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Concepts, Sense, and Ontology

What Happened to the Sense of a Concept-Word?

Carlo Penco

Abstract

In this paper I shall outline a short history of the ideas concerning sense and reference of a concept-word from Frege to model theoretic semantics. I claim that, contrary to what is normally supposed, a procedural view of sense may be compatible with model theoretic semantics, especially in dealing with problems at the boundary between semantics and pragmatics. A first paragraph on the paradox of the concept horse will clarify the attitude concerning the history of ideas that I assume in this paper. In the second paragraph I will discuss some misunderstandings in the shift from the sense/reference distinction in Frege to the intension/extension distinction in model theoretic semantics. In the third I will show how a particular interpretation of the Fregean sense of a concept word (and of cognitive sense in general) may be of interest for model theoretic semantics.

Introduction

Discussion on concepts both in philosophy and psychology have produced so many new ideas on the topic, that it becomes difficult to make any comparison between contemporary debates and the Fregean worries. After recent criticism of concepts as natural kinds (Frixione 2007, Macherie 2009) cognitive scientists, philosophers and psychologists are proposing new ways of treating different aspects of cognition in humans and other animals; are concepts developed from a prelinguistic ability to classify? How do they develop in children? If we do not define concepts as natural kinds, shall we define them as functional kinds? shall we define them epistemically, semantically o by their origin? (see for instance Sainsbury-Tye 2011). Although some Fregean "problems" are still confronted, the contemporary debate on concepts seems to go far away from the original terminology used by Frege, that attracts mainly exegetic confrontation (we have excellent examples in Künne 2010 and Textor 2011). A possibility to find new suggestions in Frege's analysis of concepts may take two trends: on the one hand we may work on how his complex distinction of "levels" of concepts present psychologists and computer scientists with new problems (cf. Brandom 2009); on the other hand we may work on the history of ideas[1] and

1 Following Dummett's distinction between history of ideas and history of thinkers given in Dummett's *Origins of Analytic Philosophy*.

look inside the development of semantics after Frege, trying to reconstruct some of Frege's ideas in a new setting. I will follow the second trend, pointing out a blind spot in contemporary semantics, due to a failure to engage with the Frege's conception of the sense of a predicate—or in his terminology, a "concept-word" (*Begriffswort*).[2]

In this paper I will try to show the compatibility of a procedural interpretation of the Fregean sense of a predicate with contemporary model theoretic semantics. I don't claim that Frege cannot suggest alternative perspectives in semantics and theories of meaning; however, as Eva Picardi (2005, 35) remarks, it is difficult to accept that radically different interpretations of Frege—such as representationalist vs. inferentialist theories—"did equal justice to Frege's central concerns". Picardi 2005 has shown some difficulties of strong inferentialism to keep some basic Fregean desiderata; on the other hand most people agree that model theoretic semantics, although it has been developed on the track of Frege through Carnap, apparently abandoned some Fregean requirements on cognitive aspects. Nevertheless I think that some of Frege's most debated views on concepts are either preserved in new settings, like lambda calculus, or could be developed inside model theoretical semantics. I will then present (1) an assessment of one of the most famous problem concerning the Fregean theory of concepts as exemplifying a way to see its compatibility with develoments of logics after him; (2) a short historical presentation of the evolution of semantics after the Fregean distinctions of sense and reference for predicates in front of the "anomaly" of the original Fregean tripartite classification; (3) a use of the Fregean requirement on the sense of predicates that impinges upon the problem of the boundary between semantics and pragmatics.

1 Frege on Concepts as "Objects of a Special Kind"

Frege's original theory of concept is grounded on his analogy between concepts and functions: "what is called in logic a concept is connected with what we

2 Although Frege distinguished concept words and relational words, we may accept the traditional view according to which the term "concept" can be used as common designator for properties and relations (Carnap 1956, p. 21); analogously "predicate" can be used as expressing one-place predicates and n-places relations: "Human" is a one place predicate, but the concept of "Assassin" is expressed by a relation, that is by a two place predicate (unless we want to include other variables as suggested by Fillmore).

call a function … a concept is a function whose value is always a truth value" (FC 15) Presented in this way the theory is certainly original with respect to the past; historically, it is a generalization of the idea of function. Stripped of its prose it can be considered the origin of the "classical" view, where connectives can be considered as functions from truth values to truth values and predicates as functions from individuals to truth values: *Px* represents a function that has the value true when completed with a singular term referring to an object falling under the concept *P*, or belonging to the class denoted by *P.*

A great deal of the philosophical discussion on Frege's theory of concept has been devoted to his theory of the non-definability of (the notion of) a concept. Frege gives a semantic definition of objects and concepts as what is referred to, respectively, by singular terms (proper names) and predicates (concept words). Predicates or concept words are for Frege unsaturated expressions, i.e. patterns given by a sentence fragment that needs to be completed by a singular term, as with "… is a horse".[3] However, in natural language, we are almost compelled to *refer to* concepts using the definite article: "the concept *horse*" How can we make the connection between the expressions "… is a horse" and "*the* concept *horse*"? How can we *say* that the concept *horse* is a concept? Our grammar suggests that an expression composed with the definite article "the" (a definite description) is a singular term, whose reference is an object and not a concept; therefore we should paradoxically assert "the concept *horse* is *not* a concept".[4] This has been called "the paradox" of the concept horse. Frege (1892b: 201) concludes that concepts are "objects of a special kind", and asks the reader to accept this incongruence of natural language. Coming back on the issue years later, Frege (1906: 210) insists that grammar may mislead us, given that using a definite description to refer to concepts is "a mistake language forces upon us". However informal elucidations should be enough to clarify the intention of the writer in order to understand the sharp distinction between concepts

3 On concepts as sentence patterns see footnote 15. Concepts are also usually expressed in natural language by words with an indefinite article, like "a horse". Textor (2011, 227–229) remarks that the idea of concepts as sentence fragments or patterns (rooted in Bg §9) and as common names coexist in Frege, although different authors have chosen to favour one or the other conception to repair Frege's theory of predication. Textor (2011, 235–38) seems to favour the common name version and a revision of what he calls "sentence remainder" view. However Frege's later insistence on "unsaturatedness" of concepts is a reminder of chemical analysis and seems to be fully coherent with the early definition of concept words as patterns extracted by sentences.

4 Frege (1892b: 196) does not use the term "paradox", but speaks of awkardness of language or linguistic dissonance, or, more literally, an "indeed unavoidable linguistic hardness" (*freilich unvermeidbare sprachliche Härte*).

and objects (functions and arguments) on which the construction of his formal system is grounded[5].

Frege required "a pinch of salt" of us in order to understand the difference between objects and concepts, remarking that not everything in a formal system can be explained, and that the elucidations of the signs preceding the presentation of the formal system are informal introductions, that cannot be expressed in terms of the formal system. Among many discussions (starting with Dummett 1973 until Davidson 2005[6]) we find two extreme positions: on the one hand Crispin Wright claims that the paradox is not solvable unless we reject the application of the notion of sense and reference to predicates; on the other hand New-Wittgensteinians claim that Fregean elucidations are plain and "robust" nonsense. Both criticisms seem overstated.

On the one hand Wright 1998 claims that Frege's use of a singular term to refer to concepts clashes with his requirement for which two expressions with the same reference should be inter-substitutable in all extensional sentences *salva veritate*, and in all sentences *salva congruitate* (*reference principle*); in fact singular terms ("the concept horse") and concept words ("… is a horse") have different grammatical roles and cannot substituted *salva congruitate*.[7] There-

5 In speaking of "objects of a special kind" Frege (1892b: 201–202) refers to the hierarchy of concepts (that anticipates Russell's theory of types) and to the distinction between properties and characteristic marks of concepts. In the hierarchy we may have concepts as arguments of other concepts; this does not blur the distinction between concept and object, because the second level concepts contain as arguments first level concepts that require empty places for individuals.

6 Dummett (1973, 212 ff.) claims that, if not solved, would be "a *reductio ad absurdum* of Frege's logical doctrines"; his original solution, based on the use of second level concepts (*ibid*. p. 214), has however been challenged and it has been often considered not completely satisfying. Davidson's book on predication tries to show that what Frege wanted to say is given correctly inside a Tarskian theory of truth; therefore what Frege wanted to say with the idea of incompleteness should be reduced to the disquotational schema: "*F*" is true of *x* iff *Fx*. On Davidson's interpretation see Picardi 2008 for her subtle and clear distinctions between Frege's, Dummett's and Davidson's viewpoints. Another very significant treatment of the subject is the one given by Parsons 1986, that discusses the confusions arising from mixing informal language and formal requirements. He claims that Frege's justification for accepting that *any* phrase beginning with the definite article must refer to an object is not well grounded even in his proper view (Frege himself recognize that "the horse is a four legged animal" does not refer to an individual).

7 Concerning the conclusion drawn from the reference principle Textor (2010, 255) claims that "the argument is not persuasive, for two expressions *a* and *b* might not be intersubstitable *salva congruitate* in all sentences and yet have the same reference." The conclusion that "… is a horse" and "the concept horse" have different reference is not warranted by the argument. According to Textor, that predicates may refer is supported by inferences with existential generalization like "Hans is a man. Therefore there is something which Hans is, namely a

fore, in the end, Frege was mistaken: singular terms refer, but predicates don't[8]. Wright criticizes Dummett's attempt to solve the "paradox" finding a way to express the second order expression "concept horse", but the discussion may probably be stopped before the beginning. One problem with Wright's interpretation is that he wonders "how exactly Frege is to communicate his semantic proposals about predicates"; he asks for a "decent semantic theory" (Wright 1998, §III) while Frege explicitly considers his elucidations something where exactness cannot be attained, because—used to introducing his formal system—they are not part of it. Instead of conceding Frege to give an informal introduction to the basic concepts of his semantics, Wright looks for a formal analysis, and comes to the conclusion that Frege's basic mistake is the application of the sense/reference distinction to predicates (concept words). Wright requires a strict formalism exactly where Frege was supposing that no formal definition is required: we cannot give *definitions* for primitive elements of the system. (E.g. Frege 1906: 301; 1924: 290). Wright is correct in saying—after Frege—that singular terms and predicates behave differently, and we may refer to predicates indirectly, by giving their extension. In fact we may use extensions (classes) as the *semantic value* of predicates (as contemporary semantics does); but this does not abolish the possibility of *speaking of* concepts.[9] We touch here a point in which—as Textor (2011, 253) remarks—"reference as what we want to speak about and reference as semantic role come apart". Speaking of the reference of a predicate is not only defining a semantic value in a formal system, but also—basically—a reminder for the distinction between a function and its extension, distinction on which Frege was insisting in all his remarks on the idea of function. We might be content to claim that, in our informal elucidations, we need to refer to entities that are not objects, but concepts.

On the other hand, since the connection between Frege's "elucidations" and Wittgenstein's remarks on the unsayable discussed by Geach 1976 and

man" (p.235).

8 This conclusion is coherent with what Parsons 1986 (451) said about Frege's claim that both names and predicates refer: "names refer to objects and predicates refer to concepts. So reference must be a relation that sometimes takes an object in its second place and sometimes takes a concept in its second place, This would make reference a relation of mixed type, and such relations are not allowed in Frege's type theory."

9 Although Wright criticizes Dummett's interpretation, his conclusion is very similar to what Dummett says in 1973, 243: "Frege's attribution of reference to incomplete expressions appears in the end unjustified"; the main motivation is that this attribution cannot be shown to play any significant role in the Fregean account of the sense of a predicate: "the role of predicates is not to pick out a concept … but, rather, to say something determinate about objects" (ibid: 244). In the end predicates refer by "force majeur", which may be interpreted as saying (as Textor (2010, 249 comments) "in short: they don't refer".

later by Diamodn 1988, many authors, mainly "New Wittgensteinians", began to theorize the "ineffability" or "nonsense" of philosophical elucidations (the elucidations of *Tractatus* itself, or the elucidations in the introduction to Frege's *Begriffsschrift*). Certainly Frege was well aware that the basic concepts of the theory are not part of it and called the words "concept" and "function" with the term "pseudo-predicates", and used to speak of "nonsense" (*Unsinn*) about attempts to define primitive elements of his system until his latest writings[10]. Wittgenstein in the *Tractatus* called "object" and "function" "formal concepts"—i.e. not genuine, empirical concepts—that "show" their function in the use of the formalism. Anticipating Quine's motto, Wittgenstein used to say that the correct use of the word "object" is expressed in the formalism by a variable.[11] However, although both Frege and Wittgenstein used the term "nonsense" ("Unsinn"), it is plain that Frege used it in special cases, where the grammar of language clashes with theoretical intuitions as in the case of "the concept horse". Instead of accepting the attempt to recognize the limitations of the grammar of our natural language to express some basic ideas of the formal system, the New Wittgensteinans consider that what elucidations attempt to say always issues in plain nonsense.

Frege's aim (followed to the extreme in Wittgenstein's *Tractatus*) was more modest, and asked for informal agreement on basic concepts of his formal theory: "since definitions are not possible for primitive elements, something else must enter in. I call it elucidation. It is this, therefore, that serves the purpose of mutual understanding among investigators, as well as of the communication of the science to others."[12] I am not alone in thinking that the so

10 "what is simple cannot be analysed and hence not definied. If, nevertheless, someone attempts a definition, the result is nonsense. All definitions of functions belong to this category" (Frege 1924: 290)

11 Although we need some steps to get to Quine's "to be is to be the value of a bound variable", we can trace Quine's motto back to some suggestions given in Wittgenstein 1921 § 4.12721–2: "Every variable is the sign for a formal concept. (…) Thus the variable name '*x*' is the proper sign for the pseudo-concept *object*. Wherever the word 'object' ('thing', etc.) is correctly used, it is expressed in conceptual notation by a variable name. For example, in the proposition, 'There are 2 objects which … ' it is expressed by '($\exists x,y$) … '. Wherever it is used in a different way, that is as a proper concept-word, nonsensical pseudo-propositions are the result. So one cannot say, for example, 'There are objects', as one might say, 'There are books'. (…) The same applies to the words 'complex', 'fact', 'function', 'number', etc. They all signify formal concepts, and are represented in conceptual notation by variables."

12 Frege 1906: 301. Although in later writings Frege made an explicit distinction between metalanguage and object language, probably—in the early writings—he simply thought that what can be said in elucidations given in natural language has a pragmatic value of communication. Elucidations should therefore not to be considered part of "metalogic" (as metalogic is intended today).

called "paradox" of the concept horse is not really a paradox[13], but what the "second" Wittgenstein would have called a "misunderstanding" due to the grammar of our natural language. Even in speaking of "nonsense" we should need a pinch of salt.

In what follows I suggest an attitude where some basic Fregean ideas can be considered not only *as such*, in contrast with logical systems developed after him, but also for their value to illuminate and being illuminated by more recent developments.

A first example is what happened of the Fregean suggestion that concepts are "objects of a special kind" (he could have said "entities"). The suggestion has been developed by Alonzo Church with the lambda notation, where we may *refer* to concepts by an expression with bound variables which is formally analogous to the iota operator for definite descriptions (that Frege introduces in *Grundgesetze* § 11). In fact, facing the problem of the paradox of "the concept horse", somebody might attempt to use a second order description operator such as: ι*F*: (*x*) (*F*(*x*) iff *Horse* (*x*)), that is "the *F* such that for all *x*, *x* is an *F* iff *x* is an horse". But Frege introduced the description operator for singular terms would have not accepted it for predicates that need to be represented as insaturated expressions (see also Dummett 1973: 244). Church breaks this prohibition and invents a new kind of operator, with the expression "λ*x.* horse (*x*)" as a way to expressing the concept horse. Contrary to Frege's requirement, we have here an expression that is not literally "unsaturated", that is with a gap. Is Church's solution radically different from Frege's? Certainly it is, from the point of view of strict literal interpretation, but, nevertheless other aspects of Frege's main tenets seem to be represented, especially in lambda abstraction and lambda application, including the sharp difference between concepts and objects, at least in typed lambda calculus[14]. For Frege a concept may be expressed by a two place predicate like "kill (ξ,ζ)"; this kind of predicate is a pattern "extracted" from sentences like "Brutus kills Caesar" or "Cato kills Cato".[15] In a much

13 See for instance, for a clear argument on this, Picardi (2008, 16–17); also Weiner 2001, in her wide discussion of the use of elucidations, seems to have a more "moderate" attitude than the one entertained by "New-Wittgensteinians".

14 The distinction between object and concept finds some space in the standard interpretation of typed lambda calculus, given that an object is an element of E and a concept is a function from entities to truth values (E → T).

15 A central tenet of Frege, since *Begriffsschrift* § 9, is the decomposition of a sentence into different patterns: in the sentence "Cato killed Cato" we may see different patterns like "Cato killed x", "x killed Cato", and eventually "x killed x" and "x killed y" (the concepts of suicide and assassin). Here, as Dummett 1973 suggested, we need to consider not only the component words, but different *patterns* that permit different inferences through quantification (like "if somebody killed Cato he is to be blamed; x killed Cato; therefore x is to be

analogous way, with lambda abstraction the concept of killing is "abstracted" from sentences like "Bruto kills Caesar", and it is defined in lambda notation as a function with two arguments: $\lambda x\, \lambda y$. kill (x, y). With lambda application we may "apply" this function to particular individuals, and get the value true with those individuals that satisfy the function, as with:

$\lambda x\, \lambda y$. kill (x, y) (Bruto, Caesar), or
λx. kill (x, x) (Cato).

The procedure of lambda abstraction is ideally very similar to the decomposition of thoughts in Frege's writings, and lambda application shows a procedure which is not so different from the filling of an unsaturated predicate with a singular term to produce a sentence. A problem is given by the fact that Frege's requirement imposes that a concept be expressed by an unsaturated expression, and therefore the lambda formula (where all variables are bound to the lambda operator) seems not to fulfill this requirement. Discussing Church's notation, Burge (2005, 21) claims that the strict requirement of expressing concepts as unsaturated entities is an error on Frege's side, and his rejection of using a singular term (like "the concept horse") as standing for a concept "constitutes one of Frege's most serious mistakes". Was it a mistake or just an apparent paradox intended to throw light on a misleading aspect of natural language? A more favorable reading might say that Church, with the technique of lambda notation, has found a way for expressing the idea of "objects of a special kind" Frege was striving to realize[16].

blamed"). I will not discuss any more this relevant feature, on which—after Dummett—there are many expository texts like Brandom 2009, Penco 2010a, 77–79 (in italian), Textor 2011, 84–99. This analysis seems not so dissimilar to the strategy of lambda abstraction and lambda conversion.

16 Given that for Frege concept words have reference, but in natural language one cannot properly say what their reference is, his solution in formal language is that one cannot introduce a concept word directly, but only using the concept within a sentence, as he does introducing connectives (or the identity sign) and showing how they work. Church gives a solution in an artificial language that avoids the problem posed by natural language restrictions. Textor 2011 remarks that the standard objection to Burge (following Crispin Wright) is that concept words cannot be intersubstitutable with singular terms (see footnote 8 on the reference principle). But the argument does not support the conclusion that "… is a horse" and "the concept horse" differ in reference. This is a good point in favor of the idea that we can refer to a concept with a kind of complete expression like the one devised by Church. Burge's attitude seems too drastic in speaking of a "major mistake" in Frege. There are independent motivation (without appeal to the saturated/insaturated distinction) for avoiding proper singular terms in order to refer to concepts. The fundamental point is that "the semantics of concept words is disconnected from the notion of identity" (See on this pont Textor 2011,

This example aims to show that our way of studying the relations between Frege and his successors may not only concern the technical differences between two systems, but also how fundamental Fregean ideas can be "preserved" inside the new settings. In what follows I try to describe a piece of history of ideas that shows an internal need of model theoretic semantics to go back to the Fregean conception of the sense of a predicate as distinct both from reference and extension, recovering a tripartite distinction that has typically been rejected in model theoretic semantics, with the "disappearence" of the sense of a predicate in contemporary semantics.

2 Frege and the Intension/Extension Paradigm

The history of ideas in logic and semantics seems easy to trace, but being accustomed to contemporary ways of doing logic, memory of the transformation of common ideas is often lost in the overlapping of different systems. I will trace some comparison between old ideas and new ones, to get the feeling of how much is lost and how much is preserved of the idea of the sense and reference of a concept-word. Fregean concepts are functions from objects to truth value: the concept "Human" is the characteristic function that selects all elements of the domain and returns the True if the element is a human and the False otherwise. In contemporary model theoretic semantics the idea of concepts as functions can be presented in terms of set theory, where the domain is the set of all possible worlds and the codomain is the set of extensions (classes of objects belonging to possible worlds). Frege was not interested in modality, and therefore we may find difficulty to see the similarity of his view of concepts as functions from individuals to truth values and the model theoretic view, where concepts are functions from possible worlds to extensions. However, if this seems too distant from Frege's view, we may see the similarity if we represent intensions as functions with two arguments, a possible world and an individual at that world; a concept could then be conceived as a function from objects and possible worlds to truth values (given that P*x* is true of the object *a* at a possible world *w* where *a* belongs to the class selected by P at that world).[17]

256 and ff.). Textor's tries to show why Frege takes the paradox of the concept horse to be unsolvable, but torelable.

17 To clarify the matter it may be useful to remark that for Curry's Law we have the equivalence between functions from possible worlds to classes and functions from the Cartesian product of possible worlds and individuals to truth values: (PW → (E→T)) ←→ ((PW × E) → T).

According to Frege what corresponds to identity of objects for concepts is extensional equivalence: two concepts are the same if they have the same extension (similarly two intensions are the same if they have the same extensions in the same possible worlds). Frege's most peculiar aspect, is his tripartite division according to which predicates have a sense, a reference *and* an extension.[18] This tripartite division is lost since the first standard interpretation of Frege's semantics made by Church; in Church's classification concepts become senses of predicates and classes become their references (or "*nominata*"):

	Singular term	*Predicate*	*Sentence*
Sense	Individual Concept	Concept	Truth Condition
Reference	Object	Class	Truth value

Partly influenced by Church's classification, philosophers began to use the term "concept" to speak of the sense or meaning of a predicate. What is misleading here is the mixed terminology: Church apparently follows Tarski (who taught in Los Angeles) in considering the referent or semantic value of a predicate to be a class, while in Frege the referent of a predicate was a function, a mapping—what Frege called "concept" and Russell "propositional function". With Church a concept is no more the *reference* of a predicate or concept word, but, contrary to Frege's ideas, its sense. This simplification made people forget the original notion of the sense of a predicate in Frege[19].

Carnap seems to be uncertain on Church's treatment of the Fregean notions: on the one hand he claims that Church "is in accord with Frege's intentions"

A suggestion on these lines is given also by Nortmann 2001, 182,192, who proposes however a different solution to the sense of a predicate, on the track of Geach's function theory of the *Sinn* of predicates, criticised by Dummett 1973 and discussed by Nortmann at pages 192–94. Nortmann suggests also a possible way to treat senses of predicates in a "subjective" manner (which seems to me a Fregean rendering nearer to Chalmers' epistemic intensions) where the sense of a predicate F represents the speaker's personal criterion for F-hood.

18 The tripartite division is clearly presented by Frege in a letter to Husserl in 1891. One of the first to give attention to Frege's tripartite division is Wiggins 1984, who finds in it confirmation of Dummett's early insistence on sense and reference of predicates and the damages to equate the sense/reference distinction to the intension/extensions one. However Wiggins, proposes a repair of Frege's theory, inserting some consideration about the copula that seem very far from Frege's attitude.

19 Dummett (1981, 252) offers the standard interpretation: "the sense of a predicate determines a mapping from objects to truth values, that is to say, a concept. The sentence is true or false according as the object does or does not fall under the concept, that is according as it is mapped by it on the value *true* or on the value *false*. The mapping of objects on to truth-values is not the sense of the predicate, but its referent: the sense is, rather, some particular way, which we can grasp, of determining such a mapping"

when he regards a class as the nominatum (reference) and the property as the sense of a predicate; however he remarks—against Church—that the sense/reference distinction is *not* the *explicatum* of the traditional distinctions between comprehension and extension or between connotation and denotation (as Church claimed). Carnap claims, on the contrary, that what correspond in Frege to the traditional distinction between intension and extension is the pair function/course of value[20]. This means that in Carnap's view the proper place for Fregean concepts (as *functions* from individual to truth values) should be taken by intensions (in Kripke's semantics, functions from possible worlds to extensions), while the course of value is apparently corresponding to the extension itself. Therefore *if* we accept possible worlds semantics, the intension of a predicate seems to be a good *explicatum* of the Fregean concept–as–function, of what was for Frege not the sense, but the *reference* of a predicate, that is a concept as a mapping. A simple schema may help clarifying differences and analogies with the Fregean ideas of concepts as functions:

	Singular term	*Predicate*	*Sentence*
Intension	function from p.w. to	f. from p.w. to	f. from p.w. to
Extension	individual object	class	truth value

Here it is vividly apparent that, as Carnap insisted against Church, the intension/extension dichotomy is not an *explicatum* of the sense/reference dichotomy. To give an example: if sense were identified with intension, then Kripke's thesis that proper names have no "sense" would be false because apparently proper names have an intension (a constant function that gives the same individual as extension in all possible worlds where the individual exists). What happened then? In model theoretic semantics we have just lost sight of the Fregean *sense* of a predicate: there is no space for it; it vanishes[21]. The game

20 The discussion with Church is developed in Carnap 1956, § 29; Carnap speaks—with a Russellian terminology—of "the distinction between the value distribution of a propositional function and the propositional function itself" (Carnap 1956, 126–27). Although Frege distinguishes the course of values from an aggregate or collection of objects, there is an apparent connection between courses of values and classes or extensions. What a course of value would be? A set of ordered pairs like, for the concept "man" <Homer, the True>, <Socrates, the True>, <Helen, the False>, <Plato, the True>, <Aristotle, the True>, <Ipatia, the False>, <London, the False>, and so on. But the set of ordered pairs is just a way to give the class of men. Frege was a Platonist; also if the extension for him "derived" from the concept (from the truth function) which is the primitive notion, he probably thought that courses of values (and the corresponding class of objects) exist independently. The priority of concepts should be epistemological, not ontological.

21 This assertion is too strong, if we think to Carnap's intensional structure as an attempt to give

is different: Frege's terminology of sense and reference is no more usable here: what was in Frege the *reference of a predicate* (a concept as mapping), is now an *intension*; what was in Frege a *reference of a singular term* (an object) is now an *extension*.

Is the last simplified schema an advantage over the Fregean presentation of the different semantic roles of singular terms and general terms (concept words)? A standard formulation of the original Fregean tripartite division we have referred above (at footnote 19) is the following:

	Singular term	*Predicate*	*Sentence*
Sense	MOP of a object	MOP of a concept	Thought
Reference	Object	Concept	Truth value
Extension		Class	

("MOP" is "mode of presentation") Why did Frege insist that the reference of a concept is not its extension? Is this tripartite analysis really useful? According to Frege, yes, for at least two reasons: (1) in developing scientific hypothesis, we need to develop concepts (functions) that *might* not have extension: the concept "planet that influences the orbit of Neptun" is a concept under which no object falls; but it is still a clear and sharp concept;[22] (2) given that coextensionality is not sufficiently fine grained to make distinctions among concepts, we need a way to distinguish among equiestensional concepts. Taking Quine's example in "Two Dogmas", if we consider only coextensionality of concepts we have no way to distinguish between "having a heart" and "having a kidney" (or "chordate" and "renate"), given that all animals with a heart have a kidney and conversely. What makes these two concepts different? A first answer is that the concepts of chordates and of renates have different criteria of application: something is a chordate if it has a heart and a renate if it has a kidney. According to the Fregean distinction, the difference is given by the sense of the predicate, not by its "reference" (the concepts themselves) nor by its extension (the class of chordates coincides with the class of renates). In Frege's view we may distinguish between coextensive predicates on the ground of their cogni-

space to the answer Frege gave to lack of substitutivity in belief contexts. However Carnap's idea has not been developed inside model theoretic semantics. A new attempt to recover the idea of sense in indirect contexts is given by Kripke 2008, but it does not touches explicitly the problem of the senses of the predicates.

22 "A concept word can be logically incontestable although there is no object to which it refers through its sense and reference" (NS 135). For a wider discussion on the arguments used by Frege to claim that concept words cannot refer to extensions, but to unsaturated entities see Textor 2011, 245.

tive senses, that can be conceived as criteria of application or—basically—as *abilities to recognize instances of the concept* (using a terminology introduced by Strawson).[23]

However, if we consider concepts as intensions from possible worlds to extensions, there is no need of this tripartite analysis. Concerning (1), the concept "planet that influences the orbit of Neptun" is a function that refers (has as its extension) to an empty set in all possible worlds in which no planet causes perturbation of the orbit of Neptune. Concerning (2) possible worlds semantics reaches a similar result without the need of distinguishing reference and extension: given that—in principle—we might have possible worlds in which chordates are without kidneys, the concepts chordate and renate might be represented by two different intensions, that is two functions that give difference extensions in some possible worlds.

Apparently, therefore, what Frege did with his machinery can be done inside model theoretic semantics with more perspicuity and precision. The Fregean system presented in his *Grundgesetze der Arithmetik* can be considered just an anticipation of model theoretic semantics, with the idea of the sense of a sentence as the thought that its truth conditions are satisfied (Frege 1893, § 32). The paradigm of intension of a sentence as truth conditions, initiated by Frege and established by Wittgensein, has become a central aspect of contemporary semantics, although in a new form. The Fregean subtleties seem to evaporate in front of the complex machinery of model theoretic semantics and what Frege's logic can do seems just something that can be arranged in model theoretic semantics, that does not "betray" Frege's insights so much. In a way model theoretic semantics seems to give all is needed. And Frege's tripartite classification seems to be, as Davidson said, an idle wheel[24]. But this conclusion is misleading.

23 See Dummett 1973, 241. In fact knowing the criteria of application is knowing the characteristic marks of a concept, that are the properties of the objects falling under the concept. The suggestion that in case of simple concepts, where we cannot individuate other component concepts, criteria of application are abilities to recognize instances of the concept comes from Strawson, as remarked by Textor (2011, 250).

24 Davidson (2005, 139), with a Wittgensteinian language, claims: "If we take predicates as referring to entities we introduce a shadowy level of explanatory machinery between the expressions and the work they do. […] But if predicates have a referent , this is in *addition* to their sense and extension. This is the wheel that becomes redundant: to describe the semantic value of a predicate is not to introduce another level of explanation." This is an example of the confusion stemming from an overlapping of terminologies, especially Church's idea that the sense of a predicate is its intension—as probably Davidson is assuming here. If we consider the intension as an *explicatum* of the reference and not of the sense of a predicate—as I think to have shown reasonable—Davidson's criticism becomes empty. Therefore, what is

3 The Sense of a Predicate as a Procedure Associated to a Function

The need of developing a formal semantic framework has been realized at the cost of abandoning Frege's worries concerning the *cognitive* dimension of sense, to adhere more strictly to a *semantic* dimension, to use a distinction made by Beaney 1996. In fact, after years of studies on Frege's writings, many authors find it reasonable to accept a kind of "bifurcation of senses" that has been forced upon Frege because of different worries: on the one hand worries on the cognitive aspects linked to belief contexts where different expression may bring out *different thoughts*, on the other hand worries on the semantic aspects where *different* expressions may have the *same truth conditions*.[25] To put things in a simple way we may use Künne (2007, but also 2010:2/5): the same truth conditional thought can be articulated in different ways. Different ways of articulating the same thought pertain to the "cognitive" dimension of sense[26], partly abandoned in Frege's later system. But this cognitive dimension has always been a provocation for semantics. Semantics has been unable to give a satisfactory answer to traditional Fregean problems such as the content of belief contexts, on which Kripke 1979 concluded that our notion on "content of assertion" is still awaiting a clarification[27]. New attempts to treat belief con-

missing, and is really rejected by Davidson, is the Fregean conception of "cognitive" sense, that—using Dummett's terminology—is required for a "full blown" theory of meaning.

25 Here I refer to an almost standard interpretation of an ambiguity in Frege between a "cognitive" and a "semantic" conception of sense. See for instance Beaney 1996, but also Penco 2003, 2013 for a discussion. The term "bifurcation of sense" comes from Horty (2007, 56), who claims that "the role of stipulative definitions shows that Frege's notion of sense should be factored into two components", a semantic and a cognitive one, linked to the psychological states of language users. This solution is reminiscent of so called "bifurcation of content" between "narrow content" and "wide content", but it has more specific exegetic aspect concerning the Fregean conception, that should deprive it of the "internalist" flavor of narrow contents.

26 A warning on terminology is needed; according to Carnap—probably influenced by the Fregean contraposition between sense and tone—intensions represent the *cognitive* aspect of meaning against the *emotive* aspect. The idea of "cognitive significance" or "cognitive sinonymy" has been developed following this viewpoint. However this is not what we should understand as the "cognitive" aspect of sense in Frege, which is to be found in belief context (for Carnap hyperintensional contexts). Carnap himself recognized a further aspect of cognition with the concept of intensional structure, used to treat substitutivity in belief context. Carnap's solution did not work, but pointed out a different notion of "cognitive", stronger than the one used by him in contrast with "emotive": a cognitive aspect, more fine grined than intensions, which is required to justify substitutivity in hyperintensional context.

27 Concerning contexts of belief and the problem of substitutivity, Kripke 1979 says that "we enter into an area where our normal practices of interpretation and attribution of belief are subjected to the greatest possible strain, perhaps to the point of breakdown. So is the notion

texts—like bidimensionalism—have been developed as "deviations" from standard model theoretic semantics: bidimensionalism is directly connected with a reappraisal of the cognitive aspect of Fregean sense and its difference with the standard concept of intension, enriching the standard view with epistemic intensions.[28] However bidimensionalism suffers of relevant problems discussed in the literature, and I wonder whether the Fregean tripartite distinction between sense, reference and extension concerning predicates—vanished in all formal models—may suggest a different development.[29]

Yet we have no clear idea of what the sense of a predicate may be represented in a formalism, nor which are its criteria of identity. Although Frege never explicitly discussed the question, in a very few places, especially concerning the mathematical realm, he gave some hints towards a solution. Working on some of Frege's quotations, Dummett suggests that equivalence in sense is something less than synonymy and more than material equivalence: two concepts have the same sense if they are "provably coextensive"[30]. This means that we need a proof, or a procedure to show the equivalence. This idea is presented for logical and

of the content of someone's assertion …" Both descriptivism and direct reference theories are unable to tackle the problem.

28 Chalmer's version of bidimensionalism is strictly linked to his view on the difference between the concept of intension and the Fregean conception of sense. As it is clearly stated in Chalmers 2002 his *epistemic* intentions are conceived as an *explicatum* of Fregean senses as semantic values. Some of the motivations behind Chalmer's view are shared in the present proposal, especially the idea of the limited amount of information as relevant for our access to extensions (see Chalmers 2002, 144–145). However I derive different conclusions from this feature, developing some ideas from the attempt to understand Frege's ambivalence on the conception of sense from the point of view of bounded rationality (see Penco 2003b).

29 Marconi 2005 claims that bidimensionalism cannot overcome what he calls "the articulation problem", that is "Semantic values must be assigned in such a way that no expression gets more than one value of the same type, unless the expression is to be regarded as ambiguous (i.e. unless it *is*, intuitively, ambiguous); at the same time, values must be assigned so that each theory can display its full explanatory power, i.e. each expression must be evaluated in conformity with *both* theories; for it is by assigning semantic values in a certain way that a theory has explanatory efficacy". For other kinds of criticism of Chalmers see also Recanati 2010 and Stanley *forth*. The first sees a weak point of Bidimensionalism in its commitment to internalizations of acquaintance relation (p.155). I don't think the internalization of acquaintance relation be necessary, but I think it might be part of semantics; we cannot be concent that there being a communication chain is sufficient: we need to express the presence of a communication chain in semantics, even if this communication chain is not internalized. Stanley points out differences with standard neo-fregeanism (Evans, Peakocke) and criticizes the notion of "structured intensions", pointing out that Carnap's answer to some Fregean worries was not intension but intensional isomorphism, and points out the need in Chalmer's approach of two different kinds of contents beyond standard intensions: epistemic intensions *and* the contents of propositional attitudes.

30 Dummett, 1991, 32. See also Picardi 2005.

mathematical concepts, but I think we might use this idea also for other kinds of concepts. Senses, both for singular and general terms, can be conceived as proofs or procedures[31]. Many Fregean examples are coherent with this idea: just think of the different procedures of getting the centroid of a triangle (the point of intersection of the medians *a* and *b*, and of the medians *b* and *c*) presented as a prototypical case of same reference and different senses in "Über Sinn Bedeutung", or the different calculation procedures represented by the two sides of a mathematical equation, examples abundantly used in Frege's *Nachlass* and in his letters to Russell and Peano. But we may also think of the example of 'Afla' and 'Ateb', as two names of two mountains that have been baptized by two different persons following two different routes, that may be conceived as two different procedures (anaphoric or causal chains) through which the same referent is given. If we think of different senses of singular terms as different modes of presentation of an object, we can think of different senses of predicates as different modes of presentation of a concept, corresponding to different procedural abilities to recognize instances of the concept. If we want to represent these procedures in a formal system of a model theoretic semantics, we may think of algorithms which compute the functions, and there may be different algorithms to compute the same functions[32].

But, wait a moment. Procedural theories of meaning of different kinds are completely at odds with model theoretic semantics: on the one hand we have intensions as functions from possible worlds to extensions and on the other hand we have procedures or algorithms given in lambda calculus[33]. Procedural theories like the ones developed in artificial intelligence and in lambda calculus, or any kind of theory where sense is conceived as a justification proce-

31 See also Penco-Porello 2010

32 A nice example is given by Pavese (forth.) who shows two different algorithms to compute the Fibonacci function as a metaphor of different ways of being given of a task. But what is a task? It is normally something that can be expressed by a predicate (run, play, drink, …). The topic treated by Pavese is very specifically pointed to the debate on practical knowledge and concerns tasks and their practical modes of presentation. But the solution proposed is very similar to the one proposed here. It is the basic point according to which a function can be computed by different algorithms. A function is a mapping; but a mapping without a method to compute it is an idle wheel, unless we were in a Platonic realm where God has the ability to understand all mappings with his powerful eye.

33 This is also true for the interpretation of Frege's logic, to which many authors in procedural semantics refer. Procedural semantics originates also from early artificial intelligence systems (that used LISP), where the meaning of an expression was given by the procedures to get to the referent of that expression. A strong criticism of procedural semantics has been given by Fodor, but Fodor's criticism seem to be overcome by more sophisticated approaches (see Horty 2007).

dure, or inferential role, are typically opposed to model theoretic semantics, because they have different semantic values: intensions on the one hand and procedures on the other, so that expressions with the same intensions may have different "procedural" semantic values. But this is exactly the problem I want to deal with: my claim here is that we are not obliged to choose between procedural semantics and model theoretical semantics. This "received view" on the supposed alternative between procedural semantics and model theoretical semantics partly depends on the influence of Evans' criticism of Dummett's interpretation of Frege. Criticizing Dummett's verificationist account of meaning Evans reacted against a procedural vision of sense; but he has thrown out the baby with the bath water. Following Evans, most authors abandoned *any* procedural aspect of the notion of sense. I think that this abandon is not necessary and we may preserve and develop model theoretic semantics, taking care of a procedural aspect, heir of the cognitive notion of senses. The Fregean bifucation of cognitive and semantic sense must be preserved, not abandoned.

I am suggesting that cognitive senses can be reinterpreted as a further semantic level where senses are (represented formally as) procedures, connected with a predicate, that may compute in different ways the mappings given by intensions in model theoretic semantics. In this setting the intension of a predicate is a function from possible worlds to extensions; but this highly abstract generalization of the concept of function represents a view from above, the point of view of logic that ideally assigns different extensions to different possible worlds. What happens if we wanted to know *how* this function is computed? We need to make the function run in order to give the extension needed: we need a procedure attached to the function, and a function may have different procedures to get the same result.[34] In this perspective, the Fregean cognitive sense of a predicate could be represented as an algorithm or procedure attached to the the predicate; procedures formally represent different ways of computing the function, therefore exemplifying different abilities to recognize or identify the objects falling under the concept. This sounds a little too much *a la* Millikan, but it is a formal rendering of Strawson's interpretation of Frege's senses as criteria of application of concepts. A "transformation" of the Fregean original tripartite semantics into model theoretic semantics might be schematized as such:

34 We might use here also the distinction made by Stalnaker 1977 between foundational semantics and descriptive semantics. Descriptive semantics should assign semantic values to the expression of language, and in order to assign semantic values it needs to use algorithms to show which semantic value is assigned to a specific expression of an utterance in a context. After this assignement, pragmatic module begins to work.

Sense	procedures attached to intensions		
Intension	functions from possible worlds to extensions		
Extension	objects	class	thruth values

A Fregean might be worried by the the disappearance of the term "reference". This is easily justifiable: as we have seen before, Fregean reference can be identified neither with intension nor with extension. We may save the intuitive concept of "reference" as what we speak about, and distinguish it from the theoretical concept of semantic value. Taking Fregean senses as procedures we deprive them of the "mysterious" aspect so much criticized, for instance, by Stalnaker 2012 against the dubious idea of "mode of presentation"; in this setting a mode of presentation is something that can be represented formally as an algorithm belonging to descriptive semantics, that should perform the need to fix the semantic values of expressions (in context).

The necessity of a third level in semantic analysis is an emerging need[35]. For instance Kaplan 2012 suggests the following: "cognitive significance is not foreign to semantics. For the maximum explanatory power, our semantic theory should countenance cognitive content, objective content, and extensions." (Kaplan 2012, 141). This seems to suggest a tripartite analysis of the kind once envisaged by Frege for predicates: speaking of cognitive content, objective content and extension is a reminder of the distinction between sense, reference and extension. The difference is that, while Kaplan recognizes the need of a tripartite semantics, the cognitive content that he proposes to insert in the third level of semantic analysis is made of "ways of having in mind": the cognitive aspect is linked essentially to the psychological. On the contrary, the "recovery" of the idea of Fregean cognitive senses interpreted as procedures associated to functions (intensions) suggests that the third level of semantics needs to be distinct from the psychological aspect, against a tendency of what may appear as a neopsychologistic turn in semantics. In contrast, the idea of senses–as–procedures may suggest something more objective, that could be expressed by a formalism and implemented by an intelligent system. The point is not just to describe psychological ways of having something in mind, but algorithms representing objective ways in

35 I have already hinted (footnote 33) at the similarity with the topic of analysis of practical knowledge; in Stanley-Williamson 2001, the reduction of "knowing how" to "knowing that" requires a layer of analysis beyond propositions; this layer is the analysis of the ways in which a proposition is presented, a "practical mode of presentation". The link with the cognitive aspects of Fregean sense is apparent.

which we use a concept, or abilities to recognize instances of the concept (also if we probably need studies about the psychological plausibility of these representations).

To give an example of how procedural senses could be used in semantics (in determining the truth conditions of a sentence) we may take a much debated example by Travis: how to interpret the sentence "the leaves are green"? Frege[36] would have said that we need a context to understand the thought or the truth condition expressed: we need the time of the utterance and the place, and we need to know which tree is referred to in uttering that sentence. What about the concept "green"? The concept GREEN is a function whose associated procedures are procedures to recognize a certain wave length, formalizing our ability to recognize green surfaces. But imagine that we are speaking of a Japanese maple tree whole leaves are naturally red and are now painted green. For a photographer interested in the color of the leaves the utterance is true; for a botanist, interested in the natural color of the plant, the utterance is false. What about the concept GREEN now? According to the different viewpoints (the photographer's and the botanist's) we should use different procedures, grounded on the one hand on the lexical meaning of the term ("color of surfaces of a certain wave length") and on the other hand grounded on the context of utterance, including different presuppositions (presuppositions concerning different goals or interests). These procedures, given the context, may produce different extensions: different objects may fall under the concept "green" depending not only on the procedure used to recognize wave length of any surface in normal light, but also on specification of which surface is the relevant one to be considered (the visual surface or the original surface?). Does this mean that GREEN is a vague concept? Not really; simply the procedures associated to the function GREEN guide competent speakers to search contextual information for what are the relevant surfaces, relevant conditions of illumination and ways of being coloured, and for relevant standard of precision[37]. Assuming—for the sake of simplicity—to represent the botanist's and the photographer's points of

36 Frege discusses the example of the sentence "This tree is covered with green leaves" in "Der Gedanke" (p. 76).

37 In a similar vein, but with the solution of classifying the use of the predicate "green" as referring to two different properties, Vignolo (2013, p. 67) says: "in the photographer's context 'green' picks out the property of having lamina that look like stereotypical green spots in daylight illumination, ignoring the colour of petioles, venation and small marks and stains. In the botanist's context 'green' picks out the property of having lamina that look like stereotypical green spots in daylight illumination and are due to natural pigmentation, ignoring the colour of petioles, venation and small marks and stains. The result is that (1) is true for the photographer and false for the botanist."

view as two different possible worlds, we might say that the intension of "green" has different associated procedures that, depending on contextual information, give different extensions in different possible worlds.

In the schema proposed above, and differently from the original Fregean schema of the tripartite division only for predicates, the idea of senses-as-procedures as a further level of semantic analysis works not only for predicates, but also for singular terms. In this case, the idea of procedures associated to the intensions of different kinds of singular terms helps to clarify problems of cognitive dissonance among speakers, which is one of the traditional problems of model theoretic semantics.[38]

Summarizing, in possible worlds semantics intensions may be considered as connected with different procedures that take information from lexical meaning and context as input and give extensions as output. In Kaplan's view, context plus character gives an intension; but nothing is said about the way in which character "activates" contextual information.[39] I suggest that we should define kinds of procedures that use the inferential power of the lexical meaning[40] applied to elements of context (domain restriction, viewpoint and standard of precision[41]).

What I have done in this paper is an attempt to show that a particular interpretation of the Fregean sense of a predicate may point to a gap in model theoretic semantics: semantics typically assumes to have semantic values given to predicates and singular terms, including indexicals; however there is no specification on *how* we get those semantic values: procedures attached to intensions should fill this gap.

38 An attempt in this direction concerning definite descriptions can be found in Penco 2010b and Vignolo 2012.

39 It seems to me that a similar point is made by Kripke (2008, 195) when he remarks that the features of Kaplan's theory "are general directions for the referents in the language, no matter when and by whom they are uttered. One does not, it would seem, need anything more. However, in any particular case, to determine the reference one needs a *specification* of the speaker, the time, or both" (my underlining).

40 Assuming weak molecularism, according to which it is necessary to share inferences connected to predicates, but there are no necessary inferences we need to share (as Perry and Marconi have shown against semantic holism; se also Penco 1999). Picardi 2005 claims that Frege is more compatible with a molecularist position than with an holistic stance as the one entertained by Brandom.

41 The idea to select these three general procedures aspects to be applied to lexical meaning does derive from the contemporary discussion in philosophy of language, but from artificial intelligence. They are general enough to enclose different pragmatic processes discussed in the literature. For a short discussion see Penco-Vignolo 2005, but also Penco 2011 and Vignolo 2013.

Last but not least, procedures are compositional: if the intension of a sentence is its truth conditions (a function from possible worlds to truth values) the procedural sense of a sentence is composed by the procedures attached to the intensions of singular terms and predicates. Is the procedural sense of a sentence a Fegean thought? Hard to say; we are now in a different theoretical environment, where new worries impinge on the boundary between semantics and pragmatics. We are in a logical environment where artificial intelligence and intelligent systems have been developed suggesting new possibilities for the *application* of logic. Still, in this attempt to see what is needed in formalizing natural language, the old suggestions of the sense of a concept word requires a re-appraisal of our intellectual history, to avoid too rigid contrasts between different paradigms.[42]

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