Logical and Epistemic Modality

Hasen Khudairi

1 Introduction

This essay examines the interaction between the project of metamathematics and the cognitive science of concepts. The aim of metamathematics, i.e., the methodology of the deductive sciences, is to provide a precise characterization of the concepts figuring in rigorous, formal theories (Tarski, 1930/1983).[1] The concepts at issue include those familiar from metalogical inquiry – e.g., completeness and categoricity – as well as formal concepts admitting material conditions on their satisfiability, such as the concept of truth. The philosophical significance of the examination is that – while many formal properties can be readily accommodated within cognitive-scientific theories of mental representation, by, e.g., enriching the expressive resources of the languages in which the properties are defined – there are some ineliminable metamathematical concepts, the ontological commitments of which would appear to be more demanding. Whereas most concepts in the methodology of the deductive sciences are consistent with the extensional characterization of the mathematical languages at issue, the latter, ontologically inflationary metamathematical notions can be distinguished e.g. by their constitutively intensional properties.

In this essay, I examine, in particular, the interaction between the philosophy

[1] See also the project of reverse mathematics pursued by Friedman (1975/ms) and Simpson (1999).

and psychology of concepts and the modal characterization of the deductive – rather than inductive and non-monotonic – concept of logical validity.[2] The concept of logical consequence on which I will focus is model-theoretic, where the concept records the property of necessary truth-preservation from the premise of an argument to its conclusion, as well as the condition that, in the class of all possible worlds in which a premise is true, a consequent formula or succedent class of formulas is true, as well.[3] A formula which figures among the relata of the relation is a logical truth if and only if its truth is preserved under every assignment of values to the variables comprising the formula's syntactic form (cf. Tarski, 1936/1983; Sher, 1991: ch.3, 2001; Rayo and Williamson, 2003). Focusing on the case of logical necessity, I argue that the ability to account for the modal properties of concepts places a desideratum on the explanatory adequacy of theories of mental representation. I outline, then, five approaches to the nature of concepts pursued in philosophy and cognitive science, and argue that the above desideratum does not appear to be satisfiable by the candidate proposals, which take concepts to be structured and, on one of the approaches, to have a constitutively perceptual format. Given the limits of the foregoing, I appeal to a sixth approach – namely, cognitivism about epistemic modality, which takes epistemic possibilities to be intensional functions in a language of thought, consistent with an externalist conception of the etiology of information states – and endeavor to demonstrate how the proposal can sufficiently account

[2]The contention that both cognitive science and the nature of concepts comprising the language of thought are pertinent to the analysis of computation, reasoning, and validity is anticipated by Shapiro (1994). For a thorough examination of the nature of higher-order logical consequence and the role of models and modality therein, see also Shapiro (1998). Khudairi (ms_1) examines the modal profile of higher-order logical consequence, by targeting, in particular, the nature of generic absoluteness and necessary truth-preservation expressed by the relation of Ω-logical consequence in set theory (cf. Woodin, 1999, 2010; Bagaria et al., 2006).

[3]For further characterizations of model-theoretic logical validity, see Shapiro (1998: 132). Shapiro notes that there are modal and epistemic interpretations of model-theoretic validity, though does not endeavor to account for their convergence.

for the modal profile of the concept of logical consequence, and thus provide the resources necessary for a metamathematical analysis of the target notion in the setting of cognitive science.

In Section **2**, I provide a taxonomy of the extant approaches to the nature of mental representation, and argue that only the externalist information-theoretic approach, which converges with a cognitivist approach to epistemic modality, is sufficient for satisfying the possession and understanding conditions for the concept of logical consequence.[4] In Section **3**, I examine the challenge from Curry's paradox to the analysis of logical consequence as necessary truth-preservation, and demonstrate how epistemic modal algebras can be marshalled in order to provide an epistemicist resolution to the paradox, which enables the retention of both classical logic as well as the introduction and elimination rules for the truth predicate. In Section **4**, I examine alternative approaches to the epistemology of logical laws, arguing that the epistemic modal approach here advanced is consistent with approaches to logical knowledge which proceed via considerations of abductive methodology, and fairs better in response to challenges to which non-cognitivist, entitlement-based views risk being susceptible. Finally, I demonstrate how epistemic modal algebras can countenance the modal profile of rational intuition – where the notion of intuition-that is treated as a modal operator – and thus provide rigorous foundations for the premonition that the justification of logical laws might partly and defeasibly consist in their instantiation of a 'compelling' phenomenal property which presents their content as being true. Section **5** provides concluding remarks.

[4]See Edgington (2004: 10), for an incipient suggestion that formal and informal validity ought to be characterized as epistemic necessity, i.e., as an apriori consequence relation.

2 Logical Necessity and the Nature of Concepts

In the philosophy and psychology of concepts, concepts have been countenanced as terms whose reference relations may fall under at least five different types.

According to the first two approaches to the nature of concepts, concepts are abstract mental representations, which have the syntactic structure of nominal signs. The first approach takes the relation between concepts, expressed by the lexical signs, and the objects and properties that fall in the extensions thereof, to be direct. Thus, instances of mental concepts are taken directly to refer to their extensions, where the relation has been interpreted as being causal and subserved by nomological laws (Fodor, 1975; 1980; 2008).

The second type of concept construes the latter as compositional functions, which may be concatenated in order to take a form assessible for truth or falsity. Whereas the designation relation in the first approach delineated above takes it to be immediate, such that the nominal signs are, as it were, labels for the entities that they designate, the second approach takes the latter to include both a sense and a reference (Peacocke, 2008: ch. 4). The distinction between sense and reference has traditionally been witnessed by cases in which two formulas have the same extensions – e.g., '$x^2 \wedge x = 2$' and '$2x \wedge x = 2$' – yet the different modes of presentation for the formulas express a difference in their senses, i.e. their cognitive significance (cf. Frege, 1892/1997). The second type of concept records, then, the foregoing distinction between cognitive significance and reference, by identifying senses with the interpretations of the syntactic signs at issue.

The third and the fourth approaches to the nature of concepts derive from experimental psychology and cognitive science. The third approach to the nature of concepts construes the latter as prototypes. Prototypes are interpreted

as being models of classification, where the condition on inclusion in the target class is a statistical similarity relation (Machery, 2009: 84-92). Deployments of the foregoing concept-type might concern, e.g., the categorization of an entity in one of two categories on the basis of its geometric configuration.

The fourth approach countenances concepts as types of 'examplars', where examplars are bodies of information whose extensions are individuated by acquaintance with unique, paradigmatic instances. Conditions on falling under the examplar-based concept-type are taken to be implicitly measured by distance metrics between the value of the apparent property and its normalized, optimal value (op. cit.: 93-100). Exemplars may also be expressed as nominal terms – e.g., the concept, 'desk' – where instances of desks figure as the paradigms constitutive of the optimal values in the relevant distance metric.

Finally, a fifth approach to the nature of concepts takes concepts to be 'proxytypes'; i.e. a collection of information states that are unique to a sensory modality, and which are thus constitutively perceptual (Barsalou, 1999; Prinz, 2002: ch. 6; Machery, 2006).

Of the foregoing proposals concerning the structure of mental representations, only the first, direct reference approach and third, prototype-based approach would appear to be constitutively related to modal properties. The first approach takes mental representations to be defined as computational functions and syntactic lexical signs which satisfy conditions on compositionality, and whose semantic content is either implicit or externally individuated via causal and nomological relations to the agent's environment. The nomological relations between the syntactic lexical items comprising the formulas in the language of thought and the objects and properties in the agent's environment that the lexical items denote have a modal profile. Thus, for a syntactic sign, 'π', and an

object, o, there is a reference relation, R, such that it is a nomological (i.e., physical) necessity that 'π' refers to o: $\forall\pi,o\exists$R[$\Box$(Rπ,o)]. The third approach, by contrast, defines prototype classification on the basis of a statistical similarity relation, where the probability function required to define the relation will be defined on a state space, i.e. a set of empirical, i.e. physical, possibilities.

The modal properties of the first and third approaches would appear, however, to be orthogonal to the modal profile of logical consequence; i.e. necessary truth-preservation and the truth of the conclusion in all possible models in which the premises are true. The modal properties of the first approach target nomological modality – i.e., physical necessities – unique to the relations of reference between signs in a language of thought and objects in the agent's environment. Thus, the first approach targets the wrong interpretation of the modal operators. The nature of the truths at issue are crucially distinct: While logical truth is defined as the preservation of the truth of a formula under every assignment of values to the formula's variables, it is a platitude that the modal profile and possible truth of physical equations depend on what values are taken by the variables of which those equations are comprised.

One might endeavor to accommodate modality within a syntactic account of concepts by arguing that there are 'semantic requirements or necessities', which figure in the determination of 'de re semantic facts' (cf. Fine, 2010: 66, 80). With $\Box_s$ a semantic necessity, it would then be 'required of an object' – e.g., Cicero – that '$\exists$x[x = Cicero and $\Box_s$('Cicero' refers to x)]' (op. cit: 76). The rigidity with which 'Cicero' refers to the corresponding individual in all possible situations can then be grounded in its being a de re semantic necessity of the individual that the name refers to them. In the setting of concepts rather than natural language expressions, however, it is unclear (i) why it ought to be a de

re necessity of any individual whether they fall in the extension of any concept, and (ii) whether the de re modal properties of objects – beyond the necessity of identity [$\Box\exists$x(x =x)] and the necessity of being [$\forall$x$\Box\exists$y(x = y)] (cf. Fine, 2009: 19, for the example)– can suffice for the treatment of the necessity of logical validity.

The modal properties of the third approach target a probability distribution over the possible points comprising a state space. However, the existence of a probability distribution over a set of worlds is similarly insufficient to countenance the modal profile of the notion of logical validity, because – while one point in the space might necessitate the obtaining of a second point in the space, such that the second point is accessible to the first and thus true in whatever models in which the first might be – it is still metaphysically possible for the physical laws to have a different value. The possibility of entraining a distinct semantic value would thus belie the requirement of necessary truth-preservation, as a constitutive condition on the relation of logical consequence.

Despite the limits accruing to the foregoing approaches, a sixth proposal may yet be advanced, which construes models of mental representation as being unstructured. Because the sixth approach targets abstract models of cognitive architecture which are implemented in physical systems, it may be referred to as 'cognitivist'. The sixth approach takes propositions which comprise the language of thought to fall under the scope of epistemic modal operators. The cognitivist approach to epistemic possibility models the structure of mental representations as consisting in a set of epistemically possible worlds. The epistemic possibilities can be identified with an agent's states of information, broadly construed. The states of information can further concern what is conceivable or imaginable to the agent ($\diamond$), or they might concern what is apriori to the agent ($\Box$). Classical

duality axioms may then be specified, such that an epistemic state is possible or conceivable to the agent ($\diamond$) if and only if nothing rules it out apriori ($\neg\Box\neg$), and an epistemic state is apriori for the agent ($\Box$) if and only if it is inconceivable for it to be false ($\neg\diamond\neg$).

According to a uniquely algebraic approach to epistemic modality, the elements of the algebra elide formulas with terms. Frames thereof are comprised of both a set of epistemic possibilities and an accessibility relation thereon. In topological Boolean-valued modal algebras, the relation is interpreted as concerning the interior region of a space; the possibilities accessible from a designated point in the space are said to be open; and the points in the space may be mapped to corresponding regions of the algebra. In virtue of the foregoing elision in the elements of modal algebras, the modal operators taking scope over the algebra's formulas may further be identified with the monotonic, intensional functions on the algebra's terms. Crucially, then, epistemic modality can take the form of intensional functions. By reconstruing epistemically possible worlds as monotonic, intensional functions, epistemic modality may thus be argued to comprise the constitutively semantic, syntactic constituents of a language of thought, such that the epistemic intensions can thereby comprise the constitutive structure of formulas in hyperintensional contexts. The states of information over which the possibilities take scope are further amenable to external individuation [see Khudairi (ms_2)].

The empirical adequacy of epistemic modal algebras can be witnessed by their role in modeling: (i) modules in the visual system, in the computation of visual constancies – i.e., the likelihood that one among a set of psychophysical possibilities is the actual world, which places an accuracy condition on the at-

tribution of properties such as boundedness and volume to distal particulars[5]; (ii) the speech acts, such as imperatives, which have been modeled by modal operators in the program of natural language semantics (cf. Kratzer, 1977; Stalnaker, 1978; Heim, 1992); (iii) quantum information theory, where the elements of the algebra are spin-state vectors whose inputs are qubits and whose kinematic values are assigned by probability functions defined over a distribution of broadly epistemic possible worlds (cf. Ruetsche, 2011; Bub, 2011; Timpson, 2013); and (iv) propositional attitudes, such as the types of intentional action [cf. Khudairi (ms_3)]. By availing of the resources of the univalence axiom from the homotopy type-theoretic foundations of mathematics – where the latter collapses isomorphism with the identity relation – an abstraction principle for the computational intensional functions at issue can further be specified, and thus establish a conduit for knowledge thereof [cf. Khudairi (op. cit.)].

The dual representations of Boolean-valued algebraic structures are coalgebras (cf. Venema, 2007: 417). A coalgebra is a labeled transition system, comprised of both a set of possibilities and a functor, referred to as a carrier, on categories (op. cit.: 390). Coalgebras provide a natural setting in which to define modal logics (406). A coalgebraic modal logic can be defined as a model, M, to be comprised both of a set of possibilities and what is termed a Kripke functor, σ_i, which combines the accessibility relation from the frame over which the model is defined with the valuation function from the target model (409). A modal coalgebraic automata is defined, then, as a tuple, $\mathbb{A} = \langle \mathrm{A}, \mathrm{C}, \sigma, \mathrm{F} \rangle$, such that A is the state space of the automaton $\mathbb{A}$; C is the coding for the automaton's alphabet, mapping numerals to properties of, e.g., the natural numbers; σ: A X C $\rightarrow$ A is a Kripke functor, i.e. transition function, and F$\subseteq$ A is the

[5]Cf. Mamassian et al. (2002); Burge (2010); Rescorla (2013).

collection of admissible states, where F maps A to {1,0}, such that F: A $\rightarrow$ 1 if a$\in$F and A $\rightarrow$ 0 if a$\notin$F (407).

Epistemic modal algebras and their dual representations in the guise of modal coalgebraic automata are the remaining candidates for modeling mental representations, in a manner that might able to account for the modal profile of the concept of logical consequence. The relation of logical validity can be defined in epistemic modal space, such that – when one advances an argument in the course of, e.g., decision-making – necessary truth-preservation may be readily modeled in the space of epistemically possible worlds. Logical validity can then be defined as the Kripke functor, i.e. the transition function, of modal coalgebraic automata, such that there is no drop in truth value between the inputs to the function and its outputs. Taking, then, the relation of logical validity to have the form of the material conditional, consequence may be defined via the model-theoretic specifications of the logical constants which comprise the relation. Thus:

$\phi \Vdash \psi \iff \neg\phi \vee \psi$,

$\langle \mathbb{A}$,F$\rangle \Vdash \phi$ iff $\phi\in$F,

$\langle \mathbb{A}$,F$\rangle \Vdash \neg\phi$ iff $\phi\notin$F iff it is not the case that $\langle \mathbb{A}$,F$\rangle \Vdash \phi$,

$\langle \mathbb{A}$,F$\rangle \Vdash \phi \vee \psi$ iff $\langle \mathbb{A}$,F$\rangle \Vdash \phi$ or $\langle \mathbb{A}$,F$\rangle \Vdash \psi$;

$\langle \mathbb{A}$,F$\rangle \Vdash \phi \equiv \emptyset \Vdash$ '$\langle \mathbb{A}$,F$\rangle \Vdash \phi$'.

Because the present approach to the epistemology of logical laws relies on the recursive clauses in the model-theoretic definitions of the logical constants, it converges with approaches to logical knowledge which are both propositional (cf. Besson, 2010), as well as meaning-constituting (cf. Peacocke, 1992; Boghossian, 2003). As noted, however, the approach diverges from the above, by being able to characterize the modal profile of the concept of logical consequence via the

approach to concepts and cognition which stipulates that mental representations take the form of epistemic modal algebras and the modal coalgebraic automata which comprise their dual representations.[6]

In the following section, I examine a challenge to the claim that a necessary condition on logical consequence is the property of necessary truth-preservation. In section **4**, I compare the present approach to competing proposals that have been advanced in the literature, and argue that the cognitivist approach to epistemic modality is able to circumvent issues that might beset non-cognitivist views of logical justification, while being consistent with theories which draw on considerations from both abductive methodology as well as rational intuition.

3 Curry's Paradox

In this section, I examine an argument that Field (2008) marshalls against the identification of logical consequence with necessary truth-preservation, in virtue of Curry's paradox. The paradox takes several forms, although I will focus in what follows on the version proffered in Field (op. cit.: 83).

The paradox runs as follows. Given the normal truth rules, Truth-Introduction ($\phi \vdash \mathrm{T}\phi$) and Truth-Elimination ($\mathrm{T}\phi \vdash \phi$):

$\forall \neg \mathrm{T}\phi$:

1. $\phi \iff (\mathrm{T}\phi \rightarrow \bot)$.

By T-Introduction,

2. $\mathrm{T}\phi \iff (\mathrm{T}\phi \rightarrow \bot)$.

[6] For a non-algebraic, epistemic-modal approach to countenancing the invariance of logical constants and the modal profile of logical consequence, see McCarthy (1981: 516-523; 1987: 439-441). For a defence against a counterargument proferred by McCarthy (1981: 514) – to the effect that knowledge of metalinguistic structural rules for the logical constants is aposteriori – see Sagi (2015: 165-166). Sagi (op. cit.) argues that grasp of metalogical structural rules is akin to the sociological acquisition of natural language, where the latter plays an enabling – rather than evidential – role, consistent with implicit apriori knowledge.

Left-to-right,

3. $T\phi \rightarrow (T\phi \rightarrow \bot)$.

By importation,

4. $T\phi \wedge T\phi \rightarrow \bot$.

By contraction,

5. $T\phi \rightarrow \bot$.

From 2, right-to-left,

6. $T\phi \rightarrow \bot \rightarrow T\phi$.

From 5 and 6,

7. $T\phi$.

From 5 and 7,

8. $\bot$.

Field presents the foregoing as a dilemma with regard to the classical analysis of logical consequence. Given that, by the normal truth rules and classical reasoning, one can derive a contradiction from a false premise, one ought then either to eschew of one of the normal truth rules or revise one's logic. However, the horns of the dilemma may be circumvented by taking an approach to the paradox which resolves it by availing of the sources of epistemic logic. In particular, Khudairi (ms_3) demonstrates that line 5 in the derivation of the paradox provides a counterexample to modal axiom K: $\Box(\phi \rightarrow \psi) \rightarrow [(\Box\phi) \rightarrow (\Box\psi)]$. The relevant instance is then:

(*) $K[T\phi \rightarrow \bot] \rightarrow [K(T\phi) \rightarrow K(\bot)]$.

Because only truths can be known (as recorded by modal axiom T: $\Box\phi \rightarrow \phi$), $K(\bot)$ is false. Thus, the conditional proposition in the consequent of (*) has a false consequent. So the conditional proposition comprising the consequent in (*) is false. Because (*) has a false consequent, yet its antecedent $[K[T\phi \rightarrow \bot]]$ is

true in virtue of the derivation of the paradox, (*) is itself false. Because modal axiom K is the formal characterization of the notion of epistemic closure, and line 5 provides a counterinstance thereof, the derivation of Curry's paradox is thus not closed under known entailment, and thus exhibits epistemic indeterminacy. So, the dilemma between having either to eschew of one of the truth rules or restrict the laws of classical logic can be avoided.

4 Variations: Entitlement, Abduction, and Modal Rational Intuition

There are three, further approaches which converge with the approach to logical knowledge here advanced. The first is a non-cognitivist approach which makes implicit appeal to the notion of possible worlds. The second is an approach to the epistemology of logical laws which proceeds via an abductive methodology. The third argues, finally, that logical laws impress as being true, because their proof-theoretic characterization exhibits a unique phenomenal character.

4.1 Non-cognitive Epistemic Entitlement

The fundamental epistemic notion pursued by Wright (2004,a) is the notion of epistemic entitlement, i.e. a default justification, in principle explicable by an agent, for the truth of a proposition that is accepted by that agent. The primary type of entitlement that Wright outlines is referred to as an entitlement of cognitive project (op. cit.: 188). The entitlement has the form of a default presupposition, and is defined as follows: 'P is a presupposition of a particular cognitive project if to doubt P (in advance) would rationally commit one to doubting the significance or competence of the project' (2004,b: 163).

The foregoing notion of epistemic entitlement, as a default apriori warrant to pressupose or 'rationally trust' that a proposition is true (2004,a: 194), is anticipated by Wittgenstein's (1969: 344) notion of what he terms 'hinge' propositions, where the justification to believe hinge propositions is precluded from doubt and serves as an epistemic foundation for various domains of inquiry. Wittgenstein's examples of hinge propositions include, e.g., propositions about the concrete existence of objects over non-negligible intervals of time (92, 138) and Wright applies his rendition of the notion to the consistency of the theories in which Gödel's (1931) first incompleteness theorem is defined (1995: 91, ftn.9); knowledge of induction (2004,a: 183-184); perceptual knowledge of the external world (2004,a: 186; 2007); default apriori justification to believe the truth of abstraction principles in the abstractionist/neo-logicist foundations of mathematics (Hale and Wright, 2009); and to knowledge of logical laws, such as modus ponens (ϕ, $\phi \rightarrow \psi \vdash \psi$) (2004,b: 167).

On Wright's account, the source of the default apriori entitlement to accept that the propositions crucial to the foregoing are true is, as noted, an agent's non-cognitive 'rational trust' (2004,a: 194).[7] The rational trust which sanctions the default apriori presupposition that the propositions in the target domains delineated above are true is 'non-cognitive', by being in some sense owing to the behavioral and sociological proclivities of the agent, rather than owing to the agent's cognitive architecture. In Wright (2014), he further elucidates the notion of rational trust, by defining it within the setting of decision theory. Rational trust is claimed to consist in the expected epistemic utility for an agent in believing that a proposition is true (op. cit.: 241-242).

In the setting of decision theory, beliefs are codified by subjective probability

[7] Wright notes of the notion of belief that it comprises a 'sub-case' of the 'more general attitude' of acceptance (op. cit.: 177).

measures and desires are codified by utility functions. An agent can define utility values with regard to the outcome of their actions, and a preference relation can then be defined on the space of outcomes. Evidential expected utility is calculated as the product of the agent's subjective probability measure – conditional on the agent's action – and their utility function. Causal expected utility is calculated, by contrast, as the product of the agent's subjective probability measure – conditional on both their act as well as background knowledge of the causal efficacy thereof – and the agent's utility function. A final distinction concerns pragmatist or preference-based, by contrast to alethic or dominance-based, approaches to the notion of epistemic utility. Pragmatist approaches to epistemic utility proceed by defining a representation theorem for an agent's preferences, from which the agent's probability and utility functions can then be derived (cf. Ramsey, 1926; Savage, 1954; and Jeffrey, 1965). Alethic approaches to epistemic utility eschew, by contrast, of representation theorems, and instead measure the accuracy of a partial belief via a distance metric between the agent's subjective probability measure and a vindicated, ideal credence that they ought to adopt (cf. Joyce, 1998; Leitgeb and Pettigrew, 2010; Moss, 2011; and Pettigrew, 2014). Epistemic utility consists then in the minimization of alethic inaccuracy, i.e. endeavoring to lessen the distance between the vindicated credal state and one's own. However, if the agent's credence measure does not satisfy the Kolmogorov axioms, then it will always be the case that a distinct credence will be closer to the ideal vindicated credence than one's own.

In virtue of the foregoing, the notion of expected epstemic utility that is targeted on Wright's approach is presumably the so-called evidential, rather than causal, interpretation, given the role of entitlements in accounting for the epistemic status of abstraction principles in the philosophy of mathematics, as

well as the epistemic standing of logical laws. Furthermore, Wright's approach elides the pragmatist and alethic approaches to epistemic utility, in order to analyze the notion of rational trust (2012: 484; 2014: 240): On Wright's proposal, rational trust consists in an agent's preference to reduce their gradational alethic inaccuracy.

While the general contours of the foregoing project are sufficiently defined, it remains to be shown, however, how to apply Wright's account of rational trust to the epistemic entitlement with which an agent is possessed to cases of perceptual, logical, and mathematical knowledge. In what follows, I will focus, in particular, on the logical case.

An agent can be said rationally to trust that a logical law is valid if and only if and because: (i) there are no salient grounds for doubting the truth thereof; (ii) they have an overriding preference in the utility of the logical law being valid; and (iii) they have an overriding preference in reducing their epistemic inaccuracy with regard to the truth of the logical law.

Potential issues for the non-cognitivist approach would appear to concern the second and third conditions. With regard, e.g., to condition (ii): A preference in one's presuppositions being correct, given the utility thereof, is neither necessary nor sufficient for the actual correctness of those presuppositions. With regard further to condition (iii): A preference in the minimization of alethic inaccuracy about the validity of logical laws is similarly neither necessary nor sufficient for the actual reduction of the risk of inaccuracy. One issue concerns how to determine the ideal vindicated world with regard to which particular logical laws are valid, provided only the resources of an agent's preferences, credences, and utility functions. In an examination of the alethic approach to epistemic utility, Pettigrew (2012) argues, for example, that the ideal vindicated

credence that one ought to adopt might be identical to the objective chance that a proposition will occur. However, the contention that there are objective probabilities – e.g. frequencies or chances – with regard to which logic is correct requires an explanation of how abstracta such as logical theories and laws can be sensitive either to the evidence pertinent to the calculation of frequencies, or might satisfy the required compatability between chance and necessary truth-preservation. A second issue concerns the nature of the mechanisms which might enjoin an agent to reduce their epistemic inaccuracy. According to the alethic approach to gradational accuracy first advanced in Joyce (1998: 580, 593), the credence of a non-ideal agent is supposed to satisfy the Kolmogorov axioms, on pain of always being farther from the ideal credence that they ought to adopt than a credence distinct from their own. However, a higher-order sceptical issue concerns the epistemic standing of the very axioms of inductive logic, i.e., the Kolmogorov axioms, on which the epistemic credentials of the agent's subjective probability measures are presumed to rest. If – in order to minimize their epistemic inaccuracy – the expected epistemic utility in which epistemic entitlement consists depends upon the satisfaction by an agent's credence function of the Kolmogorov axioms, then it is unclear what resources would be left available for the non-cognitivist to appeal to, in order to explain their justification to believe that the higher-order Kolmogorov axioms which govern the conditions on their rational trust are themselves valid. While the approach to logical necessity via epistemic modality is consistent with defining a probability measure on the latter space, nothing in the epistemic-modal approach to the concept of logical consequence relies upon an antecedent presupposition in the truth of the axioms of inductive logic, and the epistemic status of the laws of deductive formal systems are precisely what the characterization of logical validity via

the model-theoretic definitions of the logical constants within models of modal coalgebraic categories is intended to secure.

4.2 Abductive Methodology

An alternative approach to the epistemology of logical laws avails of an abductive methodology, such that the true logic will be the one which satisfies conditions on theory choice; namely, strength, simplicity, and compatability with what is known. Priest (2014, ms) argues that the *logica ens* ought therefore to be dialetheism, according to which there can be true contradictions; while Williamson avails of a similar methodology both in order to argue that the modal logic of metaphysics is S5 as augmented by the Barcan formula and its converse (2013), and that – despite, e.g., the alethic paradoxes – the use of classical logic throughout the sciences and mathematics vindicates the retention of classical logical laws (cf. Williamson, forthcoming). Russell (2015) argues that logical laws are justified by processes of reasoning, where the management of one's rational economy and belief set will be governed by similar criteria on theory choice; and she takes the foregoing to adduce in favor of a pluralist approach to one's choice of logic.

The approach here proffered concerning the nature of concepts and its bearing on the possession and understanding-conditions for the concept of logical consequence is consistent with the above abductive methodology. The reasons for which to prefer an approach to the structure of mental representation which models the latter as epistemic modal algebras is that such models are, as noted, availed of in both cognitive and visual psychology. A further reason adducing in favor of the cognitivist approach to epistemic modality is that it possesses the resources necessary for the project of metamathematics in a manner that

distinct approaches in the philosophy and psychology of concepts are unable to replicate; namely, the analysis and explication of metalogical concepts, which in some cases might be possessed of a uniquely intensional profile.

A final abductive virtue adducing in favor of the present approach is its neutrality with regard to which logic ought to be adopted, as well as the flexibility of the semantics in being consistent with both classical and non-classical logics. It thus remains an open question which logic ought to govern one's reasoning; what the correct logic might be with regard to the various interpretations of the modal operators; and what the correct logic might be for metaphysical theorizing. Given, for example, the possible inconsistency of one's belief set, the correct logic for reasoning might be a relevance logic, which invalidates the classical law of disjunctive syllogism: i.e., $\forall\phi,\psi[[(\phi \vee \psi) \wedge \neg\phi] \rightarrow \psi]$ iff $\forall\phi,\psi[[\phi \wedge (\neg\phi \vee \psi)] \rightarrow \psi]$.[8] Relevant validity is defined via a ternary relation, such that $[\![\phi \rightarrow \psi]\!]^{\alpha} = 1$ iff $[\![\phi]\!]^{\beta} \leq [\![\psi]\!]^{\gamma}$ and $\mathrm{R}(\alpha,\beta,\gamma)$, where the parameters, α, β, and γ, range over epistemic possibilities. Then the irrelevant entailment, $\phi \wedge \neg\phi \rightarrow \psi$, can be avoided by setting $[\![\phi]\!]^{\beta} = 1$; $[\![\phi]\!]^{\gamma} = 0$; $[\![\psi]\!]^{\beta} = 0$; while $[\![\psi]\!]^{\gamma} = 1$. So, $[\![\phi]\!]^{\beta} = 1$; $[\![\phi]\!]^{\gamma} = 0$; and $[\![\psi]\!]^{\beta} = 0$. However, the foregoing comes at the cost of mental states bearing inconsistent semantic values, given that the values of ϕ and ψ are, respectively, both 0 and 1. The correct logic for epistemic modality might be KDGL, satisfying the axioms of epistemic closure (i.e. modal axiom K); seriality, which links knowledge to belief ($\Box\phi \rightarrow \Diamond\phi$); and the provability axiom $[\Box(\Box\phi \rightarrow \phi) \rightarrow \Box\phi]$. The correct logic for metaphysical modality might be necessitist S5, i.e. KTE as augmented by the Barcan formula and its converse, where E states that $\neg\Box\phi \rightarrow \Box\neg\Box\phi$ and the Barcan formula conjoined with its converse states that $\Box\forall \mathrm{x}\phi\mathrm{x} \iff \forall \mathrm{x}\Box\phi\mathrm{x}$. Finally, considerations with regard

[8] Cf. Routley and Meyer (1972,a,b; 1973).

to the verifiers for facts might adduce in favor of the adoption of an intuitionistic logic for metaphysical theorizing, which restricts the law of excluded middle and thus redefines the negation operator such that the classical law of double negation elimination is rendered invalid (cf. Fine, 2013); whereas considerations concerning the intensionality of the mereological parthood relation might adduce in favor of eschewing of the classical law of mereological idempotence; namely, if x is a part of y, then the sum of x and y is identical to y: $x \prec y \vdash (x \cap y = y)$ (cf. Cotnoir, 2015).

4.3 Intrinsic Necessity and Rational Intuition

Finally, the approach here advanced concerning the possession and understanding-conditions for the concept of logical consequence is able to elucidate a suggestion proffered by Peacocke (1992) concerning the justification of the logical laws. According to Peacocke's approach, the proof-theoretic characterization of the logical constants can be justified in virtue of having a 'compelling' phenomenal character (6-7). The suggestion that the phenomenal character of a state can afford justificatory import to believe that the state is veridical is supposed further to advance an approach to the justification of the logical laws which takes them to be self-evidently true (Leibniz, Frege, Koopman, Gödel); true in virtue of logic (Frege); and necessary (Leibniz, Gödel).[9]

The present approach converges with the above rationalist traditions by being able to model the modal profile of the 'rational intuition' to which appeal is made in various remarks by Gödel (op. cit.). The notion of rational intuition can be provided a rigorous foundation, by being treated as a modal operator, governed by a bimodal logic comprised of a hybrid between dynamic and prov-

[9] Cf. Leibniz (1704/1981: 406, 412-414); Frege (1884/1980: 90; 1893/2013: VII); Koopman (1940); and Gödel (1951: 323; 1953,V: 359; 1947/1964: 261, 268).

ability logics [cf. Khudairi (ms_4)]. The rational intuiton that a logical law is true satisfies provability logic, by satisfying axioms K (if one has an intuition that ϕ entails ψ, then if one has the intuition that ϕ then one has the intuition that ψ), 4 (if one has the intuition that ϕ then one can in principle intuit that they have the intuition that ϕ), and GL (if one intuits that the intuition that ϕ is factive, then one intuits that ϕ). Consistently with GL, the intuition-that a logical law is true is fallible, and thus does not satisfy axiom T, which states that one has the intuition that ϕ only if ϕ is true (cf. Parsons, 1993: 233). The role of the dynamic logic within the bimodal logic for rational intuition, which construes intuition-that as a modal operator, is that it enables an explanation of the manner in which rational intuition can further be related to conceptual elucidation (cf. Gödel, 1953,III: 353,fn.43; 1961: 383). Following Fine (2005, 2006), an interpretational modal operator can be defined within the setting of propositional dynamic logic (PDL), where $\langle\pi\rangle\phi$ abbreviates that 'some execution of π from the present state entrains a state bearing information ϕ', and the dual operator is $[\pi]\phi$, which abbreviates that 'all executions of π from the present state entrain a state bearing information ϕ'.[10] Fine avails of the dynamic interpretational modality in order to account for the possibility of reinterpreting domains of quantification. Uzquiano (2015) generalizes the account, such that the interpretational modality can be availed of in order to explain the possibility of reinterpreting the intensions of mathematical vocabulary, as well, such as the set-theoretic membership relation (cf. Gödel, 1947/1964). Treating the notion of intuition-that as a modal operator governed by dynamic provability logic can thus account for how rational intuition is constitutively related to processes of conceptual refinement, via the dynamically possible reinterpretations of both

[10] Cf. Blackburn et al., 2001: 12-14. A semantics and proof-theory for PDL are outlined in Hoare (1969); Pratt (1976); Goldblatt (1987: ch. 10; 1993: ch. 7) and van Benthem (2010: 158).

quantifier domains and mathematical concepts.

5 Concluding Remarks

In this essay, I have examined the philosophy and psychology of concepts and the modal profile of the concept of logical consequence. I argued that an approach which countenances the structure of mental representations on the model of epistemic modal algebras is both empirically adequate – by being availed of in visual psychology, natural language semantics, quantum information theory, et al. – and is unique in being able to account for the modal profile of the concept of logical validity. I examined, then, the limits of approaches to logical knowledge whose source is broadly non-cognitive, such that it is not based on the concepts and cognitive architecture of agents, and I provided a novel means of circumventing the apparant dilemma presented by Curry's paradox, by demonstrating how there are counterinstances to modal axiom K in the derivation thereof. I endeavored, finally, to demonstrate how – beyond providing a novel epistemicist solution to Curry's paradox – the role of epistemic modal algebra in characterizing the modal profile of the concept of logical validity is consistent with an approach to logical knowledge which is based on an abductive methodology on theory choice, and is able, furthermore, to provide a rigorous, formal foundation for an approach to the justification of logic which appeals to the modal properties of rational intuition. In virtue of the foregoing, cognitivism about epistemic modality arguably comprises the most viable approach to the metamathematical analysis of the concept of logical consequence, and to the definability of the concept's distinctively intensional profile.

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