

Coherence & Confirmation: The epistemic limitations to the impossibility theorems

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Abstract

It is a widespread intuition that the coherence of independent reports provides a powerful reason to believe that the reports are true. Formal results by Huemer (1997), Olsson (2002, 2005), and Bovens and Hartmann (2003) prove that, under certain conditions, coherence cannot increase the probability of the target claim. These formal results, known as ‘the impossibility theorems’ have been widely discussed in the literature. They are taken to have significant epistemic upshot. In particular, they are taken to show that reports must *first* individually confirm the target claim *before* the coherence of multiple reports offers any positive confirmation. In this paper, I dispute this epistemic interpretation. The impossibility theorems are consistent with the idea that the coherence of independent reports provides a powerful reason to believe that the reports are true even if the reports do not individually confirm prior to coherence. Once we see that the formal discoveries do not have this implication, we can recover a model of coherence justification consistent with Bayesianism and these results. This paper, thus, seeks to turn the tide of the negative findings for coherence reasoning by defending coherence as a unique source of confirmation.

The idea that the coherence of a body of information provides a powerful reason for accepting that information has a long history. Carneades appealed to coherence reasoning in the context of medical diagnosis. Sextus Empiricus gives the following example of Carneades’s use of coherence reasoning:

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Just as some doctors detect the genuine fever patient not from one symptom, such as an excessive pulse or a severe high temperature, but from a cluster, such as a high temperature as well as pulse and soreness to the touch and flushing and thirst and similar things, so too the Academic makes his judgment as to the truth by a cluster of appearances.¹

Alexius Meinong appeals to coherence and provides the following metaphor of how coherence may function. “One may think of playing cards. No one of them is capable of standing by itself, but several of them, leaned against other, can serve to hold each other up.”² Otto Neurath held that coherence plays a crucial role in justifying scientific knowledge.³ Coherence reasoning is also commonplace. People often accept or reject a claim on the basis of its coherence or lack of coherence with a body of information.⁴

The possibility that coherence can increase the justification of a claim has come under attack from results in formal epistemology known as ‘impossibility theorems’.⁵ These theorems aim to show (a) coherence cannot increase the probability of a claim and (b) more coherence doesn’t imply higher probability. The first claim focuses on the coherence of a single body of information and whether coherence can increase probability, whereas the second claim focuses on two bodies of information and whether the more coherent body of information is more probable. My goal in this paper is to focus on the first claim proved by Huemer (1997), Olsson (2002, 2005), and Bovens and Hartmann (2003).⁶ For ease of discussion, I’ll focus initially on Huemer’s result.

The discussion over coherence as a source of confirmation begins with the idea that the coherence of multiple reports should offer positive confirmation even if the reports individually do not offer positive confirmation. The formal results then show that a necessary condition for coherence to offer positive confirmation is that the reports individually offer positive confirmation. This has been interpreted to have epistemic upshot: *coherence cannot increase rational credence unless rational credence is first moved by the individual reports alone*. It is this upshot that I dispute. All the theorem shows is that a necessary condition for coherence is that the reports

¹Sextus Empiricus (2005, 37).

²A. Meinong *Über Möglichkeit und Wahrscheinlichkeit* (1915), p. 465.

³Neurath (1932)

⁴See Slaughter and Gopnik (1996); Thagard (2000); Roche (2013); Koscholke and Jekel (2016)

⁵For an overview see Olsson (2017)

⁶While there has been recent work on coherence that shows that under different conditions coherence justification is possible (See Huemer (2011)), there is a more natural move to make in response to the earlier results.

are individually relevant. The theorem does not show that one's credence must *first* be moved by the individual reports in order for one's credence to be moved by coherence. One's credence function could be such that, prior to learning that the evidence is coherent, one's credence is unmoved by individual reports but then, posterior to learning the fact of that the individual reports cohere, one's credence function is moved by individual reports. The project of this paper is to argue for this claim and to provide a model of coherence justification consistent with Bayesianism.

The paper is structured as follows. I begin with a discussion of the nature and epistemic role of coherence which will put us in a position to see both (i) how coherence is thought to function in epistemology and (ii) how the formal results rely on questionable epistemic interpretations. I then move to explain the impossibility theorem proved by Huemer and to highlight the epistemic interpretations of result. Third, I offer an initial defense of the power of coherence. Fourth, I consider Olsson's attempt to more adequately model the effect of coherence in terms of hypotheses about witness reliability. I then situate my approach to coherence justification within Olsson's witness reliability models. Finally, I expand on the progress we made in the fourth section to provide a model of coherence justification that is consistent with standard Bayesian approaches. These models provide a clear way to demonstrate the possibility of coherence as a source of confirmation.

1 The nature & role of coherence

A longstanding complaint against friends of coherence is that the concept of coherence is unclear. Coherence is often understood in terms of a body of information that "fits" or "hangs together". But what is this notion of "fit"? A.C. Ewing in 1934 channeled a common complaint against the use of coherence reasoning that apart from a clear account of coherence, the theory is "reduced . . . to be mere uttering of a word, coherence, . . . rob[bing] it of almost all significance."⁷

C.I. Lewis in his 1946 book *An Analysis of Knowledge and Valuation* makes headway on clarifying the notion of coherence by proposing what has been termed the 'witness agreement' model of coherence.⁸ Lewis's key observation is that coherence is best seen in the case in which multiple witnesses report that the same event has occurred. In the case in which the witnesses are independent, Lewis thinks that the

⁷Ewing (1934, 246)

⁸Lewis (1946). He also provides a formal definition of coherence. A discussion of his formal definition is beyond the scope of this paper. See Roche (2013) for a discussion of Lewis's definition of coherence, among others.

agreement of the reports provides a powerful reason to accept the report. Lewis explains,

Imagine a number of relatively unreliable witnesses who independently tell the same circumstantial story. For any one of these reports, taken singly, the extent to which it confirms what is reported may be slight. And antecedently, the probability of what is reported may also be small. But the congruence of the reports establishes a high probability of what they agree upon.⁹

Lewis's witness agreement model takes coherence to be agreement in content. Two reports cohere with each other when they report the same event. Lewis holds that agreement in content is epistemically powerful when (i) the reports are independent and (ii) individually, the reports have some positive, but small, bearing on the content of the claim. Under these conditions Lewis thinks that the coherence of the reports can bestow a significant probability on the claim thus supported, even if any single report has little effect.

Laurence Bonjour appeals to Lewis's witness argument model in his defense of a coherentist account of empirical justification. Bonjour aims to defend epistemic coherentism on which coherence is the ultimate source of justification. He sees in Lewis's witness agreement scenario an illustration of the power of pure coherentist reasoning. He writes,

What Lewis does not see, however, is that his own example shows quite convincingly that no antecedent degree of warrant or credibility is required. For as long as we are confident that the reports of the various witnesses are genuinely independent of each other, a high enough degree of coherence among them will eventually dictate the hypothesis of truth telling as the only available explanation of their agreement.¹⁰

BonJour posits that the positive bump in credence that any individual report provides is not essential to the power of coherence. If the reports are independent from one another then coherence alone provides a powerful reason to think the content is true. Bonjour's thought is twofold: (i) rational belief may not be moved by individual testimonial reports but even so (ii) rational belief may be moved by the coherence of individual testimonies.

⁹Lewis (1946, 346)

¹⁰BonJour (1985, 148)

Whether or not BonJour is right is a crucial question for the viability of epistemic coherentism. Can coherence increase the justification of a body of claims without first requiring that those claims have some justification independent of coherence? Michael Huemer’s (1997) paper attempts to answer this question by interpreting BonJour’s intuition as formal constraints on a probabilistic model. Let us turn to Huemer’s theorem and its interpretation.

2 Huemer’s theorem

Michael Huemer’s (1997) paper illustrates a fruitful engagement between Bayesian epistemology and traditional epistemology.¹¹ Huemer interprets BonJour’s intuition as formal constraints on a probabilistic model. He then proves that there are no probabilistic models satisfying those constraints. This negative result is claimed to show that BonJour’s intuition, thus formalized, is provably false. In this section I explain this result and the conclusions that are drawn from it.

2.1 Setup

Let us begin with some terminology. Let $W_{i,A}$ indicate that witness i reports A . BonJour’s claim that the witness reports need no antecedent degree of warrant or credibility may be understood thusly:

$$\text{No Cred: } P(A | W_{i,A}) = P(A).^{12}$$

In contrast Lewis’s model assumes that the witness reports have, at least, some small degree of credibility. That is,

$$\text{Cred: } P(A | W_{i,A}) > P(A).$$

In these claims, and throughout the paper, we should understand probability as rational credence.¹³ **(No Cred)** specifies that a single witness report does not move one’s credence. The idea is that one lacks any relevant information about whether the witnesses report truthfully, and so for any individual report one’s credence does not change.

¹¹Huemer (1997)

¹²There’s a tacit universal quantifier ranging over witness i . This condition should be read “for any witness i the prior probability of A is unmoved by i ’s report that A .” The other conditions below should be read similarly.

¹³See Maher (2006, 2010)

The next feature of BonJour’s intuition is that the witnesses are genuinely independent of each other. If the witnesses are independent, then, BonJour thinks, the coherence of their reports provides a powerful reason to believe that the reports are true. We might attempt to capture witness independence in terms of stochastic independence. This requires that the probability of one event does not influence the probability of another event. Events X and Y are stochastically independent if and only if $P(X | Y) = P(X)$. This is represented as $X \perp\!\!\!\perp Y$. Following this line, BonJour’s idea that the witnesses are independent becomes,

Stochastic independence: $W_{i,A} \perp\!\!\!\perp W_{j,A} =_{df} P(W_{j,A} | W_{i,A}) = P(W_{j,A})$.

This interpretation of witness independence rules out the possibility that there can be evidential relationships between witness reports. It may be that one’s credence that a second witness will report A given that a first witness reports A is higher than one’s prior credence that the second witness reports A. This would be the case if the individual witnesses are responsive to the truth. If my son reports that there is a leak in the bathroom then this rationally increases my credence that my daughter will independently report the same thing. The interpretation of independence should not foreclose this possibility.

A better way to capture BonJour’s intuition is that the individual witness reports are conditionally independent. This is,

Conditional Independence:

$$(1) P(W_{j,A} | W_{i,A} \wedge A) = P(W_{j,A} | A)$$

$$(2) P(W_{j,A} | W_{i,A} \wedge \neg A) = P(W_{j,A} | \neg A)$$

(1) specifies that one’s credence that j will report A is responsive to A alone; another witness testimony is screened off by A . (2) specifies the same thing with respect to $\neg A$. (1) and (2) are what we’d expect for witnesses who are causally independent of each other.¹⁴ In the above case, my son’s and daughter’s reports about the leak are conditionally independent in virtue of the fact that they both track the same state of affairs.

BonJour’s intuition can then be understood as saying that under the conditions of no individual credibility and conditional independence, the agreement of multiple witness reports provides powerful evidence that the reports are true. That is,

BonJour’s Formal Intuition: It is possible that $P(A | W_{i,A} \wedge W_{j,A}) > P(A)$ even if (i) $P(A | W_{i,A}) = P(A)$, (ii) $P(A | W_{j,A}) = P(A)$, and (iii) the reports are conditionally independent.

¹⁴See Olsson (2002, 262) for a discussion on the reasons for conditional independence.

This is intended to capture the thought that the coherence of reports can provide a reason to believe a claim even if the reports individually do not first provide such a reason.

2.2 Huemer's theorem

The problem with BonJour's intuition, thus formalized, is that it is a theorem of probability that under these conditions, the agreement of multiple reports does not change the relevant prior probability.

Huemer's theorem: $P(A | W_{i,A} \wedge W_{j,A}) = P(A)$ when (i) $P(A | W_{i,A}) = P(A)$, (ii) $P(A | W_{j,A}) = P(A)$, and the reports are conditionally independent.

Let us briefly track through the major moves in the proof of this theorem.¹⁵ We've seen above that conditional independence requires these conditions

$$1 \ P(W_{j,A} | W_{i,A} \wedge A) = P(W_{j,A} | A)$$

$$2 \ P(W_{j,A} | W_{i,A} \wedge \neg A) = P(W_{j,A} | \neg A)$$

(**No Cred**) maintains

$$3 \ P(A | W_{i,A}) = P(A).$$

Lastly the idea that there is a coherence justification requires

$$4 \ P(A | W_{i,A} \wedge W_{j,A}) > P(A).$$

Huemer's theorem shows that (1)&(2)&(3) imply $\neg(4)$.

While (**No Cred**) is crucial for this result, it has some surprising implications. In effect, it specifies that any single witness testimony that A is stochastically independent from A , that is for all witnesses i $W_{i,A} \perp\!\!\!\perp A$. The relationship between a witness testimony and the testified fact is like the relationship between flips of a fair coin. So, (**No Cred**) implies that there is no evidential influence between A and $W_{i,A}$. There is something funny about this because coherence is supposed to provide evidence that the witnesses are indeed testifying to the truth. Recall that (**No Cred**) is intended to capture the BonJour's thought that individual claims prior to coherence do not provide evidence; the evidence that the claims provide comes from the coherence of the claims. To fill this out, let us examine two implications.

First, (**No Cred**) implies that learning that A occurs does not change one's credence that witness i testifies that A . That is, the following is true:

¹⁵I follow Olsson's terminology. See (Olsson; 2017)

$$(\dagger) P(A | W_{i,A}) = P(A) \iff P(W_{i,A} | A) = P(W_{i,A}).^{16}$$

(\dagger) is to be expected when one lacks any knowledge about whether witness i tracks A . If one's prior credence in A is unmoved by a report that A then one ought to think that independently learning A doesn't change one's credence that i reports A . Similarly, if learning that A doesn't change one's prior credence that i reports A then learning that i reports A doesn't change one's credence that A . Thus, (\dagger) implies that i 's report that A is like background noise with respect to A .

Second, (**No Cred**) implies one's credence that i reports A is the same conditional A or $\neg A$. That is,

$$(\ddagger) P(A | W_{i,A}) = P(A) \iff P(W_{i,A} | A) = P(W_{i,A} | \neg A).^{17}$$

A natural way to understand (\ddagger) is that individual witness reports are not responsive at all to the relevant facts. Rather the relationship between the individual reports and the relevant facts is the same as the relationship between individual flips of a fair coin. Just as there is no evidential relationship between the 1st flip of a fair coin and the 2nd flip of a fair coin, there is no evidential relationship between i 's report that A occurred and A 's occurrence. This is an important consequence of (**No Cred**) which I return to below.

Given (\dagger) and (\ddagger), Huemer's theorem is easily proved.¹⁸ The crucial question, though, is how we should understand the result. I turn to this question in the next section.

2.3 Interpreting Huemer's theorem

While the theorem is indisputable, its interpretation is not. The theorem is taken to be bad news for coherentism. Huemer titles the section in which he proves the theorem 'Coherentism refuted.'¹⁹ After proving that the coherence of two testimonies doesn't change the probability of the content of the testimony, Huemer draws the following lesson:

We see that if X receives no confirmation at all from either A or B individually, then X receives no confirmation at all from A and B together. If neither witness has any independent credibility, then the correspondence

¹⁶This is a well-known result but for the uninitiated see §6 in the appendix for a proof.

¹⁷This too is a well known result. See appendix §7 for a proof.

¹⁸See Appendix §8

¹⁹Huemer (1997, 468).

of the two witnesses' testimony provides no reason at all for thinking that what they report is true.²⁰

This result, he claims, is not surprising because "it is highly counterintuitive that one can manufacture confirmation of a hypothesis merely by combining a sufficient number of pieces of evidence that are, individually, *completely irrelevant* to the hypothesis."²¹

Erik Olsson's interpretation of this theorem coheres with Huemer's. Olsson explains,

BonJour's central argument for his coherence theory is manifestly false. Contrary to what he thinks, coherence cannot generate credibility from scratch when applied to independent data. Some reports must have a degree of credibility that is prior to any consideration of coherence, or such agreement will fail to have any effect whatsoever on the probability of what is reported.²²

Olsson takes the upshot of Huemer's theorem to be that the BonJour's central coherentist intuition is provably false. As Olsson understands it, coherence generates credibility only if the individual claims already have some degree of credibility.

In the rest of the paper I dispute Huemer's and Olsson's interpretation of the consequences of this theorem. For the reasons I lay out in the next section the theorem fails to undermine BonJour's original intuition. The theorem specifies a mathematical relationship between individual items of evidence, the mass of evidence, and target claim required for a boost of credence from a mass of evidence, viz., that such a boost requires that the individual items of evidence are relevant. But that mathematical relationship doesn't have the methodological lessons that are drawn from it. At best, the theorem shows that a body of information cannot be both individually evidentially irrelevant and yet evidentially relevant upon reaching some critical mass. The theorem specifies a necessary condition for coherence justification; but the necessary condition has no implications regarding whether one's credence must *first* be moved by individual reports before being moved by the coherence of the reports. All the theorem shows is that if coherence confirms then the individual reports confirm. But that is a far cry from showing that coherentism is false. It could be that upon learning that the mass of evidence is coherent, the individual claims become evidentially relevant; but not otherwise. I turn now to develop this idea.

²⁰Huemer (1997, 469-70).

²¹Huemer (1997, 470).

²²Olsson (2005, 69).

3 The power of coherence

It's time for a story. Several years ago, there was a curious incident in my house. My son, about two years old at the time, had a bedroom between my room and the kitchen. In the middle of the night if I was peckish, I'd tiptoe through his room dodging various objects in the room in hopes not to disturb a sleeping toddler. One night, as I stealthily crossed his room, I saw a curious shadow. It could be a number of things, but, on second look, nothing was there and so I dismissed it. A day later my son was playing in his room and came out to say 'snake' or something like that in toddler speech. As best I could make out, he was being playful and so we played with legos for a while. A few days later my daughter around six years old at the time was playing with my son and says 'Oh, it's a snake'. Now, my kids make-believe much and it often focuses on animals. It's entirely common to hear make-believe speech like 'Oh, it's a shark' or 'Oh, it's a horse'. So, in an isolated context, my six-year old's speech act isn't evidentially relevant. But, in this particular context, when my daughter said that, the three events all clicked and I began to give some credence the possibility that a snake was in the house. If you've ever tried to find a snake in a house, it's not easy. Snakes don't make noise and they prefer dark places that are hard to reach. But the coherence of these reports made salient the chance that a snake was in the house that a search commenced. After removing all the furniture from my son's room, a snake was found. The story ends well for all. The snake was returned to the woods, my son's room was safe for a toddler again, and I learned to take more seriously shadows in the night and playful toddler speech.

The story illuminates a key point about the role of coherence: *coherence can make information evidentially relevant to a hypothesis when it is otherwise evidentially irrelevant*. Let's reflect on the epistemology of this curious incident. The first item of evidence, a_1 , is a perceptual experience that, by itself, lacks a clear upshot. My prior credence that there was a snake in the house was unmoved by that experience. Let S abbreviate that *there's a snake in the house*. Then my credences were as follows: $c(S | a_1) = c(S)$. Why think this? The experience was compatible with a wide range of hypotheses most of which were non-serpentine hypotheses. Similarly, for my toddler's snake 'report', a_2 . His report of a snake lacked any indication that there's a snake in the house. If you doubt this, offer to babysit a few toddlers for an evening while the parents get a needed evening out. So here to we have the following: $c(S | a_2) = c(S)$. Finally, my six year old's apparently make-believe speech that there's a snake a_3 doesn't move my prior credence that there's a snake in the house. As with the other items of evidence, it is unclear what exactly the 'reports' are reports of. As quote marks indicate, the intentional object of the report

was unclear. Coherence often plays a role in interpreting speech. E.g., what did she mean to say? She had to mean thus and so. The ‘had’ indicates an inference is being made and that inference is supported by the rest of what we take to be true. So the three reports were, *in isolation from each other*, not evidentially relevant.

But when the third event occurred, it became apparent that all three events were, in fact, indications of a snake in the house. The coherence of the reports led to a realization that the events were indeed evidentially relevant to the presence of a snake in the house. It’s natural to describe the change as follows. By themselves the reports were not evidentially relevant but the coherence of the reports showed that they were evidentially relevant. The coherence of the report changed their individual evidential role. Once I saw that the three reports were indications of a snake, the individual reports then had power to move my prior credence.²³

I take it for granted that each item of evidence is determinate. In each case there is an item of evidence whose significance can be interpreted in a number of ways. Coherence fixes an interpretation which boosts the probability of the serpentine hypothesis. A different way coherence can confirm a hypothesis is when it is unclear *what the evidence is*. This latter way conflicts with Bayesian models which assume that the evidence is determinate.

In this story the evidential relevance of the information changes after learning that the information is coherent. Prior to learning that the evidence is coherent, my credence function includes (\ddagger), that is $c(a_i | S) = c(a_i | \neg S)$. But after learning that the evidence is coherent, my credence function changes so that $c(a_i | S) > c(a_i | \neg S)$. The coherence of the reports leads to the realization that the individual reports are indeed indications of the relevant fact. Thus, Olsson’s remark that “Some reports must have a degree of credibility that is prior to any consideration of coherence, or such agreement will fail to have any effect whatsoever on the probability of what is reported”²⁴ is not sensitive to the way that coherence can change the evidential relevance of information. Similarly, Huemer’s remark one cannot manufacture confirmation by combining individually irrelevant information does not hit the intended mark. It is correct that irrelevant information doesn’t magically become relevant in sufficient mass. But it’s wrong that the coherence of otherwise irrelevant information has no effect; in fact, it can lead to the realization that the information is indeed relevant.

²³There is a parallel here in ancient legal tradition. In the Hebrew Scriptures, a conviction cannot be secured on a single witness report; convictions require multiple witness reports. Cf. Deuteronomy 19:15 “One witness is not enough to convict anyone accused of any crime or offense they may have committed. A matter must be established by the testimony of two or three witnesses.”

²⁴Olsson (2005, 69).

So, what is wrong with Huemer's interpretation of BonJour's intuition? Huemer is correct that **(No Cred)** appears to characterize the idea that one's credence is unmoved by an individual report. But the error is that **(No Cred)** is treated as a constraint on a probabilistic model according to which it must characterize a subject's credence at all times, both *before* learning that the information is coherent and *after* learning that the information is coherent. But if coherence has the power to change evidential relevance then **(No Cred)** shouldn't be understood as a constraint on a probabilistic model. Rather it should be taken as a feature of a credence function at a time which new information may overturn.

The variability of evidential relevance fits with BonJour's original intuition. In the passage quoted above note the significance BonJour places on the way coherence can 'evidentially dictate the hypothesis of truth-telling as the available explanation of their agreement.'²⁵ If we take **(No Cred)** as a constraint on probabilistic models then we lose the ability to come to see that the individual reports are indeed responsive to the truth, i.e., we lose the ability to capture the correct conclusion that the individuals are telling the truth. When we come to the conclusion post-coherence that the individuals are reporting the truth then we take their individual testimony to be evidence for the relevant claim. It's no longer true that the individual reports are like background noise with respect to the relevant claim. Coherence turns what is otherwise noise into a harmony of individual voices.

When presenting this story and my epistemic analysis of it, I have been greeted with two responses: a coherence friendly response and a coherence hostile response. The friendly response agrees that one's rational credence may be unmoved by the individual reports prior to learning that the information is coherent and that after learning the evidence is coherent one's credence is moved by the individual reports. The hostile response has focused on pushing back on the idea that rational credence is unmoved by the individual reports prior to learning that the information is coherent. It is pointed out that, for example, an ideal Bayesian reasoner may be moved ever so slightly to the serpentine hypothesis by playful toddler speak.

I have two responses to this line of thought. First, while an ideal Bayesian reasoner *may* be so moved ever so slightly, the anti-coherence response requires that a rational Bayesian reasoner *must* be so moved. The purpose of this paper is to show that this is false; there are Bayesian models that capture this dynamic and cohere with the story I want to tell in this section. Second, it may be possible to use the recent literature on imprecise credences to model the story I tell in this section. In this case what happens is that one has imprecise credences on the relevant priors and likelihoods in the story and then when one learns that the evidence is coherent, those

²⁵BonJour (1985, 145).

imprecise probabilities become precise and thus support the serpentine hypothesis.²⁶

4 Coherence and witness reliability

The idea that coherence can change the evidential relevance of information is similar to the idea that coherence can change one's credence in the reliability of a source of information. Erik Olsson has pursued this latter idea in connection with a criticism of Huemer's model of coherence justification. Huemer's model does not include different hypotheses about witness reliability which is crucial for both Lewis's and BonJour's intuitions about the case. As both Lewis and BonJour see it, the surprising agreement of witness testimonies increases one's credence that they are speaking the truth. Olsson builds a coherence model that incorporates different hypotheses about witness reliability and then proves that coherence does not increase credence apart from individual reports increasing credence. In the following I explain Olsson's result and situate my approach to coherence justification in the context of a Lewis model.

4.1 Olsson on Lewis models

In Lewis's case there are two hypotheses concerning the witnesses; either the witnesses are perfectly reliable or the witnesses randomly generate their reports. The fact that witnesses report the same event is surprising on the assumption that they randomly generate their reports. This surprising fact, it is claimed, lends credence to the reliability hypothesis. Olsson proves that even in this setup the surprising fact of coherence does not increase the probability that the reports are true apart from the reports possessing some individual credibility. Let us review this result and discuss its significance.

4.1.1 Initial setup

Consider the following scenario in which there are two groups of witnesses to a crime.²⁷ One group is far away and another group is near by. We do not know which group any given witness belongs to. What we do know is that the nearby witnesses are perfectly reliable and the faraway witnesses are perfectly unreliable (i.e., their reports are generated randomly). Let's take two witnesses Jones and

²⁶I lack space to develop this further. I hope to return to it in a future paper. Thanks to Julia Staffel for this suggestion. For some recent literature on imprecise credences see Rinard (2017); Schoenfield (2017); Builes et al. (forthcoming)

²⁷The following paragraphs follow Olsson's discussion closely.

Smith. Consider these two hypotheses which we stipulate are mutually exclusive and jointly exhaustive:

(R) Jones and Smith are completely reliable.

(U) Jones and Smith are completely unreliable.

Consider the proposition (F) = Forbes is the culprit. Let $E_{j,F}$ indicate that Jones says that Forbes is the culprit and let $E_{s,F}$ indicate that Smith says that Forbes is the culprit.

Given our assumption that reliable witnesses are perfectly reliable, the following claims are true.

$$(i) P(E_{j,F} | F \wedge R) = 1 = P(E_{s,F} | F \wedge R).$$

$$(ii) P(E_{j,F} | \neg F \wedge R) = 0 = P(E_{s,F} | \neg F \wedge R).$$

For agreement to be surprising we stipulate that there are many more witnesses further from the scene of the crime than close to the scene. We also stipulate that if the witnesses are unreliable then they generate a report that Forbes did it with the same probability that Forbes is the actual culprit. So we have the following two claims.

$$(iii) P(E_{j,F} | F \wedge U) = P(F) = P(E_{s,F} | F \wedge U).$$

$$(iv) P(E_{j,F} | \neg F \wedge U) = P(F) = P(E_{s,F} | \neg F \wedge U).$$

If, for instance, there are n suspects and exactly one of the suspects committed the crime then, on the assumption of unreliability, the probability that Jones identifies Forbes as the culprit is $\frac{1}{n}$. This probability is the same regardless of whether Forbes is the actual culprit.

Next we introduce the independence assumptions. If they are reliable then this is trivially satisfied on account of the fact that the witnesses are perfectly reliable. That is,

$$P(E_{j,F} | F \wedge R) = 1 = P(E_{j,F} | F \wedge R \wedge E_{s,F}).^{28}$$

²⁸Mutatis mutandis, for each witness.

We need to specify that the witnesses are conditionally independent on the assumption that they are both unreliable. The following accomplish this.

$$(v) P(E_{j,F} | F \wedge U \wedge E_{s,F}) = P(E_{j,F} | F \wedge U)$$

$$(vi) P(E_{j,F} | \neg F \wedge U \wedge E_{s,F}) = P(E_{j,F} | \neg F \wedge U)$$

Then we stipulate that the reliability profiles of the witnesses are independent of the guilt or innocence of Forbes.

$$(vii) P(R | F) = P(R) = P(R | \neg F)$$

$$(viii) P(U | F) = P(U) = P(U | \neg F)$$

4.1.2 Coherence in a Lewis model

We are now in a position to determine the probability that Forbes is the culprit given coherent testimonies. By Bayes's theorem,

$$P(F | E_{j,F} \wedge E_{s,F}) = \frac{P(E_{j,F} \wedge E_{s,F} | F) \times P(F)}{P(E_{j,F} \wedge E_{s,F})}$$

We determine the right-hand side as follows. First, start with the evidence that we have two concurring testimonies. By the theorem of total probability and our setup we have this:

$$P(E_{j,F} \wedge E_{s,F}) = P(E_{j,F} \wedge E_{s,F} | R)P(R) + P(E_{j,F} \wedge E_{s,F} | U)P(U)$$

Given our assumptions, this is equivalent to the following.

$$P(E_{j,F} \wedge E_{s,F}) = P(F)P(R) + P(F)^2P(U).$$

The next quantity to determine is the probability of the evidence given Forbes's guilt. We expand this using the theorem of total probability for conditional probability.

$$\begin{aligned} P(E_{j,F} \wedge E_{s,F} | F) = \\ P(E_{j,F} \wedge E_{s,F} | R \wedge F)P(R | F) + P(E_{j,F} \wedge E_{s,F} | U \wedge F)P(U | F) \end{aligned}$$

Given our assumptions, this we get the following:

$$P(E_{j,F} \wedge E_{s,F} | F) = P(R) + P(F)^2 P(U)$$

So, plugging these expressions into Bayes's theorem and cancelling $P(F)$ in the numerator and denominator, gives us:

$$P(F | E_{j,F} \wedge E_{s,F}) = \frac{P(R)+P(F)^2 P(U)}{P(R)+P(F)P(U)}$$

We've assumed that R and U are exclusive and exhaustive hypotheses concerning the reliability profiles of Jones and Smith. We can thus see by simple algebra that, if $P(U)=1$ then

$$P(F | E_{j,F} \wedge E_{s,F}) = P(F).$$

This result is not surprising. If we know that both witnesses are randomizers then their reports have no evidential value and so the prior of Forbes's guilt is unchanged. Moreover, the concurring testimonies increases the probability of F only if $P(U) < 1$.²⁹ Putting this together, in a Lewis model, concurring testimonies confirm F if and only if $P(U) < 1$.

Now we can show that if $P(U) < 1$ and $0 < P(F) < 1$ then $P(F | E_{j,S}) > P(F)$. This result proves that increase in credence provided by concurring witness testimonies under the above conditions requires that the individual testimonies provide confirmation.

Given the set up, $P(F | E_{j,S}) = P(R) + P(F)P(U)$.³⁰ Given that R and U are mutually exclusive and exhaustive hypotheses concerning reliability, we can express this as follows: $P(F | E_{j,S}) = (1 - P(U)) + P(F)P(U)$. Suppose that $P(F | E_{j,S}) = P(F)$ and that $P(F) < 1$. Then we have the following:

$$\begin{aligned} (1 - P(U)) + P(F)P(U) &= P(F) \\ \text{iff } (1 - P(U)) &= P(F) - P(F)P(U) \\ \text{iff } (1 - P(U)) &= P(F)(1 - P(U)) \end{aligned}$$

If $(1 - P(U)) > 0$ then we divide both sides by $(1 - P(U))$ to reach $P(F) = 1$ which contradicts our stipulation that $P(F) < 1$. So $(1 - P(U)) = 0$ and $P(U) = 1$.

The upshot is this. If we assume that the testimonies are individually evidentially irrelevant, that is $P(F | E_{j,S}) = P(F) = P(F | E_{s,F})$ and it's not certain that Forbes

²⁹See appendix §11 for a proof.

³⁰See appendix §10 for a proof.

is guilty then $P(U) = 1$. In this case, there is no change in credence from concurring testimonies. Conversely, if there is a change in credence from concurring testimonies then there is a change in credence from the individual testimonies. In a Lewis model, coherence increases the probability of a claim only if the individual reports increase probability.

4.2 Lewis models & coherence justification

The Lewis model of coherence justification illustrates the interplay between hypotheses about witness reliability and coherence justification. The model shows that coherence is a source of confirmation if and only if the individual evidence is a source of confirmation. If a subject's credences are unmoved by individual reports then the Lewis model shows their credence in unreliability must be 1. In this case my approach to coherence justification requires that once one learns that the evidence is coherent one's credence changes so that the hypothesis of reliability receives some positive credence and hence the evidence becomes individually relevant. This view is not as radical as may first seem for it is supported by clear intuition and relates to standing difficulties with Bayesian updating.

Let's start with the clear intuition. Often in the context of reasoning, we take for granted some statements. When my spouse asks me to pick up our daughter from ballet, I take for granted that she is not intentionally misleading me about our daughter being at ballet. Consider a probabilistic example. I take a coin at random from a jar of change I've collected. The coin looks to be perfectly normal currency. I take it for granted that the coin is fair. That is, it is simply not up for short run empirical verification that the coin has some bias. So each flip of the coin is taken to be probabilistically independent of the other flips. But suppose that after flipping the coin for 1000 times we get 723 heads. This is overwhelming evidence for a bias. But note that if it is taken for granted that the coin is fair then there's no way by Bayesian updating to get the right intuition that 723 heads out of 1000 flips provides strong evidence for a bias without each individual flip providing some small bit of evidence that the coin isn't fair. The proof here runs exactly parallel to the witness reliability model in Olsson. What happens in the coin flipping case is that after a long sequence of flips, the mass of evidence calls for a reevaluation of what is taken for granted, viz., the coin is fair.

The coin case and the coherence case can be treated alike by finding a Bayesian model for assumptions. In the following I'll treat an assumption as having credence 1.³¹ As is well known if one's credal state includes assigning unit probability to

³¹This can be modeled in Titelbaum's Certainty Loss Framework (see Titelbaum (2013)). Titel-

some empirical claim then one cannot acquire new evidence such that by Bayesian updating one comes to have less than complete confidence in that empirical claim. This is a problematic feature with Bayesian models because there are cases in which empirical discoveries (e.g., the curvature of space-time) rationally leads one to revise entrenched assumptions (e.g., that light travels in a straight line).

This is not news for the Bayesian. The core problem is that if one's credence function is mistaken in any number of ways then the rule of conditionalization updates probabilities in accord with the mistakes. To solve this problem it requires replacing the rule of conditionalization with a different rule for updating probabilities. Christopher Meacham (2016) discusses this problem and proposes a different rule that updates in accord with *ur-priors*. Let us consider Meacham's proposal and how it may be applied to coherence reasoning.³²

Bayesianism is characterized by two core commitments: probabilism and conditionalization. Probabilism specifies that one's degrees of belief (one's credences) should satisfy the axioms of probability calculus. Conditionalization specifies that when a subject acquires new evidence, E, her degree of belief, say in p , should change to what her old degree of belief in p is conditional on E. Thus, if $cr_{e^-}(p) = n$ then, upon learning E, one's new credence $cr_{e^+}(p) = cr_{e^-}(p | e)$. Conditionalization is often taken to be the *only* way one's credence changes over time. Bayesianism then offers the following counsel: start with initial credences that satisfy the axioms of probability and then change credences by updating on new information in light of those initial credences.

Meacham reviews standard complaints with conditionalization. Let's consider three. One complaint is that conditionalization doesn't properly handle cases in which a subject has updated incorrectly.³³ Suppose a subject at t_0 has credences cr_0 and then acquires new evidence e_1 at t_1 . She then updates on this evidence incorrectly. She assigns $cr_1(\cdot) \neq cr_0(\cdot | e_1)$. She then acquires some new evidence e_2 at t_2 . What should her new credences be? If she follows conditionalization then her new credences should be $cr_2 = cr_1(\cdot | e_2)$. But this doesn't seem correct because cr_1 encodes a mistake. A better approach is to first correct the error in cr_1 and then update accordingly. But, as Meacham argues, there's no understanding of conditionalization that allows for this.

baum provides a Bayesian model that allows for claims to go from a credence of 1 to a credence less than 1. My proposal is to extend this to the role of assumptions.

³²The discussion here provides a way of capturing coherence reasoning within a commitment to precise credences. The literature on imprecise credences may provide another way that doesn't require *ur-prior* conditionalization.

³³See Meacham (2016, 449-51).

A second complaint is that conditionalization doesn't properly handle changes in one's views about the relevant merits of the theoretical virtues.³⁴ Following Meacham, consider two theories T_1 and T_2 both compatible with the available evidence. Suppose that T_1 better satisfies some theoretical virtues, while T_2 satisfies other theoretical virtues. Suppose one starts off thinking that T_1 is more plausible on account of the fact that one thinks the virtues that T_1 satisfies are more important. Accordingly one's credence in the first theory is higher than one's credence in the second. But then suppose that on reflection one comes to switch one's view about the relative importance of the theoretical virtues. In this case conditionalization cannot capture this change in credence because it's not a case of updating on new evidence; rather it's a case of changing one's beliefs about the theoretical virtues. In this case change in credence requires changing in accord with a new credence function that mirrors one's new beliefs about the theoretical virtues.

A third complaint about conditionalization is that it cannot handle cases in which one learns about a new theory.³⁵ Suppose there are exactly three theories, T_1 , T_2 , and T_3 that could explain some evidence E . One has never conceived of T_3 . Yet when E occurs, one puzzles over it and comes to realize for the first time that a hitherto unknown theory T_3 would explain E . One then comes to think that $T_1 \vee T_2$ is less plausible given E . The trouble is that conditionalization cannot capture this change of belief. The reason is that prior to learning E one had the following credence: $c(T_1 \vee T_2 \mid E) = 1$.

Meacham observes that one may respond that conditionalization is intended to describe ideal subjects. But Meacham points out that being unaware of a theory doesn't imply that one violates probabilism or conditionalization per se. But if we do take this to be a case for non-ideal subjects then it becomes a case of error correction.³⁶

Meacham's solution to each problematic case is to appeal to ur-prior conditionalization. The core idea is that each change—error correction, change in theoretical virtues, or learning about new theories—is modeled as changing one's existing credence function to a credence function that represents 'ur-priors.' Meacham considers three different ways of understanding an ur-prior function: a subject's initial credences, a function that bears the right relations to a subject's credences and evidence over time, and a function that represents a subject's evidential standards.³⁷ I'll work with the last option; the ur-prior function represents a subject's evidential

³⁴See Meacham (2016, 452-53). See also Levi (1980).

³⁵Meacham (2016, 456-58).

³⁶Meacham (2016, 457).

³⁷Meacham (2016, 464).

standards.

Let us consider briefly how ur-prior conditionalization answers the above problems. We start with the idea that ur-prior conditionalization requires a move from one's existing credence function c to a new credence function c' that more accurately reflects the subject's evidential standards.³⁸ In the case of error correction, upon learning e_2 one ought to change to a credence function that corrects for the earlier mistake in updating. In the case of changing theoretical virtues one's credence function changes to reflect the new evaluation of the priors for theories. In the case of learning about new theories, when one learns E and realizes that T_3 would explain E , one's credence function ought to change so that $c'(T_1 \vee T_2) < 1$.

Ur-prior conditionalization can be invoked to model the way coherence may rationally change a subject's confidence in some claim. We've seen that in a Lewis witness model the following claims are true: (i) if one's credence that the witnesses are unreliable is less than certain then individual witness reports have some credibility and (ii) if the individual witness reports have no effect on the content then one is certain that the witnesses are unreliable. Olsson claims this is a formal refutation of Bonjour's intuition that coherence can confirm even if the evidence does not individually confirm. An appeal to ur-prior conditionalization can make Bonjour's intuition compatible with Olsson's discussion on Lewis models.

Let us begin by introducing a new atomic sentence 'C' which states that the evidence is coherent. When one acquires multiple lines of testimony to the same event, one thereby learns C . Let's suppose that prior to learning C , one's credence function includes that the reports are randomly generated. In terms of Lewis's model, this amounts to $cr_{C-}(U) = 1$ or in terms of Huemer's original model it amounts to **(No Cred)**. However, upon learning C one is rationally moved to a new credence function on which $cr_{C+}(U) < 1$ or that **(No Cred)** is false. Once one learns that C , the evidential standards counsel a change in credence function to a ur-prior function on which one has a non-zero credence that the reports are of the relevant fact.³⁹

This illustrates how learning C has significant epistemic upshot while also granting that the impossibility result shows that individual credibility is a necessary condition for coherence justification. Coherence rationally leads one to revise the evidential significance of individual pieces of evidence. We can then revise Bonjour's formal intuition to get a statement consistent with the impossibility result. Huemer and Olsson formulated Bonjour's intuition thusly:

³⁸These standards can be constrained by normative requirements. I ignore this extra complication.

³⁹This does require that ur-priors are not regular, that is, that one may have unit credence to a non-logical truth.

BonJour’s Formal Intuition: It is possible that $c(A | W_{i,A} \wedge W_{j,A}) > c(A)$ even if (i) $c(A | W_{i,A}) = c(A)$, (ii) $c(A | W_{j,A}) = c(A)$, and (ii) the reports are conditionally independent.

We can appeal to a change in credence function and to the idea of learning C to get this:

BonJour’s Formal Intuition (revised): It is possible that $cr_{C^+}(A | W_{i,A} \wedge W_{j,A}) > cr_{C^+}(A)$ even if (i) $cr_{C^-}(A | W_{i,A}) = cr_{C^-}(A)$, (ii) $cr_{C^-}(A | W_{j,A}) = cr_{C^-}(A)$, and (ii) the reports are conditionally independent.

BonJour’s formal intuition thus revised recognizes that learning that C has significant epistemic effect. It changes one’s credence function from one in which the evidence has no confirmatory effect to one on which the evidence does have confirmatory effect. In the end this should not come as a surprise; for the power of coherence lies in it showing that the individual items of evidence are indeed indicative of reality.

5 A Garber inspired model of coherence

We have seen that coherence may function to change the evidential relevance of individual evidence. It turns out that we can develop this idea to secure confirmation by coherence without ur-prior conditionalization. The move is to introduce an atomic sentence ‘ C ’ that represents that ‘the evidence is coherent’. This move parallels Garber’s Bayesian solution to the problem of old evidence which involves introducing as an atomic sentence the relevant logical fact that is learned.⁴⁰ This Garber-styled model also avoids a deeper problem with coherence proved by Huemer in 2011. This is the problem of negative coherence justification. I’ll begin with an explanation of this problem and then move to the Garber inspired model of coherence.⁴¹

5.1 Confirmation by negative coherence

Huemer’s latest paper on coherence proves a theorem that minimal commitments about the role of coherence imply the surprising result that negative coherence provides confirmation. The theorem is as follows:

⁴⁰See Garber (1983)

⁴¹Thanks to Branden Fitelson for the suggestion to pursue a Garber inspired model of coherence.

$$cr(H | E_1) = cr(H | E_2) = cr(H) \ \& \ cr(H | E_1 \wedge E_2) > cr(H) \implies \\ cr(H | \neg E_1 \wedge \neg E_2) > cr(H).^{42}$$

The theorem says that if the evidence has no individual credibility and yet the evidence is relevant in mass then the falsity of the evidence confirms H. Here's the basic idea. The individual evidence is probabilistically independent of H. Baby logic implies that $e_1 = (e_1 \& e_2) \vee (e_1 \& \neg e_2)$. Since e_1 is independent of H, $(e_1 \& e_2) \vee (e_1 \& \neg e_2)$ is probabilistically independent of H. But by confirmation by coherence the first disjunct increases the probability of H, so the second disjunct decreases the probability of H. Since e_2 is independent of H so is $\neg e_2$. But $\neg e_2 = (\neg e_2 \& e_1) \vee (\neg e_2 \& \neg e_1)$. We've just seen that the first disjunct must lower the probability of H, so the second disjunct must raise the probability of H.

This is a surprising result. It is driven by the assumption of no-individual credibility. If coherence functions to change the evidential relevance of the evidence then this theorem isn't relevant for the coherentist. Both the appeal to ur-prior conditionalization and the model below avoid the problem of negative coherence.

5.2 Learning about coherence

Recall BonJour's core coherentist intuition is that isolated evidence does not confirm but coherent evidence does confirm. When one learns that an isolated item of evidence is true, one learns something logically stronger than just e_1 ; one learns that e_1 is true and that the evidence is not coherent. That is, one learns $e_1 \& \neg C$. When one acquires multiple items of evidence that cohere with each other, one learns not only that $e_1 \& e_2$ but also one learns C .

We can then represent BonJour's claim that isolated evidence offers no confirmation as this: $P(H | e_i \& \neg C) = P(H)$. Then coherence justification requires the following: $P(H | e_i \& e_j \& C) > P(H)$. We can also add a condition that negative coherence confirmation is false. This yields: $P(H | \neg e_1 \wedge \neg e_2 \wedge \neg C) < P(H)$. We then add conditional independence as before.

Confirmation by coherence is possible if the following conditions hold.

1. $cr(H | e_1 \wedge \neg C) = cr(H)$
2. $cr(H | e_2 \wedge \neg C) = cr(H)$
3. $cr(e_1 | e_2 \wedge H) = cr(e_1 | H)$

⁴²Huemer (2011)

4. $cr(e_2 \mid e_1 \wedge H) = cr(e_2 \mid H)$
5. $cr(e_1 \mid e_2 \wedge \neg H) = cr(e_1 \mid \neg H)$
6. $cr(e_2 \mid e_1 \wedge \neg H) = cr(e_2 \mid \neg H)$
7. $cr(H \mid e_1 \wedge e_2 \wedge C) > cr(H)$
8. $cr(H \mid \neg e_1 \wedge \neg e_2 \wedge \neg C) < cr(H)$

The first two conditions capture no individual credibility formulated with the additional sentence $\neg C$ representing that when learns a single item of evidence and that it is not coherent with other evidence then one's credence in the target claim is unmoved. Conditions 3-6, as before in the earlier discussions of coherence, specify that the evidence is conditionally independent, Condition 7 specifies confirmation by coherence. Given the individual evidence and its coherence, one's credence in the target claim is increased. But condition 8 specifies that negative coherence confirmation is avoided. Given the individual evidence is false and that it is not coherent, one's credence in the target claim is reduced.

PrSaT returns a model for these conditions.⁴³

E1	E2	H	C	var	Pr
T	T	T	T	a ₁	$\frac{10}{43}$
T	T	T	F	a ₂	$\frac{1}{63}$
T	T	F	T	a ₃	$\frac{1}{185}$
T	T	F	F	a ₄	$\frac{1}{111}$
T	F	T	T	a ₅	$\frac{2}{15}$
T	F	T	F	a ₆	$\frac{3}{26}$
T	F	F	T	a ₇	$\frac{3}{25}$
T	F	F	F	a ₈	0.122249
F	T	T	T	a ₉	0.000423625
F	T	T	F	a ₁₀	$\frac{1}{999}$
F	T	F	T	a ₁₁	0.005801
F	T	F	F	a ₁₂	0.00786501
F	F	T	T	a ₁₃	$\frac{1}{999}$
F	F	T	F	a ₁₄	0.000425269
F	F	F	T	a ₁₅	0.00147749
F	F	F	F	a ₁₆	0.228193

⁴³For a discussion of PrSaT with applications see Fitelson (2008).

The crucial result demonstrates that coherence justification is possible under the conditions of no individual credibility and coherence justification. It shows that for an interpretation of C and for a probability function, it can be the case that a mass of coherent evidence can increase credence under conditions that capture BonJour’s original intuition. The upshot is that the formal results have not closed the door on the possibility of coherence. Moreover, this model is one in which the problem of negative coherence justification is avoided.

This model upholds the intuitive conditions of no-individual credibility and conditional intuition. Huemer’s 2011 article ‘Does probability theory refute coherence?’ attempts to secure confirmation by coherence by replacing both no-individual credibility and conditional independence with different probabilistic conditions.⁴⁴ Erik Olsson comments on this move:

Whatever merits Huemer’s new conditions might have, their standing in the literature is hardly comparable to that of the original conditions. Conditional Independence, for instance, is an extremely powerful and intuitive concept which has been put to fruitful use in many areas in philosophy and computer science, the most spectacular example being the theory of Bayesian networks (Pearl, 1988). Similarly, the Nonfoundationalist condition [i.e., no-individual credibility] is still the most widely used—and many would say most natural—way of stating, in the language of probability theory, that a testimony fails to support that which is testified. Thus, it would seem that coherentism is saved at the price of disconnecting it from the way in which probability theory is standardly applied.⁴⁵

A primary virtue of the approach I’ve articulated in this essay is to maintain the connection between coherentism and probability theory.

There are two natural questions about this model of coherentist justification. *First*, does the new characterization of no-individual credibility combined with conditional independence imply coherence justification? That is, do the first six conditions imply the 7th (i.e., coherence justification)? They do not. PrSaT returns a model on which the first six conditions are true and there is a decrease in the credence of H given C and e_1 and e_2 .⁴⁶ There are models of the first six conditions in which $cr(H | e_1 \wedge e_2 \wedge C) > cr(H)$ is true, models in which $cr(H | e_1 \wedge e_2 \wedge C) = cr(H)$ is true, and models in which $cr(H | e_1 \wedge e_2 \wedge C) < cr(H)$ is true.

⁴⁴For details see Huemer (2011, 41–44).

⁴⁵Olsson (2017).

⁴⁶See Appendix 12 for a model.

Second, how we should understand C ? For the purposes of the model C is an uninterpreted constant formally defined in terms of conditions 1, 2, 7, & 8. We can further specify a probabilistic content to C in terms of the condition: $cr(C \mid e_1 \wedge e_2) > cr(C \mid \neg e_1 \vee \neg e_2)$. This specifies that it is more likely that the evidence is coherent given the truth of the evidence than give its falsity. Adding this condition to the above eight conditions, yields the following model in PrSat.

E1	E2	H	C	var	Pr
T	T	T	T	\mathbf{a}_1	$\frac{9}{41}$
T	T	T	F	\mathbf{a}_2	$\frac{3}{68}$
T	T	F	T	\mathbf{a}_3	$\frac{7}{43}$
T	T	F	F	\mathbf{a}_4	$\frac{1}{473}$
T	F	T	T	\mathbf{a}_5	$\frac{4}{473}$
T	F	T	F	\mathbf{a}_6	$\frac{51}{4}$
T	F	F	T	\mathbf{a}_7	$\frac{37}{3}$
T	F	F	F	\mathbf{a}_8	$\frac{86}{86}$
F	T	T	T	\mathbf{a}_9	0.150112
F	T	T	F	\mathbf{a}_{10}	0.00101301
F	T	F	T	\mathbf{a}_{11}	$\frac{2}{71}$
F	T	F	F	\mathbf{a}_{12}	0.000568227
F	T	F	F	\mathbf{a}_{13}	0.0701725
F	F	T	T	\mathbf{a}_{14}	$\frac{1}{999}$
F	F	T	F	\mathbf{a}_{15}	0.0196476
F	F	F	T	\mathbf{a}_{16}	0.00179977
F	F	F	F	\mathbf{a}_{16}	0.0775593

What should we think of this result? As commented above it secures the desiderata for coherentist justification, but it does have some odd features. The model treats C as logically independent from e_1 and e_2 . See for instance rows 1 and 2. The model permits every truth-functional combination of C with the other variables in the model, subject to the explicit conditions explained above. I've attempted to impose further conditions on the model that rules out, for instance, cases in which C is true while e_1 and e_2 are false, but PrSat does not return a model when that is added in. It is area of further research to find other probabilistic conditions that may fit better with an intuitive understanding of C . But whether or not that is so, if we understand coherence in terms of explanatory relations that are not modeled truth-functionally we can make sense of this model. In the case of row 2, there would be no explanatory relationships between the evidence and hypothesis. Similar remarks hold for the other rows. Thus we have a formal probabilistic model of coherentist

justification that can be embedded within an explanatory coherentist epistemology.⁴⁷

6 Conclusion

I've argued that the formal results on the role of coherence fail to have their intended epistemic upshot. It is compatible with the former results that the individual credibility of the evidence changes in response to facts about coherence. Moreover, I have provided a Bayesian model that shows how learning that the evidence is coherence can be evidentially relevant. The core flaw in interpreting the formal results is to think that coherence has power only if the individual evidence *first* has power. The formal results are taken to show that coherence is always the cart being pulled by the horse of individual credibility. But the friend of coherence sees that coherence may be the horse pulling the cart of individual credibility.

⁴⁷See Lehrer (1974, 2000); BonJour (1985); Poston (2014)

Appendices

7 Proof of †

$$(\dagger)P(A | W_{i,A}) = P(A) \iff P(W_{i,A} | A) = P(W_{i,A})$$

Proof

$$P(A | W_{i,A}) = P(A) \iff \frac{P(A) \times P(W_{i,A} | A)}{P(W_{i,A})} = P(A) \quad (\text{Bayes's theorem})$$

$$\frac{P(W_{i,A} | A)}{P(W_{i,A})} = 1 \quad (\text{divide by } P(A))$$

$$P(W_{i,A} | A) = P(W_{i,A}) \quad \square$$

8 Proof of ‡

$$(\ddagger)P(A | W_{i,A}) = P(A) \iff P(W_{i,A} | A) = P(W_{i,A} | \neg A)$$

Proof

$$P(A | W_{i,A}) = P(A) \iff P(W_{i,A} | A) = P(W_{i,A}) \quad (\dagger)$$

$$a = P(W_{i,A} | A)P(A) + P(W_{i,A} | \neg A)P(\neg A)$$

$$a = ah + c(1 - h)$$

$$a - ah = c(1 - h)$$

$$a(1 - h) = c(1 - h)$$

$$a = c \quad \square$$

9 Proof of Huemer's theorem

Let $P(A) = \frac{1}{n}$. (No Cred) implies $P(W_{i,A} | A) = P(W_{i,A} | \neg A) = k$. Let each witness as being equally reliable. Then we have the following

Proof

$$\begin{aligned} P(W_{i,A} \wedge W_{j,A}) &= \\ &= P(A)P(W_{i,A} | A)P(W_{j,A} | A) + P(\neg A)P(W_{i,A} | \neg A)P(W_{j,A} | \neg A) \\ &= \frac{1}{n}k^2 + (1 - \frac{1}{n})k^2 \\ &= k^2(\frac{1}{n} + (1 - \frac{1}{n})) \\ &= k^2 \quad \square \end{aligned}$$

This proof demonstrates that the evidence that both i and j report A is, given conditional independence and **(No Cred)**, equal to the rate of reliability squared. Huemer's theorem then follows from this given Bayes's theorem.

Proof

$$\begin{aligned} P(A | W_{i,A} \wedge W_{j,A}) &= \\ &= \frac{P(A) \times P(W_{i,A} | A) \times P(W_{j,A} | A)}{P(W_{i,A} \wedge W_{j,A})} \\ &= \frac{\frac{1}{n} \times k^2}{k^2} \\ &= \frac{1}{n} = P(A) \quad \square \end{aligned}$$

10 Lewis models - negative result

I first prove that $P(F | E_{j,F}) = P(R) + P(F)P(U)$. Then I prove that $P(F | E_{j,F}) = P(F | E_{s,F}) = P(F)$ only if $P(U)=1$.

Proof

$$\begin{aligned} P(F | E_{j,F}) &= \\ &= P(E_{j,F} | F \wedge R)P(R | F) + P(E_{j,F} | F \wedge U)P(U | F) \\ &= P(R) + P(F)P(U) \quad \square \end{aligned}$$

Proof

$$\begin{aligned} P(F \mid E_{j,F}) &= P(F) \\ \text{iff } P(R) + P(F)P(U) &= P(F) \text{ (by previous result)} \\ \text{iff } P(R) &= P(F) - P(F)P(U) \\ \text{iff } P(R) &= P(F)(1 - P(U)) \\ \text{iff } P(R) &= P(F)P(R) \text{ by reliability assumptions} \\ \text{iff } P(F) &= 1. \text{ if } P(R)=0 \text{ then this reductio is blocked} \end{aligned}$$

11 Result

Prove that $P(F \mid E_{j,F} \wedge E_{s,F}) > P(F \mid E_{j,F})$ only if $P(U) < 1$.

By previous results and substitution, we need to prove that $\frac{a+b^2c}{a+bc} > a+bc$ only if $c < 1$.

Proof

$$\begin{aligned} \frac{a+b^2c}{a+bc} &> a+bc \\ \text{iff } a+b^2c &> a^2+2abc+b^2c^2 \\ \text{iff } (1-c)+b^2c &> (1-c)^2+2bc(1-c)+b^2c^2 \\ \text{iff } 1-c+b^2c &> 1+c^2-2c+2bc-2bc^2+b^2c^2 \\ \text{iff } c(b^2-1) &> c(c-2+2b-2bc+b^2c) \\ \text{iff } (b^2-1) &> c-2+2b-2bc+b^2c \\ \text{iff } (b^2+1) &> c+2b-2bc+b^2c \\ \text{iff } 1+b^2-2b &> c-2bc+b^2c \\ \text{iff } 1+b^2-2b &> c(1+b^2-2b) \\ \text{iff } c &< 1. \quad \square \end{aligned}$$

12 Conditions 1-6 do not imply 7

E1	E2	H	H1	var	Pr
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	a ₁₆	$\frac{1}{174}$
<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	a ₁₂	$\frac{9}{41}$
<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	a ₁₃	$\frac{3}{13}$
<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>	a ₆	$\frac{6}{29}$
<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>	a ₁₄	$\frac{5}{38}$
<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	a ₇	$\frac{1}{999}$
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	a ₈	$\frac{1}{79}$
<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>	a ₂	0.0136166
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	a ₁₅	0.0884748
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	a ₉	$\frac{1}{985}$
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	a ₁₀	0.0203863
<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>	a ₃	0.0136309
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	a ₁₁	$\frac{1}{112}$
<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	a ₄	0.0437422
<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	a ₅	0.00145686
<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>	a ₁	0.00058533

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