COMPLEXITY AND EMERGENCE

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Chapter 1

Introduction and Outline

1.1 What this book is all about

In this little book of mine I will briefly introduce the idea of *complexity*—complexity of the world around us and also of the world *within*. By the latter I mean the brain and the mind, along with associated bodily functions, where the world within is, of course, nested in the world at large. What is intriguing is the relation between the mind and the brain, and then, the greatly convoluted relation between the brain, the mind, and the world at large, where that world includes *minds of people* in social groups and formations of highly complex descriptions.

In the context of complexity, the idea of *emergence* will assume great relevance. The human brain emerges in a step-by-step process of biological evolution; the mind is an emergent function of the brain, aided by a host of physiological activities; social formations emerge in a complex process of human interactions, and the physical world around us is in an ever-continuing process of emergence. On top of all this, our phenomenal world is itself in a state of emergence from the noumenal (or the 'real') world everything else is lodged in.

The basic ideas of complexity and emergence are not precisely defined ones, and one doubts whether they can ever be defined with complete precision. All our ideas in this world involve *explicitly* defined aspects *along with implicit* ones. The problematic co-existence of the explicit and the implicit is a necessary consequence of the complexity of the world and of the way we comprehend it and communicate with our fellow humans.

Chapter 2 will be devoted to a qualitative outline of the idea of complexity, with little bits of mathematically oriented description thrown in at places—this in deference to the fact that a huge literature in *complexity theory* is being assembled at a rapid rate, where complex systems spanning an enormous spectrum of subject areas are analyzed in quantitative terms by means of giant computer assemblies (many of those based on AI). All these systems are *parts* of the world at large, where we look at their complex evolution within specified *contexts*.

Of great relevance, however, is the *complexity of the world as a whole*. Here we enter into the misty terrain of metaphysics, because we cannot be observers of that world. I will not hesitate to venture into this terrain though, because I feel it essential to extrapolate our ideas gained in complexity studies of particular systems to *reality at large*. Indeed, the essential thing in metaphysics is the extrapolation of ideas gained in our experience of particular entities of the world to reality as a whole, knowing fully well that such extrapolation is entirely 'illegal' in the eyes of Science.

Our approach in this book will be to follow the Kantian tradition, attempting to naturalize that tradition with insights gained in the study of complex systems. This, of course, is a daunting task, but we will try this out nevertheless, because therein lies the real fun and challenge. In this, we will distinguish between the *noumenal* and the *phenomenal*, trying to explain the strange relation between the two in the light of the pervading complexity of Reality. This naturally requires a fresh look at the subject-object binary—a fresh look that Kant introduced into the world of philosophy. On the whole, our attempt at the naturalization of the Kantian tradition (Kant himself was a naturalist in his time) will make essential use of the idea of emergence.

It is emergence that will occupy our attention in chapter 3, based on which we will take up the matter of the noumenal-phenomenal relation in chapter 4, where I will state, to the extent that I can, the metaphysical and ontological position adopted in this book.

The concept of emergence is a slippery one and is laden with philosophical controversy. In particular, the question is often raised as to whether emergence relates to the appearance of novel ontological entities or is more fundamentally a question of our mode of comprehending reality, i.e., of the way our epistemic processes work. Equivalently, is emergence 'real' or only apparent, being specific to our mode of perception? Is emergence consistent with the approach of reductionism? In this book we will not engage with all these philosophical deliberations, and will mostly focus on what the *network analogy* of complex systems tells us regarding the strange and ever-unfolding evolution of such systems we observe in reality where emergence, interpreted in an intuitive sense, is found to be ubiquitous. In this sense, chapter 3 will be an elementary and qualitative introduction to the idea of emergence.

As mentioned, chapter 4, which will be the only chapter in this book dealing with philosophical notions, will address the issue of the noumenal-phenomenal divide. In this, it will take a close look at the way we comprehend reality by way of *interpreting* the world. It is entirely our interpretation that lies at the root of the emergence of our phenomenal world, though that interpretation is not an empty one and is rooted in the 'real' or the noumenal reality. In a sense, the phenomenal is a continuously emerging *projection* from the noumenal, where the *reverse* projection makes no sense (this, indeed is basic to the mathematical idea of projection).

The notion of interpretation introduced in chapter 4 will be substantiated in chapter 5 by way of referring to the emergence of the human mind as a complex entity in itself, where we address the question as to how the unconscious and conscious layers of it undertake the job of comprehending the world and responding to signals received from it. In this, we will have a look at *theories* as our ultimate tool-kit in our interactions with reality. All this will tell us how the mind constructs the phenomenal world by way of partial and incomplete interpretation of the noumenal reality, thereby giving us a naturalistic view of the relation between the noumenal and the phenomenal.

1.2 What is complexity?

Intuitively, a complex system is one constituted of a large number of components, where all the components are mutually related by means of interactions *of various types and strengths*. The operative phrases are ‘large number’ and ‘mutually related’. This description, however, is imprecise. As we will see, there arise questions like (a) how large is large, and (b) can the components be considered as autonomous entities when they are mutually linked by means of a large number and variety of interactions?

Such questions cannot be answered to everybody’s satisfaction. In particular, one has to rely heavily on speculation and extrapolation when one talks of *reality at large*. On the other hand, things are more well defined in computer experiments where the systems under examination along with their ‘components’ are precisely specified and the number of components along with their interactions are precisely controlled too. The behavior pattern of a complex system depends crucially on its *context*, i.e., on the entities constituting its environment and the mode of interaction of the components with those entities. In addition, the context also includes a host of other things including, in particular, the *internal* constitution of its components. The context can be precisely set in a computer experiment while, in real-life studies, there always remains a problem with the specification of the context, which limits the temporal and spatial scales within which the conclusions of these studies remain valid.

1.3 What is emergence?

The idea of emergence is, if possible, even more elusive. Emergence involves some kind of discontinuity, or a *cut*, in the behavior pattern of a system as its degree of complexity crosses a certain level, when new modes of behavior and new structures are found to appear. Once again, the terms used in this statement are not precise. While an improvement in the precision can be achieved in certain specific cases, not much more can be expected by way of a general definition.

In other words, as in the case of complexity, the concept of emergence is qualitatively defined by way of extrapolation from a number of specific systems whose behavior can be studied, relatively speaking, with a greater degree of precision. That precision, however, cannot, in the very nature of things, be sufficient to pin down the definition of emergence unambiguously and with generality.

1.4 What is the relation between complexity and emergence?

Generally speaking, complexity entails emergence. As for the converse, emergence does not necessarily need complexity as a precondition.

This raises the question as to how to define ‘simplicity’. We will define a simple system as one whose behavior, within some well defined context, can be described in precise terms. However, like an albatross, the persistent ghost of ambiguity does not leave us in peace here too.

> An instance of emergence in a ‘simple’ system is an electronic circuit acting as an amplifier, with a negative feedback providing for stability, where a changeover to a positive feedback causes the circuit to act as an *oscillator*.

More often than not, the ‘behavior’ of a system is described in terms of a set of ‘rules’, or of what is referred to as an algorithm. This does not literally describe the behavior, though. For instance a robot is specified in terms of a set of sensors, a set of motor parts, and an algorithm (commonly an AI one). Specifying these is one thing, while predicting the exact sequence of movements of the robot in any given environment is another. Or, again, take the case of arithmetic, which is defined in terms of its axioms, and rules of inference, there being little ambiguity in the specification of these basic entities. However, the actual set of ‘truths’ or theorems about numbers is a different cup of tea altogether. Finally, take the case of the structure of a molecule of a given composition, to be predicted from the Schrödinger equation in quantum theory. Suppose that the Hamiltonian operator for the given constituents is well defined. The Schrödinger equation itself is then well defined too. But whether these two will unambiguously give us the structure of the molecule is not at all certain, especially when one takes into account the quantum mechanical symmetry properties of the many-body wave functions (I do not refer here to the lack of precision resulting from the representation of the wave functions or energies in terms of convergent power series).

The distinction between an algorithm and the actual set of 'events' resulting from an application of that algorithm, constitutes the subject of algorithmic complexity. The qualitatively specified notion of complexity introduced above has deep (but vaguely defined) connections with algorithmic complexity theory.

The phenomenon of emergence refers, in numerous situations of interest, to the actual behavior pattern of a system as it unfolds from an algorithmic description of it.

> The unspoken assumption of *scientific realism* is that there lies an 'algorithm', or a 'law' underlying the behavior of entities of nature. This book, while subscribing to the fundamental tenet of realism, holds a somewhat different view regarding 'laws' underlying natural phenomena.

However, once the actual behavior pattern is recognized, one needs to address the behavior of a new 'system' whose emerging behavior pattern generally requires a new algorithm for an economical description of that 'system' in a new context—whether such an algorithm can be found is an entirely different matter altogether.

The relation between complexity and emergence is pithily expressed in the Hegelian-Marxian dictum (explained and made widely known by Engels)—*quantity changes to quality* (see [6]; [7] includes a brief introduction to the aphorism; see also [1]; [16] traces it to Heraclitus and to the Chinese Daoist philosophy.).

1.5 Is it worthwhile to proceed further with all this vagueness hanging around?

When one comes to think about it, precision and lack of ambiguity is a rare thing in life. Very few things, if any, in real life are known precisely and explicitly, while vagueness and implicitly known aspects of entities and events are ubiquitous, and inherent in the way we comprehend this world of ours and act back on it. Complexity and emergence are real-life phenomena and are not too amenable to simplification and precision. While there is a large body of literature devoted to complexity *science* and complexity *theory*, it is still not clear as to whether complexity—and emergence too—can be neatly addressed in scientific terms. On the other hand, complexity is not of the same level of generality as philosophical discourse—it is there everywhere around us, and cries out to be comprehended with as much of clarity as possible. In this book, we adopt the position that complexity and emergence are topics in *mesophilosophy*—hovering somewhere in between science and philosophy. This is all the more understandable when one notes that features of complexity and emergence are to be found everywhere in systems spanning a staggeringly wide spectrum, where quantitative formulations of these features are possible to a considerable extent in numerous situations of interest.

Trying to understand complexity and emergence in depth and in quantitative terms as far as possible, seems to me to be a *very* worthwhile endeavor indeed. Of course, extrapolation of ideas relating to complexity to *reality at large* is a different matter altogether, but that is also something that I set my eyes on (see chapters 4, 5), in a spirit of exploration. This last is, evidently, related to metaphysics, but metaphysics is not something to be dismissive of.

The sciences, by their very nature, tend to confine themselves to precisely defined systems in precisely defined contexts. This takes away much of the complexity that is ubiquitous in nature, though the results arrived at in scientific explorations are very pertinent in understanding real life phenomena in a large number of situations of interest. Still, in the absence of ideas relating to complexity and emergence, the horizon within which science looks for ‘truth’ remains rather limited, where the ‘bigger picture’ often goes unnoticed in the pursuit of precision. What is more, scientific investigations, when pursued with the awareness of the bigger picture that includes complexity and emergence, are likely to be more lively and fruitful.

1.6 What resources does this book draw upon?

This book is based on ideas and concepts in complexity and emergence to be found in current literature—ones that I have presented my way as I understand those. But it does not stop there. It puts forward a number of ideas that have been assembled essentially by way of interpretations of mine, where I have taken care to ensure that those interpretations do not run counter to available literature on the respective topics. This, for instance, is the case in a number of sections of the text dealing with the relation between the noumenal and the phenomenal reality and the way we construct the latter as it emerges from the former.

I lay no claim to originality, simply because my interpretations are not ones that have gone through any rigorous cross-examination. I share these with my prospective readers with the hope that they may find these to be interesting and worth taking seriously, as indeed does the hope flicker in the bosom of all authors.

Much of the content of this book overlaps with what I have already written in a number of monographs and articles (see, in particular, [18], [19]). However, much else is ‘emergent’ and warrants a new text.

Chapter 2

Complexity: basic ideas

We begin by repeating that a complex system is one made up of a large number of components, with a large number of interactions (or *correlations*) between them, the correlations, moreover, being of a large number of types and strengths. The phrase 'large number' occurs repeatedly in this statement but that, generally speaking, is how it is.

For background to the present chapter refer to [14], [26], [33], [15], [34].

2.1 A complex system and its context

In real life, we look at a complex system and its behavior *within some context*. The context is made up of all entities not included in the system under consideration, among which only some are considered relevant in comprehending and describing its behavior, either in detail or in terms of a set of regularities or rules that can be used as an algorithm to generate that behavior. In the case of a spatially extended system, the context may, at times, be specified in an approximate sense in terms a set of *boundary conditions*, such as those in the case of a fluid in motion.

The context may also be taken to include the set of *initial conditions* on which the subsequent evolution of the system depends. In the case of a fluid in motion the initial condition is described in terms of the so-called *velocity field* over the region initially occupied by the fluid particles.

The context, as a matter of fact, involves much more. For instance, an essential part of it is made up of the *horizon* within which all our observations relating to the system are confined. Thus, in the case of an everyday macroscopic system such as a piece of stone, the description of its behavior is limited within the space-time horizon of *classical theory*, where the relevant *action* variables are large as compared with the value of the Planck constant. Further, the context is also set by the current scope of our interpretations, concepts, and theories.

Finally, the context is also determined by the *depth* referred to in our description of the system under consideration. For instance, in describing and explaining the behavior pattern of an individual, we do not (and, generally speaking, need not) refer to the individual neurons in her brain, though large scale neuronal aggregates may be relevant in the attempted explanation.

Referring back to the set of entities in the world that are considered irrelevant in determining the behavior of the system under consideration, these may, at a subsequent stage of theory building appear to be relevant within the context of a different space-time *scale*. For instance, the process of *quantum mechanical decoherence* of the state of a macroscopic object over infinitesimally small time scales may depend on an enormously large number of minute environmental fluctuations impacting on it. Such impacts were not considered relevant before the advent of the quantum mechanical theory of environmental decoherence. As another instance, drops of rainwater falling on a large piece of stone may not be of relevance to its history on a relatively limited time scale, but may cause significant erosion in a larger span of time.

In the following, when talking of the context we may refer to one or more of the aspects indicated above, keeping the others implied. The intended meaning is to be read from the setting where it is mentioned.

2.2 How autonomous are the components of a complex system?

When we say that a complex system is made up of a large number of components with multiple interactions between them, we have to be careful as to the independence, from an ontological point of view, of the categories referred to.

For instance, how legitimate is it to talk of the 'components' independently of the system as a whole? Because, as soon as we speak of a component without reference to the system and to the other components in the latter with which it is in interaction, we commit the violence of looking at that entity without regard to its setting, like looking at a pearl by prizing it away from the necklace in which it was set along with its fellow pearls. Strictly speaking, the components, their interactions, and the system as a whole (not to forget the context as well) are to be considered as an entire single package, each implicitly defining and determining the others. It is thus like a set of implicit equations in mathematics—everything determining everything else, where no single variable has a meaning of its own considered in isolation from the others.

In physics, one often talks of particles interacting with one another. Seemingly, each particle is an autonomous entity on its own. In reality, each such particle is implicitly involved in interactions with *fields* that permeate entire space-time—indeed, the particle is itself a dynamical state of a field. In other words, underlying the interactions between particles are space-time dependent fields. And what seems to be a particle as an autonomous entity is actually what is referred to as a *dressed* one as opposed to a *bare* particle. *Moral*: an entity taken out of its own place in a complex system has a very tenuous existence, if at all—a free or completely autonomous entity has a very limited significance within strictly limited space-time scales since it wastes no time in getting hooked to some complex interactions with other entities in its surroundings (the *neutrinos* seem to constitute an exception of sorts).

When dealing with complex systems, one always has to look at *what lies beyond*—in looking beyond particles we get at quantum fields. What looks like an autonomous entity is actually involved in interactions of untold depth, and is not autonomous at all.

2.3 Saving the autonomy of entities: spatial and temporal scales

The idea of everything being implicated with everything else is a deep one. However, while true in a literal sense, it can be ignored in practice *within specified space-time scales*. We do have a good intuitive idea about time scales in real life. What appears not to change and not to interact with other entities of the world within a short span of time does interact and get altered over a longer span—think once again of a piece of stone that retains its shape, size, and hue for days but gets corroded by impact of environmental particles over months.

A complex system can be looked upon as being composed of autonomous components whose autonomy is adversely affected over a time scale characteristic of its interaction with other entities in the system. But those interactions are commonly of many and varied types—the component in question interacts relatively strongly with some other components, and weakly with some others, while there is likely to exist still others with which it interacts very weakly. This implies the existence of *multiple time scales* over which the component in question evolves. In other words, over a very short time scale it can be looked upon as an autonomous entity while over longer scales its time evolution gets more and more entangled with that of other components in the entire complex of systems. Ultimately, a large number of components get entangled in a mutually coupled behavior pattern when the coupled system as a whole acquires a kind of autonomy—there results an *emergent behavior* of the complex system under consideration.

Thus, entities of the world have an autonomous identity only in a relative sense, with characteristic features that remain intact over limited time spans. That autonomy gets lost over longer time scales when a large number of entities get locked in an entangled pattern because of their mutual interaction.

More often than not, entities interacting with one another are spread out in space, though that space need not be the three dimensional one familiar to us. Consider, for instance, the totality of concepts lodged in the mind of an individual. These do form a complex system since there exist very intricate interactions and correlations among the concepts, due to which (and also due to signals received from the outer world) they evolve in a complex manner. However, the concepts do not reside in the three dimensional physical space but in an abstract one. Similarly, a set of quantum mechanical particles interact within a *linear vector space* that is an abstract one too.

When the components of a complex system interact within some space (it is helpful to think of our familiar physical space so as to keep things simple) they form emergent *structures* of varying description in that space, where each such structure is characterized by some *spatial scale*. When looked at over a short range of space, a component may appear to have a certain autonomous identity while over larger scales, a collective identity of interacting components may make its appearance.

In the context of all this, the basic fact remains that the constituents in an interacting complex system are *never truly autonomous*. An electron belonging to an interacting system appears not to differ from a free electron, but in reality, the 'bare' electron gets 'dressed' by interactions that *renormalizes* its charge and mass. For a weakly interacting system there exists a certain correspondence between states of a bare constituent and those of the dressed (i.e., interacting) constituent, but as one enters into the domain of relatively strong interactions, the correspondence ceases to hold, and the states of the system (or of some relevant sub-system in it) are no longer described by variables having a correspondence with the state variables of the bare constituents—*collective variables* make their appearance (refer to section 3.10 in chapter 3). Some of these collective variables are analogous to the bare ones while some others are truly synergistic in nature. For instance, consider the enormous number of state variables pertaining to a human body. Among these can be identified the variables that denote the states of individual cells of the body—ones that are similar to variables describing an isolated living cell. But closer scrutiny reveals a host of differences between the *in vivo* and *in vitro* cells. On the other hand, truly collective variables pertaining to a living human being are her body weight, height, body temperature, and so on.

2.4 Complex systems: nested hierarchies

Complexity generates nested hierarchies in virtue of the phenomenon of emergence.

Consider, for instance, a living organism such as a human being. It is a highly complex system made of organs interacting by means of pathways of which little is known even now. An organ is a complex system in its own right, no less inscrutable in its structure and function and is, in turn, made of organelles and cells—entities no less complex. A living cell is composed of macromolecules, the latter made up of smaller molecules and atoms—complex objects over again. One can go on to nuclei and then to protons and neutrons, to quarks, arriving finally to a land unknown—maybe to one where fields of unknown description are engaged in eventful interactions. All these are, taken in succession, complex systems nested within one another, and each is referred to as residing in a level 'lower' than the preceding one. Likewise, the organism in question is itself engaged in interactions with other organisms, thereby constituting an ecosystem at a 'higher' level of complexity, where the ecosystem, in turn, resides within a complex environment made up of a large number of equally complex components. In other words, complexity is ubiquitous, with 'levels' of complexity nested one within another, where different such hierarchies interact and are entangled with one another in untold ways.

For instance, think of the hierarchy made up of neurons, the brain, the mind, and epistemic, political and religious communities constituted of minds of men. What is of great significance is the manner in which these human communities interact with the bio-geological hierarchy referred to above.

Nature is an enormously vast web of complexity where complex systems are nested and entangled all round. What is more, this infinitely complex tangle of complex systems is generated in an eternally unfolding process of emergence.

2.5 What can the role of science be within this all-pervading complexity?

Science seeks for regularities in Nature—ones that make the world predictable and allow us to act back on it to our advantage. Added to this, science is said to be *truth-seeking*. The seeking of regularities is implicitly assumed to be tied to our journey toward some final truth about nature amid pervasive complexities and irregularities. For truth is implicitly assumed to reside in regularity and harmony—one that transcends all conflicts and turmoil.

This is slippery ground indeed, for seeking regularities in the behavior of *parts of reality* may be utterly different from looking for an ultimate truth about *reality as a whole*.

Complex systems have very complicated patterns of evolution in time and space, and they pass through alternating and intertwined regimes of stability and instability. At times, a complex system is caught in a stable and regular behavior pattern, but juxtaposed to that there may exist unstable and irregular patterns as well, and even some seemingly weak influence may cause the system to make transitions between various regimes of behavior of such contrasting types.

Can there be some *ultimate explanation* behind such complex transitions between contrasting behavior patterns that science can help us seek out? One can indeed try to painstakingly untangle the intrinsic and extrinsic causes underlying such erratic and complex behavior, and also identify spatial and temporal scales separating behavior patterns of diverse types – which is precisely what science does. But is this process of untangling of knots one after another destined to come to an end with one final knot remaining to be untied?

> Science studies particular systems essentially by a process of simplification and idealization, where weak influences on a system are ignored and its behavior is analyzed within a limited space-time horizon. At times, even relatively strong influences are similarly ignored, simulating the latter by means of appropriately specified context effects.

Nature is not a succession of knots but is one enormous *tangle*. Science is great at *locally* loosening some part of the tangle and creating a comprehensible picture of some specific part or other of this world of ours. Whether it is destined to untie the entire tangle is not for anyone to predict—as for me, I do not entertain hope on this score (strangely though, I do not refer to myself as a pessimist and I think highly of the scientific endeavor of mankind—science is an ongoing journey, just as life at large is, with no known final destination). However one may look at it, reality is never divested of complexity.

To make things worse, local success of science in understanding parts of reality does not imply even a remote possibility of *global success*, i.e., success in arriving at an ultimate truth describing the regularity and harmony inherent in *reality as a whole*.

But this is something that we will repeatedly come to in later sections of this book.

2.6 Complexity harbors conflicts

The components within a complex system constantly exert push and pull on one another. The term ‘conflict’ is commonly used to mean ‘opposition’. Referred to a complex system, it means ‘out of harmony’—generally speaking, that is.

Consider, for instance, the vast web of beliefs spun within the mind of an individual. These are usually assumed to form a logically *consistent* system, in which any inconsistency is believed to lead to belief *revision*. However, nothing can be further from the truth than the idea that our beliefs follow a logically consistent pattern. In reality, beliefs can and do form a highly contrary system since many of these are tied together not by force of logic but by *emotions*. Such beliefs are revised only under an emotional upheaval of sorts.

> Our mind works by following two distinct approaches—that of *affect* and that of *reason* (refer to chapter 5). However, though distinct, these two operate in close association with each other.

Or again, consider a dynamical system composed of particles, whose states can be jointly depicted in a *phase space* of an appropriate number of dimensions. Generally speaking, the state of any one particle is influenced by those of all the other particles taken together by means of interactions of various types and weights. In this, the effect of some chosen particle, say A, on another, say B, arises along many channels—one of these is the direct effect of A on B while others arise in virtue of the effects exerted by means of chains of intervening particles (such as A-C-D-$\cdots$-B, where C,D,$\cdots$ make up an intermediate chain). All these influences on B are, more often than not, uncorrelated with one another and do not make a coherent pattern. This constitutes another instance of lack of harmony within a complex system. A particular example is found in the case of a *disordered lattice* in condensed matter physics where the lack of harmony is referred to as *frustration* (see, for instance, [32])—a phenomenon having far-reaching effects in making possible the existence of *multiple phases* of the lattice.

As another instance, consider the set of preferences of an individual, generated by her affect system (see [19]), in virtue of which she may have a strong preference for a certain beverage. On passing by a cafe with a friend one evening and being invited by the latter to share a drink, she is caught in a dilemma—she has left her baby girl at home with her nanny and is anxious to return so as to take charge of the girl, but is equally eager to share a leisurely drink of her favorite beverage with her friend. Real life is full and flooded with such contrary pulls and pushes, small and big, that makes our existence so burdensome and yet so challenging.

The vast repertoire of conflicts within a complex system makes for a very complex evolution of the system as a whole, involving alterations between regimes of stability and instability, and of regularity and irregularity—all these regimes being characterized by corresponding *time scales* (or, more generally speaking, spatio-temporal scales). As a regime of instability is crossed from one stable configuration to another (refer to section 3.6)—where a stable configuration may, generally speaking, involve a time variation as in a state undergoing periodic oscillations—there occurs an episode of emergence (see section 3.11).

> The lady in dilemma referred to above may exhibit strange behavior: she may enter the cafe in the happy company of her friend but then, in sudden consternation, may turn round and start running back home.

Complexity is almost the same thing as the prevalence of conflicts all round—complexity goes hand in hand with *contrariness*.

2.7 Complex systems and networks

A complex system is, at times, conveniently depicted in terms of a *network* (see, for instance, [33]), the latter being made up of a set of *nodes*, with *links* of various types and strengths connecting the nodes— the former are supposed to represent the components of the system under consideration and the latter the interactions or correlations between the components. The network evolves as the system itself evolves under the interactions between the components— new nodes appear, some get deleted, and the configuration of links gets updated in the process.

At times the correspondence between a system under study and a network with some specified structure is not established explicitly, and the idea of a network is used more as a metaphor than an actual representation of the system, this being especially true when one speaks of general features of complex systems. Some systems are too complex to be explicitly represented by networks (consider, for instance, the vast set of concepts residing in the mind of an individual, in which case the network idea may constitute a useful analogy, but not a literally valid representation; alternatively, one often speaks of a *conceptual space* [5]).

Networks are used to describe simpler systems as well (networks are studied in a branch of mathematics, where they are referred to as 'graphs'), but their application to the study of complex systems is rapidly gaining ground. A network representing a complex system is characterized by a number of features shared across a wide spectrum of systems of various types, ranging across physical, chemical, biological, ecological, geological, meteorological, and social contexts, not to mention many more.

One such feature is that of *wide-ranging connectivity*—most pairs of nodes chosen at random are connected by links of various types, where the connection may be either direct or through a succession of other nodes—numerous such routes being possible for an arbitrarily chosen pair of nodes. The fact that there commonly exist links of various different *types* in a complex network is possessed of great relevance—one says that the network is *multi-layered* [33] , where a layer corresponds to links of some definite type. What is more, the links, in addition to being of different types, can differ in terms of their quantitative strengths or weights. The weights may, moreover, differ in their *signs* too (think of the synapses between neurons in our nervous system, where a synapse may be excitatory or inhibitory).

Based on this brief introduction to the idea of complex networks we will, in this book, make repeated reference to networks in subsequent sections.

2.8 Complexity involves feedback

Many of the links in a network may be *causal* in nature. A spike generated in a neuron within a neuronal assembly may cause a spike to be generated in some other neuron—this constitutes an instance of a causal link. Importantly, causal effects can flow in both directions. A node, say, 'A' in a network may exert a causal influence on some other node 'B' while, B may also exert a causal influence on A, maybe through the intermediary of a chain of a distinct set of links in the network. This mutual effect is referred to as a *feedback*.

Strictly speaking, a cause results in an effect at a later instant of time. However, two nodes in a network may be locked in a feedback *loop* without discernible delay, as is the case of numerous electronic circuits in their respective steady states. The latter are instances of relatively *simple* networks with feedback. However, even simple systems with feedback exhibit the phenomenon of *emergent behavior*, as mentioned earlier.

Speaking generally, the same basic phenomenon of feedback leads to the one of emergence in complex systems, where emergence is not generated by design (as in an electronic circuit) but routinely in virtue of the phenomenon of wide-ranging connectivity (refer back to sec. 2.7)—components of a complex system are constantly engaged in a vast number of interactions and correlations of various types, owing to which causal effects flow in diverse directions between various pairs of components.

> While causal correlations are mentioned here for the sake of concreteness, correlations of many other types may exist in complex systems. For instance, words in a dictionary are correlated through their connotations—one word may be linked to several others by means of its connotation. It may appear that a word is linked to only a few others in this manner, but chains of such links connect every word with almost every other in the dictionary, which makes the words explain one another in an implicit manner—there is no basic set of words in terms of which all others are explained.

Related to the phenomenon of feedback, is that of *feedforward*. A feedforward is some kind of a triggering interaction transmitted from one node to another that alters the state of the latter so as to make it respond in some novel way when subsequent signals reach it. Feedforward is of particular relevance in the functioning of complex adaptive systems where it can lead to diverse types of emergence.

The phenomenon of emergence will be addressed in chapter 3. It may be mentioned here that it is the phenomenon of emergence that is responsible for the generation of complexity by making possible the appearance of novel structures and functions in interacting systems. In other words, complexity and emergence are reciprocally related features of many-component interacting systems.

2.9 Behavior patterns of complex systems: CPS and CAS

Complex systems appear to follow the 'rule', *the whole is different from the sum of the parts*. This is not a very precise statement but appropriately sums up a number of features observed in the behavior patterns of actual systems, and refers to intriguing traits associated with complexity and emergence.

For instance, consider a system made up of three components (or *subsystems*, as they are often referred to), say, ‘A’, ‘B’, and ‘C’. If the behavior of the combination of ‘A’ and ‘B’ is known, along with the behavior of each of the combinations ‘A’,‘C’ and ‘B’,‘C’ (at times, the three combinations behave in analogous manners), then one cannot infer the behavior of the combination of all three taken together from the properties of the pair-wise combinations and from known properties of the individual components under consideration. In other words, the presence of additional components in a multi-component configuration makes a notable difference—a fact of great relevance in the phenomenon of emergence. Suppose ‘A, ‘B, and ‘C’ are three persons of known temperament and mental disposition, and also suppose that the behavior of ‘A’ in the presence of each of ‘B’ and ‘C’ is known. This may prove to be utterly inadequate in explaining the behavior of ‘A’ in the presence of ‘B and ‘C’ taken together (‘A’ may exhibit friendly or neutral behavior toward ‘B’ but may show loving considerations toward ‘C’; on the other hand, ‘A’ may be found to be seething with suppressed emotions in the presence of both ‘B’ and ‘C’, and may even exhibit some degree of belligerence towards ‘B’ because of ‘C’ apparently ignoring the presence of ‘A’).

> Referring to the field of physics, this may sound like ‘three-body interactions’ dominating over ‘two-body’ ones. However, even in the absence of three-body interactions (of which no convincing evidence has been obtained so far), the behavior of a system of three particles may be quite intractable, looked at in terms of the interactions considered pairwise. For instance, three particles with pairwise gravitational interaction may exhibit chaotic dynamics (see sec. 2.10).

Described in general terms, the behavior of a complex system made up of numerous subsystems turns out to be *non-trivial in a major way*. In this context, one distinguishes between *complex physical systems* (CPS) and *complex adaptive systems* (CAS), as highlighted in [14].

A CPS is made up of elements or subsystems that have fixed properties — the molecules of a gas, the spins in a magnetic lattice, or the parts of an automobile. A subsystem in this case can be in any one of a fixed set of states, where a state can change under the interaction with other subsystems belonging to the CPS — often the ones that, in some sense, are ‘close’ to the subsystem under consideration. For instance, the position and momentum of any particular molecule in a gas get modified by interactions with other molecules in its close vicinity, while the effects of distant ones are usually small.

> **A note on ‘remote causes’**.
>
> Considering any specified molecule in a gas, distant molecules exerting a negligible effect on it constitute instances of what may be referred to as *remote causes*. The hall-mark of a complex system is that, as mentioned earlier, such remote causes become relevant beyond some characteristic time scale, i.e., *remote causes cannot be ignored for long*. For instance, molecules lying at a large distance from some specified molecule in a gas will ultimately come close to it and influence its motion, i.e., remote causes are relevant in the behavior of the gas as a whole. *Complexity*, in other words, is, to a large extent, generated by the operation of remote causes that assume relevance in the long run.
>
> Analogous to remote causes are ones that may be referred to as ‘underlying causes’. When a system is looked at from the point of view of interactions among its components, a phenomenon may appear inexplicable. However, when probed down to a deeper level in the hierarchy of complexity, the same phenomenon may appear to be less of a mystery. For instance, the behavior of atoms and molecules gets explained to a large extent when considered in the context of electrons and nuclei constituting them.

In contrast to CPS, the properties of components making up a CAS get changed in the presence of other elements and of other systems interacting with these. For instance, the ability of a gene to express itself as a sequence of amino acids may change under the influence of some other macromolecules around it. The components of such a system — commonly referred to as *agents* — ‘learn’ or ‘adapt’ themselves as they interact with other agents.

The ability of the elements of a CAS to adapt themselves leads to quite amazing behavior exhibited by such systems — often in the nature of *goal-directed* processes, such as the self-replication of genes, or the making of *decisions* by the human mind. To be sure, a CPS may also behave in a 'purposeful' manner, such as a cellular automaton devised in early days by Von Neumann that could be made to replicate itself, and a vast number of cellular automata designed subsequently. The difference between such CPS with strange behavior and CAS with adaptive elements often lies in the way these systems are generated — while the purposiveness of a CPS may be given to it by some kind of human intervention ('programming'), a CAS usually evolves in virtue of its own dynamical characteristics where, at some level deep down the hierarchy, CPS elements (complex molecules, for instance) may be found to play a crucial role. In other words, the learning or adaptive abilities of a CAS may be looked upon as *emergent properties* of assemblies of CPS (example: biological evolution emerging from pre-biotic evolution), and not as fundamentally mysterious ones.

> 'Goal-directed' evolution is, as a matter of fact, ubiquitous in nature, in CPS as well as in CAS. A large system isolated from its surroundings invariably evolves towards an equilibrium configuration—in a manner of speaking, such a system 'seeks out' the equilibrium state. However, fluctuations exist at all scales within the system, and relatively small subsystems continually move out of equilibrium, driving one another into patterns of rich behavior. In nature, one finds endless instances of *driven* systems moving away from equilibrium configurations, eventually returning to equilibrium when the driving gets turned off. A living organism constitutes such an instance, thriving on supply of matter and energy from external sources, before it eventually dies. The entire life-history of such an organism consists of a series of 'purposeful' activities. 'Purpose' lies as much in the eyes of the beholder as in the beholden.

It is difficult to exhaustively categorize — item by item — the extremely rich and diverse behavior patterns of complex systems. Even the more notable ones like the appearance of emergent properties become somewhat elusive when one attempts to pin these down to precise formulation. This does not mean that the various behavior patterns themselves are figments of imagination — the very complexity of the systems prevents an unambiguous and universally valid characterization of these behavior patterns.

The rich and intricate behavior patterns of a complex system often appear in the form of impenetrable mysteries in the *cause-effect* relationship that it exhibits. A 'small' or insignificant 'cause' often leads to quite dramatic 'effect' . Likewise, as an instance relating to a CAS, a 'small' change in environmental conditions leads to the eventual emergence of a new species in biological evolution. Commonly, a small or 'negligible' cause is found to lead to notable effects because of the role of factors hidden in the depths of complexity of the system under consideration, or of context effects (erroneously) assumed to be of no consequence. Thus, a few grains of sand added to a sand-pile may cause the latter to collapse because of the fact that it was close to *criticality* to start with. Analogous intricacies and puzzles are met with in respect of *emergent properties* of complex systems. More of this later.

2.10 Complexity and non-linearity

In addition to the feature of feedback (sec. 2.8), complex systems commonly involve the one of *non-linearity* too. This, once again, goes with the fact that a complex system entails wide-ranging interactions (and,more generally, *correlations*) of diverse types among its components. From the mathematical point of view, an interaction can be a linear or a nonlinear one, though such mathematical description does not necessarily apply to all types of correlations in a system. Among all possible linear and nonlinear interactions, the latter are, generally speaking, overwhelmingly preponderant in occurrence.

Nonlinear dynamical systems provide us with very useful and relevant ideas relating to how complex systems evolve in time.

More generally, systems evolve in time *and* space where the space in question need not be our familiar three dimensional physical space. As mentioned earlier, this will mostly remain implied in future references to 'space' in this book, though our discussion will often focus on evolution of spatially extended systems in the three dimensional physical space.

Further, in referring to time evolution, we will mostly talk of the temporal aspect of the change of state of a system. The aspect of evolution of distributed components of the system under consideration in their respective state spaces will again be left implied.

Numerous complex physical systems (CPS; recall that the subsystems making up a CPS are not adaptive in nature) are described in terms of *differential equations* where these equations are, generally speaking, of the nonlinear variety (as mentioned above, linear systems are, in a sense, exceptional though these are familiar, well-studied, and useful too).

1. Nonlinear differential equations are also of use as models describing numerous features of CAS, such as the behavior of subsystems of biological organisms. Examples are to be found in the propagation of electrochemical pulses ('spikes') along a nerve axon, the dynamics of the human heart, and predator-prey dynamics in ecosystems.
2. Apart from and in addition to differential equations, *mappings*, or difference equations, also constitute useful paradigms in the area of dynamical systems. In an early and influential paper by Robert May ([24]), one encounters complexity in apparently very simple systems (idealized biological populations) evolving in discrete time (successive generations, assumed to be non-overlapping) through a succession of notable changes in the pattern of time evolution (referred to as *bifurcations*, see below). The parameter whose value controls the bifurcations in this system was related to the rate of production of offspring from one generation to the next. Evidently, this parameter is determined by a large number of factors relating to the life-cycle and reproduction of the species under consideration, the details of which is ignored in the simple set of nonlinear equations describing the population.
3. Nonlinear equations do not conform to the *principle of superposition*, and serve as illustrations of the rule expressed qualitatively as 'the whole is different from the sum of parts'. No general principles exist for the construction of solutions of nonlinear differential equations, and the infinite diversity and variety in the time evolution of systems described by these equations remains largely unexplored. Nonetheless, deep insights have been developed regarding various *types* of behavior that these systems follow. The *qualitative theory* of nonlinear systems was developed by Poincare and other great mathematicians in the first quarter of the last century. Their investigations were carried forward in large strides by others during the second half of the century, resulting in a highly developed theory that is far beyond the scope of the present book.

In describing the various types of behavior of a system represented by a set of nonlinear differential equations, one generally looks at the *large time* regime, i.e., the one in which the *transient* behavior, if any, is not of relevance, when the system exhibits a behavior pattern that is termed 'asymptotic'. Speaking schematically (i.e., not entering into a precise classification, which is fraught with difficulties anyway), this long-term or asymptotic pattern may correspond to a time-invariant state, an oscillatory state, a quasi-periodic state, or to *chaotic* behavior.

> A quasi-periodic state is a generalization of a periodically varying one, where the time-dependence of the relevant state variables involves several frequencies, incommensurate with one another. While a simple periodicity is symptomatic of a relatively simple state of conflict involving only a few relevant variables of a system, quasi-periodic or chaotic behavior is indicative of more pervasive role of conflicts.

There exist several quantitative indicators of chaotic time evolution. In a manner of speaking, there may be numerous different *types* of chaos. The indicators of chaos are mostly based on various *entropy* measures (see, for instance, [33]; see also [12]). It seems likely that the generic behavior of nonlinear systems involves chaotic time evolution.

In a chaotic time evolution, either the whole of the phase space or some part of it is explored (by the point representing the state of the system under consideration) in a random manner. In contrast, time-invariant, periodic, and quasi-periodic behavior patterns are referred to as *regular* ones.

A nonlinear system is commonly characterized by one or more *parameters* that may be looked upon as setting the context in which it evolves. If it happens to interact weakly with other systems that exert some degree of influence on it, then the parameters themselves change slowly. The question then arises as to how the pattern of time evolution of a system gets altered as the parameters are set at various different values. The answer to this question is fascinating: the pattern of time evolution goes through a multiplicity of qualitatively different scenarios, such transitions in the nature of evolution being referred to as *bifurcations*. For instance, there may occur a transition from a stationary configuration to a periodic one, from a periodic to a quasi-periodic one, or even from a regular to a chaotic one.

What is more, for a given set of parameter values, the phase space may be partitioned into a number of regions of quite intricate structure such that distinct regions correspond to qualitatively distinct patterns of motion. Such patterns are defined in terms of the geometry of bunches of trajectories in the phase space initiated from various initial points. Indeed, one observes the remarkable phenomenon of *sensitive dependence on* *initial conditions* where trajectories initiated from points in the phase space close to one another veer apart and get separated by relatively large distances after sufficiently large time intervals. Added to this is the above phenomenon of a sensitive dependence on the context (qualitative changes in the pattern of time evolution as a set of control parameters are modified to a small extent), referred to as bifurcations. The two phenomena taken together make for an extremely rich and complex repertoire of behavior traits of even quite simple-looking nonlinear systems.

2.11 Complex time evolution

The possible patterns of time evolution of a complex system can be grasped by referring to the behavior—briefly sketched above—of relatively simple-looking sets of nonlinear differential equations (or else of nonlinear mappings as well). Remarkably, such systems can be self-determined, i.e., governed solely by well-defined rules of evolution and yet *non-determinable*, i.e., unpredictable as far as their behavior patterns are concerned.

A complex system is characterized by wide-ranging correlations of multifarious types among its subsystems, among which nonlinear interactions and feedback loops feature prominently, and are of great relevance in the generation of its exquisitely intricate patterns of time evolution. As is commonly observed in the case of nonlinear dynamical systems with feedback, a complex system is characterized by a sensitive dependence on *initial conditions* and a sensitive dependence on the *context*, where novelties abound at every turn of the evolutionary process.

It is this bountiful generation of novelty, attended with a similarly remarkable scarcity of predictability, that is the hall-mark of the time evolution of nonlinear systems and, more generally speaking, of complex systems too.

> I repeat that the time evolution of a system commonly involves the generation of *spatial structures* in temporal succession, i.e., the term 'time evolution' is more appropriately referred to 'spatio-temporal evolution'. In the case of a spatially distributed system, one has to include appropriate *boundary conditions* in specifying the context within which the evolution takes place.

In general conformity with the description of the time evolution of nonlinear systems sketched above, a complex system evolves through a succession of time scales, alternating between regimes of stability and instability, where a stable regime exhibits a complex pattern of coexistence of regular (predictable) and irregular (chaotic or unpredictable) patterns of behavior.

> Above, I have distinguished between sensitive dependence on initial conditions and sensitive dependence on the context. More generally, one can combine the two into one broad category and speak of the *context effect* in complex space-time evolution, because both arise due to the wide-ranging interactions among the components of a complex system. Within this broader category, the dependence on initial conditions may be referred to as the *intrinsic* factor while that on the external systems ('context' in the narrower sense) as the *extrinsic* influence. The two are distinguished by the difference in time-scales over which they operate. As commonly observed, intrinsic factors operate on a shorter time scale while extrinsic ones modulate the intrinsic effects over a relatively longer span of time.

2.12 Complex systems: instabilities are mostly local in nature

The dynamical evolution of a complex system that may span a number of time scales, can be notionally represented by a trajectory in a state space (at times referred to as a 'phase space'; strictly speaking, though, the term 'phase space' applies to Hamiltonian systems in mechanics). Such a representation is notional because more often than not, state variables for a complex system cannot be defined in precise and quantitative terms. We will, in this book, use the term 'phase space' or 'state space' as a convenient means of visualizing the behavior pattern of a complex system where, moreover, the analogy of nonlinear dynamical systems serves as a useful paradigm. The state space representation acquires validity for model systems defined in mathematical terms.

The state space to be considered for a complex system is necessarily of a large number of dimensions since each of the large number of components of the system has to be described in terms of state variables of its own (this also goes for collective or 'macroscopic' variables describing emergent processes since, notionally speaking, the collective variables are functions of the large number of microscopic state variables). At any point of time, the 'trajectory' representing the evolution of the system spans all the large number of dimensions in the state space.

Continuing to invoke the analogy of nonlinear dynamical systems, one refers to the *Lyapunov exponents* at each point of the state space. A Lyapunov exponent (typically denoted by the symbol λ) at some point P in the state space compares the trajectory initiated at P with that at a neighboring point P′, where P′ is chosen to be at a short distance from P in some particular direction in the state space (one obtains a different λ along a different direction, appropriately chosen; for a N-dimensional state space, there exists N independent Lyapunov exponents at each point P)—λ tells us how the separation between the two trajectories gets altered with time. A positive value of λ implies that the two get more and more separated (*sensitive dependence on initial conditions*) while a negative value indicates that the two come closer with the passage of time.

A progressively increasing separation between the two trajectories under reference is indicative of an instability, but more often than not the instability is confined to some low-dimensional subspace of the phase space. What is more, as the trajectory initiated at P′ diverges away from the one starting at P, nonlinear effects come in so as to abate the divergence, in consequence of which the separation between the trajectories does not grow unboundedly. In other words, the instability resulting from a positive value of λ can be described as a 'local' one, though more severe instabilities are also possible.

In general terms, a complex system, in spite of the phenomenon of sensitive dependence on initial conditions, has the ability to 'heal' itself in the event of a 'small' disruption in its smooth time evolution—*a cut on the skin on my body heals in no time; a cyclone, even after causing much havoc, finally dies down.*

Not all complex systems, however, possess the power of self-correction—only those that do continue to exist (with relatively minor alterations in their characteristic features) over considerably large spans of time. A piece of stone, when corroded by atmospheric effects, disintegrates since it does not possess the ability to heal. As a human being in the company of fellow humans, if I do not introspect and substantially revise my socially detrimental beliefs and practices by bringing in my moral and spiritual values, I run the risk of going insane and even of being rejected by society.

The analogy between the evolution of nonlinear dynamical systems and that of complex systems of diverse types will be recalled in the next chapter in section 3.6.1.

2.13 Network structures: an introduction

Complex networks are, generally speaking, highly inhomogeneous structures in respect of the nature of nodes and links, and their distribution. There does not exist any single indicator of how the nodes and links are distributed with reference to type and connectivity—which is simply a consequence of the complexity and inhomogeneity of a network.

An incomplete indication of the structural complexity of a network is obtained by looking at how well connected its nodes are—this can be done by referring to the *connectivity* ([33]) of an individual node and also to the average connectivity of all the nodes taken together, or even the average connectivity of a chosen subset of nodes. A related measure is the one that describes the numbers of nodes with various specified values of links connected to those. One can also look at how well connected the neighbors of some specified node are. Another important indicator refers to *clusters* or communities of nodes—ones that are well connected among themselves but relatively sparsely connected to nodes or clusters not belonging to them. There is a second notion of clustering based on the likelihood that any two neighbors of a given node will also be neighbors of each other.

In all these measures relating to the degree of connectedness of nodes, one can include the weights associated with the links so as to arrive at an improved measure of the importance of nodes in influencing other nodes around it. A quantitative indicator of connectedness of nodes in a network is in terms of the so-called *adjacency matrix*. Such a matrix can be used in a self-consistent manner to express the mutual importance of nodes in the network by means of the mathematical concept of what is termed *eigenvector centrality*—a concept that relates to a *collective* feature of the structure of a network and is useful in the study of emergence. Other measures of the relative importance of nodes in a network include the shortest distance (closeness) between any chosen pair of nodes, expressed in terms of the smallest number of links between these.

In this context, an individual component in a complex system, or a node in its network representation, will be termed a *microscopic* element, in contrast to the system as a whole or a relatively large and well connected subsystem or cluster in it, which will be referred to as being in the nature of a *macroscopic* entity—this is in analogy with the practice in thermodynamics, a subject with which the quantitative study of complex systems has close ties. Here the term 'well connected' refers to connections among nodes within a cluster. Macroscopic clusters (or, simply, 'clusters') in complex networks have special roles to play in the behavior patterns of systems within space-time scales that often have major relevance from a practical point of view. For instance, macroscopic neuronal aggregates in the brain, and interactions between them, are responsible for various psychological states and processes in the human mind.

Large and well-connected clusters (i.e., ones of the macroscopic type) in a network can often be characterized in terms of a set of *collective* features (or variables) in so far as their role in the network properties is concerned, where these collective features assume relevance in the context of the phenomenon of emergence.

2.14 Complexity knows no central control

A complex system (call it 'C'), strictly speaking, can have no subsystem whose dynamics determines to a major extent the dynamics of the system (C) as a whole. More specifically, the controlling subsystem (call it 'S'), even if it exists, cannot be of a size small compared to that of C. For, if such a subsystem (S) were to exist, then C would no longer be a complex system itself, since the dynamics of S (a simple system, to all intents and purposes) would be sufficient to describe and explain that of C. Put differently, a truly complex system (C) is *irreducible*—its behavior pattern cannot be explained or understood in terms of that of a simpler subsystem (S).

As an example, we consider the human brain which is an enormously complex system by all standards. The *mind* is an emergent mode of functioning of the brain (see chapter 5 for more detailed considerations) and is a complex system itself, comprised of an *unconscious* and a *conscious* layer, where none of the two can be described as a controlling subsystem. It is commonly supposed that the unconscious mind is based on parallel distributed processing of a large number of neuronal aggregates, while the conscious mind is, to some extent, analogous to a computing system with a Von Neumann architecture (refer to sec. 3.15.2 in chapter 3). More appropriately, however, even the conscious mind does not have anything like a central processing unit—it is based on a large number of neuronal aggregates much like the unconscious mind, the major difference compared to the latter being a large-scale *integration* between these aggregates. In other words, the conscious mind does *not* have anything like a controlling subsystem since large-scale integration is fundamentally distinct from control.

It may so happen that a system (C) is made up of two parts, say, C_1 and C_2, where one of these (say, C_1) has a simpler controlling subsystem (S), while C_2 has no such subsystem. In that case, it is not the composite system made up of C_1 and C_2, but the one made up of S and C_2 that can be described as a complex system (with the added observation that S does not act as a controlling subsystem od C_2).

Put differently, *all* of the large number of dimensions of the (putative) state space of a truly complex system C have to be essentially necessary for describing and explaining its behavior pattern, where no subspace of a smaller number of dimensions can serve the purpose.

2.15 Complex systems: statistical description

The wide-ranging connectivity of multiple types resulting in a highly complex time evolution of a complex system, and the associated intricacies relating to its structural features, makes an exact and unambiguous description of those features and of its behavior pattern quite impossible and meaningless. Instead, one has to resort to *statistical descriptions* of various types, such as those relating to characteristics of its microscopic elements and also ones concerned with features of macroscopic clusters, where the network as a whole may also be referred to. A statistical approach becomes necessary for both a *static* and a *dynamic* description of network features where, instead of the exact value of some relevant variable (say, s), one looks at its probability distribution over some appropriate range of values, specified by means of a *distribution function* (say, $P(s)$, at times referred to as, simply, a 'distribution') satisfying a normalization condition (total probability of all possible values has to be unity).

A large variety of distribution functions have been found to be relevant in respect of features of complex networks, among which the *power law distributions* ([33]) are of particular significance. A power law distribution contrasts with *exponentially* falling ones that obtains for systems involving a large number of almost independent participating components—$P(s)$ falls off extremely rapidly for values of s corresponding to *unlikely* events. In a power law distribution on the other hand, $P(s)$ falls off less rapidly, owing to which such a distribution is referred to as a *fat-tailed* one—the 'fat' tail is symptomatic of the high degree of correlation between the network components, where *remote causes* assume relevance in the asymptotic probability distributions (i.e., ones at large enough times).

The time evolution of a probabilistic system (such as a complex network) is described in terms of *stochastic dynamics*, where the time-variation of the probability distributions ($P(s)$) is described and solved for by making use of a number of mathematical techniques. These involve solving for the time dependent distribution functions and also space-time dependent *correlations* of various orders between physical variables pertaining to the microscopic and macroscopic subsystems of the complex system under consideration.

2.16 Regular and random networks

From the practical point of view, the dynamics of a network can be described by specifying how new nodes are incorporated in it, how some nodes established earlier get removed, and how links are established into and removed from the network structure—a useful approach is to describe how the totality of weights characterizing the links varies with time. The elementary processes involving the setting up and disappearance of nodes and links have a pronounced effect on the overall structure of the network (at various points of time and, specifically, at large times) expressed in terms of numerous connectedness and closeness measures.

An important class of networks includes those whose nodes and links are randomly distributed ([33]). Such random networks contrast with ones where nodes and links follow a regular pattern.

Instances of random networks (a more appropriate description is in terms of a network *ensemble*) are the Erdös-Rényi (E-R) network and the Gilbert network. An E-R network, in particular, is one with a fixed number of nodes (N) and links (L), where all networks (with the given numbers of nodes and links) in the ensemble are equally probable (with probability, say, p; the Gilbert network is closely related). Interestingly, an E-R network shows certain structural 'phase transitions' when one or more of the parameters N, L, p are made to change.

Networks of an intermediate nature, incorporating both random and regular features, are of great relevance—these are the ones that, strictly speaking, can be referred to as *complex* networks. Generally speaking, a regular network is characterized by high degree of clustering and large separations between nodes. A complex network with a relatively small degree of randomness incorporated within a regular structure exhibit the phenomenon of a high degree of clustering with a *low* average separation between nodes. This effect, discovered by Strogatz and Watts, underlies the so-called *small world* phenomenon observed in many social networks and also in numerous other areas such as in the architecture of the human brain ([31]). In a completely random network, say of the E-R type, one generally observes low clustering and short distances.

2.17 Multi-layer networks and co-evolution

Complex networks are more often than not, of the *multi-layer* type ([33]), where links of various distinct *types* connect the nodes. In terms of the complex systems represented by the networks, it means that there exist various distinct types of correlations and interactions between the system components. The multi-layer structure of networks results in an exquisite complexity in the time evolution of complex systems.

In particular, complex systems exhibit the phenomenon of *co-evolution*, a term in frequent use in the area of evolutionary dynamics. In general terms, co-evolution implies the joint evolution of the number and types of nodes, along with the number, types, and weights of the links in the various layers in the representative network—in short, *everything evolves along with everything else*. In this complex picture of co-evolution, there takes place, in particular, a continual alteration in the *macroscopic* structure of the network, where large clusters of various size emerge in the background of the microscopic distribution of nodes, with the attendant emergence of *collective interactions*—the emergence and dissolution of macroscopic structures are regular features of complex time evolution.

Within the scenario of co-evolution, the behavior patterns of all the components of a system (or some sub-system within it) are *mutually determined in a self-consistent manner*, and involve a *multitude of space-time scales*.

2.18 Complexity and truth

In a complex system, *truth* assumes vastly complex proportions for, in reality, truth is a *multi-faceted* entity.

> Truth is unambiguous and absolute only in simple systems—ones that can be described precisely and, in a sense, *axiomatically*, though, even in such systems, it is usually enormously difficult to *arrive* at truth. Even an

> axiomatically defined system is not always a simple one since its consistency is not guaranteed automatically. For instance, I may entertain the idea that all my beliefs are internally consistent and are, moreover, consistent from a social point of view—I may even think that these are generated from a set of core beliefs akin to axioms, but in reality beliefs are treacherous objects—they are often inconsistent and are tenaciously held together by *emotions*. An apparently simple system may be a deceptive thing since it may have internal inconsistencies and external links that cannot always be ignored.

A complex system is more often than not lodged in a complex environment and is generated (commonly, by way of emergence) from a substratum of complex systems at a lower level of a nested hierarchy. It is impossible to consider or describe a complex system without simplifications or idealizations, and such descriptions are always limited in scope, however justified the simplifications may appear to be. As mentioned above, a complex system is more often than not a part of a bigger complex system and is generated from some underlying complex system too. The effect of systems in the environment on the one under consideration is commonly taken into account in terms of some specified context. The effect of underlying systems is more difficult to account for—the behavior of a system at a higher level, though emergent from those at a lower one, is properly described in terms of collective properties (refer to sec. 3.10.1 in chapter 3) complementary to those of the systems at the lower level. However, the two sets of properties continue to remain coupled—something that is somewhat rarely apparent in the behavior of the emergent system since the latter interacts with other emergent systems mostly by means of collective properties.

In summary, the description and explanation of the behavior of a complex system is never a closed exercise—one that is self-contained and precise. Which is why every explanation and every assertion pertaining to that behavior always involves ambiguity. In particular, an altered context reveals a different facet of the system that has only partial overlap with what some other context divulges. This is so elementary a fact that its import is often overlooked in weighty philosophical discourse. *I find a neighbor extremely patient, humorous, and kind-hearted in his dealings with my family, but that same person behaves in an abominably rude manner when in communication with aged persons of our locality seeking his favor*: he has—in common with his fellow humans—exquisitely complex and intricate mental traits that reveal distinct and even contrary facets in diverse contexts. A complex system harbors contradictions and conflicts (sec. 2.6), as a result of which every statement that appears to be true from one perspective reveals itself to be false (or having contrary aspects) when looked at from another.

Within some specified and well-defined context, truth is unambiguous, but it is extremely rarely that the context in which a complex system is located can be specified precisely and completely. The great and fundamental implication of this is that *truth is rarely free of ambiguity*. Every assertion of truth is potentially associated with incongruous and even contrary truths. This brings us to another great Hegelian-Marxian aphorism—*the unity of opposites* ([25], [6]).

Truth relates to the world we perceive, but that is a world we construct in bits and pieces by means of *interpretation* from a reality that is too complex to perceive and comprehend as a whole. And, our perception of a situation is not unique—it varies from person to person, from one social group to another. What is more, what is perceived and reasoned as truth today gets altered dramatically as new contexts of experience and perception are opened up before us. *A friend of yesterday, whose loyalty and faithfulness was apparent as a matter of rock-solid truth, may turn into a fiend today about whom the only truth that now applies relates to his diabolical infidelity.*

Truth, of course, has two aspects to it—the ontic and the epistemic. The two are recognizably distinct in the case of simple systems—the issue of existence of truth is distinct from the one of how we arrive at it (however, this too is not free of controversies: is truth Platonic or is it constructed from our perception of reality?). For complex systems, on the other hand, truth can be defined only with reference to our perception and interpretation because there is no way to describe and comprehend a complex system *as it is*. Needless to say, simple systems are idealizations set up in our imagination and are not to be found in real life, though they are relevant in that they act as guides in our engagement with reality.

In summary, complexity entails ambiguity and multifacetedness in truth. The only truth that is unambiguous and absolute is one asserted for a precisely defined system within a precisely defined context, which is a rare thing indeed. We, however, ignore the ambiguity in truth for practical purposes in our journey through life. But the ambiguity persists, as revealed, for instance, in court proceedings that appear never to end.

2.19 Complexity and the cause-effect relation

Complexity makes the cause-effect relation a highly ambiguous, entangled, and opaque one.

I have a bad headache this evening. My wife assures me that it is caused by my anxiety over my son's rather ordinary school report that he brought home in the afternoon—a diagnosis I agree with. On the other hand, my neighbor tells me that the headache is caused by the worrisome atmospheric pollution, which also seems to be a reasonable assertion. Can my headache have multiple causes?

Multiple causes are certainly possible for an event pertaining to a complex system, fundamentally because of wide-ranging interactions among its components and with other systems having some influence on it. Another factor contributing to such multiplicity relates to systems residing at a lower or higher level in a hierarchy of complexity. My headache, for instance, may be caused in part by a lack of balance in my neurotransmitter system that generates a foul mood in me. It may, for all I know, also be caused by the report I read in the papers this evening that two global superpowers are poised for a military confrontation, thereby heating up the global political atmosphere.

Multiple causes are also possible in simple systems, though in such systems one can, in most situations, meaningfully identify the *proximate* cause of an event. Consider, for instance, a gas made up of identical particles, confined in a closed container—this can arguably be taken to be a simple system provided that one makes a number of simplifying assumptions about the particles and their interactions. Consider the event of a particle A being at a position P at some time instant t_1. This event can be termed an effect of the same particle having been in a close vicinity of some other particle B at the position Q at time t_2 $(< t_1)$, provided that A does not have a close encounter with some other particle in the intervening time. However, proximate causes are not always meaningful in real life. Even in the case of the simple gas, it may so happen that P is close to Q (i.e., the effect of B on A brought to bear at time t_2 is, in some sense, a small one) and an earlier encounter with a third particle C with A being at the position R at time t_3 $(< t_2)$ causes A to move from R to Q, close to P (i.e., the effect of the cause at time t_3 is *dominant* over that at time t_2).

Proximate causes and dominant causes allow us to make good use of the cause-effect relationship in our journey in life while another type of cause is also relevant in practice—a cause that can be *invoked*, or resorted to, so as to produce a desired effect with relatively less effort—we refer to those as *effective* causes. For instance, *as my son continues to receive rather ordinary grades at school, I mentally review how to make him do better, and conclude that his teachers and classmates would not be effective in the immediate future, and I request a friend of mine—one having a charming personality—to spend time with my son every evening, trying to generate in him an interest in his studies. This is seen to work wonders and I thank my friend profusely for the efforts he put in for my son.*

Added to all this, we recall the existence and role of *remote* causes and *underlying* causes (refer back to sec. 2.9) so as to appreciate that the cause-effect relationship can indeed be a complex one in real life.

However, such classification of causes is of little value in complex systems nested within a hierarchy of complex systems all round (refer to the case of my headache that I am at a loss as to how to cure) because, in such systems, *cause-effect relations flow in all directions with all possible strengths.*

Referring to the network representation of a complex system, almost all nodes interact with almost every other either directly or through intervening series of nodes where, in addition, the interactions can be of diverse types and strengths. Thus, there is a likelihood that pairs of components get locked in *mutual interaction* or there exist more complex feedback loops, producing implicitly generated effects of untold variety that simple or idealized systems cannot even approximate. In other words, while we can identify cause-effect relationships with little ambiguity in a simple system and can put it to good use in real life—real-life systems can be modeled as simple ones within specific space-time bounds—such identification turn out to be utterly inadequate over space-time scales that differ by several orders of magnitude.

In summary, complex systems are highly tangled bundles of cause-effect tie-ups.

However, all this ambiguity and opacity in the cause-effect relations in complex systems does not run counter to the *principle of causality*—every cause *precedes* any of its effects by a time interval that may be large or small, depending on the *signal* that has to flow from the cause-event to the effect-event so as to produce the influence on the latter exerted by the former. The fact of cause preceding the effect is independent of the observer. Regardless, the intricacies of the cause-effect relationship continue to be pervasive in complex systems.

Chapter 3

Emergence: basic ideas

For background to the present chapter, refer to [15], [30], [34].

3.1 Emergence: introduction

Philosophical debate abounds on the topic of emergence. Is emergence ‘real’, or is it a matter of how we perceive things—is it ontological or epistemological? Does it involve the appearance of novel structures or just of novel behavior patterns of the pre-existing structure of a complex system described in terms of its components (or subsystems)? Can emergence be understood in terms of the properties of the underlying components without recourse to additional and independent concepts?

In the present book, we will not engage directly with these philosophical issues, and will instead work on a ‘common-sense’ basis, at times referring to the network representation of complex systems. Imagine a complex network evolving in virtue of the multifarious interactions between its components, and also in response to the influence of external systems on it, where the latter will be referred to as the *context* effect—these are commonly referred to as ‘intrinsic’ and the ‘extrinsic’ effects, where the two are generally distinguished by the time-scales over which they operate. In the course of its complex evolution (recall the idea of co-evolution and of the intricate behavior pattern of a complex system in its process of co-evolution; refer back to sections 2.11 and

2.17), there arise diverse configurations of the network differing in details of the way the components of the system get involved in interactions with one another.

> For instance in one configuration the interaction between nodes, say, A and B may occur predominantly through the intermediate nodes C, D, while in another, the A-C-D-B chain of interactions may be much weaker than the direct A-B interaction.

Emergence involves the formation of special configurations of the network where the nodes in some large ('macroscopic') cluster are strongly linked with one another through mutual interactions while their interactions with nodes lying outside the cluster and with external systems are relatively weak, being determined by the configuration of the cluster as a whole—more precisely, a cluster interacts predominantly with other clusters in the network and with external systems through what can be termed 'collective variables'. This we explain further in section 3.2 below.

3.2 Clusters and collective variables

It is in virtue of the configuration of the cluster as a whole (we call it C for the sake of brevity) that it interacts in novel ways with systems beyond its confines, i.e., with other subsystems of the complex system under consideration (call it S) and with external systems forming the environment of S. Here the term 'subsystems' refers to individual components of S as well as to clusters belonging to it. The idea is that the cluster C under consideration retains its configuration (i.e., the relative disposition of components belonging to it, including the multi-layered structure of links correlating the components, and hence including all the complex interactions among these components) over time scales of interest, and interactions with subsystems and systems external to C do not substantially alter this configuration—in other words, C attains a distinct stable identity of its own, which other clusters belonging to S do not necessarily enjoy.

What is important to note here is that, within some time scale of interest, the configuration of C, which has been assumed to remain almost constant, acts as a *constraint* on the components constituting its nodes, and their interactions with external subsystems and systems *under the constraint* differs markedly as compared with the interaction that would have obtained, had the constraint been not there. In this, a cluster attaining an identity of its own resembles the formation of a *bound state* made up of a number of particles. For instance, the interaction of an atom (say, A) with another atom or molecule (B) is endowed with distinct features as compared with the joint interaction of the protons, neutrons, and electrons making up A, all considered in their unbound states, with the same particle B. In this sense, the bound state appears as an emergent entity when the protons, neutrons, and electrons are made to interact with one another in some appropriate manner. And, it is in this sense that one commonly states that the behavior pattern of an emergent system cannot be known from the behavior of its constituents considered independently of one another, or—to use a philosophically oriented term—cannot be *reduced* to the properties of its constituents considered as independent entities.

1. To be sure, the properties of the atom A, including its interactions with B, *ultimately* depend on the properties of the protons, neutrons, and electrons making up the structure of A. As far as our knowledge of atomic physics goes, the properties of the atom A (including its interactions with B) can, to a considerable extent, be traced back to those of its constituents, but that is because one is dealing with only a few constituents and their interactions (already, the case of the many-electron atom becomes nearly intractable), assumed to be known in some given context—in the case of a complex system made up of a large number of interacting constituents, there arise *large explanatory gaps* as one tries to understand the properties of the bound entity in terms of its constituents alone. This is clearly the situation in the case of a solid, for which multitudes of its properties (expressed in the way the solid responds to external perturbations) are explained in terms of *quasi-particles* (see, for instance, [21], see also [29]) rather than of its basic constituents, namely the nuclei and the electrons.
2. A large part of the polemics between the reductionist and emergentist points of view can be circumvented as one realizes that the two camps are often talking at cross-purposes, since many of the terms over which the polemics unfolds are *interpreted* differently by the the respective proponents involved. This, of course, does not mean that the polemics is devoid of content (in this context, see [9]; Coleman refers to the relation between the two points of view as an 'awkward alliance'; more specifically, he sees the two as 'intertwined' with each other)—participation in the polemics does certainly shape our world-view, and wold-view certainly matters in our perilous journey through life.
3. Is the idea of emergence compatible with the scientific point of view which seems to be a largely reductionist one? An insightful commentary is to be found in [1]. Significantly, Anderson accepts that the description of one level of reality can be reduced to that of a lower level, but points to the futility of trying to *construct* the former from the latter. Solid state theory can, to a large extent, be reduced to molecular and atomic physics but cannot be constructed from the latter.

It may be noted that the emergence of clusters with distinctive features is not a necessary phenomenon in a complex system but is a contingent one in the course of its co-evolution (in which *everything evolves*, including the number and types of components and their interactions), though the vast complexities inherent in the process of co-evolution make it almost a certainty that, unless the system itself perishes from the face of the earth and merges into its environment, some instance or other of emergence has to appear in it in some sufficiently large span of time. It is, of course, in the very nature of emergence that one cannot determine or predict when and how that phenomenon of a cluster acquiring a distinct identity of its own occurs. The conditions necessary for the appearance of a cluster C as a distinct entity having an autonomy of its own in a complex system S are: (a) its (relative) stability as a configuration of components locked in mutual interaction, (b) the requirement that its interactions with external systems (clusters in S other than C itself, and systems external to S) be determined by *collective variables* or parameters characterizing the configuration of C—these collective variables are fundamentally determined by the parameters describing the states of the individual components in C considered independently of one another, but the nature of that dependence of the 'macroscopic' variables on the 'microscopic' ones is largely indeterminate.

> In addition, the emergence of a cluster (a 'macroscopic' subsystem of a complex system) whose interactions are described in terms of collective variables, depends crucially on the context, i.e., on the configuration of the environment—for instance, the emergence of certain forms of life from certain earlier forms required the right proportion of oxygen in the atmosphere.

In a manner of speaking, the phenomenon of emergence involves *an intimate mix of the ontic and the epistemic*. Further considerations are to be found below in sec. 3.4

3.3 Emergent structures and emergent processes

The phenomenon of emergence involves both *structures* and *processes*, though both are aspects of one and the same course of co-evolution of a complex system. The distinction between the two is manifest in time-scales of different magnitudes. The various rock formations that can be identified in a geographical region may be looked upon as emergent structures because they appear to have stable features over time spans of several years. Over longer spans of time, however—ones that merge with geological time scales—the rock formations change dramatically, with some disintegrating, some others showing strange transformations, and still other formations appearing as novel ones.

While one may focus on an emergent structure in some particular context, one has to keep in mind that such structures are nothing but aspects of ongoing processes of co-evolution in complex systems.

3.4 The ontology and epistemology of emergence

We take here a brief look at the issue of ontology and epistemology since a more complete understanding will have to await considerations on the divide between the noumenal and the phenomenal, which we will focus on in chapter 4.

The fact that a cluster emerges in the course of co-evolution of a complex system as a result of increase in the size of the system and also of the interactions among its components, is indicative of the ontic roots of emergence. But, as we saw in sec. 3.2, the bare fact of co-evolution is not sufficient for emergence, since the latter requires the formation of a cluster that is stable, in a relative sense, over a certain time-scale of interest where, in addition, the interactions of the clusters with systems external to it has to be determined in terms of a set of collective variables distinct from (but originating in) the ones describing the states of individual components in it, considered independently of one another.

> However, interactions between individual components continue to exist—those between the components within the cluster and the ones between the internal and external components. It is in the backdrop of all these microscopic interactions that the macroscopic interactions (say, the interaction between clusters C_1, C_2 within the network representing a complex system S, and also the one between a cluster C in S and a system, say, S' external to S) take place, being determined by the collective variables of the respective macroscopic subsystems, represented by clusters.

> The issue here is not one of the continuing existence of interactions between components, but that of the

parameters or variables in terms of which these interactions are fruitfully described.

The epistemic aspect of emergence relates to the *indeterminateness* of the phenomenon of appearance of a cluster meeting the above two requirements—any attempt at the determination of when and how such a cluster emerges is destined to fail because the large size of the complex system under consideration (and also of the diversity of the interactions between its components) reduces such an attempt to one of solving an intractable problem. The intractability is fundamentally related to the sensitive dependence of the course of evolution of a complex system on initial conditions and on the context because it is the phenomenon of this sensitive dependence that all the exquisite intricacies of co-evolution are ultimately rooted in.

Related to this is the added intractability of determining the relevant collective variables in terms of microscopic ones.

Evidently, the ontic and the epistemic are inextricably intertwined in the phenomenon of emergence.

3.5 Digression on the term ‘properties’ of a system

We digress here to a consideration of what we commonly refer to as the ‘properties’ of a system—the intrinsic and the extrinsic aspects to these.

The *internal* constitution of a system determine its properties in the ontic sense—the properties are determined by intrinsic factors independently of how we perceive them. However, we perceive those properties by noting how the system interacts with *external* ones—such interactions provide the epistemic basis of a description of the properties of the system. It is generally assumed that the two descriptions—the intrinsic and the extrinsic, or the ontic and the epistemic—coincide. However, that is far from the case in real-life systems. Both the ontic and the epistemic descriptions involve infinite regress. In the intrinsic description, one can go on to deeper and deeper levels of determination—all constituting a hierarchy—with no guarantee that the levels ever come to an end. In the extrinsic description, on the other hand, one has to consider the behavior of the system in an infinite variety of external conditions so as to know its true nature. What is more, the intrinsic and the extrinsic may prove to be ultimately related by means of *fields* that know no space-time bounds.

What is commonly accepted as an intrinsic determination refers to the properties as determined by the immediately preceding level in the hierarchy (for instance, the properties of a solid as determined by the ensembles of electrons and nuclei, without referring to the internal structures of the nuclei). This, strictly speaking, is a context-dependent determination of the properties of the system—one limited within the context defined by a certain level in the hierarchy. In the extrinsic determination, on the other hand, we look at how the system in question (say, C) behaves in the company of external systems, where those external systems may come in an infinite multitude of configurations. It is devoid of meaning to look at some particular set of configurations of external systems and declare that it is sufficient to note the behavior patterns of C in interaction with configurations belonging to that particular set, so as to conclude what the 'properties of C' are. It is generally accepted that, if we look at a sufficiently large set of configurations of external systems, then a reasonably good description of the properties of C are arrived at. Needless to say, such a description is contextual too.

There now arises the question of reconciling the properties determined intrinsically with those arrived at extrinsically. This, precisely, is the job of science. In the physics of solids one undertakes an endless quest at understanding how the response of a solid to external influences of various descriptions relates to the properties of its internal constituents—that quest never terminates. And, what is of fundamental relevance in this context is the following: in one's efforts at correlating the response of the solid to external influences of diverse types with the known interactions of the nuclei and the electrons in the solid, one repeatedly comes across situations where it is not the nuclei and electrons themselves, but *quasi-particles* of various descriptions that acquire relevance, signifying emergent formations in the solid.

The physics of solids does succeed to some extent in correlating the quasi-particles to the interactions of the nuclei and the electrons, but one finds that the characteristics of the quasi-particles are, to a large extent, independent of the specific solid under study (for instance, Cooper pairs of a universal description arise in various different superconducting metals, and the low-lying energy levels of a Fermi liquid have a number of universal features resembling those in helium-3 [29]). Various emergent formations and processes in complex systems share common features across a wide span of situations of diverse types, regardless of the specific features of the components of systems where they appear: for instance earthquakes and neuronal spikes share the feature of a slow build-up and a rapid discharge beyond a certain tipping point. Put differently, the relation between the properties of the constituents of a system and those of the system as a whole is largely indeterminate.

In summary, the intrinsically and extrinsically determined properties of a complex system are of distinct origin, and may be looked upon as constituting ontological and epistemological descriptions of it—the two are intertwined since extrinsic descriptions are correlated with intrinsic ones, though an irreducible gap exists between the two because of the phenomenon of emergence (further considerations are to be found in sec. 3.16). Emergence, in other words, is all about a gap, or a *cut* in our perception of what keeps on happening in a complex system, that gap being the result of the phenomenon of sensitive dependence on context (understood in a broad sense as mentioned in section 2.11).

> The intrinsic and the extrinsic can be assumed to have a common origin if all entities in nature are constituted of the same fundamental constituents and the properties of all those entities are determined by these basic constituents. The search for ‘fundamental’ constituents, however, may involve an infinite regress, or may end up in an infinitely complex interplay of fields, or even may be linked to entities and correlations hitherto unknown. In other words, the definition of what we refer to as ‘properties’ of an entity may not be as simple as it appears on first sight.
>
> The devil is in the details!

With this brief digression, we introduce two fundamental aspects of the complex evolution of systems, namely, *self-organized complexity* and *self-organized criticality* where the latter is, in a sense, an important special instance of the former, both being processes basic to emergence.

3.6 Emergence as self-organized complexity

Emergence appears by means of intrinsic interactions in a 'macroscopic' subsystem of a complex system (a large cluster in the network representation) as it evolves across a regime of instability, with extrinsic factors modulating its intrinsic evolution, in virtue of which it passes through alternating regimes of stability and instability. This ubiquitous aspect of emergence is referred to as self-organized complexity.

3.6.1 Looking back: lessons from nonlinear differential equations

We recall a number of features of time evolution of nonlinear systems (refer back to sections 2.10, 2.11) that act as very useful paradigm in understanding where emergence finds its place in the exquisitely intricate scenario of complex time evolution.

To start with, we refer to phase transitions in physics that indicate transformations in *equilibrium configurations* of systems. More generally, complex systems undergo *dynamic* phase transitions involving qualitative changes in their behavior patterns. Such changes are brought about by the joint effect of the intrinsic interaction among the components of a complex system and of changes (commonly, small and slow alterations) in the state of its environment. Referring again to a notionally defined state space (commonly referred to as the 'phase space') of the system, dynamic transitions can be described as changes in the (geometrical) patterns of sets of trajectories in that space.

1. We repeat that the above description of dynamic phase transitions is a notional one in that a state space (or, more specifically, a phase space) cannot always be defined in precise terms, to say nothing of trajectories in that space or of the geometrical pattern of bunches of trajectories. Precise and rigorous definitions exist for non-linear ordinary differential equations with 'reaction functions' (i.e., ones representing the rates of changes of relevant variables) belonging to certain acceptable types. Nonlinear partial differential equations are also amenable to precisely formulated analysis to some extent. Such analyses are, in the main, applicable to complex physical systems (CPS). Behavior patterns of complex adaptive systems (CAS), on the other hand, are more commonly described in qualitative and non-mathematical terms. While computer-generated descriptions and simplified mathematical models are available in abundance for adaptive systems as well, the results obtained in the theory of non-linear differential equations act as powerful analogies for broad classes of complex systems.
2. There is no general method of solution to nonlinear differential equations. However, the so-called *qualitative theory* for such equations is a highly developed one, where the topological features of trajectories

> in the phase space are of central interest. This theory is the result of remarkable contributions from mathematicians, physicists, and engineers over the last hundred years.

Continuing to refer to the analogy with nonlinear differential equations, one refers to sets of *Lyapunov exponents* (we use the symbol λ to denote a Lyapunov exponent; a complete set of Lyapunov exponents will be denoted by Λ) in various distinct regions of the phase space. Imagining a trajectory initiated at some given point (say, P) in the phase space, a positive λ indicates that a trajectory initiated from a point (say, P′) slightly shifted from P in some particular direction in the phase space will deviate progressively from the trajectory initiated from P (*sensitive dependence on the initial condition*). A negative value of λ, on the other hand, implies that the two trajectories will progressively come closer.

The complex dynamics of the system under consideration depends crucially on how Λ varies from point to point throughout the phase space, since this determines the patterns traced out by bunches of trajectories in the various regions of that space. As a result of this variation, the behavior pattern of a complex system, as revealed in the disposition of bunches of trajectories in the various regions of the phase space, differs markedly (and often spectacularly) in the respective regions, constituting what may be termed a ‘complex behavior pattern’—for instance, in some part of the phase space, there may exist a regular configuration, around which trajectories reveal a periodic or quasi-periodic behavior, while in some other regions the behavior may be chaotic. In the latter situation, the behavior may be termed ‘stable’ in that the chaotic trajectories may get confined to a ‘strange attractor’— a region (having a complex structure) in the phase space such that all trajectories initiated in it eventually stay inside.

Added to this complexity depending on initial conditions in the phase space, there arises the fascinating complexity resulting from an alteration in the *context* where the context is commonly taken into account in terms of boundary conditions and, additionally,in terms of sets of parameters characterizing the evolution of the system under consideration (for instance, the Rayleigh number or the Reynolds number in the case of fluid flow—a host of such parameters characterize the flow under various different physical conditions). Here the paradigmatic phenomenon is *bifurcation* that reveals a change in the geometrical disposition of trajectories in the various regions of the phase space as certain threshold values of the relevant parameters (including those specifying the boundary conditions) are crossed.

> I repeat that this entire mode of description of the evolution of a complex system is based on the analogy with what one finds in the qualitative theory of nonlinear differential equations (referred to as the theory of *dynamical systems*), which acts as the paradigm in much of our current view on complex dynamics. The mathematical notion of the phase space or, more generally, of the 'state space'—the term 'phase space' applies to the special case of Hamiltonian systems—does not, strictly speaking, apply to any and every complex system. Once again, we use the term 'phase space' in the sense of a paradigm. The term 'Lyapunov exponents' then refers to factors in the system behavior that tend to amplify or to suppress small deviations in its state in the course of its subsequent evolution.

In summary, the dynamical evolution of a complex system may be of an exquisitely intricate nature, involving co-existence of regular and irregular behavior patterns of various kinds and transitions between stable and unstable regimes of evolution where an instability implies that the behavior pattern may change markedly and new modes of behavior make their appearance. Here the term 'behavior pattern' refers to both the temporal evolution and *spatial* structures describing the system under consideration.

3.6.2 Self-organized complexity

The transitions between distinct behavior patterns induced by instabilities in the system may often take place over a relatively small time scale as compared with the intervals over which stable modes of behavior remain dominant. Since, over such short time scales, it is the intrinsic interactions that dominate over the extrinsic ones, the generation of novel modes of behavior through instabilities in complex systems is referred to as *self-organized complexity*. The role of extrinsic factors in this context is to *steer* the system to the *edge of instability*. Since the instability often leads to an irregular ('chaotic') behavior pattern, the point of occurrence of the instability is correspondingly referred to as the *edge of chaos*.

Self-organized complexity may involve varied and diverse types of bifurcations in the system behavior, where I repeat that the term ‘bifurcation’ is used as an analogy, by invoking the paradigm of nonlinear differential equations that may or may not be of strict applicability to the system under consideration.

One such bifurcation scenario in nonlinear differential equations and nonlinear mappings (analogues of differential equations, where ‘time’, or, more generally, the independent variable, is assumed to vary in discrete steps) is referred to as *intermittency*. The pattern of intermittent and recurrent transitions to some novel mode of behavior has been found to occur over a wide range of real-life phenomena involving complex systems and is referred to as *self-organized criticality* (SOC).

Self-organized complexity (which includes self-organized criticality, see sec. 3.6.3 below), is characterized by ‘punctuated equilibria’ , i.e., approximate equilibrium configurations punctuated with short-lived phases of transitions across non-equilibrium regimes. In addition, there is found the co-existence of distinct behavior patterns, any of which can be seen to occur, depending on initial conditions.

3.6.3 Self-organized criticality

As a system is pushed to the edge of instability by the operation of extrinsic factors over a relatively long time scale, it makes a rapid transition to a new regime of stability where, however, the state of stability is slowly altered by the operation of extrinsic factors that once again steer the system to the edge of instability, thereby inducing another precipitous transition to a new—slowly evolving— stable regime. This repeated pattern of slow evolution to the edge of stability, succeeded by a rapidly changing phase of instability (‘punctuated equilibrium’), is observed over a fascinatingly wide range of real-life situations, such as the stick-slip process in dynamic friction, the recurrent generation of neuronal voltage spikes, forest fires, earthquakes, and the generation of *avalanches in sand-piles*. Such recurrent alteration of a system configuration between stable and unstable regimes involving two distinct time scales is the hall-mark of self-organized criticality (SOC). In other words, the occurrence of SOC can be described as a recurrent phenomenon involving a slow 'accumulation of strain' in a system, followed by a rapid 'relaxation' or 'burst' when the strain gets released.

The *sand-pile* has been widely studied as a prototypical model of SOC ([33], [15]). As grains of sand are slowly added (extrinsic factor) to a growing sand-pile, nothing noticeable happens at first (slowly evolving stable regime) till some critical height is reached when an avalanche is generated (rapid transition induced by instability) and part of the sand-pile collapses. If the process of addition of sand grains is continued, then a similar collapse occurs once again, and the avalanche gets repeated in a recurrent but indeterminate manner.

SOC is typically characterized by the occurrence of *power law distributions* in the statistics of numerous physical features relating to the dynamics of the process. In the sand-pile example these include the duration of an avalanche relaxation event, and the size of the avalanche. The occurrence of the power law distribution is independent of the actual speed of addition of sand grains to the pile, provided only that it is sufficiently slow—hence the name *self-organized* criticality. The occurrence of power law distributions is indicative of the absence of any particular *scale* associated with the critical transition, analogous to the case of a critical state in an equilibrium phase transition of a thermodynamic system (the so-called ferromagnetic transition in the Ising model constitutes a particular instance). In contrast to equilibrium phenomena, SOC represents a non-equilibrium process where, in the critical transition, no particular spatial or temporal scale is favored by the system, i.e., in a large number of occurrences of the transition, spatial and temporal structures (e.g., the avalanche size and the duration of avalanches in the case of a sand-pile) at all scales can be identified.

> However, *after* the transition, some particular scale is chosen by the system, depending on the context. The scale-free transition is essential for the system to be able to make this 'choice'.

As mentioned above, the transition at the critical point in SOC is one to an approximately stable or steady configuration that evolves slowly and eventually approaches a critical state over again. More generally, the transition may be one to a time-dependent state, either periodic or quasi-periodic or even to a state with chaotic time dependence. Such transitions are common in nonlinear systems and are observed in real-life phenomena such as the Rayleigh-Benard convection in a fluid (see sec. 3.10.4 where we look at symmetry-breaking transitions, of which the formation of convective rolls constitutes an instance) and transition to fluid turbulence. The spatial and temporal structures emerging in all these more general situations can be considered as instances of self-organized complexity.

3.7 Digression: what does the term 'interaction' mean?

In continuation with the content of sec. 3.5, where we examined the relation between the intrinsic and extrinsic determinants of the properties of a system, we will look here at what we mean by the term 'interaction', since the properties of a system are determined by its interactions of various descriptions. As indicated in the section referred to, properties are commonly assumed to be generated by interactions intrinsic to it while, from the epistemological point of view, we perceive the properties by means of interactions with external systems.

The question remains as to *where the interactions originate from*. For instance, the interaction between two atoms or molecules can, to a considerable extent, be seen as originating in the constitution of atoms (or molecules) in terms of electrons, protons and neutrons. But then—one can keep asking—where do the interactions between these constituents originate from? This, indeed, is a deeply metaphysical question, to which the final answer may remain unknown for ever. However, the quest may prove to be a misplaced one unless the relation between the noumenal and the phenomenal (see sec. 3.8 below; for further considerations, see chapter 4) is understood properly because (as per the point of view adopted in this book) therein lies the elusive metaphysics.

More specifically, 'interactions' between objects in this world are rooted in correlations of various kinds, and are constructed to explain their behavior observed in experience. In other words, perceived interactions, associated with causal effects between entities may, in the ultimate analysis, be the result of a projection from the noumenal to the phenomenal, the ‘real’ nature of which may remain ever unknown.

3.8 The noumenal and the phenomenal: a quick glance

Viewpoints in metaphysics can never be acceptable to all. The idea underlying the noumenal-phenomenal divide too may not be accepted by all, though this book is based on the fundamental supposition of that duality. However, the divide between the two worlds is not of divine design but is solely due to the infinite complexity of the noumenal reality and to the way we approach that reality by *interpretations* of tiny patches here and tiny fragments there.

Al our experiences and all our concepts make up our phenomenal reality and are confined to that reality alone where the phenomenal, in turn, is rooted in the noumenal. Though the phenomenal reality is also an infinitely complex one, it is generated within the womb of the noumenal. The latter is a reality that is self-determined and is a closed one. While the phenomenal is defined within the context of the noumenal, the latter has no context to be defined in.

In a manner of speaking, all our experiences and concepts are *projections* from an infinite-dimensional noumenal world, akin to a mathematical projection from a higher dimensional space to one of a lower dimension. As mathematical projections go, the reverse passage from the target space to the source space makes no sense—while the phenomenal is rooted in the noumenal (in the sense of a projection), one cannot reconstruct the latter from the former, not even as an approximation.

The statement that the noumenal reality is a closed one means that it is not defined in terms of an external enveloping entity, but implicitly defines itself. The implications of this (supposed) fact is staggeringly deep. For instance, it is likely that cause-effect relations close upon themselves, and that causality loses its familiar meaning.

To conclude: their descriptions notwithstanding, the phenomenal and the noumenal are both real—neither of the two being illusory, divine, or transcendental—where the former constitutes a minuscule part of the latter which, in the very nature of things, includes in itself space-time scales of all magnitudes and cannot be accessed *as it is*.

3.9 A brief note on the terms ‘interaction’ and ‘correlation’

The term ‘interaction’ has a causal implication—the interactions between the components of a complex system cause an immensely complicated time evolution undergone by it and, at the same time, lead to emergence. The context effect (in the narrower sense of the term ‘context’; refer back to sec. 2.1) also involves interactions between the components of the system in question and external systems making up the environment. Indeed, both the terms ‘interaction’ and ‘causation’ are commonly thought to bear an ontological association, since both define the *being* of an entity—its interactions with other entities and its causal powers associated with the interactions.

However, all these associations and implications pertain to the phenomenal world, and do not have analogs in the noumenal reality that we may hope to know.

> The relation between the noumenal and the phenomenal has a fully naturalistic interpretation, and will be indicated in chapter 4.

As noted earlier, the projection from the noumenal to the phenomenal cannot be traced back from the latter to the former. It is likely that there exist aspects of the noumenal that project to experiences and concepts in the phenomenal, though it is entirely inscrutable as to what those ‘aspects’ are. Referring to any event, experience or concept, say P, in the phenomenal world we will refer to the purported noumenal aspect projecting on to it as noumenal-P though we have no way to specify what this noumenal-P is—in other words, ‘noumenal-P’ is an entirely notional entity.

In other words, the interactions and their causal associations that we perceive cannot be extrapolated back to the noumenal reality, and what one may refer to as noumenal-interaction and noumenal-causation may not have any resemblance at all to our perceived and known features of interactions and causal powers. It is in this sense that one can interpret the philosophical skepticism of David Hume when he affirmed that categories like causality have no ontological existence in 'reality' (see, for instance,[17])—Hume did not distinguish between the noumenal and the phenomenal as Kant did, and so his language was different too (see, for instance, [28]). As for Kant, he identified space and time, and causality too, as categories pertaining to the perceiving mind and quite rightly did not refer to the noumenal-space, noumenal-time, or noumenal-causality (see chapter 4 for background).

Our perception of interactions having causal power differs from that of *correlations*, since a correlation between two entities need not have any causal asymmetry associated with it. Correlations are ubiquitous, while causal interactions are not—the idea of the former does not essentially need the notion of flow of time. In this sense we will, at times, assume that the notion of 'correlation' can be carried over to the noumenal universe, i.e., we will, as a matter of convenience, abbreviate 'noumenal-correlation' to just 'correlation'. Put differently, the noumenal world 'evolves' as does a sea in turmoil, involving vastly pervasive correlations.

1. Of course, whatever one says about the noumenal universe is notional and is in the nature of an 'illegal' extrapolation from the world of our phenomenal experience. In other words, (noumenal-)correlation is no less hypothetical than noumenal-interaction, both referring to purported aspects of the noumenal world that project on to observed features of the phenomenal. Likewise, the term 'evolution' has only a symbolic meaning when applied to the noumenal world since the unidirectional flow of time is, strictly speaking, not meaningful in it. More of this in chapter 4.
2. Any description pertaining to the noumenal reality is, by the very nature of it, incorrigibly symbolic. However, that world itself is not a fictitious one since it generates our familiar phenomenal reality—metaphysics is to be distinguished from fiction. Metaphysics is induced from experience, though not uniquely.

Returning to the question as to how interactions are generated in the first place, the answer has to be that these are ultimately of noumenal origin—aspects of noumenal-evolution generate our perception of interactions between phenomenal entities. It may seem to be abominably anti-scientific to throw everything ultimately into the black-box of the noumenal, but there is no way out of the infinite regress into which we find ourselves while asking questions about origin that seem never to end. As far as metaphysics goes, we have to recall that the noumenal is no less real than the phenomenal reality, since the latter is rooted in the former—only, it is a self-determined whole that is beyond our ability to comprehend by way of extrapolation from the phenomenal.

3.10 Emergence and collective degrees of freedom

Back to the concrete from the metaphysical!

But, of course, all our considerations ultimately rest on our metaphysics—the latter persists, like an albatross.

3.10.1 Collective behavior in emergence: a brief overview

Emergence is essentially about the popping up of novel behavior patterns of a complex system in a way that cannot be predicted or explained in precise terms in spite of the fact that such unfolding of novelty is determined by the mutual interactions of its constituents, subject to appropriate context effects. It is the large number of components of such a system and the wide-ranging and diverse interactions (or, more generally, of *correlations*) among those that is principally responsible for the phenomenon of emergence.

However, emergence occurs for systems of relatively small size as well under appropriate conditions (example, cellular automata such as the self-replicating ones designed by Von Neumann), essentially because of the interdependent response patterns of the components, involving feedback loops. As instances of systems of intermediate size, one may refer to the flocking of birds flying in V-shaped formations where the formations, on being broken up, get reassembled again. Compared to the large number of degrees of freedom of each individual bird, the V-shaped formation is determined by a relatively small number of collective degrees of freedom. As a flock encounters strong wind or some other obstacle, its subsequent course is described in terms of these few variables.

The relatively small number of relevant variables in the description of emergent behavior is indicative of a *coherence* that may be structural, or temporal, or even an intricate combination of the two. Structural coherence, for instance, is manifest in the shape of a deformable solid. Under applied forces generating stress in the body, it gets deformed in a combination of a few relevant modes (elongation, shear, bulk deformation), where the vibrational modes of the large number of molecules in it are not of relevance. Shape emerges in an aggregate of molecules when a large number of those interact under appropriate conditions (specified in terms of parameters such as temperature and pressure).

> The description, in terms of collective variables, of emergent behavior in a complex system differs fundamentally from the behavior pattern of its 'microscopic' components, despite the fact that the collective variables are determined by the microscopic ones (see sec. 3.16 for more complete considerations). This, indeed, is where the novelty in emergence lies.

Temporal coherence is expressed in terms of only one or a few frequencies characterizing the temporal behavior of a system. This phenomenon is commonly referred to as *synchronization* ([15], chap 7; of great relevance in chemical and biochemical reactions is the process of *autocatalysis*, where synchronization occurs routinely [34], sec. 4.5). The synchronous lighting of a large number of fireflies distributed over a region constitutes a notable example. There takes place a large number of periodically varying physiological processes in the human body, constituting instances of synchronous functioning of large populations of cells in numerous organs and tissues: the circadian rhythm, the heartbeat rhythm, the respiratory cycle, and rhythmic waves of excitation in the brain detected by EEG recording are a few of the more well-known among such synchronous phenomena.

A vast literature exists on emergent structures and processes in condensed matter studies and in studies on chemical processes, these being relatively more precise and mathematical in nature. On the other hand, studies on emergence in complex systems are growing in number at a rate comparable to the occurrence of an explosion, in fields of biology, ecology, evolutionary dynamics, sociology, psychology, weather sciences and meteorology, geology, epidemiology, economics and finance, and business studies. Though heavily dependent on computation and statistical analysis, these are also fast acquiring a measure of precision based on systematic analysis, unraveling emergent phenomena almost at every turn of investigation in each and every chosen area. What is more, it appears that there exist classes of complex systems and complex processes where emergence possesses *universal features* when considered in the context of some particular class.

We conclude this section by referring to *embryogenesis* and the developmental biology of the human body, which is a stupendous phenomenon involving a large succession of episodes of emergence—it has the appearance of an orchestrated process of enormous splendor and complexity, starting from the zygote, developing into a fetus, and finally ending up with the exquisitely complex and organized human body functioning in a coherent manner. At every stage of the process there occurs spatial and temporal self-organization so that, in the end, the human body emerges with its unique spatial disposition of tissue and organs with their interdependent functioning involving a huge array of synchronized processes. The latter interact with one another so as to generate weak low-dimensional chaos in the temporal functioning of the organism—the admixture of chaos is indicative of the robustness and flexibility of the various synchronized processes that survive their mutual interaction. A notable early attempt at understanding the dynamics underlying the process of embryogenesis (a special case of what is referred to as *morphogenesis*, or the emergence of patterns in macroscopic systems) was that of Turing ([34], sec. 4.2; [15], sec.2.5) in his study of *reaction-diffusion systems*. Reaction-diffusion systems are involved in the emergence of what are referred to as *dissipative structures* in a wide array of evolving complex systems.

> In a development complementary to the one initiated by Turing, C. H. Waddington introduced the idea of *epigenetics* in the developmental process. The idea of epigenetics has subsequently had far-reaching implications ([26]), especially in the field of genetic network and gene regulation. Waddington, incidentally, coined the term *homeorhesis* in depicting stable dynamical trajectories in the evolution of a complex system (see, for instance, [23]).

3.10.2 Emergence in condensed matter physics: the XY model

As a concrete instance of emergence that has been studied in detail, we refer briefly (and qualitatively) to the formation of *vortices* in the so-called *XY model* that describes the statistical mechanics of a two-dimensional array ('lattice') of rotors—the microscopic elements of the model—each rotor being a vector (an arrow) of length unity that can point to any and every direction in a two dimensional plane (described in terms of X- and Y-coordinates of points). This simple description of the model notwithstanding, the emergence of vortices is found to be relevant in understanding a wide array of systems in physics [15] illustrating the phenomenon of *universality* in large classes of systems.

In the (classical) XY model the interaction energy between two rotors juxtaposed to each other is proportional to the cosine of the angle between them, while distant rotors do not interact. Between two nearest neighbors, the interaction energy is minimum when they are parallel, and maximum when they are anti-parallel. In the equilibrium state of the system at any given temperature, any specified configuration of rotors in the lattice is characterized by a certain probability of occurrence given by the *Boltzmann factor* ($e^{-\frac{E}{k_{\mathrm{B}}T}}$, where E stands for the energy of the configuration, T for the temperature, and k_{B} for the so-called Boltzmann constant). Making use of this formula, one can calculate the macroscopic properties of the system in the equilibrium state at temperature T.

As the temperature is made to increase gradually from $T = 0$, there does not at first appear any notable change in the macroscopic properties since the model does not show an order-disorder phase transition as seen in the one dimensional *Ising model*. Instead, the *free energy* of the system increases gradually, till one encounters a *discontinuity* at a certain temperature T_{KT}, referred to as the Kosterlitz-Thouless (KT) transition temperature. This transition is indicative of the phenomenon of emergence in the system that can be understood by referring not to the individual rotors (the microscopic constituents) but to composite structures referred to as the vortices.

A vortex is a structure centered around a rotor, in which the orientations of the neighboring rotors undergo an integer number of complete cycles (or *windings*) as a complete circuit is described around it. Depending on the direction of the winding direction with respect to the sense in which the circuit is described, one distinguishes between a vortex and an *anti-vortex*. At low temperatures the statistical mechanics of the system is conveniently worked out in terms of vortex *pairs* where a vortex and an anti-vortex form a bound configuration, whose energy increases (i.e., the binding energy decreases) with the separation between the two. At temperatures above $T_{\rm KT}$, the pairs break up and disperse due to the decrease in their binding energy, and a significant change comes about in the macroscopic properties of the system. Conversely, as the temperature is made to decrease through $T_{\rm KT}$, there occurs the formation of the vortex-antivortex pairs, with the average separation between the members of a pair decreasing as T decreases, giving rise to an attendant modification of the macroscopic properties of the system (such as the so-called *spin stiffness*). In so far as the macroscopic properties of the system below $T_{\rm KT}$ are determined by variables pertaining to the vortex pairs rather than by those specifying individual rotors, this constitutes an instance of emergence.

1. A mathematical indicator of the phenomenon of emergence at the transition temperature $T_{\rm KT}$ is obtained from the *correlation length* that gives an estimate of the distance over which the vortices are correlated at any given temperature. At $T > T_{\rm KT}$, the correlation decays exponentially with distance because of the nearest neighbor interaction between the rotors while, for $T = T_{\rm KT}$, the correlation length diverges (the statistical correlation between vortices decays logarithmically with their separation). This divergence is indicative of the fact that the details of the nearest neighbor interaction between the rotors is not important for the macroscopic behavior of the system, because it is the long range configuration of rotor aggregates, determined by the distribution of the vortex-antivortex pairs, that is of relevance. In other words, emergence is a consequence of the long range configuration of the lattice, where the indirect interaction between rotors mediated by intermediate chains of intervening rotors is overwhelmingly important as compared to the direct interaction—in concrete terms, the long range interaction is a consequence of the formation of the vortex-antivortex pairs.
2. The KT transition is not an *order-disorder transition* in the usual sense of the term, and is not associated with a *symmetry breaking* (briefly outlined in sec. 3.10.4), in contrast to numerous other systems studied in condensed matter theory.

Despite the fact there does not exist an exact solution for the two dimensional XY model, the phenomenon of emergence in the model is amenable to a good deal of quantitative analysis. This is due to the fact that the model, even in the limit of infinite size, is a precisely defined one. In contrast, complex systems in real life are not precisely defined, both in respect of the nature of the microscopic constituents and in respect of the diverse types of interactions between those.

The fact that it is the large scale configuration of the lattice, and not just the nearest neighbor interactions, that leads to emergence, has an important consequence: numerous systems where the direct interactions between the microscopic constituents differ in details, exhibit emergence with *analogous features*. This is the phenomenon of *universality* within classes of systems that share common features characterizing emergence. For instance, the Kosterlitz-Thouless transition, as seen in the classical XY model, is also observed in liquid crystals, thin films of superconductors, and thin films of liquid helium, where structures analogous to vortices make their appearance. Additionally, the XY model is equivalent to ones describing crystal growth and the roughening transition on surfaces, under appropriate conditions. A closely related two dimensional model, which is *exactly solvable*, shows the KT transition with features as outlined above.

The feature relating to the existence of universality classes in emergence pertaining to systems of diverse descriptions tells us that, in our attempts at describing and explaining natural phenomena, we can conveniently distinguish between *scales*—generally speaking, scales pertaining to *space* and *time*—such that phenomena in some particular scale appear as being independent of those at some other scale. As a related phenomena, in describing a given system within some appropriate scale, one may, at some juncture, be needed to reckon with a scale of a different order of magnitude since there may occur episode of emergence characterized by *large scale correlations* in the system. On the other side of the emergence, the newly emerged formation may need a different but some definite scale for the description of space-time processes in it.

1. The various scales may appear in space or time or even jointly in space-time. A segregation among scales may also take place in some other abstract state space such as in the phase space of a multi-particle dynamical system or, say, in the conceptual space in the human mind.
2. One may then analyze and explain some class of phenomena in some particular scale without regard to other scales pertaining to the system under study such that, within the chosen scale, various different systems belonging to some particular class are described in terms of analogous features and analogous behavior patterns in the borderline zone of emergence.

While the XY model illustrates the occurrence of the phenomenon of emergence at the level of equilibrium configurations, the idea of emergence itself is of more general applicability to non-equilibrium dynamical configurations describing behavior patterns of complex systems. More specifically, emergence is based on self-organized complexity, as outlined in sec. 3.6.

We *summarize* by recalling that central to the idea of emergence is the appearance of collective degrees of freedom where the behavior pattern of a system is determined by the *overall configuration* of its constituents in some macroscopic subsystem (i.e., the configuration of nodes in some cluster in the network representation), and not by the individual constituents themselves. The 'macroscopic' collective variables and the 'microscopic' variables pertaining to the constituents make up two distinct *levels* of descriptions, segregated in distinct scales of space and time.

> Universal features in emergence are related to *symmetry breaking*, a phenomenon of great relevance, to be briefly introduced in sec. 3.10.4.

3.10.3 Emergence and the divergence of correlation length

Emergence in a complex system involves a transition from one stable regime to another in the course of its time evolution across an edge of instability, where the latter represents a tipping point that opens up new regimes of spatial and temporal correlations among the components making up the system. Before the tipping point is arrived at, the system is characterized by a certain *scale* over which correlations persist among the components, whereas a new scale emerges beyond the edge. In between, the tipping point is characterized by large scale correlations that straddle space- and time- scales of distinct orders of magnitude so as to 'enable' the system to 'choose' the scale that would ensure the stability of the emerging space-time structure.

Figurative statements apart, the *correlation length diverges* ([15], chapter 6) as one approaches the edge of instability. We consider spatial correlations here for the sake of simplicity and concreteness, as seen in the case of an equilibrium phase transition, say, in a magnetic lattice—there exists a mathematical definition for the correlation length in terms of the *order parameter* characterizing the magnetized phase that emerges from the non-magnetic one as the temperature is lowered past the transition point in the absence of a magnetic field. Temporal correlations similarly reach out to large time separations, as revealed in the phenomenon of *critical slowing down*

> The order parameter constitutes an instance of the *collective variables* mentioned in above paragraphs. It constitutes a central concept in *Landau's theory of phase transitions* (see sec. 3.12 below)—a ground-breaking one in the context of emergence in general and of phase transitions in particular.

Incidentally, it is important to recall the distinction between interactions and correlations (refer back to sec. 3.9) since the latter includes the former but is more general in scope. Long range correlations between systems or subsystems can be realized by means of *indirect* causal interactions (a system, say, A may not directly interact with another system B, but may do so via the intermediary of a chain of other systems such as, say, C, D,$\cdots$). In the case of emergence a certain macroscopic configuration of the components of a complex system may acquire stability (in a relative sense) when indirect interactions through numerous long series of intermediaries acquire relevance in generating the large scale correlations.

There exist a large number of indications outside of physics that long-range space-time correlations are indeed generated under special conditions in complex systems when compared with the range of direct interactions among their components. In the case of emergence, length scales of distinct orders of magnitudes are selected on the two sides of a tipping point. Thus, in a flock of birds flying in a collective formation (commonly a V-shaped one), individual birds interact with only a few neighbors, while velocity correlations among the birds in the flock have a range that scales with the overall size of the flock.

As another instance, correlations in the neuronal activities in human brain range over large spatial and temporal separations, as revealed by the EEG rhythms (commonly referred to as $\alpha, \beta, \gamma, \delta \cdots$) and as found in fMRI recordings ([15], sec. 6.5). What is of added interest is that these correlations are segregated into groups of differing magnitudes, indicating that structures and functions in the brain are the result of a series of emergent phenomena occurring in biological evolution as well as during the developmental history of an individual. Indeed, the brain seems to operate close to criticality, being characterized by a large spectrum of scales, associated with structural and functional alterations by means of *neuroplasticity* (see [35] for background) and with the establishment of long-range neuronal connectivity made possible at frequent intervals in the brain, which is why the latter is considered to be an exceptionally adaptive organ in the body.

Briefly, the phenomenon of emergence involves long-range correlations among the components of a complex system, where the term 'long-range' need not mean large spatial separation, though correlations over large spatial distances *do* acquire relevance in numerous instances of emergence. In the network representation of the system, clusters involving large numbers of nodes (with large separations between distinct clusters) become marginally stable so that with a slight change in the context, a stable conglomerate of nodes makes its appearance, the size of the conglomerate being larger by at least one order of magnitude compared to the separation between individual nodes in the network. Instances of networks where separation does not necessarily mean spatial distance are the network of concepts lodged in the mind of an individual and a social network where separation means the difference in social status between individuals.

What is the physical agency that establishes the long-range correlations between constituents of a system close to emergence? The answer is: *fields*. Fields are distributed entities in space (I repeat that the 'space' need not the three dimensional one familiar to us) capable of establishing long-range correlations and the emergence of structures that acquire stability under given contexts. In most cases, the links responsible for interactions between constituents close to one another themselves make up a field—though only neighboring constituents get correlated by the direct interactions, interactions arise between remote elements too by means of successions of intermediate links (refer back to section 2.7, where the feature of *wide-ranging connectivity* of networks representing complex systems is introduced)—the links themselves may be looked upon as a distributed system in space.

In numerous situations, fields of more specific descriptions bring about the long-range correlations in emergence. For instance, fundamental fields such as those underlying various elementary particles, and the gravitational field too may well be responsible for long-range quantum mechanical correlations —such correlations mediated by fields may provide a likely explanation of observed features of *quantum measurement* processes where one makes use of classical measurement apparatus ([20]). Fields in air responsible for auditory and visual perception are instrumental in setting up long-range correlations in myriads of situations involving complex systems. Long-range correlations in the human brain are set up by means of long neural pathways and also by fluids responsible for neuro-transmission. Likewise, the emergence of spatiotemporal patterns in the Turing model of morphogenesis owes its origin to fields of chemical concentrations of reactants. All these instances of fields giving rise to long-range correlations, however, do not differ fundamentally from the 'field' of interactions between contiguous constituents of a complex system mentioned above.

To summarize, interactions in a complex system may be local in nature, but correlations generated from these interactions can be long-range ones.

3.10.4 Symmetry breaking in emergence

The phenomenon of *symmetry breaking* has, over the decades, acquired great relevance in condensed matter theory and field theory in physics, and has since been found to be equally relevant in other areas in the natural and social science too. In a broad sense, the issue of symmetry breaking is of essential significance in philosophy and metaphysics—how fundamental is our idea of Symmetry in Nature?

In the physics of many-body systems, one tries to relate the basic interaction Hamiltonian defining a system with the actual states that it may be in. The relation is not a direct and unambiguous one even in the case of a few-body system such as the ammonia molecule (see [1]). While the interaction Hamiltonian possesses the *mirror-symmetry* and has no in-built bias towards the generation of an electric dipole moment, an actual ammonia molecule does possess a dipole moment (an instance of symmetry breaking) though, on a larger time scale, it can flip to a state of an opposite dipole moment by quantum mechanical tunneling. In the case of crystal formation out of a liquid, the rotational and translational symmetry of the Hamiltonian characterizing the molecular assembly is broken, leaving only a lower symmetry intact in the crystalline state—in principle, thermal fluctuations can 'restore' the higher symmetry, though on an astronomical time scale.

The dipolar state of the ammonia molecule or the rigid crystalline state formed out of a liquid provide instances of symmetry breaking where the state of a many-body system is found not to respect the symmetry of the underlying Hamiltonian that specifies it at the fundamental, 'microscopic' level—in this sense, the broken-symmetry state is an emergent one. What identifies broken symmetry with emergence is that the occurrence of symmetry breaking cannot be determined in a 'bottom-up' derivation from microscopic principles alone (there are a few exceptions, though), but can only be understood within the framework of a 'macroscopic' one such as a *mean field theory* where the system *as a whole* is analyzed in a *self-consistent* manner, i.e., by maintaining consistency with the underlying Hamiltonian.

As is apparent, emergence is here associated with one out of several possible macroscopic configurations being 'chosen' by the underlying intricate many-body dynamics, consistently with some appropriate context,such as the temperature of the liquid bath in which a crystal is formed. What the context presumably does is to provide for the relative stability of a symmetry-broken configuration, such that collective state variables can emerge, characterizing the latter—in the case of the crystal one such collective variable is its rigidity, while another important collective and symmetry-breaking feature is its periodic structure that breaks the continuous translational symmetry of the liquid state.

3.11 Complexity, conflicts, and emergence

As mentioned in section 2.6, complexity harbors conflicts—generally speaking, conflicts are multifarious and diverse in a complex system. Conflicts operating in the microscopic interactions may get expressed as ones at a macroscopic level. Influences operating on individual microscopic constituents are more often than not out of sync with one another because of the feature of co-evolution (‘everything evolves’ – every feature pertaining to the system is eventually correlated with every other). In a sense, emergence can be seen as the consequence of conflicts that get expressed on a macroscopic scale. The all-pervasive pulls and pushes gives rise to an intricate pattern of time evolution involving criss-crossing regimes of stability and instability. In a complex dynamical system whose evolution can be envisaged to take place in a phase space, these generate a *spectrum* of positive and negative Lyapunov exponents, where the spectrum itself varies over various regions of the phase space. The opposite signs among the set of all Lyapunov exponents constitute the expression of conflicts operating at the microscopic level. On the other hand, only one or a few of the exponents get involved in the cross-over through an instability, thereby representing the dominant source of conflict at the macroscopic level.

> The fact that only one or a few of the Lyapunov exponents get involved in the occurrence of an instability arising out of a stable state, implies that instabilities in a complex system are mostly *local* in nature (recall section 2.12). This, however, does not rule out a situation where the change of sign of only a few of the Lyapunov exponents may lead to a big reorganization in the entire phase space of a system by way of what is referred to as a ‘domino effect’.
>
> If, in a complex system, the Lyapunov exponent involved in the cross-over through an instability pertains to a collective configurational state variable of a macroscopic subsystem, then one has a situation describing a conflict on a macroscopic scale, the resolution of which leads to the occurrence of an emergence.

In any given region of the phase space, the evolving system, represented by a wandering point describing a trajectory in the phase space, is pulled away from its trajectory in virtue of one set of the Lyapunov exponents, and pushed back to it in virtue of the remaining ones. At some point of its evolution the two opposing influences—one of pulling away and the other of pushing back—on the representative point may be in relative poise against each other at some tipping point (essentially due to the cross-over of the dominant Lyapunov exponent), when a bifurcation becomes imminent, and eventually a new behavior pattern appears, perhaps leading to an emergence.

Even when a complex system cannot be described in terms of a phase space, along with trajectories, and Lyapunov exponents, its evolution pattern can be understood by making use of these as an analogy so as to provide us with a paradigm—the latter can be used to tell us how conflicts lead to emergence. The phenomenon of symmetry breaking can also be identified in such a framework as a 'choice' among alternative possibilities that the system 'adopts' as a dominant Lyapunov exponent changes sign and a macroscopic state emerges within a given context.

The above description in terms of bifurcations, symmetry breaking, and conflicts within complex systems, though of a schematic nature, helps us understand the phenomenon of emergence as the formation and stability of large scale configurations since emergence involves long range correlations in a complex system, where collective variable acquire relevance. Referring to the network representation, clusters of various sizes are formed and then get dissolved within the larger framework of the network as a whole till a sufficiently large cluster gets close to the border of stability and then acquires an autonomous existence of its own, with links that establish correlations with other macroscopic formations (subsystems or external systems), signaling an episode of emergence beyond the edge of stability. At the edge of stability, conflicts within the system get expressed on a macroscopic scale (through a dominant Lyapunov exponent) and a symmetry breaking occurs, implying that the complex system in question 'chooses' a macroscopic state that is not inconsistent with the microscopic dynamics but is nevertheless not uniquely determined by the latter.

It is this lack of uniqueness in the emergence of a macroscopic configuration—one that is at once of ontic and epistemic origin—that is the hall-mark of the phenomenon of emergence. More precisely, even if it were possible to follow the succession of intermediate microscopic configurations during the course of the dynamical evolution of the complex system under consideration, it would be impossible to formulate rules telling us exactly how and when emergence would appear since the latter is determined by fluctuations within the system under consideration. One comes across the fundamental indeterminateness in emergence in computer simulations of cellular automata and Conway's game of life.

3.12 Landau's theory of phase transitions

The macroscopic expression of internal conflicts in complex systems at the edge of stability is succinctly formulated in *Landau's theory* of phase transitions. The more general Landau-Ginzburg theory makes use of a *free energy functional* based on a long-wavelength mode pertaining to a macroscopic *order parameter* that dominates the dynamics at the edge of stability. The long wavelength mode is a slow one that separates it from all the microscopic modes described by rapid small-scale fluctuations, the latter being irrelevant in the context of emergence.

> Nevertheless, the fluctuations are responsible for the selection of the symmetry broken state.

The free energy functional contains a quadratic and a quartic term involving the order parameter that operate in contrary directions, giving rise to alternative macroscopic states that the system 'chooses' from in a symmetry breaking transition. In the Landau-Ginzburg theory, the spatial dependence of the free energy functional is taken into consideration, so as to account for the emergence of spatial structures past the edge of stability.

The epistemic unpredictability of emergence—and the associated non-uniqueness of the emergent state—relative to the microscopic dynamics finds its expression in the fact that the order parameter, whose exact origin is left unspecified in the theory, needs to be guessed at from macroscopic considerations, in a manner consistent with the microscopic dynamics. For instance, in the case of superconductivity, the order parameter relates to the phase of the collective BCS (Bardeen-Cooper-Schrieffer) wavefunction describing paired electron states, and describes the macroscopic distribution of *vortex tubes* within a type II superconductor.

We conclude this section by referring back to internal conflicts in the microscopic dynamics of a system (refer back to sections 2.6, 3.11). An instance of such conflicts is the pervasive occurrence of *frustrations* in a disordered lattice mentioned in section 2.6 that may lead to the possibility of *multiple phases*, all distinguished by different values of an order parameter such as the *Parisi* order parameter in spin glass models ([32]). Frustrations in the proper sense of the term do not occur in the one dimensional or a higher dimensional Ising model, but conflicts do exist, and give rise to emergent structures such as *topological objects* ([21]), one instance of which is constituted by the *domain walls* in a ferromagnetic material that separate small but macroscopic magnetized regions within it having distinct directions of magnetization—removing a domain wall requires a significant amount of energy. Domain walls make up a spatial pattern within a material that is formed by symmetry breaking (see sec. 3.13 below for further considerations on pattern formation).

3.13 Symetry breaking and pattern formation

Homogeneous space, or a homogeneous (and infinitely extended) material, is endowed with the highest degree of symmetry. A uniform mixture of two finely grained solids is heterogeneous on a microscopic scale but homogeneous from the macroscopic point of view. Patterns are formed when one or more symmetries of homogeneous space are broken. In the present context of emergence, we will be concerned with symmetry breaking on a macroscopic scale, though microscopic scales can also be involved implicitly (example: lack of rotational symmetry of a unit cell in a crystal lattice).

Phase transitions often involve the emergence of symmetry breaking structures in space—a phenomenon addressed in the Landau-Ginzburg theory with great success. A notable example relates to the explanation of numerous phenomena involving spatial structures in *superconductors* . For instance, it explains the spatial inhomogeneity in the distribution of current carriers in a superconductor, especially in the context of surfaces and edge effects. In addition, the Landau-Ginzburg theory has been remarkably successful in explaining the distribution of *vortices*, made up of magnetic flux lines in *type II superconductors*.

Autocatalysis (refer back to sec. 3.10.1) is another type of process that leads to pattern formation, both in space and time, where spatial patterns may be formed in spaces other than the three dimensional physical space familiar to us (for instance, a space where the co-ordinates refer to the concentrations of reactants in a chemical reaction). Speaking in general terms and referring to the network representation of a system, autocatalysis involves the formation of feedback loops where chains of correlations between successive pairs of nodes are terminated at their points of initiation. There exists a vast literature on autocatalysis in chemical reactions, where fascinating spatial patterns with intricate temporal oscillations appear ([34]) in the absence of stirring of the reaction chamber. Simple models of autocatalytic chemical reaction systems are based on the so-called *reaction-diffusion* equations, simulations of which demonstrate the emergence of patterns. Such chains of autocatalytic chemical reactions involving spatial diffusion of the reactants are believed to have been relevant in the origin of life and are likely to be of great relevance in Turing's scheme—based on reaction-diffusion systems—pertaining to the phenomenon of morphogenesis mentioned in sec. 3.10.1.

Pattern formation in emergence involves additional conflicts in a complex system in that the spatial migration of its components may interact in a complex manner with the conflicts resulting from their interactions that arise independently of the migration.

3.14 Emergence and conservation principles

As mentioned in the above paragraphs, emergence is associated with the stability of certain 'macroscopic' configurations of components of a complex system (see, however, sec. 3.17 below), where these configurations (or 'structures') arise in the course of its complex spatial and temporal evolution. Referring to the putative phase space of the microscopic components—one of an enormously large number of dimensions—the stability can be associated with all the Lyapunov exponents (again, a notional concept) being negative (or having negative real parts) in some region of the phase space that corresponds to the emergent structure. Alternatively, some particular cluster in the co-evolving representative network retains its structure regardless of causal signals arriving from distant parts of the network—while these signals tend to disrupt the cluster, the latter retains its integrity in virtue of the mutual interactions among its internal components. Exactly which cluster is destined to qualify as a stable one is thus a question of stupendous intricacy for systems made up of a vast number of components.

In this ‘microscopic’ view, stability and instability are *contingent* states of affair—in the infinitely complex co-evolution of a system, there arise interludes when some macroscopic subsystem or other acquires a stable and regular behavior pattern past a border of instability. Alternatively, it is possible to adopt a ‘macroscopic’ view in which stability is a matter of *necessity*, where a cluster, with a sufficiently weak interaction with the rest of the network, approximately obeys the symmetry inherent in its environment, while the internal interactions ‘break’ that symmetry, causing the system under consideration to ‘choose’ one of a number of alternative configurations. In this view, the Hamiltonian of the world possesses a high degree of symmetry and interactions of successively decreasing relevance break the symmetry one after another (refer back to sec. 3.10.4). A symmetry, generally speaking, implies a conservation principle and the emergence of a configuration depends of an approximate conservation principle, along with symmetry breaking that contingently selects out one among several possible macroscopic configurations.

Which of the two views is to be considered as being of ultimate relevance? The answer has to be a qualified one—a point of view can be valid only relative to a context. However, when the question of ‘ultimate’ relevance arises, the final arbitration has to be in the realm of *metaphysics*. And metaphysics can be varied, depending on how we extrapolate our experience of reality so as to get at the *foundation* of that experience.

The macroscopic view starts from a hypothetical space-time that is necessarily the repository of all the symmetry there can be—that symmetry is broken successively by interactions conjectured to exist between various parts of the universe (it is considered to be the job of science to make the appropriate conjectures, based on observations and evidence), and structures are supposed to be formed contingently in the process of symmetry breaking. This view appears to be ‘self-evident’, not founded on any arbitrary metaphysical suppositions. But metaphysics always appears to be self-evident to those who adhere to it.

The microscopic view, on the other hand, looks at the world as being infinitely contingent, with necessity arising only within limited space-time horizons generated in the infinite sea of contingency and complexity. Evidently, the two viewpoints are complementary, and it is a matter of brute extrapolation to choose a world-view where one of the two is supposed to be fundamental to the exclusion of the other. In this book, we will adopt a metaphysics of an 'ultimate' reality that is entirely self-determined and infinitely complex, with everything co-evolving with everything else—there is nothing in this reality that is determined in itself independently of the rest of the co-evolving universe, when all space-time scales are taken into account. This will receive further consideration in chapter 4.

In what is referred to above as the macroscopic view of symmetry and conservation, a principle of relevance is the conservation of *energy*. A system with only weak interactions with the rest of the universe retains its energy. If the interactions internal to the system are sufficiently strong then a small change in its configuration implies a relatively large change in energy, which the system can exchange with its environment only in a large interval of time. Thus, such a system retains its structure and is endowed with a 'rigidity', associated with one or more collective variables. In other words, a system that can be considered to be approximately isolated from its surroundings is characterized, under appropriate conditions, by a conserved energy and, possibly, by a set of collective structural variables—it is such a structure that may emerge from within the womb of a bigger complex system, distinguishing itself from the rest of its surroundings by a conserved configuration.

The principle of conservation of energy is a consequence of the homogeneity of time. No known case of violation of energy conservation has been found to date. Incidentally, symmetry breaking differs from symmetry violation in that a symmetry broken state is associated with other states generated from it by the application of the symmetry, while no such thing occurs in the case of symmetry violation (the distinction, in a manner of speaking, is relative, depending on the Hamiltonian of the system under consideration). Thus, in the case of a supposed violation of the homogeneity of time, the energy of a system may decrease with time, without any associated possibility of a state whose energy increases with time.

However, the question of homogeneity of time and energy conservation—and, for that matter, the idea underlying all symmetry principles—takes on a completely different complexion when considered for *reality as a whole*. For instance, it is highly doubtful as to whether the evolution of reality as a whole can be said to take place within the framework provided by space and time as these are known to us in all of our experience. For, that experience is always partial and fragmentary—whatever processes we observe or imagine, these always occur within a context provided by the framework of space and time. But reality as a whole knows no context. In contrast, the phenomenon of emergence as described in the above paragraphs occurs within the phenomenal world of our experience, and not in the noumenal. We will have more to say on this in chapters 4 and 5.

3.15 Evolution computation and emergence

The most conspicuous and remarkable instance of emergence in nature is biological evolution, involving the proliferation and diversification of species.

> The scientific understanding of biological evolution has itself undergone a remarkable evolution over decades. This, of course, is an instance of *emergence in the space of scientific theories*. We will have more to say on this in chapter 5. However, regardless of our state of understanding about it, biological evolution continues and will continue to be the paradigmatic instance of the phenomenon of emergence in consequence of co-evolution.

Beginning with the origin of life from certain classes of macromolecules, biological evolution has led to the emergence of countless forms of living organisms over millions of years, many of which have become extinct with environmental changes while many others have survived in virtue of their ‘fitness’. The process goes on, with an amazing blossoming of newer and newer forms of life, adapted to their respective physical environments, and with extinction and survival running ceaselessly in parallel.

3.15.1 Evolutionary processes

The driving forces underlying evolution have been identified as mutation, genetic variability among individuals, migration, and natural selection. However, starting from the days of the 'modern synthesis' when Darwinism and Mendelism found a common ground in the science of genetics, there have taken place sea changes in the fundamental ideas explaining biological evolution (see, for instance, [26]), based on evidence of diverse kinds (from the micromolecular, cellular, and biological levels, right up to the geological, meteorological, and paleontological), and the changes continue today. Biological evolution involves an amazing mix of contingent and necessary factors, as do all processes of evolution in complex systems. Mutation, of course, is a contingent factor in evolution, but there are other contingent factors at work, such as the size of a population, epigenetic processes in the developmental history of individuals interfering with their genetically inherited fitness, chance factors associated with the migration of the members of a species, and, not the least, chance factors in the unfolding of the environmental scenario, which is an enormously complex process itself.

With all these forces at play, one can only attempt to guess at the outlines of the process of biological evolution without really expecting to *understand* its fundamental mechanism in the sense of being able to predict when and how a species will emerge even when reasonably well informed of the factors involved. This is the fundamental epistemological unpredictability in the phenomenon of emergence, which is intertwined with the ontological fuzziness of complex systems—a fuzziness that goes away as a system and its context are precisely defined, but then the system no longer remains a complex one since it is based on idealization and abstraction, and the phenomenon of emergence becomes predictable, though the prediction may require great effort and ingenuity.

Regardless of the lack of predictability in the process of biological evolution as it occurs in nature, computer simulations have been attempted with a view to mimicking it, with various degrees of predictability achieved under contexts set with various levels of precision. In many such simulations, the occurrence of the phenomenon of *punctuated equilibrium* (refer back to sections 3.6.2 and 3.6.3) has been detected.

3.15.2 Emergence in computation

Computation can be based on the so-called Von Neumann type architecture or, alternatively, on the *non Von Neumann* type. Among the latter are included *neural networks* and *cellular automata* (see, for instance, [26]), both of which carry out *parallel computation*, without a central processing or control unit.

> Von Neumann himself initiated many of the basic ideas in the non Von Neumann type computational approach. In particular, his fundamental work on self-reproducing cellular automata has had far-reaching implications in and outside the science of computation, notably in biology and, indirectly, in the social sciences too. Von Neumann had in mind the foundational issue of addressing the workings of the human brain and the human mind while looking for alternative computational architectures.

In the Von Neumann architecture, the program is executed step by step, based on the results arrived at in earlier steps, in consequence of which, the computation at any given step is uniquely determined even when it involves an ‘if-then-else’ type decision making. This is analogous to a deductive inference in the human mind in which only shared rules (shared, that is, by individuals belonging to a larger human community—rules that can be formulated in more or less explicit and logical terms) are made use of in arriving at a conclusion. The question that arises in this computational approach, is whether the final result is, in some sense, an emergent one?

> Suppose that, starting from the axioms of Euclidean geometry and making use of the rules of inference, a mathematician arrives at a difficult theorem. At no stage in the process does she invoke anything beyond the axioms, the rules of inference, and the results obtained at earlier stages. How, then, can she be said to introduce ‘novelty’ that is central to the idea of emergence?

We will not enter into detailed considerations of this question here, since it has a number of subtle aspects to it. In reality, the result of a difficult derivation or that arrived at in a long process of computation can indeed be considered to be an emergent one—we assume that the ‘length’ of a computation is a well defined concept, pertaining to the shortest of all possible computational paths that can be followed in arriving at the result, where one has to take into account the presence of loops that increase the effective length of the computational path. Of relevance here are the following two issues: (a) the proliferating number of references to earlier stages in the process as its length keeps on increasing, as a result of which the steps of computation constitute a *complex system* where there gets involved (b) a proliferating number of *decision stages* that become necessary in order to go through the process.

> In the case of a deductive derivation gone through in the mind of the mathematician, one does, in a sense, have to go beyond the ensemble of axioms and well formulated rules, in that, at certain junctures of the derivation she has to decide as to which rules, axioms, or intermediate results she is to make use of in taking the step ahead—in this process she once again makes use of rules, but now of an *implicit* nature that may, in a certain sense, be said to be unique to her (otherwise all sufficiently well versed mathematicians would be able to make the derivation).

In other words, in the Von Neumann Architecture, the result in a long computation is indeed an emergent one that cannot be predicted from the code and the data without actually following the program execution to the end.

In the non Von Neumann architecture, on the other hand, emergence can be understood in more direct terms, because at every stage of program execution, the computing units, acting in parallel, change their current states in a manner determined by the states of a number of other units interacting with it (by attaching certain weights that need not be determined beforehand) and by evaluating a certain 'fitness function' thereby going through a 'learning' process in the course of program execution. In a *genetic algorithm* [26], for instance, random 'mutations' are introduced in the process so as to mimic the course of evolutionary adaptation, with quite dramatic results appearing in consequence.

In a *cellular automaton*, the way the individual cells change their current states is fixed and given. In this sense, the automaton does not 'learn', but the proliferating number of references to states of interacting units, coupled with the rules (even quite simple ones) determining the updating of current states makes its evolution complex which, in turn, makes it impossible to predict the behavior pattern of the cellular automaton just by examining the initial data and the cell update rules. Once again, for a relatively small number of cells, one *can* determine the behavior pattern independently of the actual evolution of the automaton (e.g., Von Neumann's self-reproducing automaton, even as his discovery of the automaton was a *tour de force* in itself). Similarly dramatic and unpredictable behavior patterns appear ([26], chapter 10) in Conway's *game of life* and Wolfram's one dimensional cellular automaton. In the latter system, Wolfram identified a certain class of cell update rules for which almost all initial configurations of the automaton, after a sufficiently large number of iterations (i.e., successive events of the application of the update rules to all the cells at a time) developed 'patterns' (in the so-called space-time diagram, see [26]) that exhibited

> ”...a mixture of order and randomness: localized structures are produced which on their own are fairly simple, but these structures move around and interact with each other in very complicated ways” ([26], chapter 10).

Wolfram speculated that this class of rules represents how real-life systems evolve in nature, i.e., in other words, natural processes are analogous to computations in a cellular automaton where relatively simple rules produce the strange mix of regular and irregular spatio-temporal behavior that we observe in systems all around us. It was later proved that the Wolfram rules in question could perform 'universal computation'. In other words, computations performed by a universal Turing machine can generate emergence as in natural processes—the spatio-temporal patterns in the Wolfram automaton could not be explained in microscopic terms (i.e., from the update rules and the initial conditions alone), though an intermediate level analysis (one that refers to patterns in the space-time diagram generated in the course of the evolution of the cellular automaton) could throw some light on the mix of 'order and randomness' that was to be eventually produced.

There exists a school of thought based on the speculation that natural processes are indeed analogous to computations on a Universal Turing machine or, in other words, such processes can be looked upon as computations run on physical systems ('pancomputationalism', see [27]).

On this view, emergence in nature can be at least partially understood by means of computer simulations. In the end, however, there remains the fundamental duality between the microscopic and the macroscopic descriptions of complex systems that stands in the way of providing a deterministic account of emergence, which is a feature of the complex evolution of such systems. The interacting components of a complex system (the 'microscopic' constituents) and the emergent spatio-temporal formations (the 'macroscopic' structures) span space-time scales of distinct orders of magnitude. While the interactions of the microscopic constituents are described in terms of one set of variables (the microscopic ones), those of the emergent formations are based on a distinct ones, namely,the collective variables.

> While we have looked at emergence as a process leading from the microscopic to the macroscopic, emergence is actually a *two-way* affair—one can equally well look at the 'emergence' of the microscopic from the macroscopic. We underline this aspect of emergence in sec. 3.17 below. In other words, emergence is all about the existence of space-time scales of distinct orders of magnitude that require distinct modes of perception and description on our part.

3.16 Emergence and downward causation

Emergence defies predictability, but is based on causal links operating all the way.

The interactions between the microscopic constituents of a complex system are pervasive and multifarious. These interactions establish far-flung correlations among the constituents as the system goes through a process of complex evolution. In the course of this evolution, some macroscopic structure or other emerges with modes of interaction distinct from those of the microscopic elements that operate all along as causal agents.

> However, as mentioned above, emergence is a two-way affair; the 'microscopic' emerges from the 'macroscopic' as much as the latter from the former (see sec. 3.17 below). What is important to note is that the microscopic

> and the macroscopic entities are described in terms of distinct sets of state variables and exist under distinct sets of conditions. What is more, they are stable over distinct space-time scales.

Predictability is distinct from causality. The latter is, at times, taken to be the basis of the ontic *being* of systems, while predictability appears to be a matter of our epistemic access to the world. Though we will later outline a metaphysics that denies the fundamental gap between the two, we will not focus here on this issue of the ontic-epistemic distinction.

Accepting the view that causal powers are indicative of the *reality*, or the *being*, of an entity, the question is often asked as to whether emergence is 'real'. In other words, is an emergent entity capable of exerting causal power? In particular, does a macroscopic emergent entity causally act back upon the microscopic constituents of a complex system? This, at times, is referred to as the issue of *downward causation*.

An emergent entity is certainly capable of exerting causal influence by interacting with other entities of the world. Looking at a human being as such an emergent entity, he no doubt interacts with other human beings and other objects in the world, and that interaction is no doubt causal as all interactions are. But the issue in question is, can he exert causal influence over the cells and organs making up his body? The question is answered in the affirmative as soon as it is asked. However, a sharper question that is often posed is, whether there is some ontic essence in a person *over and above* the influence that the cells and organs themselves can exert—can the causal influence wielded by the person as a whole be *reduced* to the properties of the constituents of his body?.

Arguments over the issue of reducibility are often protracted and unending, not the least because contending exponents interpret that term differently. In other words, what they mean *implicitly* by it differs from its explicit denotation, the latter being insufficient to resolve the issue.

However, the issue of downward causation essentially revolves around the following question: do the state variables of an emergent system interact with and influence the state variables of its microscopic constituents? In order to understand what this involves, let us consider a complex system, say, 'A' made up of constituents, say, 'B_1', 'B_2', $\cdots$, from among which a certain subset, say, '$B^{(1)}$', '$B^{(2)}$', $\cdots$, form a composite entity 'C' under a given set of external conditions, so that C may be said to emerge from within the complex system A (it may so happen that all the constituents B_1, B_2, $\cdots$ get involved in the formation of the composite system C, i.e., $B^{(1)}$, $B^{(2)}$, $\cdots$ is the same set as B_1, B_2, $\cdots$). The question that now arises is, whether the state variables of C can be coupled in interaction with those of $B^{(1)}$, $B^{(2)}$, $\cdots$?

This requires a careful consideration of what the state variables of C are and what meaning can be attached to state variables of $B^{(1)}$, $B^{(2)}$, $\cdots$.

In reality, C is described by a set of *collective* variables, in which are included those relating to $B^{(1)}$, $B^{(2)}$, $\cdots$, i.e., strictly speaking, the variables pertaining to $B^{(1)}$, $B^{(2)}$, $\cdots$ are not independently relevant in so far as the emergent system C is considered. In other words, *within* the emergent system C, the constituent entities $B^{(1)}$, $B^{(2)}$, $\cdots$ lose their independent identity, the identity they had in A, when C was not in the scene. As an instance, the coupled vibration of a set of particles is described in terms of a set of collective variables, technically referred to as the 'normal mode' co-ordinates, all of which are distinct from the microscopic variables of the particles considered independently of one another. Among these normal mode co-ordinates there are some (under appropriate conditions) that resemble those of the particles in the absence of interactions—the so-called 'short wavelength' ones, while some others (the 'long wavelength' ones) are truly collective and correspond to long-range correlations among the particles. In the following, we distinguish between these two types of collective variables as the 'particle-like' and the 'wave-like' ones for the sake of easy reference.

In numerous situations, the state variables of the emergent system C are calculated only approximately (an exact calculation for a large system is out of the question)—the ones, calculated in a given stage of approximation, may be imagined to interact among themselves so as to give the next stage. For instance, the normal mode variables in the so-called linear approximation may interact with one another so as to produce the so-called *anharmonic* effects (the *phonon-phonon* interaction in a solid). In principle, though, the state variables of an emergent system are all independent ones and there is no question of coupling between those—the variables that resemble those of individual constituents (i.e., the 'particle-like' variables) in isolation are no less collective than the others (the 'wave-like ones') whose collective nature is more apparent.

What is important to consider here is, once again, the space-time scales of distinct orders of magnitude. Over a relatively short span of time, for instance, 'particle-like' and 'wave-like' variables may be found to interact because truly collective variables may emerge over a longer time scale. It is this interaction between approximate collective variables (both particle-like and wave-like ones; variables of an intermediate character may also enter the picture) that one has to consider while addressing the issue of downward causation. *Additionally* (and importantly), interactions with external systems may also be responsible for interactions between various groups of collective variables of an emergent system.

With all these qualifying factors kept in mind, one may indeed conclude that there can take place interactions between groups of components within an emergent system under the impact of external systems *and* over specified space-time scales, where these groups of components are described in terms of *approximate* sets of collective variables. Within the context determined by a specified space-time scale and specified external interactions though, it may also be true that not all possible interactions between subsets of components can take place. In other words, within the specified context, certain groups of components can interact with others while retaining their relative autonomy.

For instance, in looking at the mind as an emergent system formed by interactions between the neurons in the brain, we can (and do) distinguish between various different 'components' of the mind and, for practical purposes, consider interactions between those. However, interactions between the mind and individual neurons, for instance, do not acquire relevance in practical considerations.

3.17 Emergence is a two-way affair

As mentioned in an earlier note (sec. 3.15.2), emergence is a two-way affair, though it appears to hold generally that complex systems evolve from the microscopic to the macroscopic over limited ranges of space-time scales. However, looking at the fundamental fact that the microscopic and the macroscopic are limited to distinct space-time domains, there is no asymmetry between the two—over extended space-time ranges, there may very well appear the transition from the macroscopic to the microscopic.

The commonly perceived asymmetry between the microscopic and the macroscopic is based on an apparent directionality in natural processes where the latter gets assembled from the former. This perception is reinforced by a supposed 'history' of the universe beginning with the big bang and proceeding towards the structures that we see around us, including the giant cosmic structures. Processes of dissolution of macroscopic structures and emergence of microscopic ones often remain unnoticed. Living organisms decay into molecules, heated bodies emit photons, gravitational collapse of stars produces cosmic particles, rocks dissolve into the oceans—complex evolution knows no directionality. What appears to be emergent depends on how and what we perceive. The latter in turn depends on our modes of perception—how our senses operate, and how our scientific instruments work. Beyond all this, of course, is the fact that our perception is rooted in reality—the epistemic merges into the ontic. Our perception distinguishes between space- and time- scales of different orders of magnitude across which complex evolution takes place. The complexity inherent in natural processes gives rise to unpredictability, and emergence is one manifestation of the unpredictability where some structure or other acquires relative stability on a certain space-time scale. In this, complexity has no preference for one space-time scale over another.

What is important in emergence is the transition across an instability, where the stable regimes on the two sides of the instability are characterized by distinct space-time scales—the tipping point or the instability itself being inherently scale-free. The transition across the instability can be in either of two possible directions. If one of the two directions is found to be favored over some particular time scale of observation, the other direction can be found over a distinct time scale. For instance, the time span necessary for the appearance of some particular biological species may be of a distinct order of magnitude compared to the time required for its possible extinction.

What is more, emergence is a thing of our phenomenal world (refer back to sec. 3.8; see chapter 4 for further considerations)—the noumenal world knows no emergence because space-time scales of *all* orders of magnitude are latently inherent in it.

3.18 Complexity and emergence: summary

Emergence is all around us (refer to [15], chapter 3). It is ubiquitous and knows no preferred direction, proceeding from the microscopic to the macroscopic and, equally well, from the macro to the micro. It is rooted in the reality of complex evolution and involves an unpredictability that pertains to our perception of space- and time- scales. Emergence is commonly observed in complex systems.

The fundamental thing about complexity is large numbers—large number of constituents (the 'microscopic' components) and large numbers of interactions and correlations among those. Interactions in a complex systems are, moreover, of diverse *types*, generating pulls and pushes within the system. The complexity of evolution of such a system is referred to as *co-evolution*, in which even the number and type of the components in a system evolves, along with the nature of interactions and correlations—*everything evolves*.

Emergence is a feature of this co-evolution and involves an unpredictability in what happens at certain junctures in the course of complex evolution—the fundamental thing about emergence is *distinctness of orders of magnitude*. Referring to the network representation of a complex system (such a representation is often notional) the effective number of nodes locked in mutual interaction differs greatly in the scenarios on the two sides of emergence. Put differently, suppose that the effective size of stable clusters on one side of emergence is M (in the case of nodes interacting independently of one another, one has $M \sim 1$), and that on the other, N (usually, only a few stable clusters of size $\sim N$ are formed in the system), then $N >> M$. What is more, the interactions of large stable clusters are determined in terms of *collective variables*—ones that depend on the variables pertaining to the individual nodes, considered in isolation, in a complex manner that cannot be determined with any degree of precision.

> As mentioned earlier, emergence can, however, be found to occur in 'simple' systems too. For instance, an amplifier circuit operating with negative feedback (used for achieving stability of amplifier operation) turns into an oscillator when the feedback is changed into one of the positive type—compared to the amplifier, the voltages and currents in the oscillator exhibit a distinct pattern of variation.

The perception of emergence does not run contrary to the reductionist view, but is complementary to it. The discontinuity perceived in emergence is related to the rapid transition across instabilities,—characterized by positive Lyapunov exponents in the case of dynamical systems—such instabilities occur in the evolution of all complex systems, and is characterized by large scale spatial and temporal correlations among microscopic constituents. Emergence is often discussed in the static setting such as in the comparison among symmetry-broken configurations, but the threshold relating to the instability lurks in the background.

The reductionist point of view seems to be tacitly held in the approach adopted in the sciences, since science tries to seek out the 'fundamentals'. But foundationalism often meets with a dead end. What is foundational to us is not foundational in nature, because Nature knows no foundations and is one infinitely complex whole.

> Even mathematics doesn't have a unique and solid foundation. The infinite-fold complexity of mathematics, looked at in its entirety, is sought to be captured in *category theory*, where transformations between mathematical structures are of prime relevance rather then their ordering as to which is more fundamental and which is less.

The association of emergence with the breaking of symmetry has been referred to above as the 'macroscopic' point of view. There is a tacitly held view that space-time in itself is homogeneous, isotropic, and symmetric. As we look at the evolution of matter-energy or, more fundamentally, at the *fields* (supposedly the basic stuff the universe is made of) in this space-time, more and more of the symmetries are either violated or broken, till the only symmetry left in the universe is presumably the one relating to mass-energy. In this view, the complex evolution of the fields occurs within the framework of space-time. Gravitation is the proverbial flesh in the thorn in this view of reality. Whatever the 'ultimate' reality is, it is not likely to respect the distinction between the framework provided by space-time and the stuff that evolves within this framework.

This 'ultimate' reality, however, takes us to the realm of metaphysics.

In the next chapter we distinguish between the ultimate reality and the reality that is immediately accessible to us—these are referred to as, respectively, the 'noumenal' and the 'phenomenal'.

Chapter 4

The noumenal and the phenomenal

The *noumenal-phenomenal* dichotomy is inherent in Kant's metaphysics. What is of great interest is that the dichotomy can be given an entirely naturalistic interpretation in terms of the way the human mind perceives the world and enables us to proceed along the journey through life.

> For an introduction to the naturalist in Kant, see [22]. In the present book, however, we will be concerned not so much with the naturalism to be discovered in Kant's works, as with providing a naturalist *interpretation* of the noumenal-phenomenal divide. This, of course, is a hazardous and daunting task but is still worth undertaking. Mine is a modest effort though—one that runs the risk of being summarily dismissed by more competent minds. Regardless, it constitutes the foundational position that I adopt in this little book of mine.

The noumenal world is the ultimate reality that there is. It is the repository of all the complexity that we perceive in our phenomenal world, and of much more. Among the two worlds, the noumenal is inclusive of the phenomenal, but they differ fundamentally in the extent to which we can access them. It is a matter of common experience and of more informed study that our mind perceives the world by taking in innumerable signals (some of those signals may originate in some part of our mind itself), and forming mental representations of those by a process of *integration* with *previously stored* representations of diverse types. This is the process of *interpretation* that the mind engages in while accessing the world.

But perception and interpretation are strictly limited in scope— we do not perceive reality all at a time, but receive and store signals from *parts* of that reality. Our perception is always fragmentary and transient, and we build up our world by patching up bits and pieces of mental representations emerging in experience. What we generate by way of putting together all the partial representations of reality is precisely our phenomenal world that is ever evolving as a complex system in itself. All the while, the 'real' (or *noumenal*) reality remains in the background as the remote source all our experience and of all that we build up as our phenomenal reality—remote in the sense that we cannot access it as a whole. We cannot access the noumenal precisely because of the limited range of our senses, as a result of which the reality that we do access is the phenomenal one. The noumenal, if it is to be accessed is to be done so *in entirety*, something that our senses are eminently incapable of undertaking.

How well does the phenomenal represent the noumenal? Can the latter be inferred from the former? *There is no way of telling.*

4.1 The phenomenal is a low dimensional projection from the noumenal

The phenomenal is a highly skewed partial representation of the noumenal. Both are complex and infinite dimensional but, compared to the latter, the former is a non-starter. This book is based on the fundamental metaphysical assumption that the noumenal reality is infinitely more complex than the phenomenal one and that the latter constitutes an absurdly low dimensional *projection* from the former.

A projection of an entity from a high dimensional 'space' to a low dimensional one is a representation where some essential aspects of the entity are hidden or absent. Consider, for instance, an object characterized by its color, shape, and size (a 'three dimensional' representation), and suppose that it is presented to an observer by mentioning only its color and shape, while withholding all information about its size—a projection from three dimensions to two. What is more, the two dimensions that are revealed may not represent the projected features faithfully (a consequence of how our interpretation works)—for instance, the brightly colored may appear curved and the light colored, pointed.

While we have absolutely no clue as to how the projection from the noumenal to the phenomenal works, we can adopt guesses by way of extrapolation from our experience that the nomuenal reality is infinitely more complex and infinitely vast compared to the phenomenal world generated by our senses. Aspects of the noumenal get projected into the phenomenal so as to produce all our sensations, perceptions, and interpretations, all our experiences and all our thoughts. However, *as in mathematics, the inverse projection from the phenomenal to the noumenal makes no sense*—we have no clue as to the aspects of the noumenal world that get projected to generate specific experiences and interpretations in our phenomenal reality.

4.2 From the phenomenal to the noumenal: the 'dissolution' of space-time

In particular, *space-time* and *causality* are essential preconditions of all our experience and thought in the phenomenal world, and one never knows how they are generated by projection from the noumenal. These set the necessary context to our experiences, making it possible for us to establish relations between entities and events and to generate meaning in them. The framework provided by space-time is essential for us to form categories in our mind—categories, in turn, help us in the process of *inference*, on the basis of which we *act back* on the world and get along in our journey through life. It is not that space-time and causality are purely subjective creations to serve our fancy—they emerge from an underlying reality all right, but assume the familiar features by which we perceive (or, rather, intuit) these, only in the phenomenal world.

Of crucial relevance in our intuition of space, time, and causality is the *ordering* that finds expression in space- and time- *scales* of various magnitudes. All our perceptions and experiences, including those of emergence, are located in the *context* defined by spatial and temporal scales. Episodes of instability and emergence appear striking to us because of this segregation of scales. However, the underlying noumenal reality need not distinguish between scales of different orders of magnitude. Already, as our horizon of observation and experience expands, pushing away the hitherto known frontiers of space and time—towards both the sub-microscopic and the cosmic—the segregation between the infinitesimal and the infinite appears to dissolve. What are referred to as singularities connect the two together into a strange knot—as if space-time closes upon itself. Unresolved questions of vast magnitude loom before us, having been opened up by instruments of observation that point towards the infinitesimal and the infinite not explored till recently. Concepts relating to space-time are already in a state of strange turbulence, indicating that such concepts are specific to our modes of perception of entities in the phenomenal world and are not likely to be of fundamental relevance in the noumenal.

The link between the cosmic and the submicroscopic is provided by *fields*. Fields span a vast range of space and time—fields don't respect any segregation between space-time scales, and correlate the cosmic with the submicroscopic. In particular, the gravitational field is likely to give an entirely new twist to our conception of space and time in describing the phenomenal world itself, which tells us that it may not be possible to smoothly extrapolate space-time back from the phenomenal to the noumenal. Already in the realm of non-relativistic quantum theory—fields don't respect the demarcation between the relativistic and non-relativistic domains either—the classical view of local realism is found not to be a valid one. Interestingly, the fields make it possible for quantum correlations ('entanglement') to be shared globally and, at the same time, play a dominant role in the process of decoherence (i.e., removal of entanglement) in classical objects—entanglement and decoherence militate against local realism ([20]).

4.3 Complexity and the noumenal-phenomenal divide

When we describe the behavior pattern of a complex system, or try to explain that behavior pattern in terms of known principles, we always do so *in a context*—one that may or may not be explicitly or precisely defined. Ideally, the context represents the effect of all systems other than the one under consideration while in practice, only a limited number of systems that can possibly have some influence within some relevant span of space and time, are taken into consideration. This *limited horizon* within which a complex system is described contrasts with what pertains to the noumenal reality as a whole.

All our descriptions and experiences are within the phenomenal world that is perpetually in the process of being built up piece by piece in the course of our perception of reality. Those perceptions are generated by means of an infinitude of signals of multifarious kinds received from the noumenal world but they do not result in the cognizance of the noumenal world itself, precisely because those are strictly limited in scope—perceptions and their subsequent interpretation in our mind generate the cognizance of some fragment of reality that gets added to the phenomenal world that has been generated in similar perceptions in the past.

Complex systems are ubiquitous in the phenomenal world. Since all those are rooted in the noumenal, the latter has to be regarded as the ultimate repository of all complexities—indeed we adopt the metaphysics that the noumenal reality is an infinitely complex whole that exists in itself, while revealing its aspects in fragments—and that too in a skewed fashion—to our senses.

The noumenal, however, is contextless—it has no bigger environment within which it is defined, no external systems with which it can exchange information, matter, or energy. It has an infinity of 'dimensions', i.e., aspects that generate—in the form of projections—all the features of the phenomenal reality captured in all the complex systems that we observe and describe. All the perceived features of the complex evolution of systems in the phenomenal world are the result of such projections from what we will refer to as noumenal-evolution, being aware that there is no way to reconstruct aspects of the noumenal from which specific aspects of the phenomenal arise by projection.

As an infinitely complex system itself, the noumenal world is in a state of 'evolution' that we refer to as the noumenal-evolution since there is no way we can compare its features with the evolution of systems in our phenomenal world. The latter occurs within the framework of space and time—indeed the very concept of space and time is generated as a pre-condition of our perception of systems in the phenomenal world in their being and becoming. Referred to the noumenal universe, 'space-time' and 'evolution' may not be well-defined concepts, since the noumenal does not distinguish between different space-time scales, and the directionality of time loses meaning in it—the basic laws of science do not favor any specific direction of time. Observed over an infinitely long stretch of time, the reversibility of physical phenomenon is regained. And, what is of essential relevance, the *noumenal has no observer*. Everything specific to our perception—the phenomenal world, space-time, causality, evolution—all these lose meaning in the noumenal. All our extrapolations from the phenomenal back to the noumenal have no evidence to be tested against.

1. It may seem that perception with our senses and with our instruments should progressively reveal the noumenal world to us. However, perception and observation is not a passive, but an *active* process—it results in an accretion to the phenomenal world, and that too in a skewed manner since our senses (based on our instruments) add an element of interpretation to incoming data. We do not perceive the noumenal precisely because our perception works within a limited horizon. Just as taking the profile snapshot of an individual conveys to us only a minuscule fraction of the person as a whole (including her cellular and psychological details), all attempts at perceiving the noumenal gives us something else—an accretion to the phenomenal. The noumenal is (and remains) unbounded in all conceivable ways.
2. In order to perceive the noumenal world, one needs to look at reality in all possible depths and at all space-time scales, from somewhere beyond the unlimited expanse of that world. This is referred to as 'God's eye view'. Human perception of reality can only add to the phenomenal world which is already insignificant in content compared to what the noumenal holds in store.

There is, in particular, no way to tell what the 'noumenal stuff' is, i.e., how the noumenal world is constituted—it is revealed to us in context-dependent and limited projections variously as molecules, atoms, sub-atomic particles, particles of the standard model, the underlying fields, and so on. And equally, there is no way to tell how the noumenal stuff ‘evolves’—what we may refer to as the noumenal-evolution is revealed to us in context-dependent and limited projections as all the various instances of evolution perceived in the phenomenal world. All the complexities of evolution of systems in the phenomenal world—the intricate transitions through stable and unstable regimes, the resulting behavior patterns of multifarious types (regular, quasi-periodic, chaotic, and behavior patterns where these coexist), and the sensitive dependence on context, all are but limited projections from the noumenal-evolution which, in itself, is inscrutable—it occurs without a context, with no space-time framework within which to observe and describe it. Even in the phenomenal world, one is not sure whether space-time is distinct from objects and processes (i.e., is a framework within which all processes occur), or is itself an ‘object’ at a deeper level (so that, among other things, it can be quantized)—the noumenal aspect from which it is generated by projection is even more obscure.

Our knowledge of and inferences about the phenomenal world are already in turmoil as observations far into the cosmos (notably, by means of the James Webb Space Telescope) and into hitherto unknown smallest dimensions of space-time (smallest accessed length and time interval are, respectively, $\sim 10^{-18}$m and $\sim 10^{-21}$s—even smaller dimensions are in the process of being explored) raise fundamental questions, indicating that foundational concepts regarding space-time hitherto believed to be valid need radical revisions.

This, indeed, is the normal thing in science—as the context of our observations gets changed by an expansion of the domain of observation, theories undergo quite radical modification, so much so that they appear to be *incommensurate* with respect to one another. All our scientific theories pertain to the phenomenal world, and the revision of those theories is by no means a smooth process (see chapter 5 for further considerations). Indeed, expanded domains of observation correspond to distinct projections from the noumenal to the phenomenal world—projections that hide or reveal various different and disparate aspects (‘dimensions’) of the former.

This takes us back to our earlier observation that, while radical modifications of our theories correspond to specific aspects of the noumenal world being revealed to us in our phenomenal experience (notably by means of improved techniques of observation), it is fundamentally impossible for a theory to be applicable to the noumenal reality as a whole since that would require *all* the infinite-fold aspects of the latter to be accessed and put together, not as our perceptions tell us, but as they 'actually' are in the noumenal reality. Our expanding domains of experience reveal fragments of the noumenal world in a piecemeal (and *skewed*) manner, but there always remains an infinite number of aspects beyond our horizon—all those aspects entwined in a tangle that is obscure beyond description.

In summary, there can be *no* extrapolation from the phenomenal back to the noumenal because of the infinite-fold complexity of the latter, which is a self-determined, closed, and complete world. Any incomplete perception of the noumenal lands us into the phenomenal.

All said and done, though, the human mind will never stop to extrapolate. As so many of our familiar concepts tumble and dissolve in the course of this extrapolation, what may persist are the ideas of fields, of reversible churning of the fields, and of correlations generated within this churning. The noumenal is self-determined, while evrything else is generated from it by projections of various descriptions—entities and events, interactions between entities including the strengths of these interactions, space-time as fundamental percepts, causality associated with a direction of flow of time—in short, the entire phenomenal world itself in a perpetual state of flux.

4.4 What can the noumenal world be like?

The foundational metaphysical position that we adopt in this book is that the noumenal world is an infinitely complex system in a state of perpetual turmoil for which the familiar term 'evolution' is not meaningful since our concepts regarding space-time in the phenomenal world are projections from aspects of the noumenal that we have no means to guess.

Extrapolating back from the phenomenal to the noumenal is a contradiction in terms, but we do not ever cease to extrapolate from our phenomenal experience to have a glimpse of reality beyond the horizon. While it so happens that the reality lying beyond the horizon is once again the phenomenal one, can we at least *imagine* what the noumenal would be like?

In this imaginary exploration, time ceases to be unidirectional, because the world is fundamentally reversible. Indeed, time and space would cease to have the meaning they hold for us since all yardsticks of measurement would be equivalent—differences in orders of magnitude would not be meaningful. What is more, space-time is likely to have dimensions unknown to us and may have novel topological features, being somewhat like a sea in turmoil, in which reversibility is the rule. Space-time and matter-energy-field may blend together in unforeseen ways with a self-determined dynamics in which everything co-evolves with everything else, and everything is determined by the totality in a self-consistent manner. 'Being' and 'becoming' would cease to be distinct, and the term 'evolution' would not carry the same meaning as it does for us.

This, however, is getting us nowhere, and the above few lines are meant only to make us aware as to how radically different the noumenal would possibly be when compared with the phenomenal. What makes the two fundamentally different is that the former has no context in which it is embedded, and there is no observer-observed divide in it—the question as to what the noumenal would 'look like' is essentially meaningless.

> It is commonly accepted in the sciences that observation leaves the observed largely intact, the more so if the observed is as large an entity as reality as a whole. This, however, misses the point that observation essentially entails a reconstruction—an *active* reconstruction at that—of the observed. It is the reconstruction that is interpreted as the observed entity while the actual observed entity (the thing-in-itself) continues its self-determined existence. In other words, the observer-observed duality is nothing like what is commonly supposed in the sciences.

The noumenal-phenomenal divide makes it imperative to interpret the term 'realism' in an expanded sense. It tells us that the phenomenal world is constructed by our senses out of continuing and accumulating experience but, at the same time, does not imply a negation of realism while requiring only a broadened interpretation of it. All our experience originates in the noumenal reality even as we do not know what that reality is. All our interpretations, inferences, and theories serve the purpose of making sense of our existence in a fundamentally incomprehensible and inexplicable world, because that world has no responsibility to 'make sense' to anybody. From the point of view of evolution, our senses, our continuing existence, and the world we observe are, by necessity, *mutually* compatible— we would cease to exist if our senses failed to make head or tail of the signals we receive, and a world incomprehensible to our senses would be instantly inimical to our continued existence.

Entities and events are comprehensible to us precisely because these are always perceived within some context or other, where only some aspects of these are revealed to us. But 'entities' and 'events' in the noumenal world have no context to constrain how those are perceived, and all their aspects are revealed at once—the ones perceived as the forms and motions of macroscopic objects, the incessant motions of all their atomic, sub-atomic and elementary constituents, the long-term processes involving the decay and degeneration of macroscopic objects, the cosmic processes—all these merged into one single infinite dimensional turmoil with no possibility of distinguishing between all the different 'types' of forms and processes. In summary, the phenomenal world is comprehensible only because it emerges within limited contexts of space and time, ones with which our own perceptual abilities match in a mutually consistent manner.

The point of view espoused in this book (it *is* a point of view—nothing more) has a close parallel with the one to be found in [13]. Hoffman does not explicitly distinguish between the noumenal and the phenomenal, but is very specific in distinguishing between what we perceive and the 'reality' that lies beyond. The reality in which our perceptions originate is a weird and incomprehensible one. For instance, space-time 'is doomed' in that reality, as Hoffman so significantly insists. Indeed, modern physics is in quandary in its efforts to assign a proper place to space-time in a way compatible with *gravitation* too finding a rightful place for itself in the overall framework of physics. Indications are that the idea of space-time is an *emergent* one from a 'reality' to which no comparable concepts apply ([3], [36]).

As Hoffman explains from his own point of view, biological evolution necessarily works in such a way that we perceive a *vastly compressed* data set, skimming only the surface of an enormous and infinite dimensional data space representing the true reality. While that infinitely vast data set pertains to the noumenal reality—to draw a parallel with the point of view put forward in the present book—the compressed data set relates to the phenomenal. This, indeed, constitutes the naturalization of the Kantian noumenal-phenomenal dualism.

4.5 The absence of emergence in the noumenal world

The stray observations of sec. 4.4 above are nothing but extrapolations from our perceptions in the phenomenal world. However, the noumenal-phenomenal divide belongs to the realm of metaphysics, where logic and evidentiary deductions have no place. This contrariness is not surprising—our metaphysics is based on how we extrapolate from our worldly experience—the only thing that one has to remember is not to identify this extrapolation as 'the correct' world view—distinct metaphysical views cannot be made to fight with one another. On the other hand, metaphysics does influence our mode of inquiry into reality. It is with all this kept in mind that we try to develop a plausible picture in metaphysics—because that is what constitutes the naturalization of philosophy.

If at all one can imagine that extrapolations from our phenomenal perception have some relevance, then the conclusion becomes inevitable that *the noumenal reality has no place for emergence*. Emergence is context-specific—being essentially dependent on distinctness of space-time scales— and is fundamentally conditioned by the subject-object divide. A phase transition in a system depends on the temperature and pressure, and is an objective occurrence, but needs a subject to identify it as an emergence—the basic characteristic of emergence is that it is not predictable on the basis of interactions of the constituents considered in isolation, and predictability presupposes a subject.

> Indeed, the Kantian dualism dissolves the subject-object binary into one complex whole. What is considered objective or real pertains to the phenomenal world, and the phenomenal is itself generated from the noumenal by means of human interpretation and inference—processes invariably stamped with modes of thought specific

> to individuals and to epistemic communities. Such modes of thought are fundamental preconditions to all perception and interpretation—space-time and causality, for instance, are indelibly stamped with this Kantian dualism.

The noumenal turmoil accommodates all types of processes—continuous and discontinuous, 'predictable' and 'unpredictable'. Since the noumenal is one single complex entity, there is no way to see whether some 'process'—processes, however, cannot be separated from one another in the noumenal boiling-pot—is predictable or not from the 'properties' of the constituents.

Once again, this is getting us nowhere—but once again, we extrapolate from experience and make use of our foundational metaphysical supposition to claim that the absence of emergence in the noumenal universe is not inconsistent with our metaphysics.

There is another aspect to our notional supposition that emergence is irrelevant in the noumenal universe—that of *symmetry* and *symmetry breaking* (refer back to sec. 3.10.4). As with emergence, symmetry and the breaking of it are specifically phenomenal notions. There is a tacit supposition in the sciences that the world we perceive is just an approximation to the ultimate reality and that our experience, aided by evidence and our theories, is not a fundamentally flawed guide to that 'ultimate' reality. According to this tacit supposition, the processes we experience follow rules that resemble and approximate 'simple' laws pertaining to the 'ultimate' reality—and the idea of *symmetry* is central to those laws. It is supposed that all processes in the 'ultimate' reality take place within a profoundly symmetrical framework, but that symmetry is broken as we consider specific systems with specific interactions among their constituents. The more complex the interactions are, the greater is the extent of symmetry breaking.

The noumenal-phenomenal divide, on the other hand, looks at this issue from a different perspective—the phenomenal world of our experience is *not* embedded in a profoundly symmetrical framework (space-time, perhaps) but in the noumenal world in which there is *no symmetry at all*, precisely because the noumenal is the repository of *all* the complexity there is. The approximate symmetry that is found to reside in a phenomenal process is there only because it is observed within a certain space-time domain, abstracting from all other complexities, the latter being kept out of consideration in conformity with the space-time context. Emergence occurs as some subsystem of a complex system attains stability within some space-time domain in the course of the complex evolution of the system, when that subsystem behaves as a collective entity within a complex environment. The point of view of symmetry breaking works as an effective one when the evolution of a (macroscopic) subsystem within a complex system takes place in approximate isolation from all other interacting systems and subsystems where the system dynamics possesses a symmetry that gets broken as the macroscopic state (of the subsystem) 'chooses' to be one among several possible alternatives.

Briefly stated, the noumenal reality is infinitely complex and possesses no symmetry at all. Emergence is a phenomenon confined to the phenomenal world that can be understood in terms of stability of subsystems (of complex systems), and also—in special cases—in terms of symmetry breaking. The term symmetry is relevant only for systems that are approximately isolated from the rest of the complex world.

4.6 The noumenal and the phenomenal: summary

The noumenal world is not something mysterious and remote. It is the very world we live in but don't perceive *as it is*. Our perceived world is the phenomenal—the latter is made up of fragments of the noumenal, assembled and patched up by our capacity of *interpretation*. We interpret by comparing with past experience, where each morsel of that experience is colored with our preferences and emotions while being, at the same time, judged against reason. Our interpretation is a strange blend of the implicit and the explicit that shows up in all our decisions, inferences and theories (refer to chapter 5). In other words, the phenomenal is not a composite of passively collated snapshots of the noumenal—it is a fantastically skewed mosaic that we *reconstruct* from bits and pieces of the noumenal. It is,in other words, a world where the objective and the subjective merge together.

> The implicit mode of accessing and comprehending the world, *over and above* the explicit, is a necessary

consequence of the complexity of reality.

What is of great importance to note in this connection is that the fragments and pieces of the noumenal that the phenomenal is composed of are not randomly accessed ones, but are those that are consistent with our *capacity to perceive* and our *continued existence* based on that capacity—it is the steady process of evolution that ensures the *mutual compatibility* of the perceiver and the perceived. Out of the myriads of signals originating in the noumenal world, only relatively few are perceived by us, while all the rest are ignored as being of little relevance in the space-time domain germane to our continued existence—to our 'being and becoming'. This is what has been referred to as 'data compression' in [13].

The noumenal is the ultimate reality that is by no means simple, symmetric, and harmonious. We ourselves are responsible for selecting what appears to us as simple, symmetric, and harmonious within some given context or other in our phenomenal world—indeed, simplicity and harmony is as much a matter of perception as an 'objective reality' in the perceived world. When that simplicity is found not to apply with universal success—when anomalies spring up—we seek new regularity in some expanded context that makes our picture of reality more complex. The journey continues without end.

Chapter 5

Complexity and emergence: the human mind and the phenomenal world

For background to this chapter see [19].

The phenomenal world, while being rooted in the noumenal (or the 'real') reality, is assembled by the *human mind* in patches, by way of fragmentary perception and interpretation. Whatever the mind perceives, it interprets by integrating the newly formed representations with a vast store of representations generated in the past, thereby generating *meaning* in perceptions.

> Signals received by the mind from the external and internal worlds get represented in the form of dynamical excitation patterns in neuronal assemblies.

As we speak of complexity and emergence, we cannot avoid speaking of the unfolding of two phenomena of supreme relevance—the *emergence of the mind* and the *emergence of the phenomenal reality from the noumenal reality*. The two are strangely correlated and conditional upon each other.

The human mind emerges in a protracted process of evolution, in which living organisms develop specialized capacities step by step, in a hierarchical manner—not all organisms are endowed with the same set of capacities and not all possible capacities are generated in one single species. Each capacity corresponds to some specific aspect of the complex behavior pattern of an organism, where it is by means of the behavior pattern that the latter *acts back* on reality—recall that the reality incessantly sends a multitude of signals to an organism, which the latter makes use of in generating its behavior.

> Significantly, it is the *noumenal* (or the 'real') reality that sends signals and it is on the same noumenal reality that an organism acts back. As we have mentioned earlier in this book, there is nothing transcendental (in the sense of being divine or mysterious) in the distinction between the noumenal and the phenomenal—though the former is, truly speaking, transcendental in the Kantian sense.
>
> As for the phenomenal, it is no illusion or figment of imagination either (refer back to section 3.8 and to chapter 4)—only, it is a minuscule fragment of the noumenal since our perception is, by the very nature of things, fractional and of strictly limited scope. It is confined to a limited space-time horizon while the noumenal includes within itself space-time scales of all magnitudes.

It is in this ongoing process of interaction between individual organisms and reality that specialized organs of perception play a pivotal role. In the case of a large class of species, all the signals received in perception are principally relayed to the *brain*, while in a certain subclass, behavior is generated by means of organized and structured activities of the brain that are commonly referred to as the function of the *mind*.

It is at times supposed that the mind is specific to human beings. However, the mind is itself a structured entity (based on emergent structures in the brain), and what is presumably specific to humans is the *conscious* layer of the mind. Apart from the conscious layer, human beings share with many other species an *unconscious* mind that emerges from certain structural organizations of the brain appearing in the course of evolution. However, it is likely that several species other than humans are endowed with an incipient consciousness too, and an associated self-awareness akin to those in human beings. From now on, though, we will focus on the emergence of the human mind and its role in the way we perceive the world.

5.1 The unconscious and the conscious mind

5.1.1 The unconscious mind

The mind is a structured and organized mode of functioning of the brain, based on the emergence of large neuronal aggregates, each of which functions in a collective manner, with all these aggregates forming *clusters* in the neuronal network. The unconscious mind is made up of all the clusters operating more or less independently in parallel, there being a large number of such clusters distributed throughout the extent of the brain, with little role played by connecting pathways between the structural units—the clusters function primarily by processing of signals received by organs of perception including, in particular, signals received from various organs within the body.

In relatively early stages of evolution, all these clusters are fundamentally engaged in certain essential diagnostic functions relating to the complex environment—both the external and internal environments of an individual—by way of establishing *associations and correlations* between signals. These correlations are then made use of by the body as a whole for the purpose of adaptation. Certain correlations are identified as ‘desirable’ and certain others as ’undesirable’ for the individual in question. The clusters proliferate throughout the developmental history of an individual—a process that constitutes ‘learning’.

In the case of human beings, this parallel and distributed processing by neuronal clusters reaches quite an advanced level of organization and is based on activities of certain identifiable regions of the brain, each of which is specialized to the processing of some particular type of signals. Alongside, there takes place a limited coordination between various clusters whereby distinct signal types get associated with one another for more advanced diagnostic functions. Notably, there arise numerous co-ordinations whereby signals are associated and correlated in terms of their ‘social’ relevance. In other words, the emergence of neuronal clusters resulting in the unconscious mind perform three types of diagnostics—those pertaining to the physical environment outside the confines of the body, those relating to organs within the body itself and, finally, the ones pertaining to the *social* environment.

A remarkable aspect of the unconscious mind is its capacity to generate *affect and emotions*. Most of our perceptions and experiences are marked as ‘desirable’ or ‘undesirable’, i.e., of either positive or negative psychological valuation by the activity of affect modules in the brain, the latter being supposed to have emerged at a quite early stage of biological evolution. In other words, affect generates a fundamental classification of the world of great adaptive value. Emotions, on the other hand, constitute a more complex classification of the world, being an ‘implicit language of the mind’, of great efficacy. In addition, emotions act as means of *amplification* in the mind, thereby imparting *stability and instability* to mental processes, the latter being of fundamental relevance in our mental life.

Finally, there emerges the *self* in the human mind (supposedly though, many other species are equipped with an incipient ‘self’), developing around the axis provided by affect and emotions. To start with, the self remains confined within the unconscious mind, eventually pervading the conscious mind too. The conscious part of the self is based on explicit or reasoned preferences and aversions—any experience that generates such conscious feeling in an individual gets associated with her ‘self’. The latter is thus a complex and emergent psychological structure that keeps on evolving throughout the life of an individual.

5.1.2 The conscious mind

The conscious mind involves the activities of large-scale neuronal aggregates, with more or less identifiable structures in the brain corresponding to various types of activities. What is more, the processing of information in the various aggregates is accompanied with *coordination and integration* across these, so that now the mind, equipped with its unconscious and conscious layers, is truly an information processing system with astounding capacities—the individual is now endowed with enormous possibilities pertaining to perception, *interpretation*, and behavior. The coordination and integration between neuronal aggregates is brought about by neuronal pathways and chemical transmitters.

It is the capacity for interpretation that enables an individual to generate *meaning* out of perception, whereby internal representations of signals received from various sources are associated with one another and are then integrated with representations *formed in the past*, resulting in enormously enhanced learning abilities. Of remarkable significance is the concomitant emergence of *memory* and of *emotions*, the latter being based on the *affect* system (sec. 5.1.1). Mental representations are now marked with affect-based and emotion-laden markers and stored in memory so that, on later recall, these bring back the past loaded with innumerable associations, based on which the integration between the past and the present assumes great significance, enabling the mind to generate *future* anticipations and plans, along with *communications* between individuals and, based on all this, meaningful *behavior*.

1. Of course, all these features of the mind are present to some extent in numerous species other than humans. The human mind is (apparently) exceptional in the *degree* of integration and co-ordination among various information processing modules that makes it a complex system with truly amazing potentials—this, once again is an instance of 'quantity leading to quality' (for reference, see chapter 1), which is the hall mark of emergence in complex systems. Equally, the mind is equipped with the ability for individual and social *self-destruction*—an indication of massive conflicts that the immense complexity of the human mind harbors (refer back to section 2.6).
2. Referring to the multifarious sources from which the mind receives signals which it makes use of in interpreting the world, it has the remarkable ability to *map its own activities*. This may sound like self-reference. What actually happens, however, is that one part of the brain receives signals from other parts, while that part itself is read by some others, and all the resulting representations are then integrated with one another just like those received from various other sources in the world.

The fundamental building blocks of thought that all conscious activity makes use of are *concepts*. While the unconscious mind sets up associations and correlations of strictly limited scope among percepts, the fundamental principle on which the conscious mind operates is to set up associations between percepts generated from widely different sources and tie those up into bundles that we recognize as concepts. What is more, all concepts get mutually associated and correlated into a *conceptual network* (or,conceptual *space*) of vast scope, where the network keeps on expanding with accumulating experience of individuals and of social communities of multifarious description.

> Signals received from the external and the internal worlds are represented in the form of dynamic excitation patterns in neuronal aggregates, where the patterns can be of literally innumerable varieties in so far as their spatio-temporal structure is concerned. Pathways between these aggregates lead to associations among the representations of percepts whereby concepts are formed, and further production of pathways sets up correlations among concepts. The conscious mind can be said to have emerged with the proliferation of pathways between large neuronal assemblies. The basic functional aspect that distinguishes the conscious from the unconscious mind is the formation of concepts and the potentially unlimited correlations among those.

Based on the consciously generated correlations between concepts, there emerges the capacity of the mind to employ *reason*, associated with *abstraction*. While affect establishes a relation of preference or aversion between concrete experiences, reason correlates *classes* of concepts and experiences generated by *abstraction*. Abstraction is arrived at by locating common features in concepts and experiences, 'commonness' being one of several means by which the conscious mind establishes correlations.

A boy in his adolescence often feels a vague resentment against his father's authority within family. This is an instance of an affect generating a specific correlation—that between the father and a sense of repression. As he grows into a rebellious young man, his resentment grows into a systematic defiance against all authority—a reasoned aversion based on abstraction from several instances of authoritarian repression.

Abstraction and the use of reason – that is how the conscious mind establishes correlations between concepts, a job in which it makes use of *beliefs*.

5.1.3 Beliefs

Activities of the human mind involve an amazing collaboration between its unconscious and conscious components.

> While speaking of unconscious and conscious mental activities, one has to keep in mind that these are not exclusive of each other, since the unconscious merges continuously with the conscious. These represent two

> aspects of human thought, corresponding to distinct but overlapping structural and functional organizations of the brain.

The conscious mind establishes correlations between concepts (and, in the process, generate new concepts too), mostly by means of *beliefs* where beliefs, in turn, are concepts having a complex structure.

> A large class of beliefs is lodged in the unconscious mind too, where concepts are tied together not so much by reason as by affect and emotions. Indeed, beliefs are mostly the joint product of the conscious and the unconscious layers of the mind.

In establishing correlations between concepts, beliefs act mostly as 'if–then' connectives ('*if* tiger *then* dangerous'). Beliefs play a remarkably pervasive role in the activities of the mind ([10]) and span a huge spectrum in respect of authenticity, at one end of which are beliefs that acquire the status of knowledge while, at the other end, are the ones that are not much different from superstition. From another point of view, beliefs can be either *self-linked* or*shared*—self-linked beliefs are based on affect and emotions and are often of poor authenticity, while shared beliefs are held in common by communities of individuals, are often tested against evidence (at least to some extent), and are consistent with one another (again, perhaps, to some extent). This points to a third (and related) way to classify beliefs—their content judged from the point of view of affect and emotions on the one hand and reason on the other. Most beliefs are based partly on emotions and partly on reason—of the two, one or the other may dominate in specific instances. Put differently, the content of a belief may be partly implicit and partly explicit.

5.1.4 Decision and inference: the role of self-linked beliefs

The making of decisions and inferences are emergent capacities of the mind of great adaptive relevance, and are commonly supposed to be an exercise in *rationality* where reason holds a place of eminence. In reality, decisions and inferences are based to a considerable extent on *self-linked* resources of the mind.

> Recall that the self represents an aspect of the mind associated with affect and emotions. It is mostly made up of affect-driven preferences, emotion-laden beliefs, and other psychological ingredients of the mind, most of which are repressed within the depths of the unconscious in order that major conflicts may not destabilize the operations of the mind itself. Fantasies, yearnings, cravings, guilt-feelings, deep shame, disgust, gloated pride, all these are lodged in the mind with a strong link to the self. The term 'self-linked resources' mentioned above refers to all such psychological ingredients, though we will be particularly interested in this book in the preferences and beliefs generated within the mind by the affect-emotions system.
>
> Of course, the self includes *shared* psychological resources too—ones that are held in virtue of membership to larger social groups and communities. These are the resources that are based on *reason*, associated with the capacity of the mind to make abstractions out of concepts lodged in the conscious mind.

As for the capacity of inference we note that, generally speaking, inferences are of the *inductive* type, while *deductive* inferences are also possible. A purely deductive inference is entirely reason-based, but is a rare thing in life. In an inductive inference, on the other hand, it becomes imperative to go beyond a strictly reason-based succession of intermediate conclusions, and to engage in 'logical leaps', whether small or large. It is inductive inference that is of utmost importance in life—people of all ages and all types routinely engage in inductive inference in matters small and large. Almost all inferences in real life have features of both deductive and inductive types mixed in them (see, for instance, [19]).

An inference (we will now use the term inclusively, recalling that an inference is generally of the inductive type) usually proceeds along a more or less complex course, punctuated with *decision* junctures, where the inferring mind has to make a *choice*. Inference, in other words, is deeply associated with the making of decisions.

In real life we constantly make decisions, big and small—decision making is an inseparable part of life. Indeed, the making of a decision is equivalent to an inference itself. Put differently, inference and decision are inextricably tied with each other. We make use of these in all our acts of perception and interpretation—performed whether unconsciously or consciously.

5.1.5 Interpretation inference and theory building

Perception, interpretation in terms of stored representations, and the making of decisions and inferences, all these finally lead to *theory* building. Theories constitute the toolkit with which we build up our phenomenal world and respond to situations of multifarious descriptions.

Theory is supposed to reveal to us the *regularities* and *mechanisms* inherent in reality. The question as to how far this point of view is tenable will be briefly addressed later in this chapter. However, one thing that can be said for certain is that theory provides us with *models* of the world. Generally speaking, models are *analogies* of specific aspects of reality, based on which we make predictions, on the strength of which we act back upon reality and depending on the results of our action, *revise* our theories.

Theories are akin to bundles of beliefs, and theory revision is a complex form of belief revision that happens every waking moment of our life. In a manner of speaking, belief revision culminates in theory revision. As we revise our beliefs, we generate a new perspective to look at the world and acquire an altered view of what reality is—that view, continuously evolving in a complex manner, constitutes for us our *phenomenal world* that emerges from *reality-in-itself*. This reality-in-itself is the very thing we have referred to as the noumenal world—the ultimate source of all our beliefs and the recipient of all our actions—the complex whole from which the phenomenal world emerges in fragments and patches.

5.1.6 The brain and the mind: self-organized criticality

The brain and the mind ceaselessly maintain their integrity but are, at the same time, constantly engaged in a high level of activity. This indicates that these operate close to criticality in respect of a large number of their respective state variables. In order to completely describe the operations of the brain or the mind one would require to specify the temporal evolution of an enormous number of state variables, most of which remain in a state of low level of activity at any given point of time, while the others show short bursts of activity and then revert back to quiescence—this pattern continues (with the quiescent and active variables now changing roles) as long as the brain and the mind remain active.

It may be recalled that the above description of the activity pattern of the brain and the mind runs parallel to how a dynamical system evolves in the phase space, the latter being in the nature of a paradigm. Accordingly we will use concepts like stability, instability, phase space (or the *state* space), and Lyapunov exponents in order to explain, by means of analogy, how the brain and the mind operate (refer back to section 2.12).

Continuing to invoke this analogy, a large number of state variables correspond to Lyapunov exponents well into the region of stability at any time in the evolution in the state space. But there remain a large number for which the Lyapunov exponents are close to instability (refer back to sections 3.6.2 and 3.6.3). As these variables cross the stability border under a small change of the context or of the initial conditions (refer back to section 2.11), they undergo rapid variation and revert back to some relatively stable state. This pattern continues as the mind (and the brain) passes through various regions of the phase space, during which process, relevant sets of state variables continuously change over from quiescent to active and vice versa.

As we pointed out in sections 5.1.1 and 5.1.2, the mind emerges by way of neuronal aggregates becoming active as collective units and, in the course of further development, interactions being set up between aggregates through neuronal pathways. In other words, referring to the individual neurons in the brain as ‘microscopic’ elements, the mind *emerges* in successive stages through the formation of ‘macroscopic’ aggregates and long-range correlations between these.

Speaking of the mind, the relevant state variables correspond to the collective variables describing the activities of the macroscopic neuronal aggregates mentioned above. Numerous studies indicate that many of these relevant variables do operate close to criticality, as seen from the algebraic decay of spatial and temporal correlations between signals collected from various different brain regions ([15], section 6.6). Such algebraic decay is indicative of a diverging correlation length (refer back to sections 3.10.2

and 3.10.3), which in turn implies *scale-free* processes in which the system in question can 'choose' any appropriate scale as it moves across an instability.

The long-range temporal correlations in the brain, corresponding to classes of mental activities, are seen in *low-frequency* signals (the so-called alpha, beta, $\cdots$ rhythms) associated with such activities and picked up, for instance, in EEG records.

5.2 The emergence of the phenomenal world

As we keep on interpreting the multifarious signals that the world out there sends to us, the phenomenal world *emerges continuously*, based on our concepts, beliefs, inferences, and theories.

1. The signals are received from the noumenal world itself, though our interpretation of those signals is in phenomenal terms. For instance, some signals from the noumenal world excite our ocular system and we interpret those as light rays, waves, or photons, depending on the context in which we perceive those.
2. Of course, the phenomenal world does not reside exclusively in our mind—it is out there as a collection of entities made of matter-energy and, *in addition*, it includes non-material things such as our concepts, memories, beliefs, our culture, and our social relations. All these are 'real' in the sense that each of these follow some definite course of evolution and that we can build concepts and theories about them, and can even test those against evidence.

   When we touch the hard surface of a table, we feel some part of the noumenal reality itself, but the nature of the thing that we feel is a matter of our interpretation and conception, and goes towards the constitution of our phenomenal world. Our interpretation does not include, for instance, the motion of elementary particles making up the molecules the table is made up of, the extremely slow process of decay and degradation of its material, and a vast number of other aspects about it. Even if we add some of these aspects to our description of the table-top, that would still constitute an account of some part of the phenomenal reality—the noumenal reality is one complex whole and any partial description can only pertain to the phenomenal world.
3. Strictly speaking, the phenomenal world differs for different individuals, since every individual perceives and interprets differently as compared to others. However, human communication generates a large overlap in our individual worlds—differences persist, though, and have to be taken into account in specific circumstances. For the purpose of this book, we ignore the differences in the respective worlds of individuals, and speak of one single phenomenal world.

As mentioned in earlier sections of this book, ideas, modes of thought, and descriptions pertaining to the phenomenal world cannot be extrapolated back to the noumenal, though all of these ideas and descriptions arise by projection from the noumenal to the phenomenal.

In a sense, therefore, the phenomenal *emerges* from the noumenal. As we keep receiving signals of multifarious types from the latter and our experiences accumulate, entities constituting the phenomenal world spring up in endless succession, each having its own collective attributes that keep on changing as our beliefs about an entity evolve, getting revised from time to time.

5.3 Truth in the phenomenal world

As we recall from section 2.18, a statement or belief about a complex system in the phenomenal world that appears to be true is not indubitably so, since it has so many contrary aspects associated with it. Truth is context-dependent, but within some reasonably well-defined context, truth is unambiguous as long as it is judged against explicitly formulated norms. In other words, within the specified context and with reference to the explicitly formulated norms, truth is something that cannot differ from person to person—this is why truth is said to be absolute and not relative—though the process to arrive at it may be a difficult and protracted one, requiring relentless application of logic and evidence, and a great deal of argument and communication between people. All this notwithstanding, *implicit perception* may go against explicitly recognized truth, because a complex system defined within a context may have so may conflicting aspects that explicit norms may be quite inadequate to pin down truth.

Truth, in other words, is infinitely elusive and is only transiently realized—that too implicitly—in all our pursuits of it.

Since entities, events, and experiences in the phenomenal world appear differently in various different contexts, there is no such thing as getting to know an entity or an event absolutely. The reality (reality within the phenomenal world, that is; recall that the phenomenal world itself is rooted in the noumenal reality) of entities, events and experiences does not run counter to these being known only relative to some context or other.

5.4 The truth of theories

Theories are the culmination of all our perceptions, interpretations, decisions, and inferences. These are made use of to make *meaning* out of the vast and chaotic influx of signals that the world incessantly sends to us in the course of its complex evolution—they let us look into the *substratum* underneath our perceived reality, discovering regularities in the world that allow us to *predict* and to *act back on reality*.

Theories are supposed to make us see the *truth* behind the apparent contrariness that seems to be ubiquitous in our experienced reality. It is commonly supposed that the process of getting at the truth is not a smooth one but is made up of a succession of attempts, each of which is part-success and part-failure—truth, in other words, is presumed to be that distilled essence of a complex and contrary world that eventually brings us face to face with the 'ultimate' reality.

But Kantian dualism tells us otherwise. Reality is infinitely complex and goes through a process of complex evolution—the latter being the projection from aspects of the noumenal reality that we refer to as constituting the noumenal 'turmoil'.

> When we speak of evolution, it is the phenomenal world that we refer to. Based on our experience of the phenomenal, it appears unlikely that the noumenal world undergoes evolution in the same sense as we perceive it in this phenomenal world of ours —aspects of the noumenal universe that give rise to evolution as we know it, have been referred to above as the noumenal 'turmoil'.

This turmoil generates all kinds of transition across instabilities and all kinds of complex behavior pattern in the phenomenal world. The noumenal is of vast expanse in time and space, and includes within itself islands of stability and regularity, generated in the course of its complex turmoil. And the phenomenal itself arises by projection as an island of relative stability and regularity, located within a limited space-time domain, that we find ourselves in. The regularity in the behavior pattern of entities within this domain has been responsible for our continuing existence and for all our perceptions. Human intellect unearths this regularity bit by bit in a long and arduous process of inference based on evidence provided to us by experience. Theories constitute our toolkit in this process of specialized inference by enabling us to look beyond facile perceptions and to dig deeper and deeper to look for explanations of anomalies that crop up every now and then in the course of our experience. With the resolution of anomalies, more anomalies crop up, more and more aspects and dimensions of the noumenal world are revealed to us by means of projections in our phenomenal experience, and the truth of our current theories is revealed as fleeting and transitory.

5.5 Why does the phenomenal world look 'simple'?

Regularities in nature and a structured reality are what the sciences look for, and our theories do seem to point to such a regular and structured reality. However, the phenomenal world we build up out of projections from the noumenal is destined to be regular, 'simple', and structured, if only because it has to be compatible with our continued existence. The myriads of signals we receive incessantly from the noumenal universe make sense to us only because we selectively perceive those in keeping with our limited perceptual capacities, and we (along with the rest of the biological world) *co-evolve* with our phenomenal world.

Had the enormous complexity of the noumenal world been revealed to us in its entirety, the continued evolution of life would not have been possible (refer, once again, to [13])—we continue to exist only because we are fortunate enough to 'make sense' of the world, and we make sense of the world only because our perception is based on a minuscule fraction of the boundless number and varieties of signals flying about in the noumenal world in all directions due to interactions between parts of that world. The noumenal world is vastly irregular, and we thrive in it only because of our ability to build a 'simplified' phenomenal world in which we live—much as a fetus thrives in the mother's womb.

In the stupendously complex reversible turmoil going on ceaselessly in the noumenal world, all kinds of transitions across unstable and stable regimes take place, with all kinds of regular and irregular behavior patterns co-existing within the vast expanse of that world. And, it is a relatively stable island of regularity within this bizarre tumult that we have evolved in, along with the phenomenal world that we have co-evolved with.

The phenomenal world of humans is not the same as that of birds, but both are relatively regular and structured for humans and birds to have evolved in.

> From a purely combinatorial point of view, one may refer to *Ramsey theory* ([11], chapter 4), results in which imply that within every sufficiently large structure, there has to exist a regular or ordered substructure. As the size of the substructure is imagined to increase the necessary minimum size of the structure increases stupendously, but we are concerned here more with the possibility than with actual numbers—what is important to note is that 'structure' is inevitably generated within the womb of randomness and disorder.
>
> In our present considerations we have focused on the dynamical evolution of networks representing complex systems, whose nodes (or vertices) denote systems that interact with one another, as represented by the links (or edges) in it. As we have mentioned, a network representing a real-life system is generally a multi-layered one and undergoes co-evolution. In this process of co-evolution that is likely to have disordered and ordered aspects built into it, the network passes through a succession of structures where, looking at the structure at any particular stage of the process, one can find substructures corresponding to 'islands' of regularity, in tune with results in the Ramsey theory.
>
> Referring to the infinitely extended and infinitely complex noumenal reality, one imagines that our scientific theories capture the order and harmony built into these islands of regularity within a vast sea of complexity. Evidently, there is nothing to guarantee that the order inherent in these islands of regularity can be extrapolated to nature as a whole.

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5.6 The emergence of beliefs and theories

As we dig deeper into reality by means of our inferences, our scope of experience also expands and ever new *domains* come up, requiring novel modes of explanation, resulting in the *mushrooming* of theories. History tells us that as our theories delve deeper, they also proliferate and form an ever-expanding mosaic. Theories corresponding to contiguous tiles of the mosaic are related in strange ways. Generally speaking, closely related theories have a common area of applicability, though they are not symmetrically related across their zone of overlap. They do not emerge simultaneously and usually the one appearing later in the day is of a broader scope. This broader theory is acknowledged as the more 'fundamental' one since one can understand concepts, notions, and consequences of the earlier theory with its help (at least the ones in the common zone of overlap), while the notions and consequences of the later theory cannot all be understood in terms of the earlier one. This is referred to as a relation of *incommensurability* between theories emerging in succession within the tiled mosaic referred to above.

The 'space' of theories, in other words, constitutes a *complex system.* In this backdrop the continuous unfolding of theories can itself be looked at as instances of *emergence* occurring in succession.

> The emergence of theories in the vast landscape of the sciences occurs incessantly and perpetually. Theories emerge in all domains of inquiry and *on all scales*.
>
> The genetics-based theory of adaptation and evolution constituted a major instance of emergence in the backdrop of Darwin's and Mendel's theories, during the fifties and sixties of the last century. Subsequently, there took place a major overhaul of that emergent theory itself which continues to this day, being based on emergent concepts on gene regulation and the genetic network (refer to [26]), along with a number of novel ideas on population genetics (complemented with ideas on the effect of contingent and emergent environmental factors on the survival of species).
>
> Recent investigations in elementary particles and field theory on the one hand and in cosmology on the other (i.e., the two farthest ends of the vast spectrum between the extremely small and the extremely large space-time scales) have exposed foundational problems on issues hitherto supposed to have been more or less thoroughly well understood. The small-scale and the large-scale theories appear to be intimately connected by means of *fields*, especially by the gravitational field that seems to be at the root of the deep puzzles now facing fundamental science. A major emergence in foundational physics looks to be on the cards wherein space-time may appear in an entirely new light (refer back to section 4.4; see also [3], [13]).

The emergence of scientific theories is accompanied with another kind of emergence, namely, the emergence of novel correlations in our *conceptual space.* Concepts lodged in our mind are correlated with one another by means of *beliefs*, the latter being complex forms of concepts themselves—the conceptual space is an absurdly complex, nested, and hierarchical one. In this conceptual space, there occurs numerous clusters, each of which are well connected within themselves, with relatively sparse correlations with other clusters in the space. Alongside, there occur certain conglomerates of concepts that are not well correlated among themselves and, at the same time, have tenuous correlations with most other clusters of concepts—clusters having notably sparse correlations with one another are referred to as 'remote' ones.

An emerging scientific theory results from correlations between such *remote* clusters of concepts. The process by which correlations are established between concepts in the conceptual space is none other than the one of *inference*. Inference generates new beliefs, new correlations between concepts, and makes the conceptual space more and more structured, though remote conglomerates of concepts keep on appearing as our experience gets broader, and ever more unexplained phenomena make their appearance. The process of inference generates new beliefs, revises extant ones, and helps us navigate through this uncertain and perilous world of ours by making use of the vast web of beliefs lodged in our mind. And, it is inference on a grand scale that results in the emergence of theories by a grand restructuring of the conceptual space where remote conglomerates of concepts get correlated with one another.

The emergence of theories, in other words, is nothing but the emergence of beliefs occurring on a *scale* of a different magnitude. Both processes can be looked at as instances of *self-organized criticality* (SOC). Indeed, a notable feature of SOC processes is that these are *scale-free* (sections 3.6.3 and 5.1.6), i.e., instances of SOC processes in a statistical ensemble are to be found on all scales.

The restructuring of the conceptual space that takes place in the emergence of beliefs and theories is a complex process, mostly because our belief system itself has vastly complex aspects to it. In the first place, beliefs and concepts are entangled with self-linked and emotion-laden ingredients of the mind. It is because of this that the emergence of beliefs and theories appear to be highly non-deterministic and 'irrational'. Indeed, the first appearance of a novel theory is often accompanied with heated acrimony since it appears irrational to adherents of the extant theory in the specific domain concerned.

What is more, the conceptual space (and the space of beliefs too—the two are intimately related) is a *multi-layered one*, i.e., concepts and beliefs are linked not by just one set of relations, but by several *types* at the same time. For instance, two concepts in a journal article may be correlated by their meanings in plain english, by their scientific connotations (in, say, some area in physics), and also by their mathematical definitions (say, in the area of group theory). In the emergence of a new theory, some specific layer of correlations may be left intact, some others may be modified to some extent, and finally, some new layer of meaning may be added so as to illuminate the concepts in an entirely new light. It is this aspect of multiple types of correlation among concepts (recall that beliefs are concepts having a complex structure) that gives rise to the feature of *incommensurability* between successively emerging theories—the extant theory doesn't have resources to interpret the freshly added layer of correlations while the new theory can interpret most of the correlations in the extant one.

In summary, the revision of beliefs and the emergence of new theories are fundamentally similar phenomena, differing only in the *scale* in the restructuring of the conceptual space. Both are generated in complex processes involving self-linked ingredients of the mind and the setting up of new layers of meaning linking the concepts.

> Incidentally, an inference is, in itself, an emergent process, commonly on a relatively small scale in the conceptual space.

5.7 Theories in succession: their significance in the scientific landscape

What do successive theories emerging in any given domain of scientific inquiry signify? The point of view commonly accepted in scientific realism is that these imply successively closer approximations to truth. Lessons from complexity and emergence tell us otherwise—thereby broadening the scope of what is referred to as realism.

Imagine a complex system, say, 'A', from the womb of which another complex system 'B' emerges. For instance, A may be an assembly of atoms and molecules and B may be a solid body or a liquid—B may have emerged from A under some specified context, such as some specified pressure and temperature. In what sense can the theory describing A be said to be closer to truth than that describing B? The point of view we adopt in this book is that even though B has emerged from A in virtue of interactions among the constituents of the latter, the theory describing the behavior pattern of B, based on state variables pertaining to it, is distinct from, and not an approximation to the theory relating to A, because the two theories make use of distinct sets of state variables and refer to distinct patterns of evolution of these variables. The fundamental thing about emergence is that the behavior pattern of B cannot be understood by referring to behavior pattern of A alone *even though* the former is dependent on the latter. As so pertinently pointed out in [1], one cannot construct the theory pertaining to B from that for A, *despite*—according to Anderson—the A-theory being, in some sense, more fundamental than the B-theory.

Which is more fundamental than which depends on the meaning we attach to the term 'fundamental'. And, speaking of *meaning*, much of it is mostly *implicit* rather than explicit in any given context – this entails the possibility of unproductive and labored communication between people. One may well say that the A-theory is, in a sense, broader than the B-theory, meaning thereby that certain aspects of the latter can be understood in terms of the former, while the reverse claim does not hold (the theory of solids makes copious reference to the quantum theory of electrons, atoms and molecules—though it cannot be constructed from that theory alone—but the behavior of electrons, atoms, and molecules cannot be understood from the theory of solids). Whether this makes atoms and molecules more 'fundamental' than solids has to be an exercise in semantics.

In the vast landscape of scientific theories, which has had a protracted history of evolution, very many theories have emerged from others, and pairs of theories emerging successively have had an asymmetrical relation like that between classical physics and quantum theory—certain classical phenomena appear as limiting cases of quantum mechanical results but the reverse claim does not hold.

> A similar asymmetrical relation holds between the ray theory of light and the wave theory. The Darwin-Mendel theory of heredity, adaptation, and evolution can be understood in terms of the 'modern synthesis' based on molecular biology, but cannot be constructed in entirety from the latter, nor can the Darwin-Mendel picture be resorted to in understanding the genetics-based one.

Theories are built to explain certain features of regularity in our experienced reality (the phenomenal reality, that is) and to make use of those explanations in predicting the behavior of chosen parts of reality and to act back on those. In rewarding us with a successful explanation, a theory can certainly claim to have made a useful guess relating to some aspect of the noumenal reality from which our experiences in phenomenal reality (including the observed features of regularity in it) are generated by projection, but *cannot* claim to have guessed correctly what that noumenal aspect is, precisely in virtue of the fact that the inverse of a projection makes no sense.

Significantly, theories are built *contingently*, based on how our experiences in the world get altered in the course of time. The course of expansion of our experiences is certainly 'progressive' in the sense that experiences gained later in time are inclusive of those gained earlier, but that does not mean we are getting closer and closer to the 'true reality'. It is certainly true that our experiences are giving us glimpses into more and more numerous aspects of reality but that is no reason why one can say that we are approximating that reality to an ever greater extent since an infinite number of 'dimensions' of that reality remain beyond the horizon of our comprehension (for instance, the two dimensional projections of two completely different three dimensional objects may look utterly alike) and, moreover, the relatively few aspects of reality that we comprehend, we comprehend in a skewed fashion—recall that we perceive and comprehend reality by means of interpretations generated in our mind.

All our experiences 'progress' contingently and in fits and starts—there is little overall design or 'logic' in how and in which direction the horizon of our experience expands. And theories emerge in an equally contingent manner—there is a sense of progression in this process of emergence of theories, but no sense in which it can be said to be approaching a culmination.

Successively emerging theories, in brief, make up a ceaselessly expanding and evolving mosaic in the scientific landscape, but there is little overall pattern or design in that evolution—our theories reflect an abounding complexity residing in the noumenal universe and do not reflect a continuing approach to the 'final truth' in that ultimate reality.

1. Thus there are two kinds of succession in which theories emerge. *One* is by way of broadening of our experience in the world wherein new areas of inquiry open up for emerging theories to explain. Such new areas of experiences appear contingently and in fits and starts. And, *the other* is by a deepening of our level of inquiry by way of an attempted look at some deeper substratum beyond our perceived reality so as to explain anomalies in the extant theory. Evidently, the two types of theory emergence are closely related, and *both* involve new 'dimensions' of the noumenal reality being revealed to us by way of projections onto the phenomenal world whereby the latter itself evolves in fantastic ways.
2. As we see, there are *three* related complex processes going on in parallel. *First*, the ceaseless evolution of the phenomenal world by means of successive projections from the noumenal; *secondly*, the restructuring of our conceptual space by way of remote clusters in that space getting connected; and, *thirdly*, the restructuring of the space of *scientific theories*, generating a progressively more complex mosaic in that space where theories are related with one another in strange ways, among which stands out the relation of incommensurability. Of the three, the one of fundamental relevance is the ever-continuing emergence of the phenomenal world by successive projections from the noumenal, since it underlies the other two.
3. Since the 'dimensions' or aspects of the noumenal reveled in successive projections are independent of one another, the theories built upon successive episodes of projection do not constitute continuing approximations to some final or ultimate truth. For instance, if the color and shape of an object is communicated to someone without information about its size, then the information so communicated does not represent an approximation towards a complete description of that object as compared to a description in terms of, say, the color alone. Color, shape, and size

> are disparate aspects of a body, and *an increase in information* in what gets revealed to us does not necessarily constitute a better approximation. This is indicative of the fundamental relevance of the noumenal since it decides the course of evolution of our scientific theories, without itself being accessible to our perception and interpretation. The noumenal is utterly real, but is known to us only through the proxy of the phenomenal.

5.8 Is there deeply hidden regularity and simplicity in Nature?

The answer to this question ultimately defines the metaphysics we adopt. Scientific realism is mostly committed to the view that there do exist deeply hidden regularities in Nature and our most well-established scientific theories are to be looked at as attempts to seek out these regularities in the form of foundational 'laws' pertaining to mechanisms inherent in reality. Successively emerging theories in science are supposed to constitute a progressive sequence of approximations to some ultimate truth about nature that lies at the bottom of all the multifarious phenomena we observe. The laws of nature are *necessary* ones, while they operate on initial conditions that, truly speaking, are *contingent*—science does not take upon itself the responsibility of looking at what lies at the bottom of the contingent. The dichotomy between the necessary and the contingent is at the heart of how scientific realism looks at the way science inquires into Nature.

Implicit in this view, commonly (and tacitly) held by working scientists, is the supposition that truth is some fundamental principle describing the innermost secret of reality, while the job of science is to unearth it: science is to *discover* the truth hidden in nature, but not to *construct* it.

We have already commented on the complexities of truth in section 2.18—truth in a complex system is multi-faceted and context-dependent. What is more, it is partly discovered and *partly constructed* by means of interpretation of the perceived world—

ideally, the aspect of construction ceases to be a problem when truth becomes universally accepted within some precisely specified context, through a protracted process of debate, discourse, and justification. And that is precisely what science is supposed to be doing.

Paradoxically, locating a system within a fixed and precisely defined context reduces it to being a *simple* one. Complexity in real life always shows up in the face of shifting context when features specific to complex evolution show up in a multiplicity of space-time scales involving transitions across instabilities, and co-existence of multiple behavior patterns—regular and irregular—in various different regions of the state space of the system under consideration.

Science, seeks out truth in the phenomenal world of ours, and that too by idealizing and isolating some part or other of the world which, in a relative sense, possesses stability on some particular space-time scale. As some different facet of the phenomenal world is revealed, a scientific theory gets revised and some broader theory is constructed to comprehend and interpret the newly revealed world. While that broader theory is presumed to be more ‘fundamental’ than the previously existing one, it is, as we have noted, in the nature of a clever guess at some facet of the phenomenal world while that facet itself is a projection from the noumenal reality whose complexity has an infinity of dimensions to it. The very fabric of the theory gets altered as more and more dimensions are revealed in the form of novel facets of the phenomenal reality, though part of the older theory remains subsumed in aspects of the newly emerging one that pertain to the dimensions already glimpsed at.

The noumenal world is in a state of seething turmoil that gets revealed in the form of the complex evolution of the phenomenal world. Successful theories of science do possess some correlation with that turmoil, which is what constitutes the basis of their success. Indeed, the projection of the noumenal that gives rise to our phenomenal world is itself in the nature of a relatively stable one since otherwise we would not have been able to make any sense out of it. To be more precise, the phenomenal world evolves in such a manner and over such space-time scales that it is intelligible to our senses, aided by our instruments of exploration. In this sense, there exists a compatibility between the phenomenal world and our capacities of perception—we have *co-evolved* in conjunction with the world that we perceive, made up of the features of the noumenal world that get revealed to us.

Can it be that there is some essential principle lying hidden in some deeply remote recess of the noumenal world, that is responsible for *all* the complexity of the latter? The theory of dynamical systems and computer simulations of complex systems indicate that very simple dynamical rules can indeed generate remarkably complex evolutions of systems. Indeed, much of our ideas on complex evolution is assembled around the paradigm of complexity being generated out of simple rules. Can it be that this paradigm applies to the noumenal reality itself?

The metaphysical position adopted in this book is not compatible with the possibility indicated in the foregoing paragraph. For one thing, the idea of simple rules generating an infinite dimensional complexity gives rise to the question as to what constitutes the origin of those simple rules. Questions of origin are notoriously bothersome in metaphysics.

However, metaphysical presumptions cannot be argued or fought over. Metaphysics is an extrapolation from the world of our experience, and extrapolations can be multifarious. What is a plausible metaphysics to one is preposterous to another. The foundational idea based on our experience with complex systems in this phenomenal world of ours is that of *co-evolution*,i.e., the evolution of diverse entities in a mutually consistent manner—it is this idea that gives rise to our metaphysics. While the concept of ‘evolution’ as a unidirectional dynamics may not be applicable to the noumenal reality, the one of ‘evolution’ as a directionless turmoil may well be. If one must spell out one’s metaphysics, ours would be: the infinitely complex noumenal reality is in a state of directionless turmoil where every one of its aspects (or ‘dimensions’) is implicitly related with and generated by all the others—the noumenal world, in other words, is a co-evolving and self-determined one.

This metaphysics, vague and undefined as it is, can nevertheless imply something more tangible in the context of the scientific process in our phenomenal world—science is the endeavor of seeking out partial and shifting truths in ever-proliferating domains of experience. Theories in science form a strange mosaic in a constantly changing landscape where one theory may be broader than another in its ability to provide an interpretation for the latter (without the reverse being true), but there is never any convergence to some foundational theory that points to some ultimate simplicity and harmony inherent in nature. Instead, there is a proliferation in the space of theories, generating a more and more complex mosaic. A transitory trend of convergence brings in a succeeding phase of divergence caused by unexplained anomalies, when the very fabric of the theory gets changed—a new theory appears in the scene, incommensurate with the old.

Is reality complex and contingent? Or, is it driven by necessity, regularity, and harmony? This little book of mine votes for the former.

> Necessity, regularity, and harmony result from our reconstruction of reality, a reconstruction based on perception, interpretation and inference.

5.9 How are successively emerging theories related? Asymptotic series and singular limits

As we have noted, successively emergent theories in any given domain of inquiry are related in a nontrivial manner, a principal feature of which is *incommensurability*. Successive theories do not constitute a progressive approximation to some ultimate truth, for which complexity makes no room. Instead, theories approach reality in a manner analogous to successive terms of an *asymptotic series*. Put differently, incommensurability has a close analog in *singular limits*. This brings us to the last two sections of this book.

5.9.1 Asymptotic series

A *convergent series* is one where one can unambiguously attach a meaning to the idea of 'summing up' an infinite number of terms. In principle, one can perform a term-by-term addition to obtain successive *partial sums* of the series, which approach as close as one wishes to a fixed number — the *sum* of the infinite series in question. Each partial sum differs from the sum of the series by an 'error term' that gets smaller and smaller as successive terms of the series are added up.

Innumerable examples exist of such convergent series representing mathematical and physical quantities of interest. One such object is the number 'pi' (π), the ratio of the circumference and the diameter of a circle. In decimal terms it is approximated by 3.14159265, but this value differs from the actual value of π by a small error term — the error never vanishes even when one fills up a large number of decimal places. There exist several convergent expansions where successive partial sums approach π at a rapid rate.

Convergent series are useful not only to represent numbers but *functions* as well. Thus, a function $f(z)$ depending on the variable z (commonly one taking up complex values of the form $a + ib$, where a, b are real numbers) can be represented by a convergent series for every specified value of z within some specified domain of convergence.

Contrasting with the case of convergent series, there exist examples of infinite series — of great relevance in mathematics and the physical sciences — that are endowed with *contrary* significance. Such a series, referred to as an *asymptotic series*, can be used to approximate a function with great accuracy but is typically a *divergent* one. Thus, a series of the form $(a_0 + a_1 z + a_2 z^2 + \cdots + a_N z^N + \cdots)$ can be used to approximate a function $f(z)$ at a point z in some neighborhood of any given point, say, $z = 0$, by evaluating the partial sum up to an optimum order $N = N(z)$ (where it is possible to estimate $N(z)$ quite accurately), but on evaluating the successive partial sums beyond $N(z)$ one finds the series to diverge. Early exponents of the power and potentiality of asymptotic series were George Stokes and Henri Poincare among others, who reinstated these divergent series in the road map of mainstream mathematics and physics following a phase when these were all but banished from respectable research programs.

5.9.2 Singular limits

The noted mathematician-physicist Michael Berry illustrated the idea underlying a singular limit by means of the following interesting observation, made in a light spirit: half the bodily remains ($\delta = \frac{1}{2}$) of a worm discovered in an apple after a big bite is more revealing (and revolting too) than a full worm ($\delta = 1$) since it indicates that the other half is now residing in your digestive tract; by the same token, say one-tenth of the remains ($\delta = \frac{1}{10}$) is even more revolting, and so on, till you discover to your delight that one of the apples in the lot does not reveal a worm ($\delta = 0$) even after several bites, because that indicates that the apple is *worm-free* (discounting the other appalling possibility). Here $\delta = 0$ is a *singular limit* since something entirely novel emerges in this limit as compared to small values of δ, close to it.

Other well-known examples of the phenomenon of singular limits in physics are: the limit of the viscosity of a liquid going to zero (no turbulence in the singular limit), the limit of wavelength of light going to zero (in relation to the size of an obstacle; no interference and no diffraction fringe), the Planck constant going to zero (in relation to the size of a typical action integral; classical mechanics: no tunneling through a potential barrier, no explanation for the hydrogen spectrum, ... no nothing).

Berry and a number of other mathematicians and physicists (see, for instance, [4], [2], [8]) have worked on what a theory looks like *close* to a singular limit because the limit itself is not smooth, and it is of great interest to know what transpires close to the limit ($\delta \gtrapprox 0$) as against the situations corresponding to $\delta = 0$ and δ substantially away from zero. This sheds much light on what is referred to as *theory reduction* — a singular limit corresponds to some limiting value of a relevant parameter (denoted by δ here), close to which a theory assumes a *complex* form. The complexity, originally hinted at by Stokes in the context of summing up divergent series, melts away as δ takes up the value zero and also as delta moves substantially away from zero where, however, the theory is of a notably different structure.

More generally, singular limits illuminate the *transition* between different *levels* of reality — they tell us how the levels differ ‘qualitatively’ and yet can be understood in terms of the continuous variation of a single parameter δ (or of a number of parameters). They tell us that the qualitative difference is the result of a certain ‘violent’ behavior (‘Stokes phenomenon’) close to the limit — a ‘violence’ that can nevertheless be understood in terms of the smooth variation of a single parameter. What is more, this ‘violence’ can typically be related to the appearance of an *asymptotic series* (sec. 5.9) describing some typical physical prediction of the theory.

Evidently, asymptotic series and singular limits are likely to have a great deal to say about emergence, theory revision, and incommensurability.

And here must end our rather dizzying (and, perhaps, somewhat delirious too?) journey in this book. Thanks for being on board.

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