perception. In another cases, an object is presented to us by being thought. Ohmori thinks that mathematical objects and purely theoretical objects in physics like electrons and protons can be presented to us only by being thought.

Among the thoughts which present objects to us, there is a special class of thoughts which have close ties with perceptions. They typically figure in imagination and remembrance. When we imagine an object, we imagine ourselves to perceive it. When we remember an object, we remember having perceived it. In these cases, the object is not actually perceived; it is only thought as being perceived or having being perceived. What we have in the cases of imagination and remembrance are the thoughts of possible perceptions or past perceptions.

Ohmori claims that in any perception of a material object there are various thoughts of possible perceptions of the same object from different perspectives beside the actual perception of its "surface", and this is the reason why we perceive an object as a three-dimensional one. In other words, any "surface" perception of a material object is always accompanied by some thoughts about the possible perceptions of the currently unperceived parts of the object.

Although Ohmori does not say it explicitly, I would like to emphasize the following fact: when we perceive a material object, in usual cases, we are not conscious that what is given immediately to us is only its "surface" seen from a particular perspective; we experience our environment as consisting of the three-dimensional objects enduring in time. It needs a conscious effort to realize that what we see in the strict sense is only a spatial part of the object facing us and its temporal slice at the present. According to our way of talking, if we are looking at an object straightaway without any obstacle, we see the whole, not the part, of the object.

3. Ohmori on perceived triangles and thought triangles

In several places of his writings, Ohmori writes about the nature of geometry and our knowledge of space. Here I can consider only a couple of remarks he made about our route to geometrical knowledge.

When we reason about some property of a triangle, we usually draw a triangle on a paper or a blackboard. In general, geometrical figures like triangles seem to be presented to us through the sense perception of some traits of physical objects like a paper and a blackboard. Ohmori says, however, that a geometrical figure itself cannot be perceived. He says that, in seeing a triangle drawn on a paper, we have an "invisible" triangle in thought. It must be invisible because it is supposed to consist of the lines that have no breadth and the points that have no extension. Of course, we cannot see it, because it is invisible. Thus, a geometrical figure like a triangle is never presented by sense perception, but presented only by being thought.

What is the main difference between seeing a material object and looking at a figure on a paper in order to solve a geometrical problem? In both cases, we actually perceive something in our environment. In the former case, it is the surface of the object facing us, and in the latter case, it is some traits of the paper or the blackboard. The difference is in the accompanying thoughts. In the former, they are the possible perceptions of the object from different perspectives. In the latter, the accompanying thoughts are not the thoughts of possible perceptions, but the thoughts of an "invisible" triangle.

I think this account is fine as far as it goes. But, at the same time I cannot help thinking that there is something missing in this account. It might be true that we have thoughts of an invisible triangle when we look at a drawing on a paper. However, what makes us possible to think such an invisible, and hence immaterial, triangle when we perceive a certain material thing? Are there some resemblances between the two? But, how is it possible that an immaterial thing resembles a material thing?

There does not exist a similar problem in the case of the sense perception of a material object. For, in that case, the accompanying thoughts are those of possible perceptions from different perspectives, which we can imagine easily. But, in the geometry case, it is a big mystery how a material thing causes in us a thought about an immaterial entity.

I believe that there is a solution to this problem. In order to explain it, I have to talk about our recognition of linguistic entities like letters and speech sounds.

4. Our recognition of linguistic types and tokens

If we accept Ohmori's account of sense perception, we might notice that there is a good analogy between our sense perception of material objects and our recognition of linguistic entities like letters and speech sounds.

Let us consider a child who is reading a simple text in English. If she is really reading, then she must be able to recognize each letter in the text when she sees it. What is involved in her recognition of a letter? First, she should be able to see a shape on the paper. Second, she should be able to judge that the shape is a token of one of the letters (in the sense of types) in the alphabet. In order for her to be able to judge the shape as a token of a particular letter, say, "a", she should be able to remember that she encountered some other tokens of the same letter before and know that she will encounter new tokens of the same letter in the future.

We can see a pattern very much similar to Ohmori's account of sense perception.

> In the sense perception of a material object, we have (A) an actual perception of a surface of the object, and (B) accompanying thoughts of the possible perceptions of the object from different perspectives.

> In the recognition of a letter, we have (A) an actual perception of a token of the letter, and (B) accompanying thoughts of the possible perceptions of the different tokens of the letter.

I would like to make a remark similar to the one I made before about our perception of a material object. Namely, when we see a letter, usually we are not conscious of the fact that what is immediately given to us is only its particular token; if you are an experienced reader, you are not aware of the particular shapes of the letter tokens or word tokens for most of the time when you are reading. Just as we see three-dimensional and enduring objects when we see our environment, we see word types and letter types when we read.

However, there is one big difference between two cases. As we have emphasized several times, a material object is three-dimensional in space and endures in time. It is a concrete entity that exists at a particular place and time. In contrast to it, a letter as a type is an abstract entity; it is not in space nor in time, although its tokens are.

Despite this difference, I would like to claim that we perceive letter types and word types when we read just as we perceive material objects when we see our environment. Hence, we perceive abstract entities when we read.

An abstract entity like a word type or letter type is not immediately given in our perception. It is perceived only through its particular tokens. But then, a three-dimensional material object is not immediately given in our perception, either. It is perceived only through its particular surface. Hence, if we can talk of the perception of a material object, we should be able to talk of the perception of a word type or a letter type, too.

You might object that we cannot perceive abstract entities because we cannot have any causal contact with entities that do not exist in space and time. It is true that the sense perception of a surface of a material object is caused by the material object. But it might be the case that causality is only necessary for a successful sense perception. I don't see any reason to suppose that causality is also necessary for the routes to knowledge other than sense perception. In this connection, it is interesting to note that the concept of causality does not play any role in Ohmori's account of perception. On the contrary, he explicitly criticized the causal account of sense perception in a paper collected in (Ohmori 1971).

In both cases the immediate objects of our perceptions are some concrete things or events, which we perceive thorough our sense organs. We may suppose that such perceptions are causal processes. The only difference between the two cases is the difference in how the direct objects of our perception relate to the, as it were, mediate objects of our perception. In the perception of a material object, the immediate object of our perception, that is, a surface of the object, is a physical part of its mediate object; in the perception of a type, its immediate object which is either some material object or event, is a token of the mediate object of the perception. If it is allowed to say that in both cases we perceive not only the immediate object of the perception but also the mediate objects of perception, then the difference in the nature of the relations between the immediate object and the mediate object of perception does not matter.

5. Shapes as type entities

One answer to the question "What is a triangle?" is that it is a concept. Although this seems to be an obvious answer, there is a serious difficulty. As Ohmori observed, a triangle that appears in geometry consists of the lines without any breadth and the points without any extensions. For example, the three vertices where the sides of the triangle intersect should consist of a single point which has no extension, which means that each of the three sides of the triangle should have no breadth. It follows that the concept "triangle" does not apply to anything that is found in our environment.

Or, "triangle" is ambiguous, and are there one predicate which applies only to abstract geometrical objects and another predicate which applies to concrete things like the figures on a paper or a blackboard? But then, what is the relation between the two predicates? There must be some connection between them, otherwise it becomes a mystery why we can use geometry in various ways in our life.

We have already met the same kind of ambiguity; when we talk of linguistic entities like letters, words, or sentences, we usually talk of them as types, but sometimes we also talk of them as tokens. There is a systematic ambiguity in a predicate like "is a letter", "is a word" or "is a sentence", which applies to both of types and tokens. Should we think the predicate "is a triangle" is also such a predicate which applies to both of types and tokens?

Moreover, if we take up this idea and regard a geometrical figure like triangle as a type, we can borrow our account of type recognition in order to explain our recognition of a geometrical figure. Let me repeat that account of type recognition.

> In the recognition of a letter, we have (A) an actual perception of a token of the letter, and (B) accompanying thoughts of the possible perceptions of the different tokens of the letter.

If we substitute "letter" with "triangle", we get an account of our recognition of a triangle.

> In the recognition of a triangle, we have (A) an actual perception of a token of the triangle, and (B) accompanying thoughts of the possible perceptions of the different tokens of the triangle.

We know, however, that present day geometry is much more complex affair than it used to be, for example, compared to the time of Kant. We all have heard something about non-Euclidean geometry and some of you may have heard Klein's program or the Poincaré conjecture in topology. In view of this, our account of geometrical figures seems to be too simple-minded to be of any use.

In spite of such a worry, I believe there are some grounds to think it worthwhile to develop an account of geometrical figures along the line I indicated. It is because we have, as it were, a naive conception of geometrical figures which we put in use frequently in our everyday transactions. It is an important task in conceptual analysis to identify the elements of such a conception of geometrical figures and clarify its relation with various developments in modern geometry. And, I believe something like the type-token distinction is at the heart of our naive conception of geometrical figures.

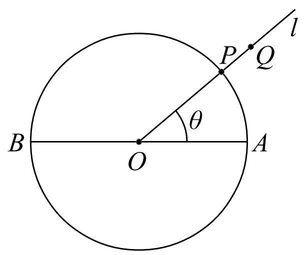
6. A problem concerning type existence

In general, the existence of a type depends on that of its tokens. As the existence of tokens is a contingent matter, the existence of types is also a contingent matter. Types are beyond space and time because they are abstract, but they are not beyond the worlds. A type which exists in this world may not exist in some other possible worlds just because it has no tokens in such worlds. There are also types which are merely possible; they do not exist in this world, but they exist in some other possible worlds in which they have their tokens. Thus, types are contingent entities.

If we accept the present account, a triangle as a geometrical object is a type entity whose existence depends on the existence of its tokens. They are physical things like a drawing on a paper or a pattern of lights on a screen, and hence are contingent beings. It follows that a geometrical object is also a contingent being.

But, is this right? Triangles do not constitute a single type. There are infinitely many types of triangles which are different from each other. There are an equilateral triangle, various types of isosceles triangles, right triangles, and those types of triangles which do not have any particular names. If there are infinitely many of them, isn't it certain that there are some types of triangle which have no tokens in our world? Then, doesn't it mean that not all the triangles exist and there are some triangles that do not exist?

Or, it may not. Some might argue like this; let us draw a circle with O as the center and OA as a radius; take any point P on the half of the circumference which ends at A on one side and consider the ray *l* from O that crosses this part of the circumference at P; then, take any point Q on *l* and form a triangle OQA; varying P and Q, you will get an infinite number of different triangles. Thus, it might be concluded that the existence of infinitely many different triangles should not preclude that each of them has a token in this world.



However, this argument is wrong, so I argue. Tokens must be some concrete entities. A typical token of a triangle is a visible and/or tangible object, although we are going to consider later the tokens of a triangle which are too huge or too small to be visible or tangible. It may be something that is made by us like a drawn figure on a paper or something that is found in nature. In the latter case, such a natural object should have some material characteristics by which we will recognize it as a token of a triangle. For a type to exist it is necessary and sufficient that it has a token; it goes without saying that a token should be an actual one and not a merely possible one; a type with merely possible tokens is not an actual type but a merely possible type.

Let us go back to the argument above. It seems to give us a device to generate a token for each of the infinitely many triangles. But, why do we think this device should work? Are we sure that we will be able to construct an infinite number of tokens that are different from each other in such a way? If we remember that tokens should be concrete things, then it is almost obvious that we can do no such thing. All we can do is to construct just a small number of different triangles, however hard and however long we might try.

A token of triangle must be something that is constructed by us or recognized by us to be a token of a triangle. Given that the human beings do not exist forever and even the universe does not exist forever, the tokens of triangles in the entire history of our universe are only finitely many. This means that there are only finitely many tokens of triangles and only finitely many different triangles have a token.

Now we have the following two claims.

> (I) The existence of a type is a contingent matter, because the existence of a type depends on that of its token, and the existence of a token is a contingent matter.
>
> (II) There are an infinite number of different triangles, but there are only a finite number of the tokens of triangles.

From these we must conclude that which type of triangle exists is a contingent matter. This means that it is a contingent matter whether a certain geometrical objects exists or not. Isn't it an absurd conclusion?

7. Geometry is the science of possible types

7.1. From actual to possible

To this worry, we can answer in the following way. The objects of geometrical investigations should not be restricted to the geometrical types which exist in this world, namely, those types which are instantiated in this world. Geometry is concerned with the totality of possible geometrical types. It seems reasonable to assume that for any geometrical type there is some possible world in which its token exists.

At first sight, this seems to give us a rather bizarre picture. Which type of triangle exists is different in different possible worlds. There is even a possible world in which there does not exist any type of geometrical object because it contains nothing that can be a token of any geometrical object.

However, I don't think this constitutes an objection to our account. In my view, geometry aims to establish the truths that hold with any possible type of triangle; and in such an enterprise, it does not matter whether a type of triangle exists in the actual world or only in some other possible world.

According to one well-known story, geometry originated in Egypt where a land survey was frequently necessary after the floods of the Nile, and such an origin is contained in the word "geometry" itself. Though I don't know whether there is any truth to such a story, it is a reasonable hypothesis that at first geometry was concerned with the features of the concrete things like the shapes of lands. In this early phase, the objects of geometry were the actual types which were instantiated in the tokens that were found in our environment. But, when geometry became a mathematical science, its concerns extended from the actual types to all the possible types.

Suppose there is a certain discipline that is concerned with some empirical phenomena. For such a discipline, one way of going mathematical is to consider systematically all the possible varieties of the phenomena whether they are encountered in reality or not. For example, linguistics is supposed to be an empirical science that is concerned with all the languages that once existed in the past, exist in the present, or will exist in the future. Still, it is concerned with only actual languages. Although "mathematical linguistics" means in reality various things now, it might have been used to indicate a discipline that tries to give a systematic account of all the possible languages whether they are real or not. The same thing can be said with geometry; if geometry is a mathematical discipline as we think nowadays, it is concerned with the totality of possible types, including those merely possible types which do not have tokens in our world.

Thus, geometry is not bound to some particular possible world like the actual world. For, it is concerned with any object existing in some possible world. A geometrical object need not exist in all possible worlds. It may not exist in some possible world. Hence, a geometrical object is not a necessary being in the sense that it exists in all possible worlds. I think this is a rather welcome consequence, because there is something fishy in the idea of an object existing in all the possible worlds.

7.2. On the idea of an object which exists in all possible worlds

What is wrong with such an idea? It is because those things which are supposed to exist in all the possible worlds are often abstract entities like properties and numbers and they do not seem to contribute anything to the "worldliness" of the world. In other words, they do not seem to be the kind of things whose existence is essential to the existence of a world.

In general, a property P is said to be an essential property of an object a, when the following holds.

(1) If a does not have P, then a does not exist.
Or,
(1') If a loses the property P, then a ceases to exist.

It seems that the existence of numbers is an essential property of the world in this sense. For, it is usually thought that the following is true.

(2) If there is no numbers in the world w, then w does not exist.

But, why do we think that (2) is true? The only reason to do so is that it is widely believed the antecedent of (2) is necessarily false. However, isn't it obvious that the truth of (2) has nothing to do with the nature or essence of a world. Moreover, it is well known that there is a philosopher who asserts the truth of the antecedent of (2) (Field 1980). Does such a philosopher conclude that our world does not exist? Obviously not. Thus, there would be no reason to believe (2).

The situation might be different with a god or the God. If you believe in such a being, then you may also believe the following.

(3) If there is no god in the world w, then w does not exist.

Why does it seem plausible to someone who believes in a god? She might hold that the world is created by the god and hence the existence of the god is necessary for the world to exist. But, if she also believes that it is impossible for anything to create itself, then she will conclude that the god cannot be a part of the world which it created and the god should exist outside of all the possible worlds.

Or, she might believe that the world is kept existing by the god. There seems to be no absurdity in holding something is kept existing by its part. But it is an entirely different matter whether there is any good reason to believe that the existence of the world is necessarily tied up with the existence of a god in that world.

There is another reason to be suspicious of the idea that there are some objects which exist in all possible worlds. It seems to make sense to ask the question "Why is there anything rather than nothing?" as many philosophers have done. But, if there are anything which exists in all possible worlds, then such a question is ill-posed because its presupposition is false. Of course, we cannot be sure that this traditional question really makes sense. For, there are many questions which seemed to make sense on the surface but turns out to be nonsense.

Or, the question does make sense and it asks whether the totally empty world is possible as far as we know. In that case the question presupposes only an epistemic possibility of the world which contains absolutely nothing. If we could find some substantial argument against such a possibility, that would be an interesting philosophical discovery. But, if we say that there is no such absolutely empty world because numbers and properties exist in all possible worlds, that will be a disappointingly uninteresting response.

Why do we suppose that geometrical objects exist in all possible worlds (although I believe that a similar argument can be advanced for mathematical objects in general, my present discussion is concerned only with geometrical objects)? I believe that it is because we suppose the following three statements are true.

(4) There are geometrical statements which assert the existence of geometrical objects.

(5) Geometrical statements are necessarily true.

(6) If a statement is necessarily true, then it is true in all possible worlds.

It must be easy to see how we come to believe that geometrical objects exist in all possible worlds if we suppose these three statements are true.

What is wrong with such an argument? I claim that if the meaning of "necessarily true" is defined in terms of possible worlds as in (6) then geometrical statements are not "necessarily true".

As I said before, geometry is concerned with all possible types. It is concerned with any geometrical type which exists in any of the possible worlds. It is not much interested in whether some geometrical types exist throughout all the possible worlds, because it is very likely there are no such types; for, there might be some possible worlds which contain nothing that can be a token of a geometrical type. Hence, a geometrical truth is not something which is true in all possible worlds; rather it is a truth which transcends them in the sense that the domain of geometrical consideration is not restricted to any one of them but the sum total of the domains of all the possible worlds.

7.3. "A geometrically perfect world"?

We have claimed that geometry is concerned with all possible types. Then, in order to see whether a certain triangle type is possible or not, we have to search for a possible world which contains its token throughout the totality of the possible worlds. Or, so it seems. Isn't there, however, a possible world in which all the possible triangle types have their tokens. Of course, we know that our world is not such a world, but if there were a world where some intellectual and immortal beings constantly engage in geometrical activities on a planet in the everlasting physical universe, there would be a token for each of the infinitely different triangles. So, instead of considering the totality of possible worlds, it might be enough to consider a single possible world in which all geometrical types have their tokens.

We may say that here is at least another solution to our problem of type existence. This is also an alternative way to conceive the ideal nature of geometry. At any case, our account that has recourse to the totality of possible worlds is only a picture. Some might object that even the everlasting geometrical activity may not produce all the necessary tokens. But whether such an objection applies or not depends on how such a "geometrically complete" world is conceived. Considering the whole extent of possible worlds does not automatically solve a similar problem, either. If we wish to be sure that the totality of possible worlds offers us enough tokens for our geometry, we should think hard about several things, starting from the total number of possible worlds.

Now we have two ways to cope with the problem of type existence. According to one, the geometry in the sense of mathematical geometry is concerned with any possible geometrical types whether they have tokens in the actual world or not. According to another, geometrical truths are actual truths in a geometrically perfect world. However, the latter claim is stronger than the claim that geometrical truths are truths holding with respect to all possible types. For example, let us suppose for the moment that the following is a geometrical truth.

(*) For any circle, there is a larger circle with the same center.

According to our original account, (*) is paraphrased thus.

> (*1) For any circle type C, if there is a world w such that C exists in w, then there is a circle type C' and a world w' such that C and C' exist in w' and C' is a larger circle than C and has the same center as C.

If we cash out what the existence of a type means, then (*1) comes to

> (*1') For any circle type C, if there is a world w such that C has a token in w, then there is a circle type C' and a world w' such that C and C' have their tokens in w' and C' is a larger circle than C and has the same center as C.

In contrast to these, the alternative account interprets (*) in the following way.

> (*2) There is a possible world G such that there exists a circle type in G and for any circle type C in G there is in G a larger circle type C' with the same center as C in G.

which is the same as the following.

> (*2') There is a possible world G such that there exists a circle type in G and for any circle type C in G there is in G a token of a circle type C' which is larger than C and has with the same center as C.

(*1) is a weaker claim than (*2) because (*1) follows from (*2) but the converse does not hold. In particular, if you accept (*2), then you should commit to the existence of a possible world which contains an infinite number of circle tokens with different sizes; (*1) does not implicate you in such a commitment; it still commits us to the existence of an infinite number of circle tokens with different sizes, but they need not exist in a single world; they may be distributed to the whole universe consisting of all the possible worlds.

In short, we may conclude that there are no reasons to favor the alternative account to our original account. First, there is no positive reason to believe that there exists a geometrically possible world. Second, there is no need to believe in such a world in order to account for the existence of the possible but not actual geometrical types.

8. How do we know a certain geometrical type is possible?

8.1. The limits of perceptual imagination

Now, a fundamental question is this: how do we know that a certain geometrical type is a possible one? Or, how do we know that there is a possible world which contains its token, if we do not have any real token of that type?

Of course, if we have some concrete drawing or picture which shows us what a type is, we have actually its token and we know that the type is possible. So the cases we should consider are those in which we have only a description of a purported type of geometrical nature. How do we know that it is a possible description of a geometrical type? If we can produce its token according to the description, then we know that it has a token and hence is a possible type. But, if we don't or can't do such a thing, how do we know that it can have a token?

The problem is concerned with the possible existence of an entity with some specification. What sort of entity should it be? As we have emphasized, a token is a concrete being that exists at a particular time and place, in contrast to a type which is abstract and does not exist in space and time. Hence, a token of a geometrical type should be a physical object or some of its physical features.

Let us take up the proposition (*) of the previous section. Why do we think this proposition is true? How are we going to justify it?

Suppose that we draw a circle on a paper. How can we be sure that there is another circle which is larger than it and has the same center? We might draw such a circle on the same paper. But, suppose that the circle we drew filled the entire paper and there is no room for another circle. We may imagine that the paper were a little larger than as it actually is and that there were another circle that is larger than the first one. In Ohmori's terminology, we have formed a thought of possible perception of a pair of concentric circles. Alternatively, in a possible world talk, we have imagined a possible world w which is the same as our world except that it contains a paper that is a little larger than the actual one and has two concentric circles on it.

Let us consider the larger of the two concentric circles. If (*) is true, then there must be another possible token of the still larger circle. We can easily imagine that there were another circle drawn on the same paper as the one in w or on some different paper which is larger than the paper in w. What we have done this time is to imagine a possible world w' which is different from our actual world and also from w.

Can we go on in this way to verify the truth of (*)? The answer is obviously "no". In some point, we have to consider an object which is too big to be perceived as a whole. Then, we can imagine at most only a part of a circle. But how do we know that what we have imagined is a part of a circle? If a circle is of cosmic size, what we imagine as a part of its circumference will not be distinguished from what we imagine as a part of some straight line.

When we are asked whether some geometrical type can have a token, we usually try to answer it by seeing whether we can draw a figure or find a middle-sized object with the specified shape. In such cases, we can be reasonably sure that it has a token if we can perceptually imagine an object which agrees with the specification given in the description of the type.

Of course, we sometimes misperceive the shape of an object just as we sometimes misidentify a speech sound or a letter. The fact that the object of our perception is an abstract object does not exclude the possibility of perceptual mistakes. It is possible to have an illusion about abstract objects; we think that we are perceiving a circle when we are looking at a many sided polygon. It is also possible to have a hallucination of them; when we think that we are perceiving a token of some complex geometrical object, there cannot be such a geometrical object in reality; if you want an example, then you can find them in many of Escher's drawings; they are full of things which are purported to be the tokens of such impossible geometrical objects.

Hence, the fact that we can perceptually imagine something which meets the description of a type does not give a conclusive evidence that there can be its token. But it gives us at least a *prima facie* reason to suppose that it is a possible type. In sum, when we try to see whether a given description of a type can have a token, we rely on some principle like the following.

(P) It is true in most cases that if we perceptually imagine an object which satisfies a description D, then D is a description of a physically possible entity.

But, if we wish to consider how the things stand in general, we should be prepared to encounter the situations which are beyond our perceptual capacities. As our power of imagination goes only as far as our perceptual ability, in such situation, we can not make use of the principle like (P) to see whether a given description is one of a physically possible object.

The things and events of cosmic scales are often beyond our perceptual and imaginative powers. Similarly, our perception and imagination fail us with respect to extremely small things and what is happening among them. Consider the following proposition which is similar to (*).

(**) For any circle, there is a smaller circle with the same center.

It is obvious that the truth of (**) cannot be ascertained by perception and imagination alone, if (**) implies that for any physically possible token of a circle there is another physically possible token of a smaller circle.

8.2. Geometry as a part of a physical theory

These are the cases where our thoughts of possible perceptions are of no use to see whether something is physically possible. What should we do then? What do we have in order to judge something is physically possible besides our limited power of imagination? An evident answer is that we have a physical theory to tell whether something is physically possible or not, that is, we should adopt a principle like this.

(T) If T is a true physical theory and the existence of D does not contradict with T, then D is a description of a physically possible entity.

Although this is a most straightforward way to determine whether a given description is that of a physically possible entity, there seems to be a problem if the description in question is purported to be that of a geometrical type. For, it seems that any physical theory requires to account for the spatial structure of our universe, and hence, a system of geometry is an essential part of any physical theory.

Now, the path we have followed seems to be a very curious one. We started with the observation that the existence of a geometrical type depends on that of its tokens, which should be physical in nature. If we wish to show some geometrical type is possible, we should somehow demonstrate that its token is physically possible. In everyday context, it can be done by imagining possible perceptions of such a token. However, this method does not work at all in the cases which are beyond our perceptual and imaginary capacities. So, finally we made an appeal to a true physical theory as the ultimate arbiter of the physical possibility. But, here we face a surprising turn of events, which is that in making an appeal to a physical theory we are also making an appeal to a system of geometry which is a part of the theory. It seems that we have come back to our starting point after all. And, doesn't this mean that we should possess the knowledge of what geometrical truths are in order to acquire that very same knowledge?

If you think that our account is circular in this way, then it is because I have not yet told a fuller story. As a matter of fact, I am going to literally tell a story. It is a fictionalized story of how our geometrical knowledge might have developed from a humble origin to a highly sophisticated discipline.

> Our interests in geometry must have originated with the various but recurring shapes of the objects surrounding us. Through such experiences and perhaps owing to our innate capacity, we acquire the ability to recognize various geometrical types and talk about them. In this early stage of the development, geometry is concerned with only actual types, that is, those types which have actual tokens. But, it should not be very difficult to extend our geometrical beliefs to possible types by having the thoughts of possible perceptions of possible tokens of them. Still, it is fragmentary and lacks a systematic character. For one thing, we cannot form any definite thought about the figures which are too large or too small for our perceptual and imaginative ability. Hence, the next step is to organize various geometrical beliefs we came to have into a certain system. It is widely believed that the Greeks invented the axiomatic method to do just this.
>
> Once we have a system of geometry, it became a tool to give a systematic

> description of the space we are in. Thus, it forms an essential part of our physical theory. Although the system of geometry now covers the entire physical universe from the extremely small to the extremely big, it must agree with our everyday experience with the shapes of the objects for most of the part. In particular, the systematic geometry should agree with most of the judgments we make by our perception of those middle-sized objects among which we find ourselves. This means that there should not be any irreconcilable conflicts between the criterion (P) of physical possibility given by our perception and the criterion (T) given by a true physical theory.

I believe there is nothing absurd in such a story, and a similar story can be told with many fields of our intellectual activity. A theoretical interest is first aroused by everyday observation and after many years of the development the resulting theory is used in turn to refine our everyday judgments.

But, there are some reasons to suspect that the story is not really completed yet, for there still remain at least two worries.

First, do we have any reason to believe that every geometrical type we encounter in geometry has a physically possible token? Although any physically possible world is also a geometrically possible world, might not there be a geometrically possible but physically impossible world?

Secondly, even if we succeeded in arguing that geometrical possibility and physical possibility coincide and that every geometrical type has a physically possible token, we could not yet tell which geometrical type is possible. It is because there are a number of empirically equivalent physical theories with different geometries. The possible geometrical types are different according to different geometries.

Although they are serious worries for our account of geometry, I believe there is a way to respond to them. To do so will lead us to discuss some of the central issues in the philosophy of mathematics and that of science. Thus, if we wish to attend to the first worry, we should adopt a certain variety of modalism in the philosophy of mathematics originally introduced by (Putnam 1967), while it must be obvious that the second worry is very closely related to the still continuing debates about conventionalism in the philosophy of science. Unfortunately, I don't have a space to pursue these issues now. I leave it for another occasion.

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