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Counting and Indeterminate Identity

1. Introduction

In what follows, I argue against a species of indeterminate or vague identity I dub 'Split Indeterminate Identity' or 'SII'. Alleged cases arise when there are objects x, y, and z such that it is indeterminate that x=y, indeterminate that x=z, but (determinate that) y≠z. Furthermore, the indeterminacy is due to the objects themselves and not to an imprecision in the language we use in denoting the objects.[1] Admittedly, the prima facie plausibility of such a position is witnessed by judgments regarding a variety of identity puzzles. Consider a wooden ship, Ksenia, which leaves port for an ocean voyage. When Ksenia is at sea, workers replace each of her parts one by one with new ones. We may see this as a 'reparation' of the ship. Meanwhile, the old parts are put back together according to Ksenia's original blue print. We may see this as a 'reassembling' of the ship. At the end, we have two distinct ships returning from the trip, but there seems to be no fact of the matter as to whether Ksenia (the ship that left port) is identical to either of the docking ships.[2] I argue that SII is incompatible with indubitable facts regarding sets and counting.

In order to simplify things, let SII to be the following claim: *It is indeterminate that a=b (a≠b), it is indeterminate that a=c (a≠c), and b≠c.* Here, 'a', 'b', and 'c' are directly referential proper names for the original ship, the reconstructed ship, and the repaired ship respectively.[3] If I show that SII is not true for the ship case, then without loss of generality, we can infer that it is not true in the general case. The argument has the following form and will require the defense of both premises.

(i) If exactly one ship left port and exactly two ships docked, then SII is not true.[4]
(ii) Exactly one ship left port and exactly two ships docked.
Therefore:
(iii) SII is not true.

[1] Throughout this paper I will only be concerned with this type of indeterminacy.

[2] Other cases abound in the philosophical literature. Consider spectrum cases where a series of minute alterations (either cross-world or temporal) originally of some object y leads to an object z that is distinct from y. There will be some object x 'in the middle' of the spectrum that appears to be indeterminately identical to y and to z. There are also non-temporal /non-modal cases. Consider a house with a main structure and carport attached to it. The house x is indeterminately identical to the main structure y (not including the carport), and it is also indeterminately identical to z: the main structure *plus* the carport. Yet, z is distinct from y. Finally, we can think of split-brain transplants this way. A man goes in for surgery. Half of his brain is put into a body and the other half into another body. The ensuing persons are psychologically continuous with the original man. However, since they both can't be identical to the original man, one might think that the distinct post-surgery persons are each indeterminately identical to the original. These are all alleged examples of SII.

[3] I will take it for granted that there is nothing problematic regarding the naming of objects that enter into this type of indeterminate identity. In this, I am simply following the literature though I think that this is a non-obvious assumption and may be legitimately challenged.

[4] I will assume that there are no other objects in the universe besides the ships a, b, and c.

After I present the argument, I will briefly discuss how this argument avoids common replies to arguments against the possibility of vague identity given by Gareth Evans (1978) and Nathan Salmon (1981). I also consider implications for strong and weak Kleene logics. I conclude that defenders of these logics cannot hold on to the position that extensionality individuates sets if they want to maintain SII. As I will explain below, this is stronger than the mere claim that some instances of Set Essence for sets are neither true nor false.[5]

There are two further features of my argument that I want to highlight now. First, at several points I will be criticizing Terence Parsons'(1987, 2000) sustained defense of indeterminate identity and SII . In particular, Parsons is mistaken about each of the following: (a) SII is compatible with Set Essence, (b) He has provided an adequate way of counting objects that enter into SII, and (c) He has given a successful philosophical argument for establishing the non-truth of (ii) above. Second, since the indeterminacy in question is due to the world as opposed to an imprecision in the language used to describe the situation, I do not consider supervaluational responses to my argument. As I will explain below, the connectives are 'truth' functional and I treat indeterminacy as a species of non-truth and non-falsity. So from ⌜it is indeterminate that Φ⌝, we may infer that ⌜it is not true that Φ⌝ and ⌜it is not false that Φ⌝. Also, from ⌜Φ⌝ we may infer ⌜it is true that Φ⌝. Finally, from ⌜it is true that Φ⌝ we may infer ⌜it is not indeterminate that Φ⌝.[6]

1. Set theoretic argument for (i)

This argument is a reductio ad absurdum of the SII claim.[7] It basically runs as follows: There exists sets F={a} and G={b,c}. Since F has cardinality 1 and G has cardinality 2, then F and G are distinct. Set Essence for sets entails that F is identical to G if and only if F and G share exactly the same elements. Therefore, F and G do not share the same elements. This, in turn, will entail that there is some element of F (or G) that is not an element of G (or F). So either it is true that a is not identical to b or it is true that a is not identical to c. But by the SII assumption, it is not true that a is not identical to b and it is not true that a is not identical to c. The contradiction then obtains by 'cases'.

1. It is indeterminate that a=b (a≠b).	Premise
2. It is indeterminate that a=c (a≠c).	Premise
3. b≠c	Premise

[5] 'Set Essence' is name of the axiom defended by Parsons (1987, 2000) and Woodruff and Parsons (1999): Set A= Set B $\leftrightarrow \forall x(x\in A\leftrightarrow x\in B)$. It is just the conjunction of Extensionality ($\forall x(x\in A\leftrightarrow x\in B) \rightarrow A=B)$), and Set Indiscernabilty ($A=B \rightarrow \forall x(x\in A\leftrightarrow x\in B)$) which is (classically) a logical truth.

[6] These are some of the rules of inference for the indeterminacy in question. They are not to be construed as valid axioms. For example, a conditional ⌜Φ → it is true that Φ⌝ is not true in all models because the antecedent could be indeterminate making the consequent false. However, the corresponding rule of inference is truth preserving since if the antecedent is true, then so is the consequent. For a discussion of related issues, see Heck (1998).

[7] Of course, when engaging in non-classical reasoning a successful reductio argument against a position P, may not imply that P is false, but rather that P is not true. In the present case, I am only interested in showing that SII is not true. This leaves open the possibility that SII is indeterminate. It would be hard to see how this option would be of any consolation to SII defenders, since the identity puzzles that motivate SII require the truth of SII (not that it is indeterminate) for their "solution".

4. $F=\{a\}$, $G=\{b,c\}$	Premise
5. F has exactly 1 element (cardinality 1)	Premise
6. G has exactly 2 elements (cardinality 2)	Premise[8]
7. ~(G has exactly 1 element)	6
8. $F \neq G$	Contrap. of LL 6 and 7[9]
9. $F=G \leftrightarrow \forall x(x \in F \leftrightarrow x \in G)$	Instance of Set Essence
10. $\exists x(x \in F \leftrightarrow x \notin G)$	8 and 9
11. $(a \in F \leftrightarrow a \notin G) \vee (b \in F \leftrightarrow b \notin G) \vee (c \in F \leftrightarrow c \notin G)$	10
Proof by Cases	
12. $a \in F \leftrightarrow a \notin G$	Case 1
13. $a \notin G$	4 and 12
14. $a \neq b \wedge a \neq c$	Contrap. of LL 4 and 13
15. $b \in F \leftrightarrow b \notin G$	Case 2
16. $b \notin F$	4 and 15
17. $a \neq b$	Contrap. of LL 4 and16
18. $c \in F \leftrightarrow c \notin G$	Case 3
19. $c \notin F$	4 and 18
20. $a \neq c$	Contrap. of LL 4 and 19
END OF CASES	
21. 14 entails 'it is not indeterminate that $a \neq b$' and also 'it is not indeterminate that $a \neq c$'	Meaning of 'indeterminate'.
22. 17 entails 'it is not indeterminate that $a \neq b$'	" "
23. 20 entails 'it is not indeterminate that $a \neq c$'	" "
24. Each case (14,17 and 20) entails something that contradicts either 1 or 2.	
25. Contradiction from cases.	24
26. 1, 2 and 3 cannot all be true.	Reductio step.

2. Non set theoretic argument for (i)

Assuming that exactly 1 ship left port and exactly 2 ships docked, we can give an argument against SII. But such an argument will require an additional premise regarding counting and indeterminacy (Principle for Counting with Indeterminacy).

(PCI) If it is indeterminate whether x has property P, then there is no determinate answer as to exactly how many things have property P.[10]

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[8] We form these sets (F and G) via the axiom of comprehension. The open formula 'x is a ship the left port' will form a set F of just one element. The cardinality claim is justified by the antecedent of (i), which we are assuming for the proof. The naming stipulation guarantees that the element in F is named 'a'. Similar remarks apply to the set G.

[9] I will discuss the legitimacy of using the contrapositive of Leibniz's law in §5.

[10] To be more precise, the consequent should read: ⌜Exactly N things are P⌝ is not true when N names an integer numeral.

This principle has great plausibility regardless of what stance one takes towards vague identity or SII. For example, if it is indeterminate whether Harry is bald, then there will not be a determinate answer as to exactly how many people are bald. It will be indeterminate whether or not to count Harry as someone who is bald.

Once we accept this principle, the argument for (i) follows. Let us assume that exactly 1 ship left port. We now have three options as to whether b left port. It is either the case that b left port, b did not leave port or it is indeterminate that b left port.[11] I will consider each of these in turn. If b left port, then we would have no reason to make a distinct judgment regarding c. The thought experiment is set up so that intuitions regarding b and c are exactly parallel. So we would have to judge that c also left port. But this would make it so that at least two ships left port since b and c are distinct. We may rule this option out because it would contradict our assumption that exactly 1 ship left port.

In the second case we assume that b did not leave port. With the further assumption that leaving port is a property, ship a would have a property that b lacks. Thus, proving that they are distinct and not indeterminately identical after all. This option leads to a refutation of SII.

For our third case, if we assume it is indeterminate that b left port, then according to PCI above, it would not be true that exactly 1 ship left port. This would contradict our assumption again, so we rule this case out. The only option that does not contradict our assumption that exactly 1 ship left port is the second one. But this one entails that SII is not true.

3. Reasons for (ii)

The strategy for showing (ii) is as follows. I note that prima facie, we have good reason to think that in the ship scenario given in the opening paragraph, exactly 1 ship left port and exactly 2 ships docked.[12] But of course this is the case. We can ask the question 'how many ships left port?' the most common and intuitive judgment is that exactly 1 ship left port. Similarly, if we ask 'How many ships docked?', the most common and intuitive judgment is that exactly 2 ships docked.

I do not doubt that these judgments are ultimately correct and unless we have some good reason to give them up, we should not deviate from them. I do agree, however, that after further careful thought some philosophers might arrive at the judgment that it is indeterminate that exactly 1 ship left port or that it is indeterminate that exactly 2 ships docked. That is, they might judge that it is indeterminate that F has cardinality 1 or that G has cardinality 2. These judgments would usually come about after they have already decided that SII is the correct description of the ship scenario. Parsons (1987, 2000) has tried to provide reasons for the indeterminate cardinality claims. My strategy here will then be to show that all of the reasons he gives ultimately fail. I therefore conclude that

[11] I ignore higher order vagueness because I can't see any feature of the ship scenario that would lead one to think that, for instance, it is indeterminate that it is indeterminate that ship b left port.

[12] I do not mean to beg the question by describing the ship scenario using such locutions as 'the ship that left port'. The definite article 'implies' that there is exactly one ship that left port. But this is precisely what is at issue here. I only use the definite article for ease of exposition since it is the most natural way of describing the events. Is it possible to give a complete neutral description of the events that does not include claims about number? Perhaps we can imagine someone visually witnessing the events and wondering whether the SII hypothesis applies to the scenario.

the conservative judgment (ii) stands. I also consider a further initially appealing argument for the indeterminate cardinality claim but show that it is also fallacious.

The following sections are organized as follows. In §4.1, I consider Parson's formal method of counting objects that enter into indeterminate identities. I conclude that these methods either entail that exactly 1 ship left port and exactly 2 ships docked or if they do not, they are inconsistent with SII. Either way, Parsons has failed to establish the indeterminate cardinalities a defender of SII would seek in order to block (ii). In §4.2, I consider Parsons' philosophical argument for the indeterminate cardinality judgments. I argue that if we combine his premise there with a truism regarding counting we get a result that is inconsistent with SII. Thus making his argument for the indeterminate cardinalities useless for a defender of SII. Moreover, insofar as we are willing to accept the very plausible counting principle I set forth, we have a direct argument against SII. In §4.3 I consider yet another argument against (ii) but show that it is fallacious.

4.1 Parsons' methods for counting

Parsons (1987) proposes a method for counting where there are exactly 2 objects satisfying 'x docked' just in case (i) there are two objects each of which definitely satisfies 'x docked' each of which is definitely distinct from each other and (ii) every object that definitely satisfies 'x docked' is definitely identical to one of them.

Given this criterion, it is (definitely) true that exactly 2 ships docked. This is because both conditions hold. The first condition holds because there are two objects (b and c) such that each definitely satisfies 'x docked', and b and c are definitely distinct. The second condition also holds because every object that definitely satisfies 'x docked' (b and c) is definitely identical to one of them (b and c). Similar reasoning will show that exactly 1 object left port.

I conclude that if we use Parsons' (1987) method for determining cardinalities, we will arrive at (ii). So SII defenders, including Parsons, cannot use this resource to reject (ii). The method instead vindicates it.

Parsons (2000) provides another method for counting objects that enter into indeterminate identities. This method corresponds to the classical way of expressing number in predicate logic except that the connectives are non-classical: There are exactly n Φs just in case there are at least n Φs and there are at most n Φs. At first blush, this method seems like a promising one for blocking (ii) because it gets the result that it is indeterminate that exactly 1 ship left port. Surprisingly, further analysis shows that this way of counting leads to a contradiction when applied to the SII case. It gets the result that exactly two ships docked and that it is indeterminate that exactly 3 ships docked! The latter, of course, entails that it is not true that exactly 2 ships docked. Thus, we arrive at a contradiction. Let us examine this in detail.[13]

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[13] Parsons adopts a strong Kleene logic except for '⇒' which is the Łukasiewicz conditional. A disjunction is true just in case at least one of the disjuncts is true. It is false just in case all of its disjuncts are false. It is indeterminate otherwise. A conjunction is true just in case all of its conjuncts are true. It is false just in case at least one conjunct is false. It is indeterminate otherwise. The Łukasiewicz conditional is true just in case either the antecedent is false, the consequent is true, or if both the antecedent and consequent are indeterminate. The conditional is false just in case the consequent is false and the antecedent true. It is indeterminate otherwise. The negation of a sentence is true just in case the sentence is false. It is false just in case the sentence is true. Otherwise, it is indeterminate. Also, '∃xΦx' is true just in case at least one

There are exactly 2 ships that docked:
$\exists x \exists y(Dx \wedge Dy \wedge x \neq y) \wedge \forall x \forall y \forall z\ (Dx \wedge Dy \wedge Dz \Rightarrow x=y \vee x=z \vee y=z)$[14]

The first conjunct is true because the following is true: '$Db \wedge Dc \wedge b \neq c$'. In order to determine whether the second conjunct is true, we first restrict our domain to {a, b, c}. We can check that if we instantiate to any combination besides all three terms, one of the disjuncts in the consequent will be a tautology. Therefore, the instantiated conditional will be true. The only interesting case to check is when we instantiate the universal to all three terms: '$Da \wedge Db \wedge Dc \Rightarrow a=b \vee a=c \vee b=c$'. Both the antecedent and consequent are indeterminate. Since Parsons is using the Łukasiewicz conditional, the conditional is true. We conclude that the universal statement is true and therefore that *it is true that there are exactly 2 ships that docked.*

Consider now,

There are exactly 3 ships that docked:
$\exists x \exists y \exists z(Dx \wedge Dy \wedge Dz \wedge x \neq y \wedge x \neq z \wedge y \neq z) \wedge \forall x \forall y \forall z \forall w(Dx \wedge Dy \wedge Dz \wedge Dw \Rightarrow w=x \vee w=y \vee w=z \vee x=y \vee x=z \vee y=z)$

This statement is indeterminate. Let us check the first main conjunct. Note that the following is indeterminate: '$Da \wedge Db \wedge Dc \wedge a \neq b \wedge a \neq c \wedge b \neq c$'. This is because none of the conjuncts are false and some are indeterminate (the first, fourth and fifth). If we fix the domain to {a, b, c}, we can check that all other instantiations in the existential matrix make it come out false. So the first main conjunct is indeterminate.

For the second main conjunct, we note that any complete instantiation (on the relevant domain of a, b and c) will have a true consequent because one of the disjuncts will be a tautology. We can finally conclude that because the first main conjunct is indeterminate and the second true, that *it is indeterminate that exactly 3 ships docked.* From this it follows that it is not false that exactly 3 ships docked. But from the previous result that exactly 2 ships docked, we can infer that it is false that exactly 3 ships docked. So we arrive at a contradiction. Parsons' method for counting, which initially had some plausibility, is inconsistent with SII. This method cannot be used by defenders of SII to argue against premise (ii).

So far we have seen that Parsons' methods for counting objects that enter into indeterminate identities either vindicate (ii) or else are inconsistent with SII. We have yet to see a good reason to give up (ii).[15]

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object satisfies 'Φx'. It is false if no objects satisfy it. Otherwise, it is indeterminate. The universal quantifier is defined in terms of the existential quantifier.

[14] We let 'D' be 'is a ship that docked'. Alternatively, we may let 'D' stand for 'is an element of the set G (from the §2 set theoretic argument). Recall that set G is just the set of all the ships that docked.

[15] Parsons (2000) gives yet another way of counting objects that enter into indeterminate identity. But this way of counting will deliver the number of ships that have determinately docked and determinately left port. The result is then two and one, respectively. This way of counting, which would seem to vindicate (ii), is irrelevant for our purposes here because I am not interested in the number of ships that determinately docked or determinately docked. I am interested in the number of ships that docked or left port (simpliciter).

4.2 Parsons' philosophical argument for the indeterminate cardinality claims

We will now examine Parsons reasons for accepting the indeterminate cardinality claims. I will argue that the crucial premise there coupled with a truism regarding counting plus SII lead to a contradiction. I conclude then that the reasoning cannot be used by defenders of SII to object to (ii). Let us consider Parsons on the topic:

> Suppose we ask how many ships left port. Certainly, the original ship left port; that is determinately true. Since it is indeterminate whether the ship with new parts is the original ship, it is indeterminate whether the ship with new parts left port; likewise for the newly assembled ship. (These inferences presume that leaving port is, or is equivalent to, a property; they also presume that if either of the ships that docked had left port, that ship would *be* the original ship. Each of these is an additional assumption that maybe questioned.) It appears then that there might be no determinate answer regarding how many ships left port. (Parsons 2000, p. 142)

One crucial assumption here is that from the claims that a left port and that a is indeterminately identical to b, Parsons arrives at the claim that it is indeterminate that b left port. The principle here appears to be the following and it is important enough to be dubbed the 'principle of indeterminate identity':

(PII) If x has property P and it is indeterminate that x=y, then it can't be false that y has property P.[16]

This principle has to be correct. If x were indeterminately identical to y and x has property P, it couldn't be false that y has property P. If it were false, then we would have a clear violation of Leibniz's law.

With this principle in hand we can arrive at the claim that it is indeterminate that b left port and indeterminate that c left port. If we add the principle for counting with indeterminacy (PCI) we deduce the indeterminate cardinality claims: that it is not true that exactly one ship left port. Similar reasoning will lead to the claim that it is not true that exactly 2 ships docked. This would then be enough to block (ii).

It turns out, however, that SII and PII are inconsistent with the following truism.

(1) There is exactly 1 ship that docked just in case there is a ship that docked and nothing other than it is a ship that docked.

I conclude from this that Parsons (or any defender of SII) cannot use PII to arrive at the indeterminate cardinality claims and thereby block (ii). But we can make a stronger claim: if we do accept these intuitive principles, we must reject SII.

The only question that could arise regarding (1) is how to read the 'just in case'. I propose that the left hand side will always share the same truth-value as the right hand side, including 'indeterminate'. This is due to the intimate relation that holds between

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[16] Given the truth of the antecedent, the consequent will sometimes hold because it is indeterminate that y has P. The other times it will hold because y has P (on its own merit). Note also that this principle will predict that 'being indeterminately identical to a' does not express a property. It can then block Evans' argument. This is precisely what defenders of indeterminate identity often hold.

what the two sides express. The relationship there is much like the relationship between being a bachelor and being an unmarried male adult.[17] It can't be the case, for instance, that though Mike isn't a bachelor, it is indeterminate that Mike is an unmarried male adult. The latter implies that it is indeterminate that Mike is a bachelor. Likewise, it can't be that though there isn't exactly 1 ship that docked, it is still indeterminate that: some ship docked and no other ship did.

Another way of seeing this is by switching to an epistemic reading of indeterminacy. If there is a question as to whether there is exactly 1 ship that docked, then there will be a question as to whether there is a ship that docked and nothing other than it is a ship that docked. Similarly if we aren't sure as to whether there is a ship that docked and nothing other than it is a ship that docked, then we won't be sure that exactly one ship docked.

I propose then that the right bi-conditional to use to capture the 'just in case' is the Łukasiewicz conditional we had already been using in accordance with Parsons work.

We may now rewrite the truism in a more perspicuous manner:

(2) There is exactly 1 ship that docked ⇔ there is an x such that: x is a ship that docked and nothing other than x is a ship that docked.

The contradiction will take a few steps. First, we know that (3) is true.

(3) There are at least 2 distinct ships that docked.

This just follows from the fact that b and c are distinct ships each of which docked. This entails:

(4) ~(There is exactly 1 ship that docked).

Combining (2) and (4) we get the following:

(5) ~(There is an x such that x is a ship that docked ∧ nothing other than x is a ship that docked).

The contradiction we are seeking is forthcoming from a combination of PII and SII. To see this, first consider the following two sentences:

(6) a is a ship that docked.
(7) Nothing other than a is a ship that docked.

SII and PII entail that (6) is not false because b docked and a is indeterminately identical to b. Similarly, (7) is not false. If it were false, then there would be a ship distinct from a that docked. But what would this be? b and c are the only ships that we can say docked. But none of them are objects 'other than' a. They are each indeterminately identical to a. Given that each of (6) and (7) are not false, we can derive the following:

[17] I am supposing that being a bachelor is just to be an unmarried male adult in a sufficiently strong sense of 'is just to be' so that it underwrites the claims that follow.

(8) It is not false that (a is a ship that docked ∧ nothing other that a is a ship that docked)

Now note that we also have the following:

(9) It is false that (b is a ship that docked ∧ nothing other than b is a ship that docked).
(10) It is false that (c is a ship that docked ∧ nothing other than c is a ship that docked)

It is easy to see why both (9) and (10) hold. For example, in (9), the last conjunct in the main sentence is false because c docked and b is distinct from c. Similar remarks apply to (10).
Finally, let us consider the right hand side of (2):

There is an x such that (x is a ship that docked ∧ nothing other than x is a ship that docked)

Restricting our domain to a, b and c we can see by attending to (8), (9), and (10) that this sentence is not false. Both b and c fail to satisfy the matrix but it is not true that a fails to satisfy it. So we can derive the following:

(11) It is not false that (There is an x such that (x is a ship that docked ∧ nothing other than x is a ship that docked))

But now (5) and the semantics for '~' gets us:

(12) It is false that (There is an x such that (x is a ship that docked ∧ nothing other than x is a ship that docked)).

(11) and (12) contradict each other. So what went wrong? Recall that we used PII, SII, and (1) to arrive at the contradiction. We may look at the result in two ways. First, assuming that (1) is true, and since Parsons uses PII in order to argue for the indeterminate cardinality claims, it follows that his reasoning cannot be used to block (ii) in a defense of SII. We have noted previously that in the absence of a reason to give up (ii) we should maintain it because of its intuitive force. Second, this is a direct argument against SII. Both PII and (1) are very difficult to doubt. Since they are inconsistent with SII, we must reject SII.

4.3 Another fallacious argument against (ii)
Finally, I want to briefly address another way that might lead one to think that the sets F and G have indeterminate cardinalities.[18] One could think that since {a, b} is indeterminately identical to {a}, and since it is indeterminate whether {a, b} has exactly 1 element, it would have to follow that it is indeterminate whether {a}(=F) has exactly 1

[18] Recall that by the way that we constructed the sets, the claim that it is indeterminate that F has cardinality 1 is equivalent to the claim that it is indeterminate that exactly 1 ship left port. Similar remarks apply to G

element.[19] This is another case where it is natural to have the judgment that {a}'s cardinality is indeterminate, but after further reflection we can see that this line of reasoning is fallacious.

If we were to accept the argument above we would have to accept the exactly parallel argument where we just replace '{a}' with '{b, c}': Since {a, b} is indeterminately identical to {b, c} and since it is indeterminate whether {a, b} has exactly one element, it would have to follow that it is indeterminate whether {b, c} has exactly one element. That is, it would follow that it is not false that {b, c} has exactly 1 element. But {b, c} has at least two elements since b is distinct from c. So it is false that it has exactly 1 element. Therefore, this line of reasoning cannot be used to block (ii) and defend SII.[20]

5. Gareth Evans and Nathan Salmon

[19] I am grateful to an anonymous *Mind* referee for bringing this line of reasoning to my attention.

[20] Is there a bijection (a one-one, onto function) between F and {∅} and a bijection between G and {∅ {∅}}? And if so, does this settle the issue of their cardinality? If we identify F with the set of ship(s) that have the property of leaving port, then given that exactly 1 ship leaves port (as I have been urging), we can see that the bijection should hold for F. Similar remarks apply for G. As it turns out, however, if we accept the SII hypothesis, counting with bijections is problematic for a defender of SII. To see this, first note that we can directly show that F is distinct from G by showing that there is no bijection between F and G. Note that there being a bijection between F and G is just for there to be a relation R between F and G that is (i) a function, (ii) one-one and (iii) onto. There are only two relations that are relevant to check. The first one can be specified as follows: aRb and aRc. Now, it is easy to see that it is false that this is a function. This is because the following is false: '∀x∀y∀z[xRy & xRz → y=z]'. One of its instantiations is '(aRb & aRc → b=c)', which is false (on any interpretation of the conditional) because the antecedent is true and the consequent is false. Since it is false that R is a function then it is false that R is a bijection. The other relation to check is: bRa and cRa. It is easy to check that this relation is not one-to-one because the following is false: ∀x∀y∀z(xRz & yRz → x=y). It is false because one of its instantiations (on any plausible reading of the conditional) is false: (bRa & cRa → b=c). Here, the antecedent is true and the conclusion is false. So having checked the candidate relations for bijections between F and G we see that it is false that they are bijections. It follows that it is false that there is a bijection between F and G. If we take bijections as a method of counting seriously, as we should, then this would show that F and G have distinct cardinalities and are thereby distinct. We can bypass then any worries about whether or not F and G have determinate cardinalities. We could recast the argument this way, and resume the argument at step 8 where it is established that F is distinct from G.

Defenders of SII, however, might reject the standard way that I have defined functions. This way leads to further unacceptable results for SII. For instance: It is not true (it is indeterminate) that there is a bijection between {a} and itself. This can be seen by considering the relation exhausted by the specification: aRa. But now it is indeterminate whether R is a function. This is because 'aRb & aRc → b=c' is indeterminate (the antecedent is indeterminate and the consequent false). This entails that the universal '∀x∀y∀z(xRy & xRz → y=z)' is not true. But this shows that it is not true that R is a function and hence not true that it is a bijection. Since R is the only candidate for a bijection between {a} and itself, it follows that it is not true that there is bijection between {a} and itself. So it is not true that {a} has the same cardinality as itself. It is clear then that a non-classical way of defining functions must be used if one wanted to preserve SII. However, an issue arises as to whether or not this new way of defining functions will deliver the cardinality *(simpliciter)* of sets or instead the *determinate* cardinality (the number of elements that are determinately in the set). At any rate, given the considerations in this footnote, the classical way of defining functions is inadequate to handle SII. In particular, it is inadequate to get us the correct cardinality judgments using bijections. The burden is on defenders of SII to define functions in such a way that they get the intuitively correct bijection relations and restore bijections as a proper way of obtaining the cardinality of sets. Parsons' set theory for indeterminate identities stops short at relations and does not cover functions. This is one reason why I choose not to speculate as to how SII defenders might define functions.

Gareth Evans (1979) and Nathan Salmon (1981) have independently put forth arguments against the possibility of indeterminate identity (not just SII). Their arguments have been subject to numerous replies. I cannot go through all of them here. I note that the arguments presented in this paper rely on facts about cardinality and thereby depart significantly in spirit from theirs.

It is useful, however, to consider some common replies to those arguments and see how they do not apply here. A controversial aspect of the Evans and Salmon arguments is that they rely on there being a property expressed by 'is indeterminately identical to d' (where 'd' is a name that enters into indeterminate identity statements) or 'λx[x is indeterminately identical to d]'. I have made no use of such a locution. Where I required properties, I have only used uncontroversial properties expressed by predicates such as 'has cardinality 1' (for step 8 in §2), 'is an element of set F (G)' (for steps 14, 17, 20 in §2), and 'is a ship that docked (left port)' (in §3 and §4). Note that the linguistic expression of the properties did not include the 'indeterminate' or 'determinate' operators.

Another related objection is that the contrapositive of Leibniz's law does not hold in all cases involving indeterminacy. However, the cases where it allegedly fails always seem to involve the use of the 'determinate' or 'indeterminate' operators. My use of the contrapositive of Leibniz's law (e.g. in steps 8, 14, 17, and 20 in §2 and in §3) do not invoke those operators. Finally, it is often said that one may not 'quantify in' the 'indeterminate' or 'determinate' operators. I have not done so.

5. Implications of the set theoretic argument for the Kleene logics.

Theorists who adopt either a strong or weak Kleene logic can deny the truth of (9) which is an instance of Set Essence. They can thereby block my argument. They can deny the truth of (9) because in these logics, the right hand side of (9) is indeterminate. It is one thing to accept the notion that (9) is indeterminate if F and G are indeterminately identical, but the problem arises when we note that F is distinct from G. If we combine this result with the claim that (9) is indeterminate, we arrive at the following conjunction: F and G are distinct sets and it is not false (nor true) that they share exactly the same elements.

The reason why this result is unacceptable is that set essence or extensionality for sets tells us *what it is* for sets to be identical: they must share the same elements. If it is indeterminate whether sets share the same elements, then it has to be indeterminate whether they are identical. The set theoretic argument puts a serious strain on those who want to advocate these logics, set theory and SII. They would have to come up with a different way of individuating sets.

6. Parsons' set theory

Terence Parsons (1987, 2000) rightly recognizes the importance of extensionality in set theory and gives an account of how set essence is true despite the possibility of indeterminate identity and SII.[21] My argument shows that he is wrong on this count.

Parsons holds that instances of Set Essence such as (9) are true just in case both sides of the bi-conditional share the same truth-value (or are both indeterminate). So, he rejects

[21] Peter Woodruff (1999) in collaboration with Terence Parsons also defends set essence with indeterminate identity.

the Kleene logics. His formulation covers mundane cases of indeterminate identity such as when it is indeterminate that x=y and so the sets X={x} and the set Y={y} are such that it is indeterminate that X=Y. In such an instance, '$(\forall z)\ (z\in X\Leftrightarrow z\in Y)$' is indeterminate. This is because although it is true that x is an element of X, it is indeterminate whether x is an element of Y. Parsons then gets the result that the universal statement above is indeterminate. It follows that this instance of set essence '$X=Y \Leftrightarrow \forall z(z\in X\Leftrightarrow z\in Y)$' is true since both sides of the bi-conditional are indeterminate. The argument against SII, however, provides a counter-example to Parsons' theory.
Consider (9) again with Parsons' logic:

$$F=G \Leftrightarrow \forall z(z\in F\Leftrightarrow z\in G)$$

We have established that F is distinct from G. However, on Parsons' view, the right hand side is indeterminate because though $a\in F$, it is indeterminate whether $a\in G$. Since such an instance of the universal is indeterminate (and none of the others are false), the universal is indeterminate. But since 'F=G' is false, this instance of the axiom is not true. So Parsons cannot, as he wishes, preserve the truth of set essence. Moreover, he faces the same problem I noted for the Kleene logics: extensionality for sets no longer tells us what it is for sets to be identical.

7. Conclusion

I have argued that we should reject SII. The argument has the feature of avoiding common objections to the classical arguments against indeterminate identity. In fact, it departs significantly in spirit because it relies on principles of counting objects. I have not, however, given a direct argument against mundane cases of indeterminate identity where it is only claimed that it is indeterminate whether x=y. However, it appears that a position that endorses the latter but not SII is unprincipled since puzzles that motivate SII are the same kinds of puzzles that motivate all indeterminate identity claims. [22]

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References

Evans, Gareth 1978: ‘Can There be Vague Objects?’. *Analysis*, 38, p. 208.
Heck, Richard 1998: ‘That There Might Be Vague Objects (So Far as Concerns Logic)’. *The Monist*, 81, pp. 274−296.
Parsons, Terence 1987: ‘Entities Without Identity’. *Philosophical Perspectives 1*: pp. 1−19.
Parsons, Terence 2000: *Indeterminate Identity*. Oxford, New York: Oxford University Press.
Salmon, Nathan 1981: *Reference and Essence*. Princeton: Princeton University Press.
Woodruff, Peter and Parsons, Terence 1999: ‘Set Theory with Indeterminacy of Identity’. *Notre Dame Journal of Formal Logic*, 40, pp.473−496.