Logical realism and the riddle of redundancy

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1 Introduction

According to an influential view,[1] when it comes to representing reality, some words are better suited for the job than others. This is *elitism*, a thesis that will be presented with more detail in section 2. It is often expected that the set of the best (or perfectly elite) words should not be redundant,[2] nor should it be arbitrarily chosen. There is a tension between these two requirements. Pushing the requirement of non-redundancy typically leads to arbitrariness, and pushing the requirement of non-arbitrariness typically leads to redundancy. The riddle of redundancy is that, under elitism, we are compelled to chose among two theoretical vices: arbitrariness and redundancy.

In this paper, I discuss the riddle of redundancy as it arises for the logical realist. Logical realism is the thesis that there are logical terms which are elite. If we are logical realists, we have, for example, to choose whether '$\forall$' is elite, or '$\exists$' is elite. Under classical logic, either choice would be arbitrary, for they are inter-definable via classical negation. If we choose both, then

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[1] David Lewis' realism about natural properties and Ted Sider's realism about structure. 'Elitism' is meant to encompass these (and other similar) positions.

[2] Notice that it is sets of words which are redundant. A word w can only be called 'redundant' inasmuch as it belongs to a set where there is either a synonym of it or a collection of other words which can jointly fulfil w's role. For example, classical disjunction is not redundant by its own, but the set that contains the classical connectives of negation, conjunction, and disjunction, is indeed redundant.

we would be picking redundant vocabulary, as both expressions would turn out to be dispensable. I say more about the riddle of redundancy in section 4, and I present the details of my solution in section 5, along with some possible objections.

In her ‘Following logical realism where it leads’, Michaela Markham McSweeney argued that ‘logical realism commits us to one of two surprising views: that the logical structure of the world is unknowable, or that it is deeply unfamiliar’ (McSweeney 2019, p. 138). This paper’s solution for the riddle of redundancy digs further on one of the alternatives: that of logical structure being unfamiliar. This is to say that the issue of identifying the logical constants which better represent the logical structure of the world (*i.e.*, identifying the *jointcarving* logical constants) will be a little bit harder than what might have been expected at first. I argue that the riddle of redundancy can be solved, at least for the case of logical vocabulary, just by using a different notation. In this notation, negation is represented as a flip or reflection over the horizontal axis: not-p is expressed as ‘$\scalebox{1}[-1]{$p$}$’. Choosing appropriate symbols for other logical constants, we can point at a single set of logical constants which is not arbitrarily chosen, nor redundant.

2 Elitist metaphysics

Could there be anything special about the vocabulary we use to describe our world? There is a persuasive, nowadays influential, line of reasoning which answers: *Yes.*

> **Elitism.** Some chunks of vocabulary are better than others.

How could this view be motivated? Consider the following scenario. The world is just fluid, as depicted in figure 1.[3] We have several ways of describing this world. We could do so by means of the chunk of vocabulary $\Sigma_1 = \{\text{red, blue}\}$, as depicted in figure 2; or with the chunk of vocabulary $\Sigma_2 = \{\text{rue, bred}\}$, as depicted in figure 3. Somehow, it seems that Σ_1 fits this world in a better way. This intuition can be made more precise by the following observation. We would like properties to be such that, if two objects share some particular property, then these objects are similar, and objectively so. By looking at figure 1, we can see that the regions q and r are similar, while neither of them are similar to p. This is accounted for by the use of Σ_1, as we can see in figure 2: similar regions, q and r, share the property of being

[3] This is an extension of an example due to Sider (2011).

red, and neither of them share this property with the region p that is not similar to them. However, when we consider Σ_2, it appears gerrymandered, inaccurate: q and p share the property of being rue while they are not similar, and two similar regions, q and r, do not share a property that accounts for this similarity. So, indeed, some words are better than others; in particular, the words in Σ_1 are better than those in Σ_2.

Elitism allows for there being a relation held between words, ranking their betterness: 'I_i is more elite than I_j'. If some word is such that no other word is (strictly) better than it, then the former is *perfectly elite.* This notion need not be defined in terms of the relation 'I_i is more elite than I_j'; however, I take this to be a good way of introducing it. Being perfectly elite is often taken to be primitive (Sider 2011, ch. 2). Now, if some collection of words, a language, accounts for all of reality, then that language is *complete.* Following Sider, the thought that a language accounts for all of reality is made more precise by demanding that any statement whatsoever can be given truth conditions in terms of that language. If all the expressions of a complete language are perfectly elite, then that language is *fundamental.* Some further principles can be phrased using these definitions:

Perfect eliteness. There is at least one perfectly elite word.

Fundamentality. There is at least one fundamental language.

Uniqueness. There is at most one fundamental language.

Some philosophers subscribe to all of them. Most notably, we have David Lewis (D. K. Lewis 1983) and Ted Sider (Sider 2011). For Lewis, elite expressions would be the predicates corresponding to *natural* properties, whereas for Sider they would be *structural* terms. There is an important difference between them. Lewis would only allow *properties* as natural, but Sider goes beyond that: for him, logical vocabulary is also susceptible of being elite (structural) or not. Roughly put, some logical constants are better than others. Indeed, his motivation for introducing the notion of structure in the first place was to claim that some existential quantifier is better than any others, and that substantive debates in ontology are phrased in terms of that quantifier (Sider 2009).

What I have been calling 'chunk of vocabulary' is simply an *ideology*, in Quine's sense (Quine 1951). Just as the ontology of a theory is the collection of objects that the theory is committed to, its ideology is the collection of (primitive) terms that it uses. It is also important to note that, for some philosophers, ideology is the *locus* of fundamentality: it is ideology which

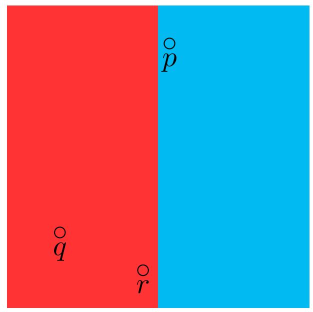

Figure 1: A liquid reality.

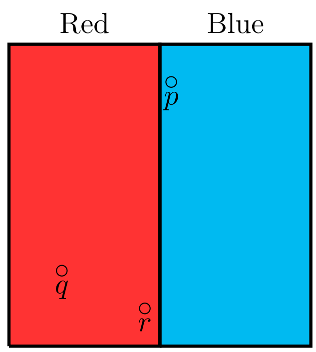

Figure 2: Good categories.

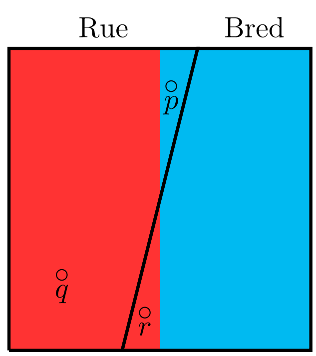

Figure 3: Not-so-good categories.

is susceptible to being fundamental. That is why I named the principle of *Fundamentality* above as I did. Due to their metaphysical grandeur, we should try to figure out which are the *fundamentalia*. However, significant complications arise for such goal under an elitist framework. I present these complications in section 4, and address them in section 5.

3 Formal excursus

Before going any further, I want to be explicit and, hopefully, very clear about how I will understand theories hereafter. A theory T is a structure $\langle \Sigma, \Delta \rangle$, such that:

- Σ is a signature consisting of:
  - a collection of variables $x_1, \ldots, x_n$;
  - a collection of relation symbols R^n, where n indicates its arity;
  - a collection of function symbols f^n, where n indicates its arity.
- $\Delta \subseteq \mathrm{Sent}(\Sigma)$, where $\mathrm{Sent}(\Sigma)$ is the set of closed, well-formed formulas, defined recursively in the usual way. Intuitively, Δ is a set of sentences, the theory's axioms.[4]

I take theories to be deductively closed under some logic. Typically, the logic is assumed to be first-order classical logic, although this is an open question. We can define the deductive closure (under some logic $\mathcal{L}$) of Δ, written '$\mathrm{Cn}_{\mathcal{L}}(\Delta)$', as:

$$\mathrm{Cn}_{\mathcal{L}}(\Delta) = \{\varphi \in \mathrm{Sent}(\Sigma) : \Delta \Vdash_{\mathcal{L}} \varphi\}.$$

To say that $T = \langle \Sigma, \Delta \rangle$ is deductively closed is just to require that $\Delta = \mathrm{Cn}(\Delta)$.

T's ideology is the subset of Σ which contains every constant in Σ that appears in Δ. Notice that, even when $T_1 \neq T_2$, they could be equivalent. Following Halvorson (2019), I will call two theories equivalent, expressed $T_1 \cong T_2$, exactly when they are *Morita equivalent*. Now, two theories are Morita equivalent when there are series of theories $T_1^0, \ldots, T_1^n$ and $T_2^0, \ldots, T_2^m$ such that:

- For each i, T_i^{j+1} is a Morita extension of T_i^j, for each j; and

[4] I do not assume that the axioms should be recursively enumerable or finite.

- $\mathrm{Cn}(\Delta_1^n) = \mathrm{Cn}(\Delta_2^m)$, both theories with signature Σ such that $\Sigma_1 \cup \Sigma_2 \subseteq \Sigma$.

Intuitively speaking, T' is a Morita extension of T to Σ' if it is a Σ'-theory $T' = \{\Sigma', \Delta'\}$ such that $\Delta' = \Delta \cup \{\delta_s : s \in \Sigma' \setminus \Sigma\}$. Each δ_s is an explicit definition of a new term or sort s that was not in Σ, given precisely in terms of Σ. If this extension does not include sorts, then it is a definitional extension, and if the theories involved share a definitional extension, they are definitionally equivalent.[5] To sum up, what is important is (i) the way in which I am understanding theories, and (ii) how to tell when two theories are equivalent. For (i), theories are a list of vocabulary Σ and a list of sentences Δ, intuitively the axioms. For (ii), two theories will be equivalent when we can map them to the same theory[6] by extending their vocabulary until they share every term.

4 The riddle of redundancy

We all know about redundancy. It is a phenomenon consisting of someone (or something) repeating themselves, thus doing extra-work and becoming rather annoying to some people. Here, I am interested in ideological redundancy. An ideology is redundant when it has dispensable terms: when a smaller ideology would do the exact same job, when it would be able to rescue the same truths. This is not a very precise definition, and I will not provide one for now. An example should clarify the notion of ideological redundancy:

Consider the theory of (classical extensional) mereology. This is a theory about parts and wholes: your hands are a part of your body, a tree's branches are parts of it. As you can see, some things are parts of others. Mereology is meant to provide an account of the relation 'x is a part of y'. Consider three theories $T_1 = \langle \Sigma_1, \Delta_1 \rangle$, $T_2 = \langle \Sigma_2, \Delta_2 \rangle$, and $T_3 = \langle \Sigma_3, \Delta_3 \rangle$ such that

[5] Morita equivalence is a generalization of definitional equivalence, insofar as it not only accounts for including new terms, but also new sorts. It might be useful to think of sorts as domains of discourse. Morita extensions allows us to find translatability relations between theories with one domain of discourse and theories that have several. For example, it allows us to claim that the standard formulation of category theory, which runs on a domain of objects and a domain of arrows, is equivalent to the formulation of category theory that uses only arrows, and therefore has only one domain. If we were limited to definitional extensions, we would not have enough resources to do this. I am skipping the details.

[6] Actually, they ought to be mapped to two logically equivalent theories. T and T' are logically equivalent if and only if $\mathrm{Cn}(\Delta) = \mathrm{Cn}(\Delta')$. But under the assumption that theories are identical with their deductive closure, if two theories are logically equivalent, then they are identical.

$\Sigma_1 = \{<, =\}$,[7] $\Sigma_2 = \{\leq\}$, and $\Sigma_3 = \{\circ\}$. I will leave each Δ_i unspecified; it is just the set of axioms of classical extensional mereology phrased in terms of Σ_i. The symbols of each Σ_i are meant to represent the following notions:

- $x < y$: x is a proper part of y;
- $x = y$: x is identical to y;
- $x \leq y$: x is a part of y;
- $x \circ y$: x and y overlap.

We can construct extensions of each T_i that become redundant. For example, starting from the theory $T_1 = \langle \Sigma_1, \Delta_1 \rangle$, we can construct $T_1^+ = \langle \Sigma_1 \cup \Sigma_2, \Delta_1^+ \rangle$, where $\Delta_1^+ = \Delta_1 \cup \{\delta_s : s \in \Sigma_1^+ \setminus \Sigma_1\}$. Every δ_s is an explicit definition of s in terms of the symbols of Σ_1. Notice that $\Sigma^+ \setminus \Sigma_1 = \Sigma_2$. So the intuitive idea is that we are taking the symbols in $\Sigma_2 = \{\leq\}$ and defining them in terms of the symbols in $\Sigma_1 = \{<, =\}$. This is achieved by incorporating the definition:[8]

$$\delta_{\leq} \equiv \forall x \forall y (x \leq y \leftrightarrow (x < y \vee x = y))$$

Likewise, we can construct an extension T_2^+ of T_2 that incorporates the symbols in Σ_1 in the same manner, simply by adding the following definitions to Δ_2:

$$\delta_{<} \equiv \forall x \forall y (x < y \leftrightarrow (x \leq y \wedge \neg y \leq x))$$

$$\delta_{=} \equiv \forall x \forall y (x = y \leftrightarrow (x \leq y \wedge y \leq x))$$

Because of the way in which classical extensional mereology is axiomatized in each of these vocabularies, it turns out that $\mathrm{Cn}(T_1^+) = \mathrm{Cn}(T_2^+)$. When they are extended in a way in which they share exactly the same vocabulary, these theories give us exactly the same formulas. This is enough to show that T_1 and T_2 are equivalent. In this sense, I take them to express the same truths with different formulas. They are different, but they can be

[7] I use the symbol '$=$' in Σ_1 just to keep things from appearing unfamiliar, but it need not be understood as another notion expressed by that symbol: logical identity. In a classical mereological context, the symbol '$=$' expresses co-extensionality: objects related by it share the same material parts.

[8] I adopt the convention of using '$\equiv$' to represent identity of formulas, understood as strings of symbols.

bridged, and thus have the same content. However, each T_i^+ repeats itself.[9] Everything it said in terms of, for example, Σ_1 is now repeated in terms of Σ_2. This can turn out to be annoying for some, like an echo, or our now familiar and most repudiated audio feedback in a video-conference.

4.1 Redundancy and arbitrariness as theoretical vices

Now that we have said a couple of things about redundancy, consider the following principle:

> **Non-redundancy.** *Caeteris paribus*, redundant theories are worse than non-redundant ones.

This principle indicates that redundancy is a theoretical vice. There are several strategies to argue that redundancy is a bad feature of theories. When David Lewis introduced natural properties in his (in)famous *On the Plurality of Worlds*, he said: 'The sparse properties are another story. Sharing of them makes for qualitative similarity, they carve at the joints, they are intrinsic, they are highly specific, the sets of their instances are *ipso facto* not entirely miscellaneous, there are only just enough of them to characterise things completely and without redundancy' (D. K. Lewis 1986, p. 60). Focus on the part where he requires natural properties to characterize it all without redundancy. We could simply say *Magister dixit* and call it a day; many metaphysicians would take it as gospel, and it would even begin to appear *intuitive* to some of them.

But an appeal to authority should not be enough to establish a particular distaste for any kind of theory. A stronger reason is needed, and there is one indeed: minimality, which will be discussed shortly. Notice first that avoiding redundancy will often lead us to arbitrariness:[10]

> **Arbitrariness.** A theory is ideologically arbitrary if it contains some word, but not some other non-synonymous alternative word, even though there is no reason to prefer the former over the latter.

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[9] This only holds if we take theories to be deductively closed. Otherwise, it could be the case that $\Delta_1^+ \neq \Delta_2^+$, even though $\mathrm{Cn}(\Delta_1^+) = \mathrm{Cn}(\Delta_2^+)$. This is easily seen in our example: $\Delta_1 \cup \{\delta_\leq\} \neq \Delta_2 \cup \{\delta_<, \delta_=\}$. They are different lists of formulas, although they have the same consequences. The problem holds because deductive closure is a standard assumption in this literature.

[10] The definition provided comes from Donaldson (2015, p. 1071).

So, for example, both T_1 and T_2 are ideologically arbitrary. T_1 contains '$<$' and '$=$', but does not contain '$\leq$'. If we were to ask someone, some metaphysician perhaps, why not, they could only shrug.[11] The choice is arbitrary. Of course, T_1^+ is not as arbitrary as T_1. So that should settle the matter, right? Wrong. It leads us back to redundancy. Besides, it is still somewhat arbitrary, as it includes '$<$', '$=$', and '$\leq$', but it does not include '$\circ$'. The reason? None. Of course, we could construct an extension T_1^* of T_1^+ by incorporating to Δ_1^+ either the definition:

$$\delta_\circ \equiv \forall x \forall y (x \circ y \leftrightarrow \exists z ((z < x \lor z = x) \land (z < y \lor z = y))),$$

or, alternatively, the definition:

$$\delta_\circ \equiv \forall x \forall y (x \circ y \leftrightarrow \exists z (z \leq x \land z \leq y))$$

We now have a theory T_1^* with signature $\Sigma_1 \cup \Sigma_2 \cup \Sigma_3$. This theory is not arbitrary, but it is redundant: it repeats itself twice. So we fall into a tense situation: there are cases in which, if we want to minimize redundancy, we fall into arbitrariness, and if we want to minimize arbitrariness, we fall into redundancy.

Furthermore, let us go back to Lewis' quote. It says there are 'just enough' natural properties to account for all of reality. This in a requirement often labeled 'minimality':

> **Minimality.** If an ideology is fundamental,[12] then there is no smaller fundamental ideology.

Notice that, when discussing natural properties, structural terms, and elite terms, we shift our attention from theories in general to candidates for total theories, or theories of everything, covering all of reality. Perfectly elite

[11] They could actually say something else. People in metaphysics are very resourceful. For example, I myself have thought that Σ_1 would be preferable over Σ_2 because the real definition of '$x \leq y$' is the disjunction '$x < y \lor x = y$'. Thus, every $\leq$-fact should be grounded by either a $<$-fact or a $=$-fact. Besides, disjunctive properties are not good candidates for fundamental properties under some views (for example, Sider's). So this would lead us to prefer Σ_1 over Σ_2. But this kind of reasoning has also led me to believe some other rather odd claims as well. For example, I once entertained the thought that the universal quantifier '$\forall$' might be taken to be more fundamental that the existential (or particular) quantifier '$\exists$' because, whereas the former can be taken to express a generalized conjunction, the latter would be taken to express a generalized disjunction, and, again, disjunctions are worse candidates for structural terms than conjunctions under some approaches to fundamentality.

[12] That is, it is complete and all of its terms are perfectly elite.

terms would be the terms figuring in the axioms in such a theory. This is the principle of ideological commitment, which takes two forms:

> **Supervaluationist ideological commitment.** We are justified in believing that a term is elite if and only if that term (or a synonym) appears in *all* of the most virtuous total theories.
>
> **Subvaluationist ideological commitment.** We are justified in believing that a term is elite if and only if that term (or a synonym) appears in *at least one* of the most virtuous total theories. (Warren 2016, pp. 2424-2425)

These approaches have different consequences. Assume T_1, T_2, and T_3 are candidates for our best total theory. Then, if we adopt the supervaluationist principle of ideological commitment, then the world would not have any mereological structure, as the signatures have no (mereological) terms in common (notice that $\bigcap_{i=1} \Sigma_i = \varnothing$). But this is counterintuitive; if all the candidates for our best total theory have some mereological term, then surely there must be at least one perfectly elite mereological term. Otherwise, each of our maximally virtuous theories would independently agree that the world has mereological structure, but their joint agreement would not indicate that the world has such structure. Our allegedly fundamental language would not be complete. Notice that, at the beginning of this paper, I characterized fundamental languages as languages that are complete and include only elite terms. Under this definition, any incomplete language cannot count as fundamental. But supervaluationism will likely make all of our languages incomplete, to the extent that there are notational variants of them for which this issue arises. If there is always a notational variant for our best total theory, there will not be a fundamental language. Endorsing supervaluationism threatens the principle of fundamentality from section 2. On the other hand, if we adopt the subvaluationist principle of ideological commitment, then there would be too many mereological expressions: at least '$<$', '$=$', '$\leq$', *and* '$\circ$'. If we take the set $\{<, =\leq, \circ, \ldots\}$ to be the signature of our best total theory, then it would be highly redundant, and it would violate the principle of minimality cited above. If, instead, we were to take the subvaluationist principle of ideological commitment as stating that each Σ_i is simply the signature of a best total theory T_i, and that each Σ_i is elite, then we would have an additional complication. This clashes with the above mentioned principle of uniqueness, which tells us that there should be at most one fundamental and complete language. Here we have several fundamental and complete languages. Things cannot be so if we endorse uniqueness.

We are left with a dilemma: we can either endorse the supervaluationist principle of ideological commitment, or we can endorse the subvaluationst version of the principle. If we go supervaluationist, then we cannot account for some terms being elite, or joint-carving. This pushes us towards redundancy via subvaluationism.

This is also a problem for logical vocabulary; in general, for any other kind of vocabulary such that there is an alternative vocabulary inter-translatable with the former. This is the case, for example, concerning $\{\neg, \wedge, \forall\}$ and $\{\neg, \vee, \exists\}$. We have the unpalatable result that only '$\neg$' is elite, but none of these are elite: '$\wedge$', '$\vee$', '$\forall$', and '$\exists$'. Sider cannot have this as he is explicitly committed to the claim that the logical connectives (or at least some, besides negation) are elite (Sider 2011, §10.1). If we go subvaluationist, then we have redundancy, and a violation of minimality. An alternative might be to drop the requirement for uniqueness. Then we could have several elite ideologies as the vocabulary of several best theories written in terms of different fundamental languages. I will not consider dropping uniqueness in this paper; Alessandro Torza (2021) has argued for pluralism about eliteness on the grounds that it solves the riddle of redundancy.[13]

We might also claim that this epistemology is confused. Sider would agree with this assessment, which I discuss in section 4.2.1. But putting that alternative aside for now, we can see that taking (b) is our best option if we don't want to eliminate too many kinds of terms as candidates for elite terms and if we want to follow uniqueness.

What about arbitrariness? Some undesirable features of arbitrarily chosen theories are related to the fact that what counts as elite would be determined by *us*. This leads to a kind of anthropocentrism about structure, which is a rather odd view. Structure (or natural properties, or elite terms, however you want to call it) is meant to stand for objective features of reality. Unless

[13]Roughly put, for each best theory T_i there is an eliteness—or structural—operator $\mathscr{S}_i$ such that, for every $t \in \Sigma_i$, it is the case that $\mathscr{S}_i(t)$. Although this alternative has a natural way of dealing with the problem currently at hand, it might be found lacking by some philosophers. Importantly, for Lewis, natural properties were supposed to be reference magnets, but this cannot be achieved under pluralism about eliteness. Also, Sider extended his notion of structure to quantifiers in order to argue for ontological realism, an argument that cannot be salvaged under pluralism. It seems that embracing pluralism about structure would undermine some of Lewis and Sider's main motivations for positing their respective versions of eliteness in the first place. This makes me hold on to uniqueness at least for now: it keeps the notion of eliteness from losing some of its theoretical utility.

we are ready to adopt some form of idealism—be it subjective, transcendental, German, or British—we might want to avoid appealing to ourselves when it comes to deciding what reality is really like. Notice that this decision would not be based on cognitive limitations or the like, but on arbitrarily finger-picking what the world is like. It is not that we have a better grasp of conjunction over disjunction, so we believe that $\wedge$-theories are better than $\vee$-theories. It is simply us deciding. There is another reason I can offer to motivate the rejection of arbitrariness, although I cannot think of any strategy to support it. It is the simple impression that the principle of sufficient reason should apply in fundamental matters. This is not to say that the fundamental ought to be further accounted for, but to say that, when we are claiming that something is fundamental, we better have a reason to do so. Arbitrarily postulating something as basic is unfortunate methodology.[14]

I might be wrong though; perhaps we have an accurate epistemology of structure, and it simply leads to these results: that *we* are in charge of deciding arbitrarily what is fundamental. But who is 'we'? You and I? What if my neighbor decides that the fundamental quantifier is '$\exists$' instead of '$\forall$'? What to do with these disagreements? The thing is that they appear nonsubstantive. Alternatively, the epistemology we are working with might be wrong. Maybe it cannot account for structure. Then structure would simply lie there and we would not be able to account for it. That is the view that Sider seems to prefer when it comes to these hard choices. However, I have provided reasons to endorse the following principle:

> **Non-arbitrariness.** *Caeteris paribus*, arbitrary theories are worse than non-arbitrary ones.

When we endorse both the principle of non-redundancy and the principle of non-arbitrariness, we get the riddle of redundancy:

> **The riddle of redundancy.** Both redundancy and arbitrariness are vicious, but you have to pick one.

The next subsection presents two strategies to deal with the riddle, and I offer reasons to be sceptical about them.

[14] Another way to argue for arbitrariness as a theoretical vice is by appealing to authority. Ted Sider seems to think that arbitrariness is indeed vicious, and that it might even trump parsimony (Sider 2020, p. 209). Gustav Bergmann before him also seemed to think that the 'constants of our ideal language' should be non-arbitrary (Bergmann 1950, p. 90). Michaela Markham McSweeny explicitly holds that we should not believe in arbitrary theories (McSweeney 2019, p. 123). But I dislike appeals to authority, so do not mind them.

4.2 Unknowable or undetermined

4.2.1 McSweeney and Sider: epistemic indeterminacy about structure

Michaela Marckham McSweeney (2019) and Ted Sider (2020, pp. 209-210) argue that realists about structure who are having a hard time with hard choices could endorse *epistemic indeterminacy about structure*. In what follows, I will write '$\mathscr{S}(t)$' to express that t is elite. Sider's view is this: it is metaphysically determinate whether $\mathscr{S}(\forall)$ or $\mathscr{S}(\exists)$, but it is epistemically *in*determinate whether $\mathscr{S}(\forall)$ or $\mathscr{S}(\exists)$. Either '$\mathscr{S}(\forall)$' is true, or '$\mathscr{S}(\exists)$' is true; but they are both unknown. In brief, one of them is actually elite, fundamental, but we do not know which.[15]

This is not McSweeney's preferred view, but it seems to be Sider's. The response is, of course, not final, as we are still allowed to look or alternatives that do not leave things unknown. Other things being equal, if a framework lets us know things that another one does not let us know, the former is preferable over the latter. So if an alternative can be found which deals with the riddle of redundancy without having to admit ignorance about important issues, then we should endorse it.[16]

Notice that epistemic indeterminacy solves the riddle of redundancy by taking the horn of arbitrariness: it is what it is, as far as the world is concerned, either $\mathscr{S}(\forall)$ or $\mathscr{S}(\exists)$. One could argue that this is a non-vicious sort of arbitrariness, as it does not have anything to do with our choices; it is simply a fact. However, something about this still appears unconvincing. This position has two costs. One is the cost of ignorance, which I already discussed in the last paragraph. The other is the cost of implausiblility. The intuition (also held by McSweeney (2019, p. 118)) remains that there should not be a metaphysical difference between a theory phrased in terms of the universal quantifier, and an equivalent theory phrased in terms of the existential quantifier.

[15] McSweeney discusses this view under the name of 'privileged'. However, it is not her preferred view. She roots for 'unfamiliar', the thesis according to which no (known) logical constant is structural, a position akin to going supervaluationist in the above-mentioned dilemma. Her main point though, is that (logical) realists *must* choose between epistemic indeterminacy and going supervaluationist.

[16] Sider defends epistemic indeterminacy about structure from detractors who argue that a notion that leads to many open, unanswerable questions, should be rejected precisely because of that. His reply is that there are cases that clearly do not abide by this line of reasoning. For example, the framework of the kinetic theory of matter allows us to pose the question of whether the total number of particles is odd or even, which we cannot know. But those would be rather wacky grounds to reject the kinetic theory of matter.

4.2.2 Torza: metaphysical indeterminacy about structure

Alesssandro Torza has taken a different approach to these difficulties. For him, it is metaphysically determinate that some quantifier is elite, although it is metaphysically *in*determinate which one is. In order to do this, we must allow eliteness to be indeterminate in some cases. In return, this lets us assert, at the same time, that:

- 'either $\mathscr{S}(\forall)$ or $\mathscr{S}(\exists)$' is true; but
- neither '$\mathscr{S}(\forall)$' nor '$\mathscr{S}(\exists)$' is true. (Torza 2020, p. 374-375)

This account generalizes. For some terms $t_1, \ldots, t_n$, we are justified in believing '$\mathscr{S}(t_1) \vee \ldots \vee \mathscr{S}(t_n)$' when, for each best total theory T_i, there is at least one $t_{j \leq n} \in \Sigma_i$. This is simply a way of rewriting the subvaluationist principle of ideological commitment in a way that incorporates Torza's indeterminacy about structure. On its own, this account of structure leads to difficulties regarding ontological realism, the thesis according to which some ontological debates are substantive. In order for an ontological debate to be non-substantive, the two quantifiers involved in the debate, '$\exists_1$' and '$\exists_2$', should be equally structural, and there must not be another '$\exists$' more structural than either $\exists_i$. But for any such quantifier, there would be an alternative universal quantifier, such that '$\mathscr{S}(\forall)$ or $\mathscr{S}(\exists)$', without ever being able to derive from this alone that $\mathscr{S}(\exists)$. So this cripples Sider's account of the substantivity of ontological debates, and thus his account of ontological realism. However, Torza offers a solution: just ask '$\mathscr{S}(\exists_1)$' to be as true as '$\mathscr{S}(\exists_2)$', and ask for there not to be any $\exists$ such that '$\mathscr{S}(\exists)$' is truer than either of the former statements.

The fact that there is no single quantifier which is arbitrarily taken as structural, for none of them is pointed out as *the* structural quantifier, and there is no need to be redundant because the threat of arbitrariness is absent, makes this a solution to the riddle of redundancy. However, Torza's defence of structural indeterminacy is abductive: it is meant to be adopted because of the results it delivers. I believe there to be an alternative proposal that would do the same job, only without any detours through indeterminacy, and without losing Sider's case for ontological realism, which was his main motivation for introducing the notion of structure.

5 Getting rid of the riddle

In this section, I present my strategy for dissolving the riddle of redundancy, along with responses to some possible objections. I speculate that the kind of solution that I offer can be generalized for other kinds of vocabulary that might lead us into the riddle of redundancy, but—as my main concern in this paper is logical realism—I only focus on logical vocabulary. Presumably, any formulation of a good theory will need to resort to words such as ‘and’, ‘or’, ‘some’, and the like. That is why it is important that we can account for this vocabulary being elite, as it is unlikely to be dispensable.

5.1 The proposal

This proposal is to be dubbed *Ramsey’s flipping negation.* It is very straightforward: we achieve our goal by messing with notation. Take $\{\wedge, \bigwedge\}$ to be our set of logical constants. Intuitively, ‘$\wedge$’ is the familiar conjunction, and ‘$\bigwedge$’ is the universal quantifier. We do have negation, although it is not a constant. We negate a formula by flipping it with respect to its horizontal axis:

$$\varphi \xmapsto{\text{Neg}} \scalebox{1}[-1]{φ}.$$

Now, let φ and ψ be formulas. The following are formulas:

- $\scalebox{1}[-1]{$\varphi$}$
- $\varphi \wedge \psi$
- $\bigwedge x\varphi$

Assuming you are familiar with the usual way of reading the meaning of the more usual set of connectives, I ask you to consider the next collection of equivalences to clarify the meaning of this new notation:

$$\begin{aligned}
\neg\varphi &\Leftrightarrow \scalebox{1}[-1]{φ} \\
\varphi \wedge \psi &\Leftrightarrow \varphi \wedge \psi \\
\varphi \vee \psi &\Leftrightarrow \varphi \vee \psi \\
\varphi \to \psi &\Leftrightarrow \scalebox{1}[-1]{φ} \vee \psi \\
\forall x\varphi &\Leftrightarrow \bigwedge x\varphi \\
\exists x\varphi &\Leftrightarrow \bigvee x\varphi
\end{aligned}$$

This notation was suggested by Frank Ramsey in order to avoid some problems related to the fact that people committed to believing that p also seem to be committed to believing an infinity of propositions like $\neg\neg p$, $\neg\neg\neg\neg p$, and so on (Ramsey 1931, pp. 145-147). Notice that, under this approach, all those formulas are identical—not only equivalent—to the initial p. To see this more clearly, compare what happens when we negate a formula twice with the regular notation, and under under Ramsey's flipping negation:

$$\textbf{(Regular negation)}\ p \overset{\text{Neg}}{\longmapsto} \neg p \overset{\text{Neg}}{\longmapsto} \neg\neg p$$

$$\textbf{(Ramsey negation)}\ p \overset{\text{Neg}}{\longmapsto} \scalebox{1}[-1]{p} \overset{\text{Neg}}{\longmapsto} p$$

In the first case, although $p \leftrightarrow \neg\neg p$, $p \not\equiv \neg\neg p$.[17] In the second case, the initial and the final formulas are both logically equivalent and identical, as we have both that $p \leftrightarrow p$, and that $p \equiv p$. This is good for Ramsey because it no longer follows that we ought to commit to infinitely many beliefs when we adopt a single belief.[18] Let us see this notation in practice a little further.

We can see that the formula '$p \vee q$' can be defined in terms of $\{\neg, \wedge\}$ as '$\neg(\neg p \wedge \neg q)$'. Let us now take this formula and change the regular negations for Ramsey negations:[19]

$$\neg(\neg p \wedge \neg q) \rightsquigarrow \scalebox{1}[-1]{$\neg p \wedge \neg q$} \rightsquigarrow p \vee \scalebox{1}[-1]{$\neg q$} \rightsquigarrow p \vee q$$

So the formula '$\neg(\neg p \wedge \neg q)$' is not only equivalent to '$p \vee q$', but also identical to it when expressed in this notation. The same thing applies to accounting for the formula '$p \to q$' in terms of $\{\neg, \wedge\}$ by means of the formula '$\neg(p \wedge \neg q)$':

$$\neg(p \wedge \neg q) \rightsquigarrow \scalebox{1}[-1]{$p \wedge \neg q$} \rightsquigarrow \scalebox{1}[-1]{p} \vee q$$

We could substitute $\neg p \vee q$ for $\neg(p \wedge \neg q)$ and it would also lead us to $\scalebox{1}[-1]{$p$} \vee q$. So any variation of the formula $p \to q$ is only expressible as '$\scalebox{1}[-1]{$p$} \vee q$' in this notation. Now let us see the case of quantifiers. We can account for '$\exists x\varphi$' in terms of $\{\neg, \forall\}$ by means of the formula '$\neg\forall x\neg\varphi$'. Similarly, they turn out to be the same formula under this new notation:

[17] Remember that I adopted the convention of using '$\equiv$' to represent identity between formulas as strings of symbols.

[18] This notation actually does not solve Ramsey's problem, as it is still the case that p is equivalent to but different from $p \wedge p$, and $p \wedge p \wedge p$, and so on. This is due to idempotence, which has an analogue for '$\vee$'. So people committed with p are still committed to believing a countably infinite amount of equivalent propositions.

[19] I use the arrow '$\rightsquigarrow$' to represent step-by-step changes of notation.

$$\neg\forall x\neg\varphi \rightsquigarrow \neg \bigwedge x\neg\varphi \rightsquigarrow \bigvee x \rotatebox[origin=c]{180}{$\neg\varphi$} \rightsquigarrow \bigvee x\varphi$$

So, as before, '$\bigvee x\varphi$', the formula standing for '$\exists x\varphi$', will be identical to the formula standing for '$\neg\forall x\neg\varphi$' in the notation of Ramsey's flipping negation. If we expand our vocabulary to include an operator '$\sqcap$' such that '$\sqcap\varphi$' is taken to mean 'it is necessary that φ', usually expressed as '$\Box\varphi$', we can account for possibility in an analogous way. In more standard notation, we would say that '$\Diamond\varphi$' can be accounted for in terms of '$\neg\Box\neg\varphi$'. We run the same substitution:

$$\neg\Box\neg\varphi \rightsquigarrow \neg \sqcap \neg\varphi \rightsquigarrow \sqcup \rotatebox[origin=c]{180}{$\neg\varphi$} \rightsquigarrow \sqcup\varphi$$

Yet again, under this notation, '$\Diamond\varphi$' can only be expressed in one way: '$\sqcup\varphi$.'

We can now take theories T_1, T_2, and T_3 with $\Sigma_1 = \{\neg, \wedge\}$, $\Sigma_2 = \{\neg, \vee\}$, and $\Sigma_3 = \{\wedge\}$, and expand each one of them to some T_i^+ with $\Sigma_i^+ = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3$ such that the T_i^+ are logically equivalent to each other by incorporating the regular definitions for new connectives. The T_i are all equivalent.[20] If a particular ideology allows for non-arbitrariness to be achieved without redundancy—and arbitrariness and redundancy are indeed undesirable theoretical features—, then this particular ideology is preferable. So this speaks in favor of Ramsey's flipping negation, concretely, in favor of T_3.

5.2 The worries

5.2.1 Just the classics

This view might appear unsatisfactory for those who would like to deviate strongly from classical logic:

- It only works if our underlying logic validates all De Morgan's laws that can be phrased in that logic's vocabulary, be it those that make use of $\{\neg, \wedge, \vee\}$, $\{\neg, \forall, \exists\}$, or $\{\neg, \Box, \Diamond\}$, among others;
- It only works if our underlying logic validates double negation.
- Any non-material conditional cannot be accounted for.

[20] I am actually skipping the details here. The notions I am using were only defined for translating theories with the same underlying logic. To see how to translate between logics, see (Dewar 2018).

Classical logic, with its material conditional, does validate all of these. However, other logics might not validate them. In intuitionistic logic, double negation is not valid; in particular, its direction $\neg\neg p \nVdash p$. Neither are some of De Morgan's laws. Some other logics that aspire to be the One True Logic use conditionals that are not material. Notable examples are Routley's Ultralogic (Routley 2019), or C. I. Lewis' $S3$ (C. I. Lewis 1918, chapter 5; C. I. Lewis and Langford 1932). So the logic of our best theory might not welcome this notation.

There is a line of reasoning that can save us from these concerns. It would, however, take us to the realm of the philosophy of logic. It basically amounts to defending classical logic for one of its intended roles: that of being the regimentation of the transmission of truth. This is a notion of logic that need not take into account concerns related to cognitive claims, or on whether a conditional is best suited to provide a model of how we actually (or ought to) make inferences. Classical logic, along with its classical implication and the material conditional, is a theory about the preservation of truth; it is a worldly theory, blind to concerns about us and how we reason (see Russell 2020, 2015). I would like to distinguish between two senses in which a logic can be said to be universal. The first one is a logic for all applications. That, I think, is the most usual sense of the phrase. I do not think classical logic is a candidate for being universal in this sense. However, I do take it as a candidate to be universal, or global, in another sense: it might still be the best framework to use for describing reality, for writing the book of the world, as Sider would put it.

Nevertheless, the status of classical logic as an appropriate logic for the book of the world should not be a reason for concern. If the One True Logic is one in which the relevant equivalences do not hold, then some of the tension between arbitrariness and redundancy is likely to vanish. For example, the argument $\neg\forall x\varphi \Vdash \exists x\neg\varphi$ is invalid in intuitionistic logic. Therefore, the ideologies $\{\neg, \forall\}$ and $\{\neg, \exists\}$ are not inter-definable. In such a case, it is not arbitrary to choose either of them, because choosing either of them would simply be wrong (they are functionally incomplete); and choosing $\{\neg, \forall, \exists\}$ would not be redundant, as neither '$\forall$' nor '$\exists$' are dispensable. Likewise for $\neg(\neg\varphi \wedge \neg\psi) \Vdash \varphi \vee \psi$, which is also intuitionistically invalid, and thus makes the ideologies $\{\neg, \wedge\}$ and $\{\neg, \vee\}$ not inter-definable nor functionally complete. So if the underlying logic of our best total theory were intuitionistic logic, then its ideology would either need to include $\{\neg, \wedge, \vee, \rightarrow, \forall, \exists\}$,[21] or

[21] Given that intuitionists typically see '$\neg\varphi$' as abbreviating '$\varphi \rightarrow \perp$', it would be better aligned with an intuitionistic spirit to pick $\{\perp, \wedge, \vee, \rightarrow, \forall, \exists\}$ instead.

a set with different connectives. We could introduce weird connectives that are functionally complete by themselves, such as Kuznetsov's analogs of the Sheffer stroke (Kuznetsov 1965, p. 275):[22]

- $C_1(\varphi, \psi, \chi, \alpha, \beta) \Leftrightarrow^{df} \varphi \rightarrow ((\psi \wedge \neg\chi) \wedge (\alpha \vee \beta))$
- $C_2(\varphi, \psi, \chi) \Leftrightarrow^{df} ((\varphi \vee \psi) \wedge \neg\chi) \vee (\neg\varphi \wedge (\psi \leftrightarrow \chi))$

It can be argued on other grounds whether the ideology $\{\neg, \wedge, \vee, \rightarrow, \forall, \exists\}$ is better or not than the ideology $\{\neg, \vee, \leftrightarrow\}$, or $\{C_1\}$, or $\{C_2\}$. For example, the latter two might be preferable on grounds of ideological parsimony, and $\{C_2\}$ might be preferred over $\{C_1\}$ because C_2 has a smaller adicity than C_1 (it is ternary instead of quinary). That is a very different scenario from the one of classical logic. In classical logic, there is no obvious reason—analogous to the ones just mentioned—to prefer $\{\neg, \wedge\}$ over $\{\neg, \vee\}$ or $\{\neg, \rightarrow\}$. My proposal is designed to counter the difficulties of that particular case. Other logics will certainly find other peculiarities, but the general proposal still stands: try to dissolve equivalences at the level of notation in order to avoid the advent of arbitrariness or redundancy.

5.2.2 Still arbitrary

I would like to highlight that, under my approach, there is no difference between the sets $\{\wedge, \bigwedge\}$ and $\{\vee, \bigvee\}$. The only difference is the orientation of the symbols, but this sort of orientation is not significant in the subpropositional level under my approach; it is only significant for formulas. So there is no pending arbitrary choice to be made among the sets: $\{\wedge, \bigwedge\}$ and $\{\vee, \bigvee\}$. They are the same set. But even then, there might be a rival view, one using either the Scheffer stroke '$\uparrow$' or Peirce's ampheck '$\downarrow$' (also known as 'Quine's dagger', using the symbol '$\dagger$'). It is easier to illustrate what these connectives mean by showing the equivalences:

$$\varphi \uparrow \psi \Leftrightarrow \neg(\varphi \wedge \psi)$$
$$\varphi \downarrow \psi \Leftrightarrow \neg(\varphi \vee \psi)$$

They are both functionally complete. Let us consider the case of '$\uparrow$':

- $\neg\varphi \Leftrightarrow^{df} \varphi \uparrow \varphi$,

[22] Both are functionally complete by themselves. The following equivalences can be corroborated: $\neg\varphi \Leftrightarrow C_2(\varphi, \varphi, \varphi)$, and $\varphi \vee \psi \Leftrightarrow C_2(C_2(\varphi, \varphi, \varphi), C_2(\psi, \psi, \psi), C_2(\psi, \psi, \psi))$. If we enriched our vocabulary by introducing negation (as a *defined* connective), we could also express the last formula as $C_2(\neg\varphi, \neg\psi, \neg\psi)$.

- $\varphi \rightarrow \psi \Leftrightarrow^{df} \varphi \uparrow (\psi \uparrow \psi)$,
- $\varphi \wedge \psi \Leftrightarrow^{df} (\varphi \uparrow \psi) \uparrow (\varphi \uparrow \psi)$,
- $\varphi \vee \psi \Leftrightarrow^{df} (\varphi \uparrow \varphi) \uparrow (\psi \uparrow \psi)$.

We have arbitrariness again, as nothing justifies our adoption of ‘$\wedge$’ over the alternatives, ‘$\uparrow$’ and ‘$\downarrow$’.

I am tempted to claim that ‘$\wedge$’, ‘$\vee$’, ‘$\uparrow$’, and ‘$\downarrow$’ can be shown to be the same connective. Notice first that they are each other’s dual:[23]

$$\neg(\varphi \uparrow \psi) \Leftrightarrow \neg\varphi \downarrow \neg\psi$$

$$\neg(\varphi \downarrow \psi) \Leftrightarrow \neg\varphi \uparrow \neg\psi$$

Given that they are duals, we can define ‘$\downarrow$’ in terms of ‘$\uparrow$’. The definition is provided by the equivalence:[24]

$$\varphi \downarrow \psi \Leftrightarrow^{df} ((\varphi \uparrow \varphi) \uparrow (\psi \uparrow \psi)) \uparrow ((\varphi \uparrow \varphi) \uparrow (\psi \uparrow \psi))$$

Strictly speaking, negation is not written as ‘$\neg\varphi$’ when our vocabulary is limited to ‘$\uparrow$’ (or ‘$\downarrow$’), but ‘$\varphi \uparrow \varphi$’ (or ‘$\varphi \downarrow \varphi$’). We will proceed as before: instead of writing ‘$\varphi \uparrow \varphi$’, we will write ‘$\rotatebox{180}{$\varphi$}$’. I show that the aforementioned definition boils down to identity between formulas understood as strings of symbols under Ramsey notation:

$$\begin{aligned} ((\varphi \uparrow \varphi) \uparrow (\psi \uparrow \psi)) \uparrow ((\varphi \uparrow \varphi) \uparrow (\psi \uparrow \psi)) &\rightsquigarrow (\rotatebox{180}{φ} \downarrow \rotatebox{180}{φ}) \downarrow (\rotatebox{180}{ψ} \downarrow \rotatebox{180}{ψ}) \\ &\rightsquigarrow (\rotatebox{180}{φ} \downarrow \rotatebox{180}{φ}) \downarrow \psi \\ &\rightsquigarrow \varphi \downarrow \psi \end{aligned}$$

Therefore, under our new notation, the definition stated above can only be written as $\varphi \downarrow \psi \Leftrightarrow^{df} \varphi \downarrow \psi$. But this is trivial, considering that $\varphi \downarrow \psi \equiv \varphi \downarrow \psi$. Just as it was the case between ‘$\wedge$’ and ‘$\vee$’, it turns out that ‘$\uparrow$’ and ‘$\downarrow$’ are the same connective, only flipped. Now I argue that ‘$\uparrow$’ and ‘$\wedge$’ are the same connective.

Notice that $\varphi \wedge \psi \Leftrightarrow \rotatebox{180}{φ} \downarrow \rotatebox{180}{ψ}$. Now, let the book of the world consist exclusively of the formula ‘$\varphi \wedge \psi$’ (for simplicity). Suppose that the book of the world is printed on acetate sheets: we can see what is printed from both sides. Then this formula looks like this ‘$\rotatebox{180}{$\varphi$} \vee \rotatebox{180}{$\psi$}$’ when we flip the sheet relative to its horizontal axis. Flipping something over its horizontal axis

[23] I introduce negation into the vocabulary to make the duality clear.

[24] This equivalence can be intuitively understood as stating: $\varphi \downarrow \psi \Leftrightarrow^{df} \neg(\neg\varphi \uparrow \neg\psi)$.

delivers, of course, its negation. So when we look at the book of the world from behind,[25] we get the book of falsities (or the book of the anti-world, or $\scalebox{1}[-1]{\text{the book of the world}}$, or however you would like to call it). Now focus on some relationships between the formulas ‘$\rotatebox{180}{$\varphi$} \vee \rotatebox{180}{$\psi$}$’ and ‘$\rotatebox{180}{$\varphi$} \downarrow \rotatebox{180}{ψ}$’. There is a relationship in terms of their truth-value: one is the negation of the other. There is also a relationship in terms of their visual similarity: they both feature the same (immediate) sub-formulas, oriented in the same direction, with a connective that has the symbol ‘$\vee$’ as a part: one of them has an additional centered vertical line that intersects with it, whereas the other one does not. Imagine if we could simply remove the ‘$|$’ part of the symbol ‘$\downarrow$’, leaving $\vee$ alone, but marking it with a dent—here represented with the symbol ‘$\bullet$’—somewhere nearby. Then we could indicate the negation of a formula ‘$\rotatebox{180}{$\varphi$} \vee \rotatebox{180}{$\psi$}$’ by writing down that very formula and making a dent on that side of the paper. Now, dents have the property of being concave on one side and convex on the other. The symbol ‘$\bullet$’ represents the concave side. The following holds:

$$\rotatebox{180}{φ} \downarrow \rotatebox{180}{ψ} \Leftrightarrow \rotatebox{180}{φ} \vee \rotatebox{180}{ψ}\ ^{\bullet}$$

Now, every time we write a page full of true sentences in the book of the world, we must be cautious. Someone might inadvertently flip the book and read each acetate sheet upside down, in which case they would be reading a page from the book of falsities![26] In order to avoid this rather undesirable scenario—better, in order to tell which side tells the truths and which the falsities—, the scribes of the book of the world ought to mark each page with a dent such that it is concave on the side of falsities and convex on the side of truths. With this at hand, the difference between the formulas ‘$\rotatebox{180}{$\varphi$} \downarrow \rotatebox{180}{ψ}$’ and ‘$\rotatebox{180}{φ} \vee \rotatebox{180}{ψ}$’ is just a difference of which side we looking at: the side of truths or the dual side of falsities. It is in this sense in which I claim that ‘$\wedge$’ and ‘$\uparrow$’ are the same expression: they are exactly the same trace, only that, so to speak, ‘$\wedge$’ belongs on the side of truths whereas ‘$\uparrow$’ belongs on the side of falsities.

Alternatively, we could find a way of fixing the orientation of the book of the world without introducing any dents, but a single symbol ‘$\top$’ in both the front and back cover. If we are reading the book of the world which contains only truths, the cover would look like this: ‘$\top$’, whereas if we were about to read the book of falsities, it would look like this: ‘$\bot$’, the *falsum*. This is beneficial for one reason. It might be argued that the concave side of a dent

[25]Under this particular rotation.

[26]This situation would be similar to that of Tactic and Tictac in (Roberto Casati 2000).

near a formula would be some kind of additional symbol for negation, in which case we would have two different ways of expressing negation (flipping formulas and placing them in dented pieces of paper). However, the only purpose of the dent is to fix an orientation for the pages: one side contains falsities, and the other one contains truths. But this orientation can be fixed by marking the covers of the book of the world in a way in which our negation is still just Ramsey negation.

5.2.3 Orientation adds complexity

One might argue, however, that the need to fix an orientation is a serious deficiency of this view. With regular negation, there is no need to do that, and that makes it preferable, or so the argument would go. Thus, regular negation is better as we do not need to complicate our meta-theory in order to indicate which formulas are negated and which ones are not.

I argue that fixing an orientation is also a problem for regular negation, as nothing about the syntax of a formal language tells us that 'φ' expresses the truth/assertion of φ, unless we conventionally stipulate it to be that way. To put it more clearly by means of an example, the judgment bar in Frege's *Begriffsschrift* (Frege 1879, §2) could express that a particular formula is judged to be *false*. It only does not because Frege intended it to be an indicator of assertion. The expression

$$\vdash\!\!\!-\, A$$

might have expressed that A is not the case, instead of expressing that A is being judged as true. However, there is a difference between the following:

$$-\!\!\!-\, A \tag{1}$$

$$-\!\!\!\!\top\!\!\!\!-\, A \tag{2}$$

While (1) does not express a negated content, (2) does. This is independent of how the judgment bar is meant to be interpreted. The main difference, I argue, is not about the interpretation of these propositions once a particular propositional attitude has been placed upon them. The main difference is better seen in contemporary notation:

$$A \tag{3}$$

$$\neg A \tag{4}$$

Whereas (3) can be an atomic formula, (4) cannot, as the appearance of '$\neg$' as the main connective will always make it complex. However, this distinction between atomic and complex formulas is fuzzy for our new notation with Ramsey negation. It is not clear whether p is atomic or complex, as it might be the result of applying negation twice. Also, what intuitively is the negation of p, 'b', only seems complex because we are more familiar with the symbol 'p', which we would take as atomic. It seems as if though we have applied some transformation to the more familiar symbol 'p', even though this symbol is but a mark of ink, as is 'b'. For all we know, 'b' could be atomic: we do not see any connectives or operators in that formula. Negation in our unorthodox notation does not increase the complexity of a formula. This is the main difference between both forms of negation. Notice something interesting about Frege's notation. According to him, if we have

$$\vdash\!\!\top\!\!\!-\, A$$

the horizontal bar at the right of the tiny, vertical negation bar is the content bar of A, while the horizontal bar at the left of the negation is the content bar of *the negation* of A (Frege 1879, §7). Frege could have done what Ramsey did years later, and take each negation bar as an instruction to rotate a formula along its content bar, rendering:

$$\vdash\!\!\!-\!\!-\, \forall$$

where the content bar would be the content bar of the negation of A. If we interpreted Frege's negation this way, it would approximate my view. Frege's negation also serves as an orientation device. The fact that its absence is meant to indicate the truth-orientation and its presence the false-orientation is only conventional. The same could be said about regular negation. The only difference between my notation and the others is that it does not obviously increase the complexity of formulas, while the other ones do. The rest is conventional, and adds the same weight to our meta-theory as placing an orientation for formulas to be read as negated or not. Consider Roy Sorensen's insight on this matter:

> Logicians have felt no need to state the orientation of their inscriptions because the orientation is a constant. The only permissible orientation to the page is the one customary to European languages. ... In ordinary arithmetical notation, $>$ expresses greater than and $<$ expresses less than. Since these inscriptions have the same shape, we must be tacitly using their orientation

> to distinguish them. Most European alphabets are orientational in this sense. ... The feeling that mere marks on the page are the whole symbol, is an illusion encouraged by the unvarying nature of the hidden relatum of orientation. A similar illusion arises for movement. Since we habitually relativize to the earth, we fail to realize that we are relativizing. (Constants become invisible.) (Sorensen 1999, p. 161)

Orientation does not add complexity because our languages, formal or otherwise, are already oriented.

6 Conclusion

I have done several things in this article. First of all, I presented under which assumptions we would have to make strange choices with regards to what is fundamental, and how these choices turn out to be problematically arbitrary or redundant under such assumptions. Second, I offered some strategies that offer us a vocabulary in which these problems do not arise. With just a change of notation, we can present our theories in an ideology that would not be either redundant nor arbitrary. Along with this, I advanced some worries that might arise around my proposal, and I offered my responses to each one of them. The worries were that my proposal does not work if logic is not classical. I answered first by motivating that there is some reason to think that the logic of the world is classical, and second by saying that, if it is not, then the riddle of redundancy would not arise, or it could be dealt with by means of a strategy analogous to mine. Another of the worries I presented was that, even if we were able to eliminate the difference between the sets $\{\wedge, \bigwedge\}$ and $\{\vee, \bigvee\}$—and thus also the need to choose between them—, we would still have to choose between, for example, $\{\wedge, \bigwedge\}$ and $\{\uparrow, \bigwedge\}$. If that were so, arbitrariness would remain, and the riddle of redundancy would still pose a problem for logical realists. However, I argued that there is a sense in which we can identify $\{\wedge, \bigwedge\}$ and $\{\uparrow, \bigwedge\}$ without adding anything new to the proposal. The final worry that I addressed is the worry that adding orientation to our language would make it more complex. I responded to this worry by noting that languages are, albeit often implicitly, oriented already. Therefore, involving orientation is not an addition to complexity.

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