Frege’s Intellectual Life As a Logicist Project

Joan Bertran San Millán

Frege: A Philosophical Biography, by DALE JACQUETTE, CAMBRIDGE, CAMBRIDGE UNIVERSITY PRESS, 2019, pp. xiv + 667, £ 35.00.

Despite the vast amount of scholarship that is produced every year on various aspects of Frege’s thought, until very recently there had been no book-length study of his life written in English. Kreiser’s monumental Gottlob Frege: Leben — Werk — Zeit (2001) and other partial accounts of Frege’s life written in German, such as the book by Gabriel, Kienzler (1997), have not been translated, whilst briefer accounts of Frege’s life in English appear at least partially outdated after new research has come to light. The posthumously published biography by the late Dale Jacquette, Frege: A Philosophical Biography, attempted to fill that significant gap in English scholarship.

The result of Jacquette’s work is a 667-page book that “aims at telling a plausible story about [Frege’s] life and the unfolding of his philosophical thought” [p. 6]. This general goal is broken down into an attempt to answer specific questions: “(1) What exactly did Frege hope to achieve in his mathematical and philosophical writings? (2) Should Frege’s efforts be considered to have succeeded or failed, and in either case for what reason and in what sense? (3) What meaning should Frege’s success or failure be understood to have for his significance in a wide-screen panorama of the history of logic and newly emergent analytic philosophy?” [p. 5]. These are relevant questions, the answers to which would definitely contribute to constructing an informative picture of Frege’s significance in the history of logic and philosophy. However, in my view Jacquette did not succeed in answering any of these questions.

Like most biographies, the narrative of Frege: A Philosophical Biography is structured chronologically. Leaving aside a short prelude and an introduction, the book is divided into fourteen chapters. Eight (Chapters 1–3,
5, 7, 12–14) are mainly biographical. Chapter 10 is devoted to Frege’s professional activities between 1894 and 1902 (the years that separate the publication of the two volumes of Grundgesetze der Arithmetik (1893,1903)): it mixes biographical details, a brief comment on Frege’s review of Husserl’s Philosophie der Arithmetik (1891) and an overview of Frege’s scientific correspondence during the period. The rest of the book endeavours to expose the main elements of Frege’s major works. Chapter 4 is devoted to Begriffsschrift (1879); Chapter 6 to Grundlagen der Arithmetik (1884); Chapter 8 to Frege’s semantical papers (1891,1892a,1892b); Chapter 9 to Grundgesetze and Chapter 11 to the Afterword to the second volume of Grundgesetze and Russell’s paradox.

Despite some historical remarks, those parts of Jacquette’s book that deal with Frege’s thought are so focussed upon Frege himself that it is difficult to glean from them a wider picture of his mathematical and philosophical environment. We only receive glimpses of those mathematical and logical trends which influenced Frege or to which he was opposed. In particular, almost nothing is said about the foundationalist movement in nineteenth-century mathematics or the opposition between the Weierstrassian and the Riemannian approaches, although they are essential to provide the context for Frege’s early work in geometry and his logicist project. Moreover, only those philosophers that play a significant role in the analytic tradition are covered at length in Jacquette’s discussion. Mathematicians and logicians with whom Frege engaged in debates concerning topics that were not directly connected with the logicist project are consistently poorly represented, if considered at all. They are never substantially covered on account of the value of their work or in virtue of their historical importance, but only mentioned insofar as Frege had something logicism-related to say about them. One significant example is Hilbert, whose view on the foundations of geometry is rendered according to a popular understanding of formalism that does not justice to the complexity of his thought [pp. 287–288]. Schröder, one of the main proponents of the algebra of logic tradition – with whom Frege had a significant debate in response to the former’s review of Begriffsschrift – and Peano, who held an extensive correspondence with Frege and also wrote a review of Grundgesetze, are barely mentioned.

Most of the biographical details in Jacquette’s book are extracted from Kreiser’s monograph, which Jacquette duly acknowledges. The biographical chapters often consist of verbose speculations upon Frege’s feelings. Instead of limiting himself to the proven facts, Jacquette spends several passages discussing how Frege could have reacted to certain events.

*teorema* XXXIX/2, 2020, pp. 127-138
While he is considering Frege’s wife death in 1904, Jacquette states that “[h]er parting may have been interpreted by Frege in moments of ill humor as yet another savage contradiction in what should have been a logically well-ordered universe” [p. 503]. Another significant example is the four-page Prelude, which dramatizes Frege’s reception of Russell’s letter dated June 16th 1902, in which Russell warns Frege of the discovery of what became known as Russell’s paradox. It is also worth noting that Jacquette seemed to have a specific interest in the psychological analyses of photographs, and devotes several pages to speculation upon Frege’s appearance in some of the pictures that have been preserved (which, surprisingly, this book does not include). Reflecting on a picture showing Frege’s family around 1860, Jacquette states that “it is tempting to imagine dour nights of dutiful romance between Auguste and Karl Alexander [Frege’s parents] in heavy linen night garments” [p. 11]. All in all, the biographical fragments of this book are too long, do not add anything substantially new that could not be found in Kreiser (2001) and a significant proportion of them is uninteresting if not inadequate.

The chapters that deal with the content of Frege’s works are almost exclusively reduced to the development of the logicist thesis. One cannot deny that logicism played a prominent role in Frege’s work but it would be too simplistic to limit his thought to the attempt to reduce arithmetic to logic. Even in the context of the analysis of Frege’s logicism, Jacquette’s account might be too restricted. He claims that “Frege can be known only through his work in symbolic logic” [p. 9] and insists that the construction of a logical calculus was instrumental to a rigorous proof that all arithmetical laws are logical. However, he spends little time describing — as I shall defend below — the nature and particularities of the two versions of the calculi Frege developed during his lifetime. More importantly, the content of Frege’s Nachlass — Frege’s unpublished writings — is not systematically considered on the basis that these writings “are a world unto themselves” and “for the most part [...] do nothing to advance Frege’s logicism” [p. 602]. Frege’s work on mathematics, and specifically on geometry, is poorly covered. Jacquette’s remarks about Frege’s dissertation (1873) and Habilitationsschrift (1874) are aimed at presenting these works as antecedents to the logicist project; their actual content and relevance as mathematical works are not assessed. In fact, even though Jacquette devotes a chapter to the years Frege spent working on his Habilitationsschrift, he does not provide a single textual reference to this work; nothing of what Jacquette claims about [Frege

\textbf{teorema} XXXIX/2, 2020, pp. 127-138
(1874) is supported with textual evidence. Besides, the content of Frege’s courses at the University of Jena is not properly considered. Likewise, Frege’s polemic with Hilbert, which has been the object of a multitude of historical studies, is barely touched upon [pp. 442-447] and, moreover, there is no connection in Jacquette’s account of this polemic with essential notions such as the completeness of a mathematical theory or its consistency.

Jacquette avoids scholarly debate in general and typically refers to the secondary literature in his footnotes (which are sometimes somewhat disconnected from the main narrative). Nevertheless, the author does not refrain from explaining his own interpretation of several aspects of Frege’s thought. Some of these expository passages are based on previous publications and always argue for heterodox theses. In this context, the lack of relevant and substantial references to alternative reconstructions of those elements of Frege’s work that Jacquette covers in detail appears a troublesome choice, especially since, as we shall see below, Jacquette’s analyses are often based on misconceptions and problematic readings of the sources.

I shall focus the following discussion on Jacquette’s account of Frege’s three major works, Begriffsschrift, Grundlagen and Grundgesetze, as well as his interpretation of the nature of Russell’s paradox.

The chapter devoted to Begriffsschrift is characterised by two main aspects: a lengthy explanation of how the logical system developed by Frege in 1879, the concept-script, could render categorical judgements; and a discussion on the similarities between the language of the concept-script and that of arithmetic. The key question in Jacquette’s account of Begriffsschrift is the fact that its language is, as stated by Frege in the very title of this book, “modelled upon that of arithmetic”.

This perspective fails to consider some of the most important aspects of Frege’s 1879 work. Considering Begriffsschrift’s reception, Jacquette affirmed that “[t]he trouble was that Frege had not sufficiently explained the logic’s intended purpose” [p. 164]. This is an interesting historical question, since it is quite clear from the reviews of Begriffsschrift that Frege’s goals in this work were not properly understood. That said, Frege explicitly stated in the Preface that the construction of the concept-script had two different goals. First, by means of this logical system, Frege attempted to establish rigorous foundations for some propositions which are relevant in arithmetic and justify the case that intuition does not play any role in their proofs [(1879), p. x]. Second, Frege aimed at using the concept-script as a tool for scientific languages, in such a way that it provid-

*teorema* XXXIX/2, 2020, pp. 127-138
ed a formal apparatus adequate for the rigorization of their proofs as well as their processes of concept formation ([1879], p. xii). Frege devoted two papers to exemplify this second goal [(1879b, 1880-1881)]. In (1880-1881), pp. 23-29, Frege showed that the logical symbols of the concept-script could be combined with the primitive symbols of mathematics for the rigorous definition of complex mathematical notions, such as the notion of the continuity of a function.

Leaving aside the genesis and motivations of Begriffsschrift, Jacquette’s account of the 1879 concept-script presents significant shortcomings. One of the most important components of this logical system is the notion of function. How to understand a Begriffsschrift function is relevant for the development of this concept, particularly bearing in mind the later notion which Frege adopted from 1891 onwards. In Begriffsschrift, Frege presented a function as that part of an expression that, unlike the argument — which can be replaced — remains fixed [(1879), §9]. Frege first referred to second-order concepts in 1884 but it took him seven years to be able to define a concept as a specific kind of a function — which no longer was defined as the component of an expression. The attribution of unsaturation to the notion of function, the introduction of truth-values as possible values of a function and the separation between functional levels were essential elements in Frege’s late conception of a function, and none of them were present in Begriffsschrift. However, Jacquette states that Begriffsschrift’s concept-script “was a logic of concept-functions, existent objects, and second-level concept-functions of first-level concept-functions” [p. 132]. This kind of historical inaccuracy can also be seen concerning Begriffsschrift’s notion of generality. According to Jacquette, “[t]he quantifier logic of the Begriffsschrift was embryonic in his Habilitationsschrift of 1874” [p. 110]. This claim is repeated several times, but Jacquette offers no evidence in support of it. The genesis and sources of influence of Frege’s first masterwork, Begriffsschrift, are a complex historical problem and bold and unsupported claims such as Jacquette’s make no contribution.

Jacquette insists that Frege’s introduction of negation was instrumental in representing indirect arguments in the concept-script [p. 124], something that set this logical system apart from Boolean logic. One element of Frege’s criticism of Boolean logic in the papers he wrote after the publication of Begriffsschrift was indeed the inadequacy of Boolean logic for the representation of all the formal steps that constitute a proof [see Frege (1880–1881), pp. 43-44]. However, in this context Frege did
not mention indirect proofs. In fact, none of the proofs included in \textit{Begriffschrift} are indirect: there are no derivations with premises in this work and no reasoning by reductio. Even more startling is Jacquette’s remark that the propositional negation introduced in \textit{Begriffschrift} “released into his system the viper that would make it vulnerable to the kind of formal logical paradox that Russell eventually discovered” [p. 124]. This claim, which is hard to understand by itself, is repeated several times throughout Jacquette’s book; he neither explains it nor offers any justification. In fact, in Chapter 11, when he considers the nature of Russell’s paradox, the purported problematic character of negation is not further developed.

As a means to presenting the axiomatic system of \textit{Begriffschrift}, Jacquette provides a list of the logical laws of the concept-script and translates them into a first-order language. No predicate variable occurs in these translations; after the author’s insistence that the concept-script is a higher-order formal system, one wonders what principles determine the use of second-order quantification. Jacquette offers none. In fact, we are only given a passing remark about the inference rules of the concept script [p. 144], but no list or enumeration is given. This is particularly unfortunate since there has been a scholarly debate concerning the presence of a substitution rule in \textit{Begriffschrift}.

In the second place, I turn to Jacquette’s account of \textit{Grundlagen}. In the analytic tradition, this book stands out as one of Frege’s most studied works. Jacquette offers a lengthy commentary of \textit{Grundlagen} and puts the focus on Frege’s informal definition of the concept of \textit{Anzahl}. He insists that Frege’s definition of this concept is at risk of circularity [p. 251]. His main argument seems to be that this definition relies on the notion of equinumerosity, the definition of which in turn presupposes the notion of one (since two concepts are equinumerous if a \textit{one-one} correlation can be established between the objects which fall under them). Jacquette acknowledges that the phrase ‘\textit{one-one} correlation’ comes from Austin’s translation of \textit{Grundlagen}, while the original German is ‘beidseits eindeutige Zuordnung’. Hence, the question is whether the notion of equinumerosity presupposes the concept of the number one. As Kremer (2008) and Schirm (2010), p. 58, state, Frege resolved this kind of objection in \textit{Grundlagen} (1884), §68. Moreover, Frege’s informal definition of equinumerosity [(1884), §§71-72] shows that the concept of the number one is not required at all; the formal expression of this definition makes this blatant. To complicate things, Jacquette’s discussion is clouded by his erratic reference to concepts and his confusion between the notions of equality and equinumerosity. He states that “[n]umber in expressions concerning

\textbf{teorema} XXXIX/2, 2020, pp. 127-138
the number of $F$s is simply the extension of the concept “$=F$” or “equal or identical to the concept $F$” [p. 244]. This is difficult to understand: the number that belongs to the concept $F$ is the extension of the concept “equinumerous with $F$”, while the expression ‘$=F$’ is not acceptable for Frege since a concept is an unsaturated entity that cannot be the argument of an equality (i.e., a relation that holds between objects). Jacquette’s convoluted writing style pushes this confusion even further with phrases such as “[a]n identity or numerical equation such as $F=G$ (for numerical concepts $F$ and $G$)” [p. 242]. Leaving aside the enigmatic notion of numerical concepts, it must be remarked that an equation expresses the equality between numbers, not concepts. It can be said, of course, that two concepts are equinumerous, or that a concept falls under the concept “equinumerous to the concept $F$”, yet it should be clear that equality and equinumerosity are different relations that hold between different kinds of entities.

In the third place, Jacquette’s commentary to Grundgesetze is gravely unbalanced: in a forty-two-page chapter, Jacquette devotes twenty-eight pages to an almost sensationalist review of some parts of the Foreword, where Frege explained the reasons for the late publication of the book and addressed criticism to Erdmann’s Logic; eight pages to the reception of Grundgesetze; six pages to the actual concept-script presented in volume I; and, finally, one page and a paragraph to volume II.

Jacquette’s account of the concept-script presented in Grundgesetze is brief and superficial. Perhaps assuming that this topic is already covered in the chapter devoted to Begriffsschrift, the language of the 1893 concept-script is not even mentioned, save for some remarks about negation — which again push its role in the inconsistency of the system. One of the results of the intense philosophical work shown in the 1891-1892 papers is a semantical apparatus that shapes the syntax of the Grundgesetze concept-script. For instance, the characterisation of the notion of function allowed Frege to introduce the logical symbols of the concept-script as function names. This theoretical apparatus was absent in Begriffsschrift, and the nature of the language presented there reflects a substantive difference which passes unnoticed by Jacquette.

The presentation of the axiomatic system of Grundgesetze is no better. As a means to introduce the inference rules of this late concept-script, a quotation from the summary of inference rules offered by Frege in Grundgesetze (1893), §48, is given without any comment [p. 403]. Jacquette fails to mention that Frege’s list of inference rules contains...
twelve items, while the quotation is partial and only contains the first four. A similar lack of care is shown in Jacquette’s account of a concept-script proof. As an example of a derivation, Jacquette again provides a quotation without any comment [p. 404]. However, the quotation only contains an informal explanation Frege offered before the actual proof of a theorem [(1893), §54] from which the proposition indicated in the quotation would be proved. The quotation contains no proof; it does not include any explicit application of an inference rule and no exhaustive list of the logical principles used is given.

As an excuse for his brief treatment of the concept-script, Jacquette argues that “[f]ine technicalities of Frege’s proof in Grundgesetze are beyond the scope of biography” [p. 402]. This is a fair point, but it has to be remembered that he enters into lengthy analyses of specific aspects of Frege’s works such as the vulnerability of the concept-script to the paradox and the nature of the notions of Sinn and Bedeutung. He devoted whole chapters to each of these topics. All in all, it is difficult to see how a reader can get a proper grasp of Frege’s symbolic logic – which was assumed by Jacquette to be an essential prerequisite for one to know Frege [p. 9] – if it is only exemplified by means of partial and equivocal quotations.

As a result, the reader cannot but conclude that not much about Grundgesetze can be learned from Jacquette’s account. His chapter on Grundgesetze is one of the best examples of the disregard for Frege’s logic that can be found in Frege: A Philosophical Biography.

After describing in detail the contents of Russell’s letter to Frege of June 16th 1902 and analysing the nature of the class suggested by Russell from which the paradox could be generated, Jacquette argues for the suggestive thesis that “Frege’s Begriffsschrift alone or as modified and applied in Grundgesetze I and II already had all the resources it needed just as it stood to defeat Russell’s putative paradox” [p. 497]. In order to better understand Jacquette’s position, it is worth quoting his reasoning in full:

What has entered the history of logic’s hall of fame as Russell’s paradox is no more a logical antinomy than is the barber “paradox”. If Frege had responded in this way to Russell’s announcement of a contradiction in his letter to Frege of 16 June 1902, Frege would not have needed at least the same kind of anxious lament as the 1903 Afterword to Grundgesetze II. More importantly, he could have upheld Axiom V without the restrictive conditionalization of co-extensionality identity conditions for concept-functions revised in V’. He could have sustained, at least for a time, all of his Grundgesetze arithmetic and the philosophical logicism it was meant to substantiate. He could have answered Russell’s paradox instead of letting
it freeze him in his tracks […]. The question of continuing critical historical inquiry is whether there was any sound basis in the first place to suppose that Russell had found anything more than a superficial conundrum in Frege’s Grundgesetze, easily dissolved by the Russell paradox meta-
dilemma as of no greater moment for logic than the ontically deprived barber [pp. 501–502].

Jacquette’s main argument seems to be that Russell’s paradox is not really a threat to Grundgesetze’s concept-script and, thus, to Frege’s late logicism, because the class of classes that do not belong to themselves (called ‘K’) could contain elements that do not have the property of being a class that does not belong to itself. In his words, “[i]f Frege’s K belongs to itself, it does not follow logically that K does not belong to itself. It does not follow that K does not belong to K, unless K is redefined as the class not only of all those classes that do not belong to themselves, but of all and only those. Compare throughout: It may be true that all Greeks are mortal, but it does not follow logically from the fact that only Greeks are mortal, contrary to which inference countereamples are in plentiful supply” [p. 476]. It is difficult to make sense of this interpretation of the class K. After all, what are the elements of the class of Greeks and the class of classes whose cardinality is 2 but Greek human beings and classes with two elements, respectively?

Leaving Jacquette’s understanding of the paradox aside, it should be noted that Frege proved in the Afterword to the second volume of Grundgesetze that (1) the application of the class K to proposition (Vb) – a theorem obtained from basic law (V) – leads to an inconsistency [(1903), pp. 256-257]; and (2) that it is possible to prove in the calculus of the concept-script the negation of a generalisation of (Vb) [(1903), pp. 257-261]. The only possible conclusion, which Frege of course assumed, is that the 1893 concept-script is inconsistent. Frege attempted to reformulate the basic law (V) in order to avoid conclusions (1) and (2), but Jacquette’s claim is independent of this. Is the discovery of the inconsistency of the formal system that supported the whole logicist project nothing “more than a superficial conundrum in Frege’s Grundgesetze”? Jacquette’s failure to acknowledge the relevance of Frege’s formal results in the Afterword is just another example of his carelessness in appreciating Frege’s work.

Frege: A Philosophical Biography appears to have been neither fully edited nor proof-read. Several typos and errors can be found, particularly in quotations from Grundgesetze in Chapter 9.5 The logical symbols of the concept-script are, in general, poorly rendered and without consistency.
Biographical chapters seem to have received less editing: one frequently finds repetitions and a lack of thematic order. Chapter 5, devoted to the transition from *Begriffsschrift* to *Grundlagen*, is the best example of this. On pp. 190-193, Jacquette starts a section by mentioning Frege’s membership of Jena’s Literary Museum; immediately after, without further comment, he deals with the reception of *Begriffsschrift* and, specifically, Schröder’s criticism of this work; and, finally, again without transition, he concludes the section by reviewing Frege’s programmatic goals.

By reading *Frege: A Philosophical Biography*, a reader unfamiliar with the secondary literature on Frege might learn the basic elements of Frege’s life. There is a good deal of information contained in these pages, and it may be surprising to discover the lack of academic recognition that Frege suffered and the hardships of his life, especially during his last years. However, Jacquette’s account of Frege’s philosophical work is so unbalanced that it will hardly help the reader to understand the value and scope of Frege’s achievements in the foundations of mathematics, logic and philosophy. In sum, readers still lack a reliable and informative biography of Frege written in English.

*Institute of Philosophy*
*Czech Academy of Sciences*
*Jiříka 1, 110 00 Praha 1*
*Czech Republic*
*E-mail: sanmillan@fju.cas.cz*

**Acknowledgements**

I am grateful to Calixto Badesa, Berta Grimau and Ansten Klev for their helpful remarks and suggestions. Thanks also to Michael Pockley for linguistic advice. The work on this review was supported by the *Formal Epistemology — the Future Synthesis* grant, in the framework of the *Praemium Academium* programme of the Czech Academy of Sciences.

**Notes**

1 See, for instance, Bynum (1972) or Beaney (1997).
2 Jacquette could at least have referred to the relevant literature on the topic. See, for instance, Wilson (1992), Tappenden (2006).
3 Jacquette’s account of *Grundlagen* seems to be based on the introduction to his translation [Frege (2007)]. This translation, and in particular its introduction, has been critically reviewed by Kremer (2008) and Schrin (2010). Jacquette

*teorema* XXXIX/2, 2020, pp. 127-138
neither mentioned nor modified his analysis according to Kremer’s and Schirm’s meticulous criticisms.

Jacquette’s book, however, contains a lengthy treatment of the Afterword to the second volume of Grundgesetze. It can be found in Chapter 11: The Crucible of Logicism and the Crisis of Russell’s Paradox (1902–1904).

See, for example, the quotation included on pp. 399-400. No ellipsis indicators inform that there is a missing passage in the third line. The quotation contains several occurrences of exponentiations such as ‘2’; in the quote, they are rendered as ‘2\[x\]2’ instead of ‘2\^2’.

REFERENCES


— (1874), ‘Rechnungsmethoden, die sich auf eine Erweiterung des Grössenbegriffes gründen’, Habilitationsschrift, Philosophischen Fakultät in Jena.

— (1879), Begriffschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens, Halle, Louis Nebert.


— (1884), Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl, Breslau, W. Koebner.

— (1891), Funktions und Begriff, Jena, Hermann Pohle.


— (1893, 1903), Grundgesetze der Arithmetik, 2 Bände, Jena: Hermann Pohle.


RESUMEN
Analizo críticamente *Frege: A Philosophical Biography* de Dale Jacquette. En primer lugar, ofrezco una breve panorámica del libro de Jacquette. En segundo lugar, evalúo la interpretación de Jacquette de las tres grandes obras de Frege, *Begriffsschrift, Grundlagen der Arithmetik* y *Grundgesetze der Arithmetik*; y concluyo que el autor no representa fielmente su contenido. Por último, proporciono comentarios generales y de tipo técnico.

PALABRAS CLAVE: Frege, Jacquette, Begriffsschrift, Grundlagen, Grundgesetze.

ABSTRACT
I critically discuss Dale Jacquette’s *Frege: A Philosophical Biography*. First, I provide a short overview of Jacquette’s book. Second, I evaluate Jacquette’s interpretation of Frege’s three major works, *Begriffsschrift, Grundlagen der Arithmetik* and *Grundgesetze der Arithmetik*; and conclude that the author does not faithfully represent their content. Finally, I offer some technical and general remarks.

KEYWORDS: Frege, Jacquette, Begriffsschrift, Grundlagen, Grundgesetze.

deorema XXXIX/2, 2020, pp. 127-138