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Issues in the Metaphysics of Realism

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INDISPENSABILITY WITHOUT PLATONISM

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1 Introduction

Indispensability arguments used to be the only game in town for philosophers of mathematics. One had to be realist about mathematics if one was a scientific realist. After all, mathematics is indispensable to formulating our best scientific theories. And it would be 'intellectually dishonest' to be realist about the physical components of scientific theory while remaining agnostic or antirealist about the mathematical aspects of those theories.

Soon enough, however, the rot set in. Good philosophers began to have doubts about indispensability arguments. Parsons (1986) pointed out that the inferences to the best explanation mentioned in indispensability arguments didn't explain the 'obviousness' of elementary mathematical truths such as '2+2=4'. Furthermore, indispensability arguments leave unapplied pure mathematics in the twilight zone. In response, Quine dismissed such pure mathematics as 'recreational mathematics', surely a desperate move given that at any time a great deal of mathematics is unapplied.[1]

Then a strange thing happened. Even theorists in favour of the indispensability argument began to step back from embracing it wholeheartedly. Penelope Maddy led the way with her reminder that pure mathematics—such as set theory and analysis– is an autonomous discipline with its own distinctive epistemic practices and norms quite different from those employed in empirical

[1] For the debate over the significance of recreational mathematics, see Leng (2002) and the reply by Colyvan (2007).

sciences. In pure mathematics, mathematicians come to accept statements as true almost solely on the basis of *proof* from accepted axioms. It is surely an embarrassment for Quinean empiricism that it seems to get the epistemology of mathematics wrong. Its epistemological holism implied that mathematics should be tested and confirmed like the rest of empirical science. However, when an empirical theory fails to be confirmed, we don't take this failure as evidence that the mathematics used to articulate the theory is *false*. Rather, we don't even assume that the mathematics is being tested at all.[2]

It is bad enough that Quine's indispensability argument appears to distort the epistemology of mathematics. What is more scandalous is that philosophers cannot agree on the metaphysical conclusion of the argument. Sure, everyone agrees that indispensability is an argument for realism, but beyond this point the agreement ends. Quine's indispensability argument tells us nothing specific about the *metaphysical* nature of mathematical entities. It does not tell us *what* the basic mathematical entities are, or *in what way* they exist. It does not settle the ancient dispute between Platonists and Aristotelians over whether mathematical objects are abstract or concrete, particular or universal. The indispensability argument simply tells us that we ought to believe in the existence of whatever it is that mathematicians are talking about, because we are *ontologically committed* to them by our best scientific theories.

Despite brief protests to the contrary,[3] most scientific realists still assume that the conclusion of Quine's indispensability argument will involve some commitment to *abstract* entities.[4] In this assumption, realists are no doubt influenced by Quine's reluctant Platonism about classes at the end of *Word and Object* (1960: 233–70). Quine becomes a reluctant Platonist because he knows of no alternative way of construing classes and numbers other than as *abstract, other-worldly* entities. Deeper reflection on his indispensability argument shows that it is metaphysically shallow: the fact that such-and-such mathematics is useful in doing science tells us very little about the content of the metaphysics of science or mathematics. In fact, indispensability arguments are structurally as well as metaphysically neutral as regards the variety of realism we adopt: they don't tell us whether mathematical objects are abstract or concrete, lone atoms or structured complexes. (Similarly, arguments for the reality of atoms do not tell us whether they are hard particles or probability clouds.) Rather, indispensability arguments simply tell us that we ought to believe in the existence of mathematical objects, because we are ontologically committed to them by our best scientific theories.

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[2] The point is made at length in Sober (1993).

[3] On attempts to make way for a non-Platonist variety of realism, see Cheyne and Pidgen (1996). The possibility is mentioned in passing in Colyvan (2001: 142).

[4] In his paper in this volume, Stathis Psillos argues that the indispensability argument does lead one to conclude that there are 'mixed facts', consisting of an abstract, mathematical component and concrete, physical component. On the Platonist realism that results, there need be no causal interaction between the abstract and physical components of such 'mixed facts'.

The search is now on to salvage what is left of indispensability arguments. The insight is that mathematics *works*: that in some sense mathematics must contain a body of truths because these truths can be exploited to describe and predict events in the world. And those truths are expressed in specifically mathematical language, mentioning functions, groups and other specifically mathematical entities. Mathematical explanations are successful, because (we infer) they correctly describe (the mathematical structure) of reality. Furthermore, this insight is strictly independent of Quinean philosophy. All it requires is application of the general argument for scientific realism (using inference to the best explanation) to the special case of mathematics. Arguably, this was Quine's intention originally. But in any case, a proper understanding of indispensability arguments must attempt to distance itself from its Quinean heritage. It is this act that we attempt in this essay: indispensability without Quineanism. In particular, we think that indispensability arguments for realism need not incorporate these dubious Quinean theses:

A. The Quinean criterion of ontological commitment: to be is merely to be the value of a bound variable in a canonical (first-order logic) statement of a theory.
B. Mathematics is no different epistemically from the rest of science.

In this essay we focus entirely on the task of liberating the indispensability argument from (A). The really unique aspect of our rejection of (A) is that we do so from a perspective that is not anti-realist, fictionalist, or nominalist, but from the perspective of *(neo-Aristotelian) realism*. A *realist* about a theory T is someone who (a) believes that T is true, and has determinate truth-values independently of whether we are in a position to verify those truth-values, and (b) believes that T describes some features of reality, and that therefore the features that T describes 'really exist'. For example, suppose T contains arithmetic. Then the realist believes that arithmetic has truths, that these truths are true anyway (independently of our coming to know them), and that the subject-matter of arithmetic 'really exists'.[5] Thus far (a) and (b) describe commitments that any realist shares. A *neo-Aristotelian realist* is someone who adds to commitments (a) and (b) some distinctive views about the nature of mathematical existence. Neo-Aristotelians hold that (c) basic mathematical patterns and universals are instantiated in nature (whether they can be exactly perceived or not), and that in the case of huge structures that may exceed what's found in nature, such structures *could be* instantiated even if they aren't (see Franklin 2009). David Armstrong's position on mathematical universals qualifies as neo-Aristotelian (Armstrong 1997; 2004: c.9). By contrast, Platonist

[5] Of course, it is a further matter to specify what the subject matter of arithmetic is. Some would say it is the structure of the natural numbers as described by the Dedekind and Peano axioms.

realists reject (c) on the grounds that mathematical universals are not perfectly instantiated in nature.[6]

For our part, we think many (perhaps not all) of the difficulties with the indispensability argument can be traced back to Quine's philosophy. His criterion is anti-Aristotelian because 'value of a bound variable'—especially with the emphasis on first-order logic—is intended to be read so that the values can only be *particulars*. Nominalism and Platonism share a commitment to the thesis that all entities are particulars (the Platonist admitting abstract particulars, the nominalist not). Aristotelianism denies that. We will clarify later in the paper Quine's allowing quantification to range over particulars but not properties.

Quine's criterion of ontological commitment—'to be is to be the value of a variable'—is part of the standard indispensability argument. We think Quine gets the ontology of mathematics wrong in several respects, all of which can be traced back to his application of his criterion of ontological commitment. First, Quine attempts to fit theories into the procrustean bed of first-order logic. Thus at a single stroke he excludes an ontological commitment to properties. Second, his criterion of ontological commitment is geared up to an atomist metaphysics, emphasizing individuals rather than states of affairs (facts), and complexes of individuals related to one another.

We propose an alternative to this atomist metaphysics, using what we might call Armstrong's new criterion of ontological commitment, 'to be is to be a truth-maker, or a component of a truthmaker'.[7] It is then possible to run a new indispensability argument with a different outcome. Of course, much depends again on what the truthmakers are. We follow Armstrong in supposing that the basic items in reality are *facts* as well as relations and properties. Arguably, this less atomistic and more relational approach is a better fit with the attractive view that mathematics is about patterns rather than objects. Whether one agrees with the resulting view or not, it demonstrates the possibility of a non-Quinean indispensability argument.

Section 1 below explains the involvement of Quine's criterion in traditional indispensability arguments. Section 2 puts forward Armstrong's alternative proposal for ontological commitment. It explains Armstrong's complaint that Quine is biased against properties in his criterion of ontological commitment. Section 3 presents a new indispensability argument that uses Armstrong's criterion of ontological commitment. Section 4 concludes that the new indispensability argument is better than the old one.

[6]On the issue of perfect versus imperfect instantiation, see Pettigrew (2009).

[7]Thanks to Jonathan Schaffer for the phrase. To call Armstrong's suggestion 'a criterion' is perhaps to sharpen it beyond what Armstrong had in mind. However, we let it stand for the sake of parity in discussing Quine and Armstrong on ontology.

2 The standard indispensability argument and its reliance on Quine's criterion of ontological commitment (OC)

We are concerned not so much with Quine exegesis as the indispensability argument as it has come to be known in wider philosophy of mathematics circles. Colyvan (2001: 11) provides a general outline of the key indispensability argument:

> (1) We ought to have ontological commitment to all and only the entities that are indispensable to our best scientific theories.
> (2) Mathematical entities are indispensable to our best scientific theories.
> (3) We ought to have ontological commitment to mathematical entities.

Ontological commitment figures twice in the argument, once in premise (1) and once in the conclusion (3). However, we are not told how to determine the ontological commitments of a theory. Colyvan refers to premise (1) as Quine's *ontic thesis* as opposed to Quine's actual thesis of *ontological commitment.* The idea is that (1) can serve as a general and normative premise about what considerations govern our ontological commitments without providing a recipe, 'a criterion', for ontological commitment. It is clear, though, that the Putnam–Quine version of the argument specifically invokes Quine's criterion of ontological commitment (OC). This is explicit in Putnam's version (1971: 57):

> So far I have been developing an argument for realism roughly along the following lines: quantification over mathematical entities is indispensable for science, both formal and physical: therefore we should accept such quantification; but this commits us to the existence of the mathematical entities in question. This type of argument stems of course from Quine, who has for years stressed the indispensability of quantification over mathematical entities and the intellectual dishonesty of denying the existence of what one daily presupposes.

We shall focus our discussion explicitly on this *quantificational form* of the indispensability argument. It may well be that there is a better form of the argument that is not so dependent on Quine's criterion of ontological commitment. Be that as it may, in this form of the argument, Quine's criterion of ontological commitment (OC) is used to explain the meaning of 'indispensability' in the original argument. The entities that are indispensable are just those that are in the domain quantified over by the canonical statement of our best theory.

In practice, however, we still know very little about our ontological commitments until we identify a specific theory and its language. Most theories in physics make use of functions on the real numbers and thus incorporate the mathematical theory of real analysis. The very notion of measurement involves mapping a quantitative property (heat, weight, mass, length, charge etc.) onto a real number. For example, we measure an inchworm and learn that it is approximately 3.5 cm. In practice, we can measure quantities by just rounding off decimals and reporting quantities as rational numbers. However, if we suppose that there are no gaps in our field of numbers and no limit to the exactness of measurement, we end up with something like the real number structure (as captured by the axioms of real analysis). The real number structure holds out the *ideal* of infinite precision.[8]

Moreover, it looks to be the case that real analysis (or some structural surrogate of it) cannot be dispensed with in our physics. If this is disputed, consider the fact that Field's attempt in *Science without Numbers* (1980) to eliminate reference to the real numbers from Newtonian mechanics simply ends up imposing the structure of the real numbers on a collection of spacetime points. Field finds this outcome acceptable as a nominalist because he urges that spacetime points are concrete entities, not abstract. But he admits he would not attempt to pursue physics finitistically. From a structuralist point of view, though, the real number structure *is* instantiated in Field's collection of spacetime points. That means that the real numbers have not really been eliminated from physics. Rather, we should think of the real numbers as a certain structure that exists physically (or could exist) rather than conceiving of them as the referents of linguistic terms that could be eliminated from the language of our scientific theory.[9]

So it is reasonable to suppose that Quine's criterion of ontological commitment applied to contemporary physics commits us to the existence of real numbers and functions on real numbers.[10] Thus, we can consider a more topic-specific version of the indispensability argument. Stewart Shapiro (2000: 228) presents one such version:

> (1a) Real analysis refers to, and has variables range over, abstract objects called 'real numbers'. Moreover, one who accepts the truth

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[8] Is it just that—an *ideal*? Maybe. It must be admitted that realism about the real numbers is harder than realism about rational numbers and natural numbers. One of the reasons for this is our measurements are never infinitely exact. For some considerations in favour of classical realism, see Newstead and Franklin (2008), and Newstead (2001).

[9] For criticism of Field on this point, see especially Resnik (1985).

[10] We ignore for a moment the real tension between the view of space-time as continuous that we find in Newtonian mechanics and GTR with the view in Quantum Mechanics that space-time is quantised. The natural way to interpret the real numbers physically is as points in a space-time manifold. QM raises doubts about whether we should preserve this physical interpretation of the real numbers. Schrödinger himself thought that the idea of a continuum was exposed by QM as a myth. It's fair to say that the jury is still out, but that Schrödinger's view has the most support among physicists.

of the axioms of real analysis is committed to the existence of these abstract entities.
(2a) Real analysis is indispensable for physics. That is, modern physics can be neither formulated nor practised without statements of real analysis.
(3a) If real analysis is indispensable for physics, then one who accepts physics as true of material reality is thereby committed to the truth of real analysis.
(4a) Physics is true, or nearly true.

The desired conclusion is:

(5a) Abstract entities called 'real numbers' exist.

Shapiro's version of the indispensability argument urges that in accepting physics as true, we are thereby ontologically committed to the real numbers. Of course, many of those who are unmoved by indispensability arguments don't really believe in the *truth*—in some heavy sense—of scientific theory in the first place. To be sure, one need not be committed to the *exact* truth of the laws of physics. The laws are idealizations which the physical phenomena approximate. Still, insofar as physical phenomena conform to the laws approximately, the laws are true 'nearly enough'.

If the truth of the scientific theory is accepted, then it becomes a straightforward matter to see why one would assume an ontological commitment in accepting the theory as true. On many substantive theories of truth, truths carry ontological commitments with them. For this very reason, some theorists view the indispensability argument as begging the question against fictionalism and instrumentalism. Savvy fictionalists (such as Leng 2005a) simply don't grant the substantive truth of scientific theories and explanations. This effectively blocks the inference to the reality of the items postulated by scientific theories. However, indispensability arguments target those who are *already* scientific realists, and thus would accept the truth ('near enough') of scientific theory. The point of the original indispensability argument was to show that scientific realists should not exempt mathematics from their realism.

Several other features of Shapiro's version of the argument deserve comment. Plainly, the abstractness of the entities in the conclusion is a result of the abstractness having been input in the first premises—by a sleight of hand, Shapiro builds into premise (1a) a conception of the real numbers as 'abstract entities', where presumably these real numbers are to be understood as non-spatiotemporal entities. This metaphysical conception of the real numbers is actually extraneous to the main argument. The vulgar conception of abstract objects is that they exist outside of space-time as Platonic universals. However, there is no need to hold a Platonist view about mathematical objects in order to maintain the indispensability argument. According to our view, known as 'neo-Aristotelian realism', we hold that universals are instantiated in nature, in our actual physical world. To be sure, if the physical universe is not infinitely large in extent or in the number of particles of a certain type that it contains, then some infinite structures will be universals that are merely possibly instantiated rather than concretely, actually instantiated in the physical universe. Even so it helps the epistemology of mathematics tremendously if we can count on there being a basic stock of mathematical universals that are exemplified in the world. For if a basic stock of mathematical universals is instantiated then basic knowledge of the universals can be gained through active perception and imagination.[11]

It is sometimes thought to be fatal to the Aristotelian philosophy of mathematics that certain mathematical forms (such as a square) are not visibly perfectly instantiated in nature. However, we note that there is a way around this problem. For example, it may be that although our perceptual experience does not always present a perfect square, our perceptual experience suffices to trigger in us the category specification of a perfect geometrical square. Thus, our perceptual experience can stimulate formation of exact mathematical concepts (Giaquinto 2007: 28; see also Newstead and Franklin 2010). Our perception is not fine-grained enough to allow us to discriminate between a perfect square and a very slightly imperfect square. The perception of a very slightly imperfect square is enough to induce in us the concept of a perfect square. This concept can then be used to form mathematical beliefs that are reliably related to perceptions of mathematical patterns.

There is thus no reason why a proponent of indispensability arguments for realism must accept, without arguments, the presuppositions of Platonist realism. Indeed, indispensability arguments are silent on the question of which variety of realism holds.[12] The metaphysical views that one extracts from indispensability arguments will be largely a function of the metaphysical views that one injects into such arguments. One primary place for the injection of metaphysics is in the specification of a criterion of ontological commitment; another place is in the selection of a canonical form for expressing the theory.

It is surprising, then, that Quine's criterion of ontological commitment has not been much criticized in the context of his indispensability argument. One recent exception is Azzouni (2004) who argues in favour of 'the separation thesis': we can accept scientific theories as true without being ontologically committed to the entities in the domain of quantification of the theory. Azzouni, therefore, rejects Quine's criterion and uses it to reject the indispensability argument. We also reject Quine's criterion of ontological commitment. We show, however, that we can recast the indispensability argument and perhaps inject new life into the argument by using a different approach to meta-ontology.

There is the starkest possible contrast between the separation thesis and the truthmaker approach to ontology. Truthmaker theorists believe that truth is inseparable from being to this extent: the truth of statements depends on

[11] For our position, see Franklin (2009).

[12] Indeed, even Platonists have agreed. See Colyvan (2001: 142).

being (on what there is in the world). If one accepts T as true, one is *ipso facto* committed to the existence of truthmakers for T.[13]

3 Armstrong's Alternative to Quine on Ontological Commitment

David Armstrong has given us two promising alternatives to Quine's criterion of ontological commitment. First, he has suggested that our criterion for the reality of an object obeys the *Eleatic Principle* (EP): everything that is real makes some causal difference to how the world is.[14] EP comes in handy in the battle against the Platonist's commitment to abstract objects, which are notoriously causally inert and thus (it seems, on most theories of knowledge) unknowable, mysterious, and inexplicable. However, there does appear to be difficulty in defending EP as a criterion of reality for some mathematical objects. The curvature of space-time is used to explain the behaviour of objects in general relativity, but the geometrical properties of space-time are not obviously causal powers.[15] Realists want to affirm the reality of these geometrical properties. Obviously in response to this kind of example it needs to be made clear that EP cannot simply be the slogan 'everything that exists is itself a causal power'. However, as we are inclined to adopt a neo-Aristotelian outlook in philosophy of mathematics in any case, we are glad to interpret EP in a different way than this simple slogan suggests. Neo-Aristotelians can hold that if EP holds true of mathematics, it has to do so in some way that acknowledges the difference between efficient causality and what we might call 'formal causality'. No one finds it plausible to say that mathematical quantities are efficiently causally efficacious, for example, in the same way that a billiard ball's motion of striking another billiard ball is efficiently causally efficacious. Nonetheless, perhaps mathematical quantities and patterns are causally implicated in the world in some other sense: they are part of a *formal* causal explanation of the world. For example, had the constants of nature been different, then objects in the world would behave differently. If G's value were different, then objects would not attract one another with the same gravitational force that they do. Although the notion of formal causality might seem opaque, it is at least strong enough to support counterfactual claims. Thus, if x is formally causally implicated in W, then the following counterfactual holds:

[13]There are a variety of views held by truthmaker theorists on the relation between truths and truthmakers. Various proposals for the relation include: supervenience, necessitation, and grounding. For a critical survey, see Schaffer (2008). Lewis (2001) advocated viewing the relation as supervenience, while Armstrong (2004) views the relation as one of necessitation.

[14]The *locus classicus* for EP are the remarks of the Eleatic stranger in Plato's *Sophist* 247e. Reference to EP in contemporary discussions originates with Oddie (1982). In Armstrong's work, see Armstrong (1997: 41) and for the 'truthmaker version', see Armstrong (2004: 7), 'every truthmaker should make some contribution to the causal order of the actual world'.

[15]See Colyvan (2001: c.3) for objections to EP.

had *x* not obtained, then some event *e* in W would not obtain either. Mathematical quantities are clearly formally causally operative in a counter-factually sustaining way.[16] For example, if the peg had not been square, it would have fit in the round hole. If we peer this far down the road to interpreting EP, we see that the disagreement between Platonists and others over EP gives way to a debate about how to understand causal-explanatory relations.

To be sure, one can both accept Quine's criterion of ontological commitment and EP, but in practice it seems better to have EP supplant Quine's criterion with EP altogether. The tendency of Quine's criterion is to allow into our ontology every individual over which our theories range, whereas the tendency of EP is to restrict our ontological commitment to some smaller class of entities that are the real players in our theory. We cannot enter into this debate fully here, but record it as yet another approach to doing ontology that provides a distinct realist alternative to Quine's criterion of ontological commitment.

The second alternative to Quine's criterion of ontological commitment derives from the theory of truthmaking.[17] According to the theory of truthmaking, every truth has a truthmaker, where this truthmaker is some entity in the world in virtue of which the truth is true. On Armstrong's particular metaphysics, it is indeed the case that *every* truth has a truthmaker (truthmaker maximalism), and further the case that the main truthmakers are *facts* or *states of affairs*. The key intuition is that truth is grounded in reality. In the absence of truthmakers for a given truth, the truth would 'float free' of how the world is. Such 'free floating' truths strike truthmaker theorists as unacceptable.

The truthmaker approach to metaphysics is certainly appealing to realists, but doesn't suppose a particular form of realist metaphysics.[18] Someone with a basic ontology of things (rather than facts) could allow that X was a truthmaker for each truth of the form 'X exists', where X names some concrete particular (as Armstrong notes, 2004: 24). In such a world of things, the fundamental truths would all have the form 'X exists'.

Nonetheless, it is of course true that the truthmaker principle does exact some commitment to realism about the truth-values of propositions/statements. The truthmaker theory does assume a kind of bland, minimal realism about truthmakers. Truthmaker theory states that for every (basic) truth, there is some truthmaker in the world. As these truthmakers enjoy a mind-independent existence, it follows that truthmaker theory is realist about the existence of truthmakers. The key point, however, is that truthmaker

[16] For an outline of how to pursue such an approach, readers might consult Bigelow and Pargetter (1990: c.8). Recently, Aidan Lyon (forthcoming) suggests that mathematical items are part of a 'programming' explanation of how things work; that is, part of the high level description that explains why we see the transitions between given inputs and outputs that we see. This suggestion may be a more contemporary way of phrasing the Aristotelian claim about mathematical patterns and quantities being formally causally explanatory of the world.

[17] See Armstrong (2004) for a basic exposition.

[18] We have been helped by reading Cameron (2008).

theory does not identify the truthmakers for us. There is no automatic way to move from a statement to identification of the truthmaker for that statement. In particular, no amount of analysis of the logical form of a statement—without doing some serious metaphysics—is going to tell us what the truthmakers are. Russell's logical atomism made this mistake, and Armstrong (2004: 23) does not repeat it.

> The truthmaker *method* suggests, then, a very general way of doing metaphysics:
>
> 'To postulate certain truthmakers for certain truths is to admit those truthmakers into one's ontology. The complete range of truthmakers admitted constitutes a metaphysics ...'

Armstrong emphasizes that the hunt for truthmakers is as hard an enterprise as doing metaphysics itself or science. Our ontological commitments will depend on our having identified a true theory of nature. Given a disdain for purely armchair science and metaphysics, this theory of nature will be determined *a posteriori*. For example, if it should turn out that everything is made out of sub-atomic particles such as quarks and gluons, then perhaps the truthmakers for certain statements about the physical world such as 'There's a table' will be complex facts about how sub-atomic particles are arranged in a certain space. That means to a certain extent that the contemporary metaphysician must wait on science. According to Armstrong's *a posteriori realism*, science will discover and identify the basic universals.[19] At best, the metaphysician can hazard a guess about the general structure of the truthmakers that will satisfy our best scientific theories.

To remain faithful to his *a posteriori realism*, Armstrong warns that truthmaker theory is only 'a promising way to regiment metaphysics ... not a royal road' (Armstrong 2004: 22). Nonetheless, it is tempting to harden his theory into a criterion for ontological commitment. The slogan for ontological commitment on Armstrong's theory is therefore 'to be is to be a truthmaker (or part of one) for a true theory'.[20] We have borrowed this slogan from Schaffer (2009) and amended it by adding 'or part of one'.

How will our ontological commitments differ from those of a Quinean, supposing that both followers of Armstrong and Quine are assessing the same scientific theory? In particular, how will our mathematical ontology differ? We contend that following Armstrong's 'truthmaker' approach will result in a richer mathematical ontology that includes *properties, relations,* and *facts.*

Consider the statements:

[19] The term 'a posteriori realism' is used by Mumford (2007) to describe Armstrong's position.

[20] Adapted from Schaffer (2009). We would prefer 'to be is to be the value of a truthmaker or *one of its components* thereof'. Consider the statement 'This square is red'. The property of being *red* is one of the components of the fact (this square's being red) that makes the statement true. On Armstrong's view, the main metaphysical commitment is to the fact or state of affairs of *this-square's-being-red*. However, the primacy of facts doesn't undermine the real existence of its components, *this square* and the property of *being red* (which is partly instantiated in this square)).

(1) F*a*
(2) ∃x (Fx)

Quine thinks (2) makes plain the ontological commitment of the simple statement (1). If one accepts 'F*a*' as true, then one's ontological commitment amounts to this: *there is something* that is F. One's ontological commitment is to some particular with some property called 'F'—but not to some property F instantiated in some particular a: one need not view 'F' as naming a universal property, and one need not adopt a realist view of properties. If one likes one can read (2) in a functionalist manner as saying that *there is something that plays the role of being F*. If one were further committed to the reality of roles (on the grounds of the theory's being 'heavily' true in some realist way), then re-iterating the Quinean procedure would suggest one also accept:

(3) ∃x ∃F (Fx)

In (3) the commitment to the existence of an object and a property is made explicit. But Quineans do not think that (1) and (2) imply (3), because one might accept (1) or (2) as true, without being committed to the separate 'existence' (as asserted by the existential predicate) of F. This raises the spectre that one might accept the truth of a statement '*a* is F' while being deflationary in metaphysical terms about what this truth requires. Fiction is one area where we are used to this phenomenon. For example, 'Santa Claus has a beard' is true at least in the context of the Santa Claus story, but there is no bearded individual in the world that makes this statement true. However, in lieu of an argument for treating the statements of our scientific theories as fiction, the Quinean needs good reasons to block the move from (2) to (3). It seems that only a bias against second-order logic blocks the move. The bias against second-order logic, though, is mainly grounded in a distrust of properties as obscure entities lacking clear-cut individuation criteria.

Aristotelian realists such as Armstrong and his defenders argue that one needs the property F, the particular *a*, and also the fact of *a's being F*, to exist in order to make (1) true. According to (1), there is some particular that is F. This *something* cannot be a bare particular; it must have properties too. If 'F*a*' is true, then there is *something* that has the property called 'F'. In accepting (1) one is committed to there being *something* (called '*a*') possessing *some property* (called 'F').[21]

Armstrong for his part has long viewed Quine as guilty of 'ostrich nominalism': Quine thinks he can accept the truth of a statements such as '*a* is F' ('That house is red') and '*b* is F' ('That sunset is red') but not incur any ontological commitment to the property of being F (red) (Armstrong 1978: 16).

[21] But why stop here? The particular *a* and the property *F* must be related somehow, since '*a* is F' asserts that *a* has F-ness, not just the existence of *a* and *F* unrelated. Armstrong proposes we take the state of affairs (or fact) of *a's being F* as the truthmaker for '*a* is F'. One may also point out that one is committed to the components of the fact of *a's being F* which are the individual *a* and the property *F*, since facts supervene on their components.

Quine refrains from analysing '*a* is F' in such a way that it implies that there is a property of F-ness. However, in doing so Quine is left without the resources to explain in what respect individuals *a* and *b* resemble each other (as regards colour). The ancient 'one over many' argument posits universals (shared properties) as an answer to such puzzles. There are thus legitimate arguments for universals that go unanswered by Quine (e.g. Armstrong 1978: c.6; Armstrong 1997: c.3.). It will not do simply to dismiss the reality of universals (properties) by logical fiat.

As one might expect, Quine's analysis of the truth '*a* is F' offers a desert landscape: an ontological commitment to the lone individual called '*a*' that might satisfy the open sentence '_ is F'. Quine lacks the knowledge of Australians that deserts are not barren, but teeming with life. Armstrong's Australian picture of the matter is a dense, fertile landscape. The metaphysics required for the truth of '*a* is F' include an object *a*, its property *F*, and *the fact* that *a is F*.

4 The Indispensability Argument Revised

How now does the indispensability argument look if we run it using Armstrong's approach to ontological commitment? As we saw in the previous section, Armstrong's approach contains several components:

> (a) Truthmaker theory (which includes at a minimum the claim that every truth has a truthmaker together with some account of the truth-making relation).
> (b) Armstrong's own particular metaphysics, which identifies facts (states of affairs) as the main truthmakers, allowing for components of those facts (properties, relations, objects) as real existents.

We are going to apply (a) and a rather loose interpretation of (b) to the indispensability argument we considered earlier. In doing so—as is typical of the approach to metaphysics by hunting down truthmakers—we have to identify the particular truthmakers for a set of truths by examining those truths themselves and the practice in which they are found.

The old indispensability argument (1a–5a) claims that the truths of real analysis are indispensable to physics. We think the argument is correct in finding real analysis to be indispensable for physics. So, assuming that real analysis is indispensable to physics, we need to identify the truthmakers of real analysis. It is here that we go beyond truthmaker theory to offer a particular metaphysical claim about the nature of mathematical truthmakers. Our speculation is in keeping with Armstrong's metaphysics, although it is not specifically his view. Our view is that one of the main truthmakers for real analysis is the standard real number structure as found in any real number continuum. It is this structure which is described by the axioms of real analysis. These axioms include claims such as:

> (Btw) Between any two real numbers x and y, there is another real number z.
> (UB) Any set of real numbers with an upper bound has a least upper bound.
> (Archimedean Property) For any positive numbers x and y where $x < y$, there is some natural number n such that $nx > y$.

In addition to the continuum, real analysis also makes claims about functions and their properties such as differentiability, continuity, and integrability. So perhaps these properties should be taken as components of the facts that are the truthmakers for classical real analysis.

How does the indispensability argument look if we run it using Armstrong's criterion of ontological commitment? Remember that since Armstrong's 'truthmaker' criterion of ontological commitment is formal, we will need to supplement it with our preferred identification of the truthmakers of analysis. Here's how the revised argument looks:

> (1) The statements of real analysis concern truths about the real number continuum, both its subsets (sequences of the real numbers), the properties of those subsets (e.g. convergence) and all the functions that can be defined on subsets of the real number continuum, along with the properties of those functions (e.g. differentiability, smoothness etc.).
> (2) The truthmakers for statements in real analysis include sequences of real numbers and functions with the relevant properties. One who accepts the truths of the axioms of real analysis is committed to the existence of these mathematical entities. (Note that as usual reference to the real numbers is not to abstract entities called 'the real numbers' but to a structure, the real number continuum, that could be realised in space.)

The rest of the argument is unchanged:

> (3) Real analysis is indispensable for physics. That is, modern physics can be neither formulated nor practised without statements of real analysis.
> (4) If real analysis is indispensable for physics, then one who accepts physics as true of material reality is thereby committed to the truth of real analysis.
> (5) Physics is true, or nearly true.

The immediate conclusion of the argument is that we are committed to the existence of the truthmakers of real analysis. These truthmakers have been identified in step (2) of the argument as the sequences and functions of real numbers with the properties studied in real analysis (such as convergence, differentiability etc.). So the final conclusion is:

> (6) We are committed to the truthmakers of real analysis. These include (perhaps) the real number structure, real-valued functions, and the properties of real numbers and real-valued functions.

We stress that the conclusion is contingent on our having the correct identification of the truthmakers of real analysis. Moreover, identification of such truthmakers is a matter for those thinking about the metaphysics of mathematics. In doing so, one should bear in mind how the mathematics is being applied. However, we cannot expect an indispensability argument to tell us straight out what those truthmakers are.

5 One Problem with the New Indispensability Argument

We need to deal with problems that arise for our version of the indispensability argument, specifically from the fact that the mathematical theory under scrutiny is *real analysis*. While no one doubts that the ontology of classical real analysis includes an uncountably infinite real-number continuum, there are legitimate questions about the relation of the mathematical continuum to the structure of space-time. Whether space-time has a continuum-structure or a grainy structure is an empirical question. Thus far the evidence is equivocal, but leans towards suggesting the structure of space-time is grainy and not continuous (Wolfram 2002).

There are two possible solutions. Aristotle's own solution was to hold that the points of a continuum do not actually exist all at once. Rather, a point comes into being when we undertake an activity, such as dividing a line. Prior to such activity on the part of the mathematician, the point exists only potentially as the boundary of a line segment. The upshot of this view is that the truthmaker for many statements of real analysis could be a merely *possible* mathematical continuum. There is no need to be wedded to the view that there is a (physical) continuum in space-time.

Another possible solution is to revise our notion of which part of mathematics is indispensable for physics. Maybe real analysis is not indispensable, but some weaker form of real analysis is. Perhaps an exact mathematical description of the physical universe does not involve real analysis with its commitment to infinite divisibility. Instead the appropriate mathematics would be discrete analysis in which, for example, limits as Δx tends to 0 are replaced by ersatz limits as Δx tends to $\hbar$ (the size of an atom of space or time). Discrete analysis is mathematically legitimate, however, cumbersome (Zeilberger 2004). The main philosophical point, however, is that its ontological commitments are to the same kind of entities as real analysis: (discrete) functions which possess properties such as ersatz convergence and differentiability. The indispensability argument goes through with these entities and properties rather than the conventional ones. A kind of mathematical realism is still vindicated.

6 Conclusion

It is clear that running the indispensability argument with Armstrong's total approach to ontology results in a *qualitatively* richer ontology than the one offered by Quine. The mathematics that proves indispensable includes not just sets, but mathematical properties and facts about these properties and relations. But Quine's mathematical ontology is *quantitatively richer*: it allows unlimited numbers of classes. As Aristotelian realists, we would prefer to posit no more structures than we absolutely need to do the applied science: the rest might be uninstantiated structures of the sort posited by Platonism. We still think it's a gain to have one's basic structures be natural structures, however. In this way knowledge of such structures becomes less mysterious than knowledge of Platonic forms.

Our modest aim has been to delineate a possible position in logical space: realism about mathematics without Platonism, but motivated (in part) by indispensability considerations. We have shown that indispensability arguments can be run free of Quinean ontological baggage, such as Quine's criterion of ontological commitment. In its place we have suggested that the truthmaker approach to ontology might be preferable. We have tried to explain what such a view might look like, although in completing this task we needed to come up with our own preferred metaphysics of mathematics: Aristotelian realism (Franklin 2009).

We now pause to consider the peculiarity of our procedure. We have invoked truthmaker theory in our indispensability argument. But the indispensability argument is supposed to be an argument for realism on independent grounds—it shouldn't assume realism about mathematics. Doesn't insisting that the truths of mathematics have truthmakers assume realism about mathematics? We answer that it does assume semantic value realism (the truths of mathematics—guess what?!—have truth-values) but it does not assume a particular form of metaphysical realism. Truthmaker theory is itself agnostic about the identity of truthmakers for a particular theory, such as real analysis in mathematics. We have our favourite view of the existence of these truthmakers as Aristotelian realists. But our Aristotelian realism is a commitment beyond truthmaker theory, and not one that we expect everyone to share. Given our modest aim of establishing the viability of an alternative to Quine's Platonist indispensability argument, it would still be consistent with the letter of our position if all indispensability arguments were to be shown to reach the conclusion of realism by assuming realism at the outset. We don't think this would be a desirable outcome, but it is a possibility. Valid arguments can be question-begging, of course. To avoid begging the question we would want to have reasons independent of realism about mathematics for thinking that truthmaking was a good approach to determining the ontology of science.

We think that indispensability arguments provide compelling reasons to be realist, but not to be Platonist. The standard Quine-Putnam version of the argument relies on Quine's quantificational criterion of ontological commitment. It also imports a specifically Platonist version of realism in its suggestion that numbers and sets are 'abstract objects' (conceived of as existing outside of space and time). These metaphysical biases are not essential to the indispensability argument.

We suggest that another version of indispensability is preferable. We have suggested that we replace Quine's criterion with Armstrong's truthmaker criterion: 'to be is to be a truthmaker, or part of one, for a true theory'. We then tried to apply Armstrong's truthmaker approach to determine the ontological commitments of mathematical theories taking the theory of real analysis as our case study. We suggested that application of truthmaker suggests a mathematical ontology in which the fundamental items of mathematics are not lone objects, but patterns, properties, functions, facts, and relations. Such a qualitatively multifarious ontology—an Armstrongian bush, not a Quinean desert—might have advantages when it comes to maintaining a naturalistic epistemology.