The Material Origin of Numbers

Insights from the Archaeology of the Ancient Near East

By Karenleigh A. Overmann
The material origin of numbers
Gorgias Studies in the Ancient Near East

14

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This series publishes scholarly research focusing on the societies, material cultures, technologies, religions, and languages that emerged from Mesopotamia, Egypt, and the Levant. Gorgias Studies in the Ancient Near East features studies with both humanistic and social scientific approaches.
The material origin of numbers

Insights from the archaeology of the Ancient Near East

Karenleigh A. Overmann
Cover. Mathematical tablet (HS 201) from Nippur. It dates somewhere between the Ur III (2100–2000 BCE) and Early Old Babylonian (2000–1900 BCE) periods and is part of the Frau Professor Hilprecht Collection, University of Jena, Germany. The table is a list of reciprocals, an excellent example of the relational data transforming the concept of number. In 1935, Otto Neugebauer read the last line of the right column as 32 being the reciprocal of 112.5 (‘32 igi 1,52,30’, Mathematische Keilschrift-Texte, p. 10). The photo was produced by Manfred Krebernik and his assistant, and use of the image is courtesy of the Frau Professor Hilprecht Collection, University of Jena, Germany.

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To Bill: My first, my last, my always, and everything in between.
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## Conventions

### Timeline

<table>
<thead>
<tr>
<th>Period</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Late Upper Paleolithic</td>
<td>30 to 12 thousand years ago</td>
</tr>
<tr>
<td>Epipaleolithic</td>
<td>12,000 years before present to 8300 BCE</td>
</tr>
<tr>
<td>Neolithic</td>
<td>8300–4500 BCE</td>
</tr>
<tr>
<td>Chalcolithic (Copper Age)</td>
<td>4500–3300 BCE</td>
</tr>
<tr>
<td>Pre-Uruk V</td>
<td>8500–3500 BCE</td>
</tr>
<tr>
<td>Uruk V</td>
<td>3500–3350 BCE</td>
</tr>
<tr>
<td>Uruk IV</td>
<td>3350–3200 BCE</td>
</tr>
<tr>
<td>Early Bronze Age</td>
<td>3300–2000 BCE</td>
</tr>
<tr>
<td>Uruk III</td>
<td>3200–3000 BCE</td>
</tr>
<tr>
<td>Jemdet Nasr (JN)</td>
<td>3200–3000 BCE</td>
</tr>
<tr>
<td>Early Dynastic (ED) I/II</td>
<td>2900–2700 BCE</td>
</tr>
<tr>
<td>Early Dynastic (ED) IIIa</td>
<td>2600–2500 BCE</td>
</tr>
<tr>
<td>Early Dynastic (ED) IIIb</td>
<td>2500–2340 BCE</td>
</tr>
<tr>
<td>Old Akkadian (OA)</td>
<td>2340–2200 BCE</td>
</tr>
<tr>
<td>Lagaš II</td>
<td>2200–2100 BCE</td>
</tr>
<tr>
<td>Ur III</td>
<td>2100–2000 BCE</td>
</tr>
<tr>
<td>Early Old Babylonian</td>
<td>2000–1900 BCE</td>
</tr>
<tr>
<td>Old Assyrian (Old Assyr.)</td>
<td>1950–1850 BCE</td>
</tr>
<tr>
<td>Old Babylonian (OB)</td>
<td>1900–1600 BCE</td>
</tr>
<tr>
<td>Middle Assyrian (MA)</td>
<td>1400–1000 BCE</td>
</tr>
<tr>
<td>Neo-Assyrian (NA)</td>
<td>911–612 BCE</td>
</tr>
<tr>
<td>Neo-Babylonian (NB)</td>
<td>626–539 BCE</td>
</tr>
</tbody>
</table>

### Abbreviations for Museum Designators

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Museum Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Oriental Institute, University of Chicago, Illinois, USA</td>
</tr>
<tr>
<td>AO</td>
<td>Louvre Museum, Paris, France</td>
</tr>
<tr>
<td>Ashm</td>
<td>Ashmolean Museum, Oxford, England</td>
</tr>
<tr>
<td>CUNES</td>
<td>Cornell University, Ithaca, New York, USA</td>
</tr>
<tr>
<td>DV</td>
<td>State Hermitage Museum, St. Petersburg, Russian Federation</td>
</tr>
<tr>
<td>IM</td>
<td>Iraq Museum, Baghdad, Iraq</td>
</tr>
<tr>
<td>JA</td>
<td>National Museum of Syria, Raqqa, Syria</td>
</tr>
<tr>
<td>JRL</td>
<td>University of Manchester, England</td>
</tr>
<tr>
<td>MS, MT, MW</td>
<td>Private, anonymous collections in Europe</td>
</tr>
<tr>
<td>Code</td>
<td>Description</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>NIM</td>
<td>National Museum, Tehran, Iran</td>
</tr>
<tr>
<td>NMSDeZ</td>
<td>National Museum of Syria, Der-ez-Zor, Syria</td>
</tr>
<tr>
<td>NMSR</td>
<td>National Museum of Syria, Raqqa, Syria</td>
</tr>
<tr>
<td>OIM A</td>
<td>Oriental Institute, University of Chicago, Illinois, USA</td>
</tr>
<tr>
<td>Sb</td>
<td>Louvre Museum, Paris, France</td>
</tr>
<tr>
<td>T</td>
<td>National Museum of Syria, Damascus, Syria</td>
</tr>
<tr>
<td>UM</td>
<td>University of Pennsylvania Museum of Archaeology and Anthropology, Philadelphia, Pennsylvania, USA</td>
</tr>
<tr>
<td>VA, VAT</td>
<td>Vorderasiatisches Museum, Berlin, Germany</td>
</tr>
<tr>
<td>W</td>
<td>Artifact found at the ancient city of Uruk in Iraq</td>
</tr>
</tbody>
</table>
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