



GORGAS STUDIES IN THE ANCIENT NEAR EAST

14

The Material Origin of Numbers

Insights from the Archaeology of the
Ancient Near East



By Karenleigh A. Overmann



GORGAS ❖ PRESS

The material origin of numbers



Gorgias Studies in the Ancient Near East

14

Series Editorial Board

Ronald Wallenfels

Paul Collins

Aidan Dodson

Alhena Gadotti

Kay Kohlmeyer

Adam Miglio

Beate Pongratz-Leisten

Seth Richardson

This series publishes scholarly research focusing on the societies, material cultures, technologies, religions, and languages that emerged from Mesopotamia, Egypt, and the Levant. *Gorgias Studies in the Ancient Near East* features studies with both humanistic and social scientific approaches.

The material origin of numbers

Insights from the archaeology of the Ancient Near East

Karenleigh A. Overmann

Gorgias Press LLC, 954 River Road, Piscataway, NJ, 08854, USA

www.gorgiaspress.com

Copyright © 2019 by Gorgias Press LLC

All rights reserved under International and Pan-American Copyright Conventions. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise without the prior written permission of Gorgias Press LLC.

2019



ISBN 978-1-4632-0743-4

Cover. Mathematical tablet (HS 201) from Nippur. It dates somewhere between the Ur III (2100–2000 BCE) and Early Old Babylonian (2000–1900 BCE) periods and is part of the Frau Professor Hilprecht Collection, University of Jena, Germany. The table is a list of reciprocals, an excellent example of the relational data transforming the concept of number. In 1935, Otto Neugebauer read the last line of the right column as 32 being the reciprocal of 112.5 ('32 igi 1,52,30', *Mathematische Keilschrift-Texte*, p. 10). The photo was produced by Manfred Krebernik and his assistant, and use of the image is courtesy of the Frau Professor Hilprecht Collection, University of Jena, Germany.

A Cataloging-in-Publication Record is available from the Library of Congress.

Printed in the United States of America

To Bill: My first, my last, my always, and everything in between.

TABLE OF CONTENTS

Table of Contents	v
Figures and Tables	vii
Figures	vii
Tables	viii
Conventions	ix
Timeline	ix
Abbreviations for museum designators	ix
Acknowledgements	xi
Chapter 1. Introduction	1
Chapter 2. Numbers through a different lens	9
Cognition is extended and enactive	9
Materiality has agency	14
Material signs are enactive	19
Numbers through a new lens	24
Chapter 3. What's a number, really?	25
An archaeology of number and a role for material forms	30
Two useful theoretical constructs	32
Chapter 4. Assembling an elephant, one bit at a time	43
Numerosity, the sense of quantity	43
Language and numbers	46
Development and numbers	49
Categorization and abstraction	53
Finger-counting and human neuroanatomy	56
Chapter 5. Behavioral traces	65
A caveat about the historic ethnographic literature	67
Peoples count the same way	70
Questions we might ask at this point	85
Chapter 6. Language in holistic context	89
Lexical numbers	93
Grammatical number	100
Ordinal numbers	102
What to look for in the Mesopotamian languages, and why	104

Chapter 7. Ancient languages and Mesopotamian numbers.....	107
The ancient languages of Mesopotamia: Sumerian, Akkadian, and Elamite	110
Evidence of numerical language from ancient writing	112
Lexical numbers	115
Grammatical number	121
Ordinal numbers	123
Wrapping up the evidence of the ancient languages	127
Chapter 8. Fingers and tallies.....	131
The why, how, and what of finger-counting	133
Everything you wanted to know about tallies	140
Evidence of tallies in the Ancient Near East.....	146
The context for evaluating artifacts for use as tallies	152
Chapter 9. The Neolithic clay tokens	157
Tokens and numerical meaning.....	161
Issues in interpreting tokens as numerical counters.....	164
Newly catalogued token finds and their analysis	166
The complexity of tokens and token-based accounting	174
Chapter 10. Numerical notations and writing	179
From tokens to impressions.....	180
From impressions to commodity labels	184
How early Mesopotamian writing became the cuneiform script	187
The effects of writing on numbers.....	196
More effects of writing on numbers	201
Chapter 11. The role of materiality in numerical concepts	207
The sequence of material forms used for counting.....	209
Other materially influenced change	217
Distribution, independence, and other so-called <i>abstract</i> qualities.....	221
Chapter 12. Concluding remarks and questions	229
Some answers to the questions posed	232
Directions for future research.....	241
Appendix: Data tables	245
Bibliography.....	257
Index	297

FIGURES AND TABLES

FIGURES

- 3.1 Conceptual blending with a material anchor
- 4.1 Subitization and magnitude appreciation
- 4.2 The cortical homunculus
- 4.3 Topographical layout of the motor and somatosensory cortices
- 5.1 One-dimensional devices
- 5.2 Two-dimensional devices
- 6.1 Frequency of use for the lexical numbers *one* through *twenty* in American English
- 6.2 Distribution of 452 languages with analyzable number words
- 6.3 Distribution of 168 languages with lexical numbers and grammatical number
- 6.4 Ordinal frequency
- 7.1 Sumer, Akkad, and Elam in the 3rd millennium BCE
- 7.2 Lexical tablet with phonetic values for the Sumerian numbers *two* through *ten*
- 7.3 Sumerian lexical numbers
- 7.4 Sumerian ordinal frequency by writing state
- 7.5 Sumerian ordinal frequency by document type
- 8.1 Geographic distribution of the oldest sites with possible tallies
- 8.2 Worked bones from Kebara and Ha-Yonim, Israel
- 8.3 Worked bone from Ksar'Aqil, Lebanon
- 8.4 Worked bones from Jita, Lebanon and Ain el-Buhira, Jordan
- 8.5 Pace of technological invention, elaboration, and accumulation
- 9.1 Correspondences between plain tokens, numerical impressions, and cuneiform number signs
- 9.2 Clay bulla and early numerical impressions
- 9.3 Chronology of artifacts used in Mesopotamian accounting and mathematics
- 9.4 Types of tokens
- 9.5 Temporal distribution of tokens
- 9.6 Geographic distribution of early tokens
- 9.7 Temporal distribution of plain types used as numerical counters
- 9.8 The cuneiform sign ŠID
- 10.1 Magnitude ordering of numerical signs
- 10.2 Uruk V artifacts with higher-than-expected N₁₄ repetition
- 10.3 Geographic distribution of numerical tablets assigned to the Uruk V period
- 10.4 Administrative tablet (W 6066,a) from the city of Uruk
- 10.5 Representational modes of signs in early writing
- 10.6 Chronology of signs

- 10.7 Feature recognition of physical and written objects
- 10.8 The development of literacy from writing
- 10.9 Mathematical tablet (Erm 14645)
- 10.10 Change in numerical and non-numerical signs
- 10.11 Change in numerical signs
- 11.1 Chronology of material artifacts used in Mesopotamian numbers
- 11.2 The elaboration of numbers

TABLES

- 3.1 Russell's logical types
- 4.1 Categorical judgments
- 5.1 Number words in four Yuki dialects
- 6.1 Ordinal word frequency in American English, Mandarin Chinese, and Arabic
- 7.1 Sumerian ternary counting and eme-sal numbers
- 7.2 Akkadian lexical numbers
- 7.3 Grammatical number in Sumerian, Akkadian, Elamite, and English
- 7.4 Sumerian ordinal frequency by time period and document composition
- 8.1 Early tallies
- 9.1 Counting systems used with tokens
- 9.2 Tokens by country and site
- 10.1 Uruk V numerical tablets with exceeded bundling
- 11.1 Affordances and limitations of material artifacts used for Mesopotamian numbers
- 11.2 Chinese and English numbers
- 12.1 Cuneiform numbers
- 12.1 Comparison of numerical notations
- A.1 Pre-Uruk V (8500–3500 BCE) numerical impressions and tokens
- A.2 Uruk V (3500–3350 BCE) numerical impressions
- A.3 Newly catalogued tokens

CONVENTIONS

TIMELINE

Late Upper Paleolithic	30 to 12 thousand years ago
Epipaleolithic	12,000 years before present to 8300 BCE
Neolithic	8300–4500 BCE
Chalcolithic (Copper Age)	4500–3300 BCE
Pre-Uruk V	8500–3500 BCE
Uruk V	3500–3350 BCE
Uruk IV	3350–3200 BCE
Early Bronze Age	3300–2000 BCE
Uruk III	3200–3000 BCE
Jemdet Nasr (JN)	3200–3000 BCE
Early Dynastic (ED) I/II	2900–2700 BCE
Early Dynastic (ED) IIIa	2600–2500 BCE
Early Dynastic (ED) IIIb	2500–2340 BCE
Old Akkadian (OA)	2340–2200 BCE
Lagaš II	2200–2100 BCE
Ur III	2100–2000 BCE
Early Old Babylonian	2000–1900 BCE
Old Assyrian (Old Assyr.)	1950–1850 BCE
Old Babylonian (OB)	1900–1600 BCE
Middle Assyrian (MA)	1400–1000 BCE
Neo-Assyrian (NA)	911–612 BCE
Neo-Babylonian (NB)	626–539 BCE

ABBREVIATIONS FOR MUSEUM DESIGNATORS

A	Oriental Institute, University of Chicago, Illinois, USA
AO	Louvre Museum, Paris, France
Ashm	Ashmolean Museum, Oxford, England
CUNES	Cornell University, Ithaca, New York, USA
DV	State Hermitage Museum, St. Petersburg, Russian Federation
IM	Iraq Museum, Baghdad, Iraq
JA	National Museum of Syria, Raqqa, Syria
JRL	University of Manchester, England
MS, MT, MW	Private, anonymous collections in Europe

NIM	National Museum, Tehran, Iran
NMSDeZ	National Museum of Syria, Der-ez-Zor, Syria
NMSR	National Museum of Syria, Raqqa, Syria
OIM A	Oriental Institute, University of Chicago, Illinois, USA
Sb	Louvre Museum, Paris, France
T	National Museum of Syria, Damascus, Syria
UM	University of Pennsylvania Museum of Archaeology and Anthropology, Philadelphia, Pennsylvania, USA
VA, VAT	Vorderasiatisches Museum, Berlin, Germany
W	Artifact found at the ancient city of Uruk in Iraq

ACKNOWLEDGEMENTS

This undertaking would not have developed in the way it ultimately did without the massive investment of time, expertise, and encouragement by the scholars who have been my mentors, supervisors, professors, colleagues, influences, and friends, listed here in roughly chronological order: Joan Ray, Rex Welshon, Thomas Wynn, Frederick Coolidge, Chris Gosden, Andrea Bender, Steven Chrisomalis, Pierre Pica, Jerrold Cooper, and Robert Englund. Non-chronologically, as they require special thanks, are several individuals. Denise Schmandt-Besserat encouraged my interest in, and facilitating my access to, data on the Neolithic clay tokens, including her own extensive research. Jacob Dahl introduced me to Assyriology, taking me from the little I recalled from middle school—that Mesopotamia was the land between the Tigris and the Euphrates—to my present understanding. John MacGinnis and Tim Matney granted me complete access to their database of recent token finds from Ziyaret Tepe, generosity unparalleled among my experience with the many owners of such data. Two anonymous reviewers provided insight that helped me refine the thesis toward the present publication. Lambros Malafouris significantly influenced the direction of my thinking, while allowing me complete freedom to develop it as it made sense to me. Last and never least, Colin Renfrew gave me the single most useful piece of advice I received during my doctoral studies. I also thank the Clarendon Fund for the generous scholarship that supported my doctoral research at Oxford, and the European Union’s Horizon 2020 Programme for the Marie Skłodowska-Curie Actions individual fellowship grant funding my current postdoctoral research at the University of Bergen.

Finally, I could not have persisted in this endeavor without the unstinting love and support of my family: Bill, my husband and best friend of nearly four decades; my son Archie, near to finishing his six-year residency in orthopedic surgery; my daughter Barbara, newly reported to the Pentagon for a fellowship in Air Force medical administration; and my granddaughters Jaiyah and Siena, now 15 and 1. It is with keen and unending sorrow that I note the loss of our other son Will, Archie’s twin, to a long and difficult battle with alcoholism in 2016. Having the escape of a thesis to finish, along with the kindness of the Keble College community in Oxford and our neighbors Robbie and Dana in Colorado, helped me endure an otherwise life-shattering tragedy.

Karenleigh A. Overmann
Department of Psychosocial Science
University of Bergen, Norway
May 2019